THIRD EDITION

DUDLEY'S HANDBOOK OF PRACTICAL GEAR DESIGN and MANUFACTURE

Stephen P. Radzevich

CRC Press
Taylor & Francis Group

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Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

Printed on acid-free paper
Version Date: 20151123

International Standard Book Number-13: 978-1-4987-5310-4 (Hardback)

Library of Congress Cataloging-in-Publication Data

Names: Radzevich, S. P. (Stepan Pavlovich), author. | Dudley, Darle W. Handbook of practical gear design.
Title: Dudley's handbook of practical gear design and manufacture.
Other titles: Handbook of practical gear design and manufacture
Description: Third edition / Stephen P. Radzevich. | Boca Raton : CRC Press, Taylor & Francis Group, 2016. | Includes bibliographical references and index.
Identifiers: LCCN 2015045371 | ISBN 9781498753104
Subjects: LCSH: Gearing--Handbooks, manuals, etc.
Classification: LCC TJ184 .D784 2016 | DDC 621.8/33--dc23
LC record available at http://lccn.loc.gov/2015045371

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http://www.crcpress.com

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Preface to the Third Edition

This publication is dedicated to engineers who work in the field of gears and gear production.

Gearing is a specific area of mechanical engineering. Gearing encompasses the design, production, inspection, and implementation of gears, as well as some other supplementary subjects. This book is primarily focused on gear design and gear production.

This book is a revised,* updated, and enlarged version of the earlier published book by Darle W. Dudley entitled *Handbook of Practical Gear Desig*n. The book was first published by McGraw-Hill Book Company in 1984 and by CRC Press ten years later in 1994. Both editions originated from an earlier book by the same author entitled *Practical Gear Design*, which was published by McGraw-Hill Book Company in 1954. The last edition (1984/1994) of the *Gear Handbook* is highly recognized by the gear community all around the world.

As time passes, new knowledge is accumulated by the gear industry, and along with that, some of experiences summarized in the *Gear Handbook* are becoming obsolete. The obsolete data should be removed from the book, while the novel data in the field should be added.

Keeping in mind the importance of the *Gear Handbook* for practical gear engineers and practitioners, I appreciated and supported the attempt undertaken by CRC Press to revise, update, and enlarge the 1994 edition of this handbook.

Among the critical problems that arose when working on the new edition of the *Gear Handbook* were as follows:

- The new edition of the *Gear Handbook* should fulfill current demands of the gear industry. This requires a significant revision of the original text.
- At the same time, much effort was undertaken to preserve the original text of the *Gear Handbook*, in that way retaining the *atmosphere* of Dudley's text.

This contradiction between the hugely desired revision to update the book and the desired limited revisions to allow the book to be titled *Dudley's Handbook of Practical Gear Design and Manufacture* required a trade-off.

There were two options to consider.

The first option was to make significant changes to the original text of Dudley's book. Following this way, the charm and the philosophy of Dudley's book would be lost (or ruined). In this case, Dudley's book will no longer be *Dudley'*s book. It will be another book on gearing having the word *Dudley's* in the title.

The second option was to keep as close as possible to the original text of Dudley's book. In this case, Dudley's philosophy and his vision/understanding of gearing could be preserved.

Both options are worth consideration, and it was hard to make a decision which way to follow. Ultimately, the decision to make reasonable/limited changes to the original text of Dudley's book (where appropriate) was made. With that said, a limited number of repetitions are inevitable. The best was done in order to minimize the total number of the repetitions.

Prior to beginning the revision of the *Gear Handbook*, all earlier editions of Dudley's book [starting from 1954 McGraw-Hill edition, and ending with the latest 1994 CRC Press edition (including the 1962 and 1984 McGraw-Hill editions)] were carefully studied. Obsolete material was removed from the text where appropriate. Several new chapters, as well as new appendices are added. This made it possible to get the book updated.

Stephen P. Radzevich
Sterling Heights, Michigan

* The first revision of the *Gear Handbook* was published by CRC Press in 2012 (see Radzevich, S. P., *Dudley's Handbook of Practical Gear Design and Manufacture*, CRC Press, Boca Raton, Florida, 2012).

Acknowledgments to the Third Edition

The author would like to share the credit for any success of the *Gear Handbook* to plenty of discussions on the subject with numerous representatives of the gear community both domestic and international.

The contribution of many friends and colleagues in overwhelming numbers cannot be acknowledged individually, and as much as our benefactors have contributed, their kindness and help must go unrecorded.

Special thanks to Jonathan W. Plant, senior editor—Mechanical, Aerospace, Nuclear & Energy Engineering—for his idea to revise the 2012 edition of the *Gear Handbook*, and in this way to make the revised, updated, and enlarged edition available to the gear community all around the globe.

Many thanks also to Jill J. Jurgensen, senior project coordinator, Editorial Project Development, for the comprehensive support during the publication process, as well as to those at CRC Press who took over the final stages and who will have to cope with the marketing and sales of the fruit of my efforts.

My special thanks to my wife Natasha for her tolerance, support, encouragement, endless patience, and love.

Preface to the First Edition

This is a revision of the book *Practical Gear Design*, published by the McGraw-Hill Book Company in 1954. This book is now called a "handbook" because of its broader coverage of subjects pertaining to gear design and the recognition that the content of the original book made it a *lasting* reference that saw constant use in day-to-day gear work.

Although this revision is broader in scope and has more pages, the plan of the book and the way many aspects of the gear art are treated have been kept just the same. Those familiar with *Practical Gear Design* will find themselves right at home as they read the *Handbook of Practical Gear Design*. The main difference is that new things learned in the 1960s and 1970s about how to better design, build, and use gears are included in this book. Gear technology has made rapid progress, just as space technology and computer technology have made rapid progress in the same time period.

This book has been written with the general engineer or technician in mind. It is my hope that alert shop planning engineers, tool engineers, engineering students, cost analysts, shop supervisors, and progressive machine-tool operators will find this book of value and assistance when confronted with problems involving gear design or manufacture or involving failure in service.

The scope of the work includes the geometry of gear design, manufacturing methods, and causes of gear installation failures. A gear design is not good design unless it is practical and economical to manufacture and unless it is well enough thought out to meet all the hazards of service in the field. It has been my aim to show how a *practical* gear design must be based on the limitations and availability of machine tools and tooling setup.

As in most phases of engineering, there are often divergent methods advocated to gain a given result. The gearing art is no exception. For instance, in Chapter 7, equations are given to calculate the length of time required to make gear teeth by various processes. Several experts in the gear field reviewed the data and equations in this chapter. Their comments indicated that the data would be very helpful to most designers, but they also pointed out that some gear designs would be difficult to manufacture and a shop might—through no fault of its own—be unable to meet production rates estimated by the material in this chapter.

In spite of all that has been learned about how to design gears, how to rate them for load-carrying capacity, and how to use them properly in service, there is still an urgent need to do more bench testing, to make better analyses of gear-tooth stresses, and to learn more about effects of lubricants, variations in gear material quality, and peculiarities of the environment (corrosion, dirt in the oil, temperature transients, load transients, etc.) in which the gears operate.

This book refers to many standards for gears that are in current use. It can be expected that present standards will be revised and enlarged as gear technology progresses. This book, in several places, advises the reader to expect more and better standards for gear work as the years go by. The book gives practical advice about what to do, but it also cautions the reader to always consider the latest rules and data in a constantly changing array of gear trade standards.

Acknowledgments to the First Edition

The writing of a technical book covering a whole field of work is a very sizable project. The author is faced not only with organizing and writing the text material and equations but also with computing data, preparing tables and figures, and locating good reference data for a multiplicity of items.

I wish to wholeheartedly thank all of the people who have helped prepare this work.

The initial typing and equation work was done by Mrs. M. Irene Galarneau (a long-time secretary of mine), Mrs. Violet Daughters, and Mrs. Dorothy Dudley (my wife). The final typing, proofreading, and preparation of illustrations was done by Mrs. Carolyn Strickland (a technical assistant).

Many of the illustrations in the book have been furnished by companies that have also helped me get items of technical data. (Note the company names given in the figure legends.) The assistance of all of these companies is greatly appreciated.

The American Gear Manufacturers Association (AGMA) has been most helpful to me. The officers and staff are dedicated to the advancement of gear technology and have long provided forums for the decision and evaluation of gear experience. The great body of standards that have been issued by AGMA is of immense reference value. The reader will note that this book has made much use of AGMA standards.

I wish to particularly acknowledge the assistance of a few individuals with unusual knowledge and experience in the gear trade.

Dr. Hans Winter of the Technical University of Munich in Germany (Technische Universität München) has reviewed parts of this book and has helped me obtain data and a better understanding of the many aspects of gear research and gear rating.

Dr. Giovanni Castellani, a gear consultant in Modena, Italy, has helped me in comparing European and American gear rating practices and gear-tooth accuracy limits.

Mr. Eugene Shipley of Mechanical Technology Inc. in Latham, New York, has helped me—on the basis of his worldwide experience with gears in service—in many discussions about gear wear and gear failures.

Dr. Daniel Diesburg of the Climax Molybdenum Company in Ann Arbor, Michigan, was most helpful in reviewing data on gear metals and gear heat-treating procedures.

Mr. Dennis Townsend of the NASA Lewis Research Center in Cleveland, Ohio, has done unusual research work and also has brought together many key people in the gear trade under the auspices of ASME international meetings. The international ASME gear meeting in Chicago, Illinois, in 1977 was of much value to me.

Dr. Aizoh Kubo of Kyoto University in Kyoto, Japan, has directly contributed to this book in the section on dynamic loading. Indirectly, he has helped me greatly in getting to know the leading people in gear research in Japan.

While I acknowledge with much thanks the assistance of these and many other experts during this work, I must accept the responsibility for having made the final decisions on the technical content of this book. I have tried to present a fair consensus of opinion and the best gear design guidance from all the somewhat divergent theories and practices in the gear field.

Darle W. Dudley
San Diego, California

Author

Dr. Stephen P. Radzevich is a professor of mechanical engineering and a professor of manufacturing engineering. He received his MSc in 1976, PhD in 1982, and Dr(Eng)Sc in 1991, all in mechanical engineering. Dr. Radzevich has extensive industrial experience in gear design and manufacture. He has developed numerous software packages dealing with computer-aided design (CAD) and computer-aided machining (CAM) of precise gear finishing for a variety of industrial sponsors. His main research interest is the kinematic geometry of part surface generation, with a particular focus on precision gear design, high-power-density gear trains, torque share in multiflow gear trains, design of special purpose gear cutting/finishing tools, and design and machine (finish) of precision gears for low-noise and noiseless transmissions of cars, light trucks, and so on.

Dr. Radzevich has spent over 40 years developing software, hardware, and other processes for gear design and optimization. Besides his work for the industry, he trains engineering students at universities and gear engineers in companies.

He has authored and coauthored over 30 monographs, handbooks, and textbooks. The monographs *Generation of Surfaces* (RASTAN, 2001), *Kinematic Geometry of Surface Machining* (CRC Press, 2007; 2nd edition, 2014), *CAD/CAM of Sculptured Surfaces on Multi-Axis NC Machine: The DG/K-Based Approach* (M&C Publishers, 2008), *Gear Cutting Tools: Fundamentals of Design and Computation* (CRC Press, 2010), *Precision Gear Shaving* (Nova Science Publishers, 2010), *Dudley's Handbook of Practical Gear Design and Manufacture* (CRC Press, 2012), *Geometry of Surfaces: A Practical Guide for Mechanical Engineers* (Wiley, 2013), *Generation of Surfaces: Kinematic Geometry of Surface Machining* (CRC Press, 2014), and *High-Conformal Gearing: Kinematic and Geometry* (CRC Press, 2015) are among his recently published volumes. He also has authored and coauthored about 300 scientific papers, and holds about 250 patents on inventions in the field (United States, Japan, Russia, Europe, Canada, Soviet Union, South Korea, Mexico, and others).

Introduction

This book (further the *Gear Handbook*) deals with gears and gearing in both aspects, namely, gear design and gear production.

Gears are produced in enormous amounts—billions of gears are produced by industries every year. While the automotive industry ranks as the primary consumer of gears, numerous other industries also require huge amounts of gears: aerospace (helicopter transmission, and so forth), construction machinery, and agricultural machinery, to name a few. If we place cost savings in the production of every gear in the range of just ten cents, the total cost savings could reach hundreds of millions of dollars. This makes it clear that the gear design and the gear production processes require careful treatment.

There is much room for gear engineers and for gear practitioners for further improvements in cost-saving processes. The *Gear Handbook* can be helpful in solving many of the gear design problems, as well as gear production problems.

UNIQUENESS OF THIS BOOK

Many books on gears and gearing, both in design and in manufacturing areas, have been published so far. This third edition of the *Gear Handbook* is a unique one because of many reasons.

First, the *Gear Handbook* is based on practical experience of designing and production of gears. The collection of data in the *Gear Handbook* is invaluable for practitioners in both areas, namely, the gear design and the gear production areas.

Second, this third edition of the *Gear Handbook* is significantly enlarged in comparison to the earlier 2012 edition. New chapters are added, and a few more appendices are attached at the end of the book as well. This makes the *Gear Handbook* a comprehensive source of information for those who are involved in design and production of gears.

Third, the uniqueness of the *Gear Handbook* is proven by several earlier editions, starting from the 1954 edition. For decades, the earlier editions of the book were successfully used by gear experts all around the world. It is anticipated that this new revised, updated, and enlarged edition of the *Gear Handbook* will be acknowledged by the reader.

INTENDED AUDIENCE

Many readers will benefit from the *Gear Handbook*: mechanical and manufacturing engineers involved in continuous design and manufacturing process improvement and those who are active or intend to become active in the field. Senior undergraduate and graduate university students of mechanical and of manufacturing engineering are among them.

The *Gear Handbook* is intended to be used as a reference book, as well as an invaluable source of practical data for gear design engineers and for gear manufacturers.

THE ORGANIZATION OF THIS BOOK

This handbook is organized in 16 chapters followed by 6 appendices.

Gear design trends are considered in Chapter 1. Both manufacturing trends and selection of the right kind of gear are disclosed.

Section 1.1 encompasses small low-cost gears for toys, gadgets, and mechanisms. Then application gears, control gears, vehicle gears, marine gears, aerospace gear, and others are discussed.

The kinds of external spur and helical gears, straight bevel gears, spiral bevel gears, hypoid and face gears are covered in Section 1.2.

Gear types and nomenclature are discussed in Chapter 2. The discussion begins with classification of gears, which includes parallel axis gearing, nonparallel coplanar gears operating on intersecting axes, nonparallel noncoplanar gears operating on nonintersecting axes, and special gear types. The discussion is followed by nomenclature of all practical kinds of gears, which includes, but not limited to, nomenclature of spur and helical gears, both external and internal, crossed-helical gear nomenclature and formulas, bevel gear nomenclature and formulas, worm gear nomenclature and formulas, nomenclature of face gears, and beveloid gears. Advanced terminology in gear design is briefly discussed at the end of the chapter.

Gear tooth design is disclosed in Chapter 3. This includes basic requirements of gear teeth, standard systems of gear tooth proportions, general equations relating to center distance along with elements of center distance.

Preliminary design considerations are summarized in Chapter 4. Here, the stress formula is discussed in detail. This consideration is followed by data needed for gear drawings.

Design formulas are discussed in Chapter 5. Calculation of gear tooth data and gear rating practice are covered in this chapter.

Chapter 6 is devoted to the consideration of gear materials. Steel for gears, localized hardening of gear teeth, cast irons for gears, as well as nonferrous gear metals and nonmetallic gears are considered in this section of the book.

In Chapter 7, the concept of direct gear design is contributed by Dr. Alex Kapelevich. In this chapter, the geometry of asymmetric tooth gears, gear mesh characteristics, asymmetric tooth gearing limits, tooth geometry optimization, and analytical and experimental comparisons of symmetric and asymmetric tooth gears, along with the implementation of asymmetric tooth gears, are discussed.

In Chapter 8, elements of finite-element analysis of gears are discussed (this chapter is contributed by Dr. Sandeep Vijayakar). The following topics are covered in this chapter: motivation for computer modeling of gears, a short description of how the finite element works, two- and three-dimensional

element setup, selection of the order of elements, contact problem setup, calculation of bending stresses, contact pressures and load distribution calculation, examples, and limitations.

Load rating of gears is discussed in Chapter 9. The discussion includes but is not limited to main nomenclature, coplanar gears, RH—conventional fatigue limit of factor K, RF—conventional fatigue limit of factor U_L, crossed-helical gears, hypoid gears, and worm gearing. The chapter ends with an analysis of the state of the standards for load rating of gears.

In parallel to gear design, gear manufacturing methods are considered in the book. Chapter 10 is devoted in detail to the consideration of various methods for producing the gears. This includes gear tooth cutting by hobbing, shaping, milling, broaching, and others. Gear grinding methods, such as form grinding, generating grinding, and thread grinding are discussed. This is followed by the discussion of gear shaving, rolling, and honing methods. The consideration of the gear manufacturing methods is ended by a discussion of gear casting and forming methods.

The design of tools to make gear teeth is disclosed in Chapter 11. The design of shaper cutters, gear hobs, spur gear milling cutter, gear shaving cutter, punch tools, and sintering tools are considered in this section of the book.

Chapter 12 is devoted to the consideration of the kinds and the causes of gear failures. This includes an analysis of gear system problems, an analysis of tooth failures, and gear bearing failures, along with the consideration of some causes of gear failure other than excess transmission load.

Special design problems are disclosed in Chapter 13. Center distance problems, pro le modi cation problems, as well as load rating problems are considered in detail in this section of the book.

Gear reactions and mountings are outlined in Chapter 14. The discussion begins with an analysis of mechanics of gear reactions, which is then followed by consideration of basic reactions, bearing loads, and mounting types. After that, basic mounting arrangements are considered and corresponding recommendations are given. This is followed by an analysis of bearing load calculations for spur gears and for helical gears. Then mounting practice for bevel and hypoid gears along with calculation of bevel and hypoid bearing loads is considered. Chapter 14 is ended by an analysis of bearing load calculations for worms, for Spiroid, and for other gear types, as well as analysis of design of the body of the gear.

Finally, gear vibration is discussed in Chapter 15. This section of the book begins with the fundamental of vibration. Then, the measurement of vibration is considered. Examples of vibration in geared units are provided. Approximate vibration limits are outlined and control of vibration in manufacturing gears and in the eld are discussed.

Six appendices are attached at the very end of the book.

Appendix A: The Evolution of the Gear Art
Appendix B: Complementary Material
Appendix C: Numerical Data Tables
Appendix D: On the Concept of Novikov Gearing and on the Inadequacy of the Terms Wildhaber–Novikov Gearing and W-N Gearing
Appendix E: An Improved Load Equalizing Means for Planetary Pinions
Appendix F: Geometrically Accurate (Ideal or Perfect) Crossed-Axis Gearing with Line Contact between the Tooth Flanks of the Gear and the Pinion

All comments and constructive suggestions on content of the book should be sent to the publisher, CRC Press, Taylor & Francis Group, at http://www.crcpress.com.

Stephen P. Radzevich
Sterling Heights, Michigan

1 Gear-Design Trends

Gears are used in most types of machinery. Like nuts and bolts, they are a common machine element which will be needed from time to time by almost all machine designers. Gears have been in use for over three thousand years,* and they are an important element in all manner of machinery used in current times (see Appendix A).

Gear design is a highly complicated art. The constant pressure to build less expensive, quieter running, lighter-weight, and more powerful machinery has resulted in a steady change in gear designs. At present much is known about gear load-carrying capacity, and many complicated processes for making gears are available.

The industrialized nations are all doing gear research work in university laboratories and in manufacturing companies. Even less-developed countries are doing a certain amount of research work in the mathematics of gears and in gear applications of particular interest. At the 1981 International Symposium on Gearing and Power Transmission in Tokyo, Japan, 162 papers were presented by delegates from 24 nations. After the excellent world gear conferences in Paris, France, in 1977, and Dubrovnik, Yugoslavia, in 1978, the even larger Tokyo gear conference in 1981 was a high-water mark in the gear art.

Most machine designers do not have the time to keep up with all the developments in the field of gear design. This makes it hard for them to quickly design gears which will be competitive with the best that are being used in their field. There is a great need for *practical* gear-design information. Even though there is a wealth of published information on gears, gear designers often find it hard to locate the information they need quickly. This book is written to help gear designers get the vital information they need as easily as possible.

Those who are just starting to learn about gears need to start by learning some basic words that have special meanings in the gear field. The glossary in Table 1.1 is intended to give simple definitions of these terms as they are understood by gear people. Table 1.2 shows the metric and English gear symbols for the terms which are used in this chapter. The glossary in Chapter 5 (Table 5.1) defines gear-manufacturing terms. See the AGMA† standards for English and metric gear nomenclature.

In Chapter 2, Section 2.1 gives a simple explanation of some basics of gearing. Beginners will probably find it helpful to read Section 2.1 and the previously mentioned tables before they continue with the rest of Chapter 1.

1.1 MANUFACTURING TRENDS

Before plunging into the formulas for calculating gear dimensions, it is desirable to make a brief survey of how gears are presently being made and used in different applications.

The methods used to manufacture gears depend on design requirements, machine tools available, quantity required, cost of materials, and *tradition*. In each particular field of gear work, certain methods have become established as the *standard* way of making gears. These methods tend to change from time to time, but the tradition of the industry tends to act as a brake to restrain any abrupt changes that result from technological developments. The gear designer, studying gears as a whole, can get a good perspective of gear work by reviewing the methods of manufacture in each field.

1.1.1 Small, Low-Cost Gears for Toys, Gadgets, and Mechanisms

There is a large field of gear work in which tooth stress is of almost no consequence. Speeds are slow, and life requirements do not amount to much. Almost any type of cogwheel which could transmit rotary motion might be used. In this sort of situation, the main thing the designer must look for is low cost and high volume of production.

The simplest type of gear drive—such as those used in toys—frequently uses punched gears. Pinions with a small number of teeth may be die-cast or extruded. If loads are light enough and quietness of operation is desired, injection-molded gears and pinions may be used. Molded-plastic gears from a toy train are shown in Figure 1.1. Things such as film projectors, oscillating fans, cameras, cash registers, and calculators frequently need quiet-running gears to transmit insignificant amounts of power. Molded-plastic gears are widely used in such instances. It should be said, though, that the devices just mentioned often need precision-cut gears, where loads and speeds become appreciable.

Die-cast gears on zinc alloy, brass, or aluminum are often used to make small, low-cost gears. This process is particularly favored where the gear wheel is integrally attached to some other element, such as a sheave, cam, or clutch member. The gear teeth and the special contours of whatever is attached to the gear may all be finished to close accuracy merely by die-casting the part in a precision metal mold. Many low-cost gadgets on the market today would be very much more expensive if it were necessary to machine all the complicated gear elements that are in them.

* The book *The Evolution of the Gear Art* (1969) by D. W. Dudley gives a brief review of the history of gears through the ages. Consult the section Literature at the end of the book for complete data on Dudley's book and on other important references. A slightly edited version of the book *The Evolution of the Gear Art* (1969) can be found out in Appendix A.

† *AGMA* stands for the American Gear Manufacturers Association.

TABLE 1.1
Glossary of Gear Nomenclature, Chapter 1

Term	Definition
Addendum	The radial height of a gear tooth above the pitch circle.
Axial section	A section through a gear in a lengthwise direction that *contains the axis* of the gear.
Bevel gears	Gears with teeth on the outside of a conical-shaped body (normally used on 90° axes).
Circular pitch	The circular distance from a point on one gear tooth to a like point on the next tooth, taken along the *pitch circle*. Two gears must have the same circular pitch to mesh with each other. As they mesh, their circles will be tangent to one another.
Dedendum	The radial height of a gear tooth below the pitch circle.
Diametral pitch	A measure of tooth size in the English system. In units, it is the *number of teeth* per inch of pitch diameter. As the tooth size increases, the diametral pitch decreases. Diametral pitches usually range from 25 to 1.
External gears	Gears with teeth on the outside of a cylinder.
Face gears	Gears with teeth on the *end* of the cylinder.
Gear	A geometric shape that has teeth uniformly spaced around the circumference. In general, a gear is made to mesh its teeth with another gear. (A sprocket looks like a gear but is intended to drive a chain instead of another gear.)
Helical gears	Gears with teeth that spiral around the body of the gear.
Helix angle	The inclination of the tooth in a lengthwise direction. (If the helix angle is 0°, the tooth is parallel to the axis of the gear—and is really a spur-gear tooth.)
Internal gears	Gears with teeth on the inside of a hollow cylinder. (The mating gear for an internal gear must be an external gear.)
Module	A measure of tooth size in the metric system. In units, it is millimeters of pitch diameter *per tooth*. As the tooth size increases, the module also increases. Modules usually range from 1 to 25.
Normal section	A section through the gear that is *perpendicular to the tooth* at the pitch circle. (For spur gears, a normal section is also a transverse section.)
Pinion	When two gears mesh together, the *smaller* of the two is called the pinion. The larger is called the gear.
Pitch diameter	The diameter of the pitch circle of a gear.
Pressure angle	The slope of the gear tooth at the pitch-circle position. (If the pressure angle iswere 0°, the tooth is parallel to the axis of the gear—and is really a spur-gear tooth.)
Ratio	*Ratio* is an abbreviation for gear-tooth ratio, which is the number of teeth on the gear divided by the number of teeth on its mating pinion.
Spiroid gears	A family of gears in which the tooth design is in an intermediate zone between bevel-, worm-, and face-gear designs. The Spiroid design is patented by the Spiroid Division of Illinois Tool Works, Chicago, Illinois.
Spur gears	Gears with teeth straight and parallel to the axis of rotation.
Transverse section	A section through a gear *perpendicular to the axis* of the gear.
Whole depth	The total radial height of a gear tooth (whole depth = addendum + dedendum).
Worm gears	Gearsets in which one member of the pair has teeth wrapped around a cylindrical body like screw threads. (Normally this gear, called the *worm*, has its axis at 90° to the worm-gear axis.)

Note: For terms relating to gear materials, see Chapter 4. For terms relating to gear manufacture, see Chapter 5. For terms relating to the specifics of gear design and rating, see Chapters 2 and 3. For a simple introduction to gears, see Chapter 2, Section 2.1.

Metal forming is more and more widely used as a means of making small gear parts. Pinions and gears with small numbers of teeth may be cut from rod stock with cold-drawn or extruded teeth already formed in the rod. Figure 1.2 shows some rods with cold-drawn teeth. Small worms may have cold-rolled threads. The forming operations tend to produce parts with very smooth, work-hardened surfaces. This feature is important in many devices where the friction losses in the gearing tend to be the main factor in the power consumption of the device.

Figure 1.3 shows an assortment of stamped and molded gears used in small mechanisms.

1.1.2 Appliance Gears

Home appliances such as washing machines, food mixers, and fans use large numbers of small gears. Because of competition, these gears must be made for only a relatively few cents apiece. Yet they must be quiet enough to suit a discriminating homemaker and must be able to endure for many years with little or no more lubrication than that given to them at the factory.

Medium-carbon-steel gears finished by conventional cutting used to be the standard in this field. Cut gears are still in widespread use, but the cutting is often done by high-speed automatic machinery. The worker on the cutting machine does little more than bring up trays of blanks and take away trays of finished parts.

Modern appliances are making more and more use of gears other than cut steel gears. Figure 1.4 shows some sintered-iron gears from an automatic washer. Sintered-iron gears are very inexpensive (in large quantities), run quietly, and frequently wear less than comparable cut gears. The sintered metal is porous and may be impregnated with a lubricant. It may also

TABLE 1.2
Gear Terms, Symbols, and Units, Chapter 1

Term	Metric Symbol	Metric Unit	English Symbol	English Unit	First Reference or Definition
Module	m	mm	–	–	Equation 1.1
Pressure angle	α	°	ϕ	deg	Figures 1.18, 1.33
Number of teeth or threads	z	–	N	–	
Number of teeth, pinion	z_1	–	N_p or n	–	Equation 1.10
Number of teeth, gear	z_2	–	N_G or N	–	Equation 1.10
Ratio (gear or tooth ratio)	u	–	m_G	–	$u = z_2/z_1$ ($m_G = N_G/N_p$)
Diametral pitch	–	–	P_d or P	in.$^{-1}$	Equation 1.1
Pi	π	–	π	–	π 3.1415927
Pitch diameter, pinion	d_{p1}	mm	d	in.	Figure 1.18, Equation 1.5
Pitch diameter, gear	d_{p2}	mm	D	in.	Figure 1.18, Equation 1.6
Base (circle) diameter, pinion	d_{b1}	mm	d_b	in.	Figure 1.18
Base (circle) diameter, gear	d_{b2}	mm	D_b	in.	Figure 1.18
Outside diameter, pinion	d_{a1}	mm	d_o	in.	Figure 1.18 (abbreviation: OD)
Outside diameter, gear	d_{a2}	mm	D_o	in.	Figure 1.18
Form diameter	d_f'	mm	d_f	in.	Figure 1.18
Circular pitch	p	mm	p	in.	Figure 1.18, Equation 1.2
Addendum	h_a	mm	a	in.	Figure 1.18
Dedendum	h_f	mm	b	in.	Figure 1.18
Face width	b	mm	F	in.	Figure 1.18
Whole depth	h	mm	h_t	in.	Figure 1.18
Working depth	h'	mm	h_k	in.	Figure 1.18
Clearance	c	mm	c	in.	Figure 1.18
Chordal thickness	$\bar{s}$	mm	t_c	in.	Figure 1.18
Chordal addendum	$\bar{h}_a$	mm	a_c	in.	Figure 1.18
Tooth thickness	s	mm	t	in.	Figure 1.18
Center distance	a	mm	C	in.	Equation 1.7
Circular pitch, normal	p_n	mm	p_n	in.	Equations 1.8, 1.22
Circular pitch, transverse	p_t	mm	p_t	in.	Equations 1.8, 1.22
Lead angle	γ	°	λ	deg	Equations 1.22, 1.36
Helix angle	β	°	ψ	deg	Equations 1.11, 1.22
Pitch, base or normal	p_b or p_{bn}	mm	p_b or p_N	in.	Equation 1.22
Diametral, base or normal	–	–	P_n	in.$^{-1}$	Equation 1.13
Axial pitch	p_x	mm	p_x	in.	Equation 1.14
Inside (internal) diameter	d_i	mm	D_i	in.	Figure 1.20 (abbreviation: ID)
Root diameter, pinion	d_{f1}	mm	d_R	in.	Figure 1.29
Root diameter, gear	d_{f2}	mm	D_R	in.	Figure 1.20
Pitch angle, pinion	δ_1'	°	γ	deg	Figure 1.21, Equation 1.19
Pitch angle, gear	δ_2'	°	Γ	deg	Figure 1.21, Equation 1.19
Root angle, pinion	δ_{f1}	°	γ_R	deg	Figure 1.21
Root angle, gear	δ_{f2}	°	Γ_R	deg	Figure 1.21
Face angle, pinion	δ_{a1}	°	γ_o	deg	Figure 1.21
Face angle, pinion	δ_{a2}	°	Γ_o	deg	Figure 1.21
Dedendum angle	θ_f	°	δ	deg	Figure 1.21
Shaft angle	Σ	°	Σ	deg	Figure 1.21, Equation 1.21
Cone distance	R	°	A	deg	Figure 1.21
Circular (tooth) thickness	s	mm	t	in.	Figure 1.21
Backlash	j	mm	B	in.	Figure 1.21
Throat diameter of worm	d_{t1}	mm	d_t	in.	Figure 1.29
Throat diameter of worm gear	d_{t2}	mm	D_t	in.	Figure 1.29
Lead of worm	p_{zW}	mm	L_W	in.	Equation 1.35

Note: deg: degrees; in.: inches; mm: millimeters.

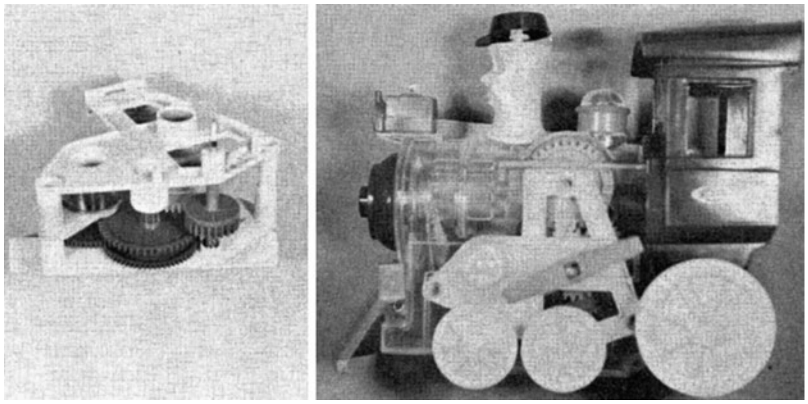


FIGURE 1.1 Plastic gears in a toy.

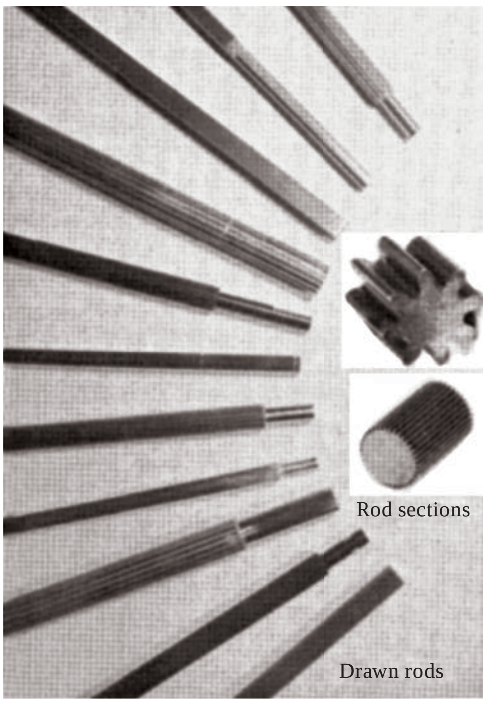


FIGURE 1.2 Samples of cold-drawn rod stock for making gears and pinions. (Courtesy of Rathbone Corp., Palmer, Massachusetts.)

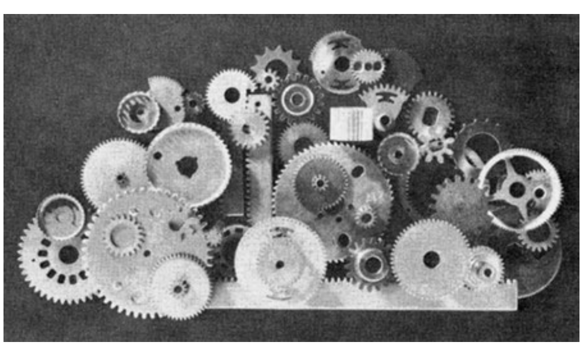


FIGURE 1.3 Assortment of gears used in small mechanisms. (Courtesy of Winzeler Manufacturing and Tool Co., Chicago, Illinois.)

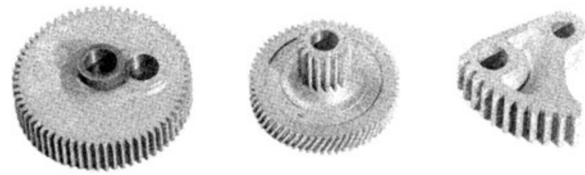


FIGURE 1.4 Sintered-iron gears used in an automatic washing machine.

be impregnated with copper to improve its strength. Gear teeth and complicated gear-blank shapes may all be completely nished in the sintering process. The tools need to make a sintered gear may cost as much as $50,000, but this does not amount to much if 100,000 or more gears are to be made on semiautomatic machines.

Laminated gears using phenolic resins and cloth or paper have proved very good where noise reduction is a problem. The laminates, in general, have a higher load-carrying capacity than molded-plastic gears. Nonmetallic gears with cut teeth do not suffer nearly so much from tooth inaccuracy as do metal gears. Under the same load, a laminated phenolic-resin gear tooth will bend about 30 times as much as a steel gear tooth. It has often been possible to take a set of steel gears which were wearing excessively because of tooth-error effects, replace one member with a nonmetallic gear, and have the set stand up satisfactorily.

Nylon gear parts have worked very well in situations in which wear resulting from high sliding velocity is a problem. The nylon material seems to have some of the characteristics of a solid lubricant. Nylon gearing has been used in some processing equipment where the use of a regular lubricant would pollute the material being processed.

1.1.3 Machine Tools

Accuracy and power-transmitting capacity are quite important in machine-tool gearing. Metal gears are usually used. The teeth are nished by some precise metal-cutting process.

The machine tool is often literally full of gears. Speed-change gears of the spur or helical type are used to control

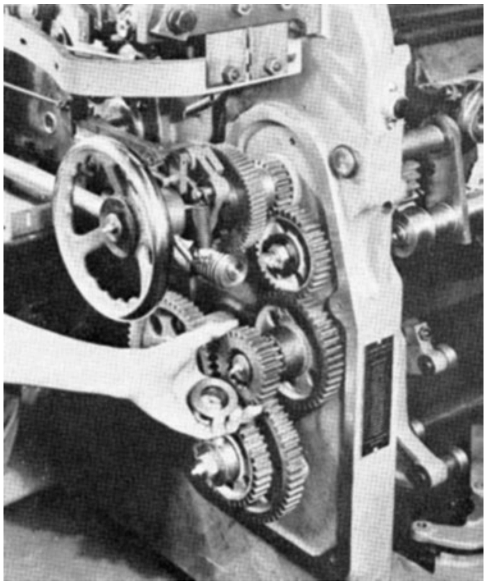


FIGURE 1.5 Example of change gears on a machine tool. (Courtesy of Warner & Swansea Co., Cleveland, Ohio.)

feed rate and work rotation. Index drives to work or table may be worm or bevel. Sometimes they are spur or helical. Many bevel gears are used at the right-angle intersections between shafts in bases and shafts in columns. Worm gears and spiral gears are also commonly used at these places.

Machine-tool gearing is often machine-finished in a medium-hard condition (250 to 300 Vicker's hardness [HV] or 25 to 30 Rockwell hardness [HRC]). Mild-alloy steels are frequently used because of their better machinability and physical properties. Cast iron is often favored for change gears because of the ease in casting the gear blank to shape, its excellent machinability, and its ability to get along with scanty lubrication.

The higher cutting speeds involved in cutting with tungsten carbide tools have forced many machine-tool builders to put in harder and more accurate gears. Shaving and grinding are commonly used to finish machine-tool gears to high tooth accuracy. In a few cases, machine-tool gears are being made with such top-quality features as full hardness (700 HV or 60 HRC), profile modification, and surface finish of about 0.5 μm or 20 μin.

The machine-tool designer has a hard time calculating gear sizes. Loads vary widely depending on feeds, speeds, size of work, and material being cut. It is anybody's guess what the user will do with the machine. Because machine tools are quite competitive in price, the overdesigned machine may be too expensive to sell. In general, the designer is faced with the necessity of putting in gears which have more capacity than the average load, knowing that the machine tool is apt to be neglected or overloaded on occasions.

Figure 1.5 shows an example of machine-tool gears.

1.1.4 Control Gears

Guns or ships, helicopters, and tanks are controlled by gear trains with the backlash held to the lowest possible limits. The primary job of these gears is to transmit motion. What power they may transmit is secondary to their job of precise control of angular motion. In power gearing, a worn-out gear is one with broken teeth or bad tooth-surface wear. In the control-gear field, a worn-out gear may be one whose thickness has been reduced by as small an amount as 0.01 mm (0.0004 in.)!

Some of the most spectacular control gears are those used to drive radio telescopes and satellite-tracking antennas. These gears are so large that the only practical way to make them is to cut rack sectors and then bend each section into an arc of a circle. Figure 1.6 shows an example of a very large antenna drive made this way. The teeth on the rack sections

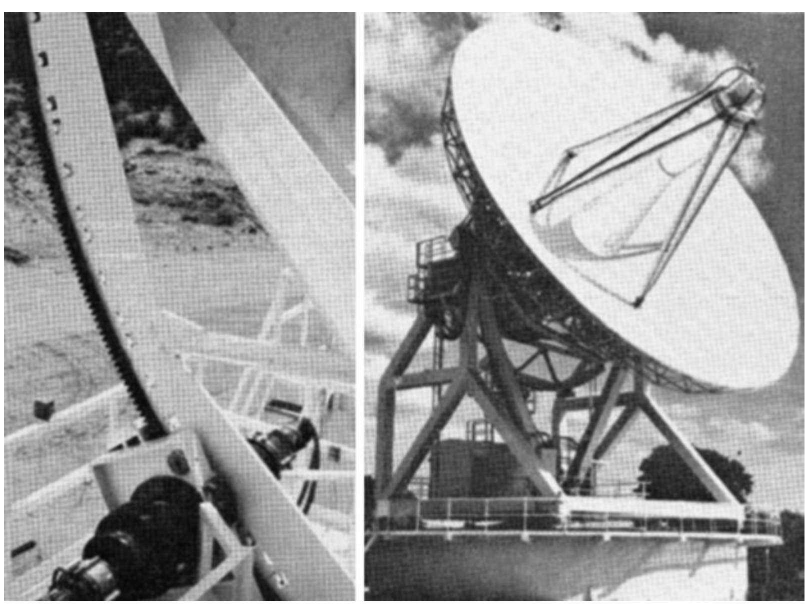


FIGURE 1.6 Gears are used to position large radio telescopes. (Courtesy of Harris Corporation, Melbourne, Florida.)

are cut so that they will have correct tooth dimensions when they are bent to form part of the circular gear.

The radar units on an aircraft carrier use medium-pitch gears of a fairly large size. Radar-unit gearing is generally critical on backlash, must handle rather high momentary loads, and must last for many years with somewhat marginal lubrication. (Radar gears are often somewhat in the open, and therefore can only use grease lubrication.)

Control gears are usually spur, bevel, or worm. Helical gears are used to a limited extent. Control gears are often in the ne-pitch range—1.25 module (20 diametral pitch) or ner. Figure 1.7 shows a control device with many small gears.

In a few cases, control gears become quite large. The gears which train a main battery must be very rugged. The reaction on the gears when the main battery is red can be terri c.

Control gears are usually made of medium-alloy, medium-carbon steels. In many cases, they are hardened to a medium hardness before nal machining. In other cases, they are hardened to a moderately high hardness after nal machining. These gears need hardness mainly to limit wear. Any hardening done after nal nishing of the teeth must be done in such a way as to give only negligible dimensional change or distortion. To eliminate backlash, it is necessary to size the gear teeth almost perfectly (or use special antibacklash gear arrangements).

Shaving and grinding are used to control tooth thickness to the very close limits needed in control gearing. The inspection of control gears is usually based on checking machines which measure the variation in center distance when a master pinion or rack is rolled through mesh with the gear being checked. A spring constantly holds the master and the gear being checked in tight mesh. The chart obtained from such a checking machine gives a very clear picture of both the tooth thickness of the gear being checked and the variations in backlash as the gear is rolled through the mesh. If the backlash variation can be held to acceptable limits in control gearsets,

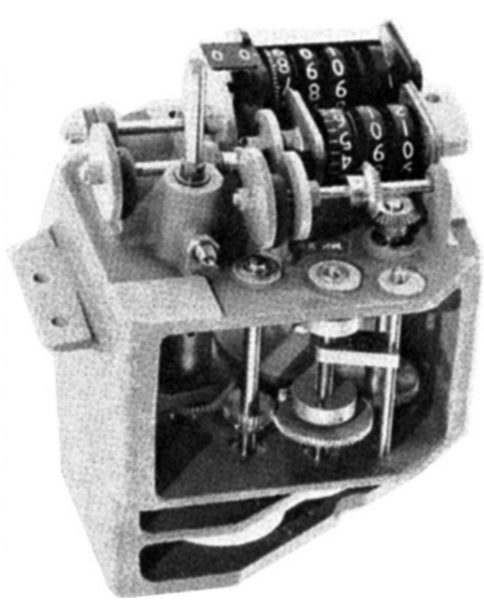


FIGURE 1.7 Control device with many small gears.

there is usually no need to know the exact involute and spacing accuracy. In some types of radar and rocket-tracking equipment, control-gear teeth must be spaced very accurately with regard to accumulated error. For instance, it may be necessary to have every tooth all the way around a gear wheel within its true position within about 10 seconds of arc. On a 400 mm (15.7 in.) wheel, this would mean that every tooth has to be correctly spaced with respect to every other tooth within 0.01 mm (0.0004 in.). This kind of accuracy can be achieved only by special gear-cutting techniques. Inspection of such gears generally requires the use of gear-checking machines that are equipped to measure accumulated spacing error. The equipment commonly used will measure any angle to within 1 second of arc or better.

1.1.5 Vehicle Gears

An automobile normally uses spur and helical gears in the transmission and bevel gears in the rear end. If the car is a front-end drive, bevel gears may still be used or helical gears may be used. Automatic transmissions are now widely used. This does not eliminate gears, however. Most automatic transmissions have more gears than manual transmissions.

Automotive gears are usually cut from low-alloy-steel forgings. At the time of tooth cutting, the material is not very hard. After tooth cutting, the gears are case-carburized and quenched. Quenching dies are frequently used to minimize distortion. The composition of heats (batches) of steel is watched carefully, and all steps in manufacture are closely controlled with the aim of having each gear in a lot behave in the same way when it is carburized and quenched. Even if there is some distortion, it can be compensated for in machining, provided that each gear in a lot distorts uniformly and by the same amount. Since most automotive-gear teeth are not ground or machined after nal hardening, it is essential that the teeth be quite accurate in the as-quenched condition. The only work that is done after hardening is the grinding of journal surfaces and sometimes a small amount of lapping. Finished automotive gears usually have a surface hardness of about 700 HV or 60 HRC and a core hardness of 300 HV or 30 HRC.

Varieties of machines are used to cut automotive-gear teeth. In the past, shapers, hobbers, and bevel-gear generators were conventional machines. New types of these machines are presently favored. For instance, multistation shapers, hobbers, and shaving machines are used. A blank is loaded on the machine at one station. While the worker is loading other stations, the piece is nished. This means that the worker spends no time waiting for work to be nished.

There is a wide variety of special design machines to hob, shape, shave, or grind automotive gears. The high production of only one (or two or three) gear design makes it possible to simplify a general-purpose machine tool and then build a kind of *processing center* where these functions are performed:

- Incoming blank is checked for correct size.
- Incoming blank is automatically loaded into the machine.

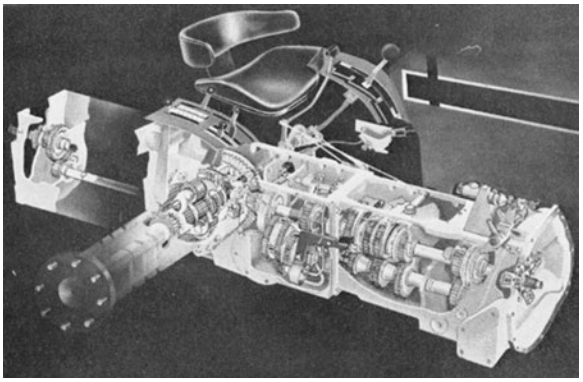


FIGURE 1.8 Tractor power unit. (Courtesy of International Harvester Co., Chicago, Illinois.)

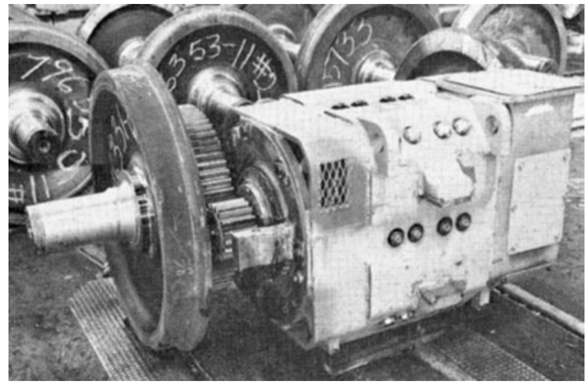


FIGURE 1.10 Railroad wheel drive. (Courtesy of Electro-Motive Division, General Motors Corp., La Grande, Illinois.)

- Teeth are cut (or   nished) very rapidly.
- Finished parts are checked for accuracy and sorted into categories of accept, rework, or reject.
- Outgoing parts are loaded onto conveyor belts.

Sometimes the processing center may be developed to the point where cutting, heat-treating, and   nishing are all done in one processing center.

The gears for the smaller trucks and tractors are made somewhat like automotive gears, but the volume is not quite so great and sizes are large. Examples of large-vehicle gears are shown in Figures 1.8 and 1.9. Large tractors, large trucks, and off-the-road earth-moving vehicles use much larger gears and have much lower volume than automotive gears. More conventional machine tools are used. Special-purpose processing centers are generally not used.

Vehicle gears are heavily loaded for their size. Fortunately, their heaviest loads are of short duration. This makes it possible to design the gears for limited life at maximum motor torque and still have a gear that will last many years under average driving torque.

Although carburizing has been widely used as a means of hardening automotive gears, other heat treatments are being used on an increased scale. Combinations of carburizing and nitriding are used. Processes of this type produce a shallower case for the same length of furnace time, but tend to make the gear tooth surface harder and the distortion less. Induction hardening is being used on some   ywheel-starter gears as well as some other gears. Flame hardening is also used to a limited extent.

1.1.6 Transportation Gears

Buses, subways, mine cars, and railroad diesels all use large quantities of spur and helical gears. The gears range up to 0.75 m (30 in.) or more in diameter; see Figure 1.10. Teeth are sometimes as coarse as 20 module (1.25 diametral pitch). Plain carbon and low-alloy steels are usually used. Much of the gearing is case-carburized and ground. Through hardening is also used extensively. A limited use is being made of induction-hardened gears.

Transportation gears are heavily loaded. Frequently their heaviest loads last for a long period. Diesels that pull trains over high mountains ranges have long periods of operation at maximum torque. In some applications, severe but infrequent shock loads are encountered. Shallow-hardening, medium-carbon steels seem to resist shock better than gears with a fully hardened carburized case. Both furnace and induction-hardening techniques are used to produce shallow-hardened teeth with high shock resistance.

Gear cutting is done mostly by conventional hobbing or shaping machines. Some gears are shave and then heat-treated, while others are heat-treated after cutting and then ground. In this   eld of work, the volume of production is much lower and

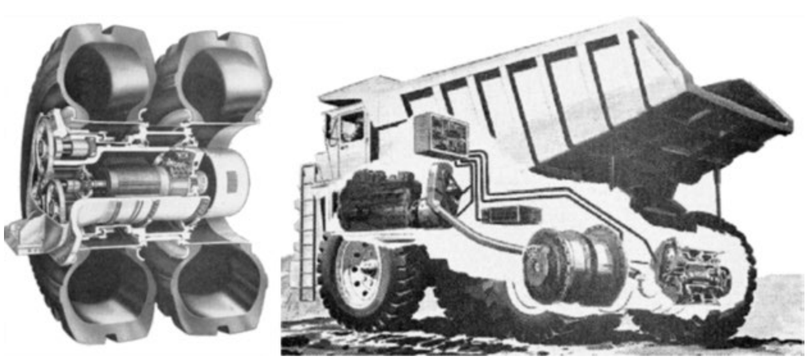


FIGURE 1.9 Motorized wheels drive large earth-moving trucks. (Courtesy of General Electric Co., Erie, Pennsylvania.)

the size of parts is much larger than in the vehicle-gear field. Both these conditions make it harder to keep heat-treat distortion well under control that the teeth may be finished before hardening.

The machine tools used to make transportation gears are quite conventional in design. The volume of production in this field is not large enough to warrant the use of the faster and more elaborate types of machine tools used in the vehicle-gear field.

1.1.7 Marine Gears

Powerful, high-speed, large gears power merchant marine and navy fighting ships. Propeller drives on cargo ships use bull gears up to 5 m (200 in.) in diameter. First reduction pitch-line speeds on some ships go up to 100 m/s or 20,000 ft/minute (fpm). Single-propeller drives in a navy capital ship go up to 40,000 kW or more of power. Some of the new cargo ships now in service have 30,000 kW (40,000 hp) per screw. Figure 1.11 shows a typical marine gear unit.

Marine gears are almost all made by finishing the teeth after hardening. Extreme accuracy in tooth spacing is required to enable the gearing to run satisfactorily at high speed. As many as 6000 pairs of teeth may go through one gear mesh in a second's time!

There is an increased use of carburized and ground gears in the marine field. The fully hard gear is smaller and lighter. This lowers pitch-line velocity and helps to keep the engine room reasonably small. Hard gears resist pitting and wear better than medium-hard gears (through-hardened gears).

Single-helical gears have long been used for electric-power-generating equipment. Double-helical gears have generally been used for the main propulsion drive of large ships. Single-helical gears—with special thrust runners on the gears themselves—are becoming fairly commonly used even on large ships.

Spur and bevel-gear drives are often used on small ships, but they are seldom used on large turbine-driven ships. (Some slower-speed diesel-driven ships use spur gearing.)

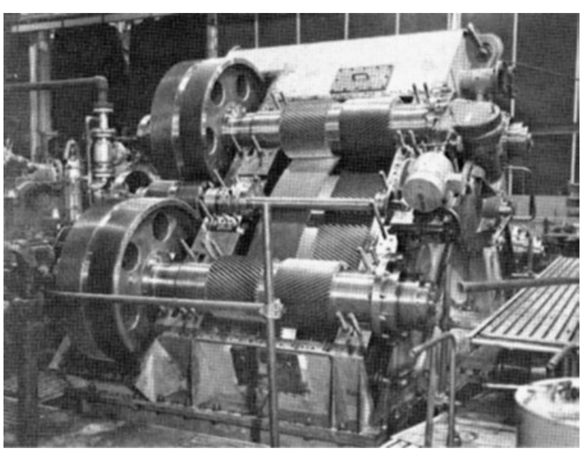


FIGURE 1.11 Partially assembled double-reduction marine gear unit. (Courtesy of Transamerica Delaval Inc., Trenton, New Jersey.)

Marine gears generally use a medium-alloy carbon steel. It is difficult or impossible to heat-treat plain carbon and low-alloy carbon steels in large sizes and get hardnesses over 250 HV (250 Brunel hardness [HB]) and a satisfactory metallurgical structure. Special welding techniques for large gear wheels make it possible to use medium-alloy steels and get good through-hardened gears up to 5 m (200 in.) in diameter with gear tooth hardness at 320 HV (300 HB) or higher. The smaller pinions are not welded and can usually be made up to 375 HV (350 HB) or higher in sizes of up to 0.75 m (30 in.).

There is a growing use of carburized and ground gears for ship propulsion. Some of these gears are now being made as large as 2 m (80 in.) in size. In a very large marine drive, it is quite common to have fully hard, carburized gears in the first reduction and medium-hard, through-hardened gears in the second reduction. Figure 1.12 shows a large marine drive unit being lowered into position.

The designer of marine gearing has to worry about both noise and load-carrying capacity. Although the tooth loads are not high compared with those on aircraft or transportation gearing, the capacity of the medium-hardness gearing to carry load is not high either. Considering that during its lifetime a high-speed pinion on a cargo ship may make 10 to 11 billion cycles of operation at full-rated torque, it can be seen that load-carrying capacity is very important.

Gear noise is a more or less critical problem on all ship gearing. The auxiliary gears that drive generators are frequently located quite close to passenger quarters. The peculiar high-pitched whine of a high-speed gear has a damaging effect on either an engine-room operator or a passenger who may be quartered near the engine room.

On fighting ships, there are the added problems of keeping enemy submarines from picking up waterborne noises and of operating the ship quietly enough to be able to hear waterborne noises from an enemy.

Quietness is achieved on marine gears by making the gears with large numbers of teeth and cutting the teeth with extreme accuracy. A typical marine pinion may have around 60 teeth and a tooth-to-tooth spacing accuracy of 5 μm (0.0002 in.) maximum. A pinion of the same diameter used in a railroad-gear application would have about 15 teeth and a tooth-spacing accuracy of 12 μm (0.0005 in.). In the comparison just made, the marine-gear teeth would be only one quarter the size of the railroad-gear teeth.

1.1.8 Aerospace Gears

Gears are used in a wide variety of applications on aircraft. Propellers are usually driven by single- or double-reduction gear trains. Accessories such as generators, pumps, hydraulic regulators, and tachometers are gear driven. Many gears are required to drive these kinds of accessories even on *jet* engines—which have no propellers. Additional gears are used to raise landing wheels, open bomb-bay doors, control guns, operate computers for gun- or bomb-sighting devices, and control the pitch of propellers. Helicopters have a considerable amount of gearing to drive main rotors and tail rotors.

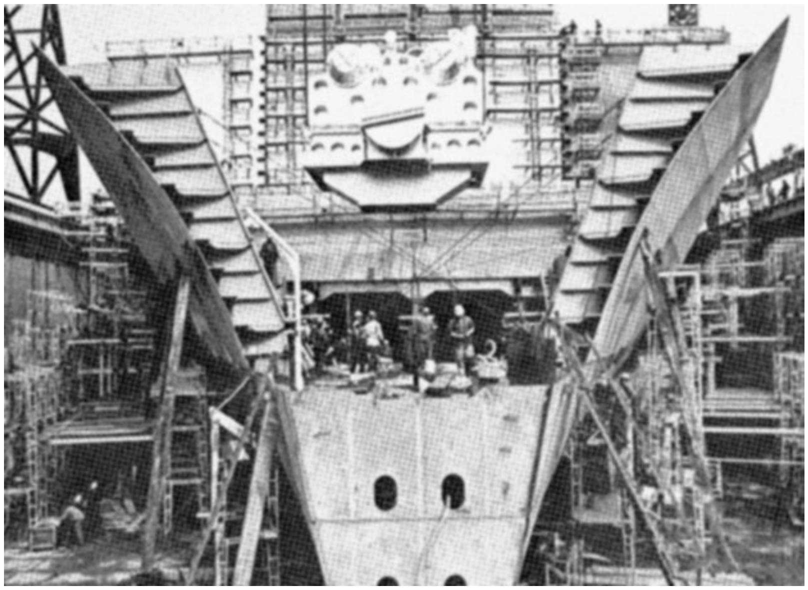


FIGURE 1.12 Lowering a very powerful marine gear unit into the engine space of a supertanker. (Courtesy of Transamerica Delaval Inc., Trenton, New Jersey.)

(See Figure 1.13.) Space vehicles often use power gears between the turbine and the booster fuel pumps.

The most distinctive types of aircraft gears are the power gears for propellers, accessories, and helicopter rotors. The control and actuating types of gears are not too different from what would be used for ground applications of a similar nature (except that they are often highly loaded and made of extra hard, high-quality steel).

Aircraft power gears are usually housed in aluminum or magnesium casings. The gears have thin webs and light cross sections in the rim or hub. Accessory gears are frequently made integral with an internally splined hub.

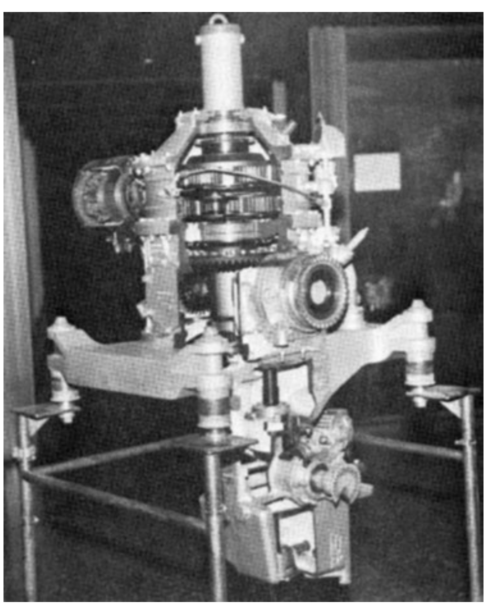


FIGURE 1.13 Cutaway view of a helicopter gear unit. The last two stages are epicyclic. The first stage is spiral bevel. (Courtesy of Bell Helicopter Textron, Fort Worth, Texas.)

Spur or bevel gears are usually used for accessory drives. The envelope clearance required to mount the accessory driven by the gear often makes it necessary to use large center distance but narrow face width. This fact plus the thrust problem tends to rule out helical accessory gears. Propeller-driven gears have wider face widths. Spur, bevel, and helical gear drives are all currently in use. To get maximum power capacity with small size and lightweight gearboxes, it is often desirable to use an epicyclic gear train. In this kind of arrangement, there is only one output gear, but several pinions drive against it.

Aerospace power gears are usually made of high-alloy steel and fully hardened (on the tooth surface) by either case-carburizing or nitriding. In some designs it is feasible to cut, shave, case-carburize, and grind only the journals. Many designs have such thin, nonsymmetrical webs as to require grinding after hardening. In general, piston-engine gearing runs more slowly than gas-turbine gearing. This makes some difference in the required accuracy. So far, many more piston-engine gears have been successfully finished before hardening than have gas-turbine gears.

The tooth loads and speeds are both very high on modern aircraft gears. The designer must achieve high tooth strength and high wear resistance. In addition, the thin oils used for low-temperature starting of military aircraft make the scoring type of lubrication failure a critical problem. Several special things are done to meet the demands of aircraft-gear service. Pinions are often made *long* addendum and gears *short* addendum to adjust the tip sliding velocities and to strengthen the pinion. Pinion tooth thicknesses are often increased at the expense of the gear to strengthen the pinion. High pressure angles, such as 22.5°, 25°, and 27.5°, are often used to reduce the base. Involute-profile modifications are generally used to compensate for bending and to keep the tips from cutting the mating part.

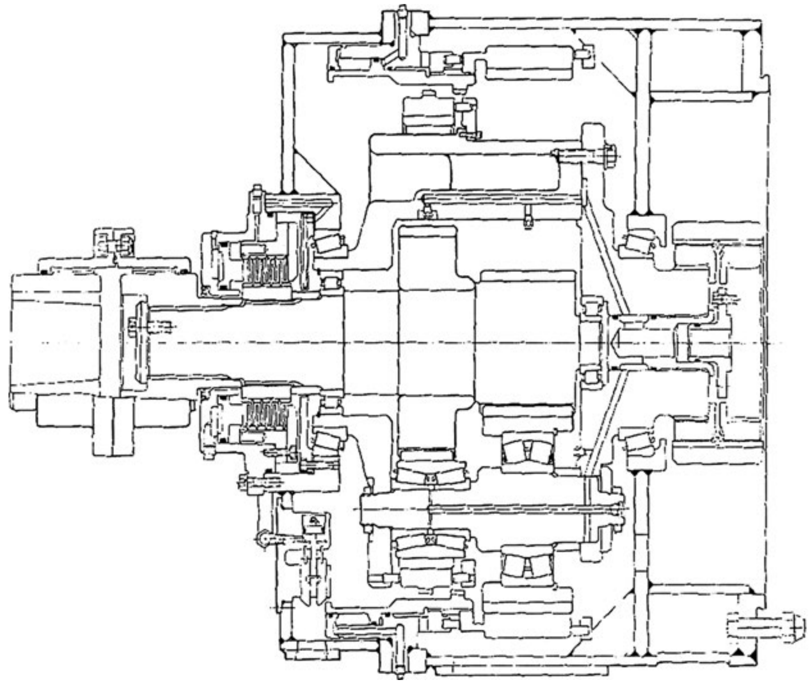


FIGURE 1.14 Truck-mounted, two-speed gear unit used to drive a piston pump for oil-well fracturing. (Courtesy of Sier-Bath Gear Co., Inc., North Bergen, New Jersey.)

The most highly developed aerospace gears are those used in rocket engines. The American projects *Vanguard*, *Mercury*, *Gemini*, and *Apollo* succeeded in boosting heavy payloads into orbit and eventually putting men on the moon. Unusual aerospace-gear capability was developed to meet the special requirements of power gears and control gears in space vehicles.

Materials and dimensional tolerances must be held under close control. A gear failure can frequently result in the loss of human life. The gear designer and builder both have a grave responsibility to furnish gears that are always sure to work satisfactorily. Extensive ground and flight testings are required to prove new designs.

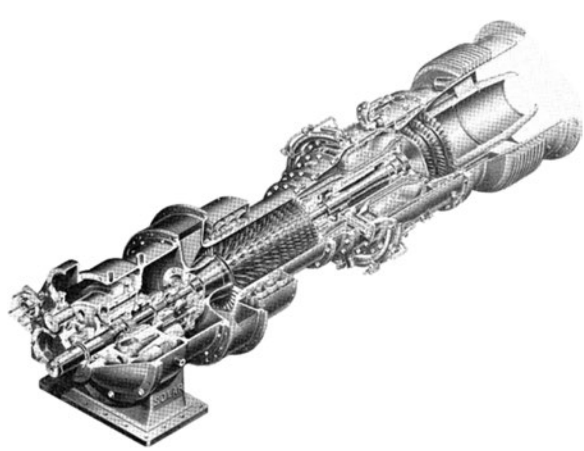


FIGURE 1.15 Two-stage epicyclic gear, close-coupled to a high-speed gas turbine. This unit is used to drive a generator. (Courtesy of Solar Turbines Incorporated, San Diego, California.)

The machine tools used to make aircraft gears are conventional hobbers, shapers, bevel gear generators, shavers, and gear grinders. For the close tolerances, the machinery must be in the very best condition and precision tooling must be used. A complete line of checking line equipment is needed so that involute profile, tooth spacing, helix angle, concentricity, and surface finish can be precisely measured.

1.1.9 Industrial Gearing

A wide range of types and sizes of gears that are used in homes, factories, and offices come under the industrial category (see examples in Figures 1.14 and 1.15). In general, these gears involve electric power from a motor used to drive something. The driven device may be a pump, conveyor, or liquid-stirring unit. It may also be a garage door opener, an air compressor for office refrigeration, a hoist, a winch, or a drive to mix concrete on a truck hauling the concrete to the job.

Industrial gearing is relatively low speed and low horsepower. Typical pitch-line speeds range from about 0.5 m/s to somewhat over 20 m/s (100 fpm to 4000 fpm). The types of gears may be spur, helical, bevel, worm, or Spiroid.* The power may range from less than 1 kW up to a few hundred kilowatts. Typical input speeds are those of the electric motor, such as 1800, 1500, 1200, and 100 revolutions/minute.

The industrial field also includes drives with hydraulic motors. The field is characterized more by relatively low pitch-line speeds and power inputs than by the means of making or using the power.

* Spiroid is a registered trademark of the Spiroid Division of the Illinois Tool Works, Chicago, Illinois, United States.

Much of the gearing used in industrial work is made with through-hardened steel used *as cut*. There is, however, a growing use of fully hardened gears where the size of the gearing or the life of the gearing is critical.

In the past, industrial gearing has not generally required long life or high reliability. The trend now—in the more important factory installations—is to obtain gears with moderately long life and reliability. For instance, a pump drive with an 80% probability of running OK for 1000 hours might have been quite acceptable in the 1960s. The pump buyer in the 1980s may be more concerned with the cost of downtime and parts replacement, and may want to get gears good enough to have a 95% probability of lasting for 10,000 hours at rated load.

1.1.10 Gears in the Oil and Gas Industry

The production of petroleum products for the energy needs of the world requires a considerable amount of high-power, high-speed gearing. Gear units are used on oil platforms, pumping stations, drilling sites, refineries, and power stations. Usually the drive is a turbine, but it may be a large diesel engine. The power range goes from about 750 to over 50,000 kW. Pitch-line speeds range from 20 to 200 m/s (4000 to 40,000 fpm).

Bevel gears are used to a limited extent. Sometimes a stage of bevel gearing is needed to make a 90° turn in a power drive. (As an example, a horizontal-axis turbine may drive a vertical-axis compressor.)

Hardened and ground gears are widely used. With better facilities to grind and measure large gears and better equipment to case-harden large gears has come a strong tendency to design the powerful gears for turbine and diesel applications with fully hardened teeth. This reduces weight and size considerably. Pitch-line speeds become lower. Less space and less frame structure are required in a power package with the higher-capacity, fully hardened gears.

1.1.11 Mill Gears

Large mills make cement, grind iron ore, make rubber, roll steel, or do some other basic functions; see Figure 1.16. It is common to have a few thousand kilowatts of power going through two or more gear stages to drive some massive processing drum or rolling device.

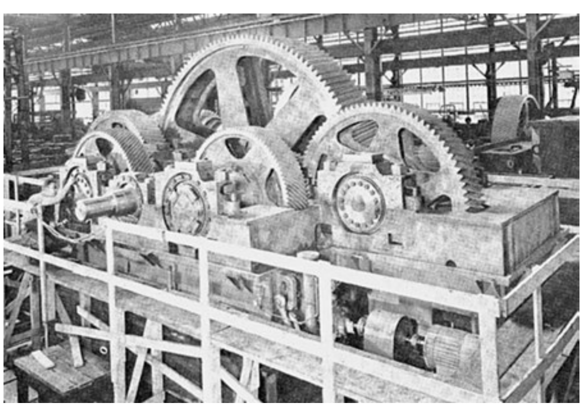


FIGURE 1.16 Large mill gear drive. Note twin power paths to bull gear. (Courtesy of the Falk Corporation, Milwaukee, Wisconsin.)

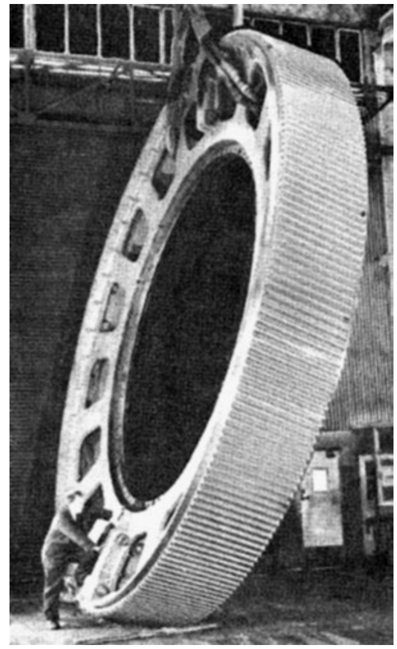


FIGURE 1.17 Jumbo-size drive gear for a large mill application. (Courtesy of David Brown Gear Industries, Huddersfield, U.K.)

The mill is usually powered by electric motors, but diesel engines or turbines may be used. The characteristic of mill drives is high power (and frequently unusually high torques).

A process mill will often run continuously for months at a time. Downtime is critical because the output ceases completely when a mill unit is shut down.

Spur and helical gears are generally used. Pitch-line speeds are usually quite low. A first stage may be doing around 20 m/s (4000 fpm), but the final stage in a mill may run as slowly as 0.1 to 1.0 m/s (20 to 200 fpm). Some bevel gears are also used—where axes must be at 90°.

The larger mill gears are generally made medium hard, but they may even be of low hardness. The very large sizes involved often make it impractical to use the harder gears. There is an increasing use, though, of fully hardened mill gears with up to gear pitch diameters of about 2 m.

Mill gears are commonly made in sizes of up to 11 m. Such giant gears have to be made in two or more segments. Figure 1.17 shows a large two-segment mill gear. The segments are bolted together for cutting and then unbolted for shipping. (It is impractical to ship round pieces of metal over about 5 m in diameter.)

1.2 SELECTION OF THE RIGHT KIND OF GEAR

The preceding section gave some general information on how gears are made and used in different fields. In this part of the chapter, we shall concentrate on the problem of selecting the *right* kind of gear. The first step in designing a set of gears is to pick the right kind.

In many cases, the geometric arrangement of the apparatus which needs a gear drive will considerably affect the selection. If the gears must be on parallel axes, then spur or helical gears are the ones to use. Bevel and worm gears can be used if the axes are at right angles, but they are not feasible with parallel axes. If the axes are nonintersecting and nonparallel, then crossed-helical gears, hypoid gears, worm gears, or Spiroid gears may be used. Worm gears, though, are seldom used if the axes are not at right angles to each other. Table 1.3 shows in detail the principal kinds of gears and how they are mounted.

There are no dogmatic rules that tell the designer just which gear to use. Frequently the choice is made after weighing the advantages and disadvantages of two or three kinds of gears. Some generalizations, though, can be made about gear selection.

In general, external helical gears are used when both high speeds and high horsepower are involved. External helical gears have been built to carry as much as 45,000 kW of power on a single pinion and gear. And this is not the limit for designing helical gears—bigger ones could be built if anyone needed them. It is doubtful if any other kind of gear could be built and used successfully to carry this much power on a single mesh.

Bevel gears are ordinarily used on right-angle drives when high ef ciency is needed. These gears can usually be designed to operate with 98% or better ef ciency. Worm gears often have a hard time getting above 90% ef ciency. Hypoid gears do not have as good ef ciency as bevel gears, but they make up for this by being able to carry more power in the same space—provided the speeds are not too high.

Worm gears are ordinarily used on right-angle drives when very high ratios are needed. They are also widely used in low to medium ratios as packaged speed reducers. Single-thread worms and worm gears are used to provide the indexing accuracy on many machine tools. The critical job of indexing hobbing machines and gear shapers is nearly always done by a worm-gear drive.

Spur gears are ordinarily thought of as slow-speed gears, while helical gears are thought of as high-speed gears. If noise is not a serious design problem, spur gears can be used at almost any speeds which can be handled by other types of gears. Aircraft gas-turbine spur gears sometimes run at pitch-line speeds above 50 m/s (10,000 fpm). In general, though, spur gears are not used much above 20 m/s (4000 fpm).

TABLE 1.3
Kinds of Gears in Common Use

Parallel Axes	Intersecting Axes	Nonintersecting Nonparallel Axes
Spur external	Straight bevel	Crossed-helical
Spur internal	Zerol bevel	Single-enveloping worm
Helical external	Spiral bevel	Double-enveloping worm
Helical internal	Face gear	Hypoid
		Spiroid

1.2.1 External Spur Gears

Spur gears are used to transmit power between parallel shafts. They impose only radial loads on their bearing. The tooth pro les are ordinarily curved in the shape of an involute. Variations in center distance do not affect the trueness of the gear action unless the change is so great as to either jam the teeth into the root llets of the mating member or withdraw the teeth almost out of action.

Spur-gear teeth may be hobbed, shaped, milled, stamped, drawn, sintered, cast, or shear-cut. They may be given a nishing operation such as grinding, shaving, lapping, rolling, or burnishing. Speaking generally, there are more kinds of machine tools and processes available to make spur gears than to make any other gear type. This favorable situation often makes spur gears the choice where cost of manufacture is a major factor in the gear design.

The standard measure of spur-gear tooth size in the metric system is the *module*. In the English system, the standard measure of tooth size is *diametral pitch*. Their meanings are as follows:

Module is millimeters of pitch diameter per tooth.
Diametral pitch is number of teeth per inch of pitch diameter (a reciprocal function).

Mathematically,

$$\text{Module} = \frac{25.400}{\text{diametral pitch}}, \tag{1.1}$$

or

$$\text{Diametral pitch} = \frac{25.400}{\text{module}}. \tag{1.2}$$

Curiously, module and diametral pitch are size dimensions which cannot be directly measured on a gear. They are really reference values used to calculate other size dimensions which are measurable.

Gears can be made to any desired module or diametral pitch, provided that cutting tools are available for that tooth size. To avoid purchasing cutting tools for too many different tooth sizes, it is desirable to pick a progression of modules and design to these except where design requirements force the use of special sizes. The following commonly used modules are recommended as a start for a design series: 25, 20, 15, 12, 10, 8, 6, 5, 4, 3, 2.5, 2.0, 1.5, 1, 0.8, and 0.5. Many shops are equipped with English-system diametral pitches series: 1, 1¼, 1½, 1¾, 2, 2½, 3, 4, 5, 6, 8, 10, 12, 16, 20, 24, 32, 48, 64, and 128. Considering the trend toward international trade, it is desired to purchase new gear tools in standard metric sizes, so that they will be handy for gear work going to any part of the world. (*The standard measuring system of the world is the metric system*.)

Most designers prefer a 20° pressure angle for spur gears. In the past the 14.5° pressure angle was widely used. It is not popular today because it gets into trouble with undercutting much more quickly that the 20° tooth when small numbers of pinion teeth are needed. Also, it does not have the load-carrying capacity of the 20° tooth. A pressure angle of 22.5° or 25° is often used. Pressure angles above 20° give higher load capacity but may not run quite as smoothly or quietly.

Figure 1.18 shows the terminology used for a spur gear or a spur rack (a rack is a section of a spur gear with an in nitely large pitch diameter).

The following formulas apply to spur gears in all cases:

$$\text{Circular pitch} = \pi \times \text{module} \quad \text{(metric)} \tag{1.3}$$

$$= \pi \div \text{diametral pitch} \quad \text{(English)} \tag{1.4}$$

$$\text{Pitch diameter} = \text{number of teeth} \times \text{module} \quad \text{(metric)} \tag{1.5}$$

$$= \text{number of teeth} \div \text{diametral pitch} \quad \text{(English)} \tag{1.6}$$

The *nominal* center distance is equal to the sum of the pitch diameter of the pinion and the pitch diameter of the gear divided by 2:

$$\text{Center distance} = \frac{\text{pinion pitch diameter} + \text{gear pitch diameter}}{2}. \tag{1.7}$$

Since the center distance is a machined dimension, it may not come out to be exactly what the design calls for. In addition, it is common practice to use a slightly larger center distance to increase the operation pressure angle. For instance, if the actual center distance is made 1.7116% larger, gears cut with 20° hobs or shaper-cutters will run at 22.5° pressure angle. (See Section 13.1 for methods of design for special center distance.)

For the reasons just mentioned, it is possible to have two center distances, a nominal center distance and an *operating* center distance. Likewise, there are two pitch diameters. The pitch diameter for the tooth-cutting operation is the nominal pitch diameter and is given by Equations 1.5 and 1.6. The operating pitch diameters are

$$\text{Pitch diameter (operating) of pinion} = \frac{2 \times \text{operating center distance}}{\text{ratio} + 1}, \tag{1.8}$$

$$\text{Pitch diameter (operating) of gear} = \text{ratio} \times \text{pitch diameter (operating) of opinion}, \tag{1.9}$$

where

$$\text{Gear (tooth) ratio} = \frac{\text{number of gear teeth}}{\text{number of pinion teeth}}. \tag{1.10}$$

1.2.2 External Helical Gears

Helical gears are used to transmit power or motion between parallel shafts. The helix angle must by the same in degrees on each member, but the hand of the helix on the pinion is opposite to that on the gear. (A right-handed [RH] pinion meshes with a left-handed [LH] gear, and an LH pinion meshes with an RH gear.)

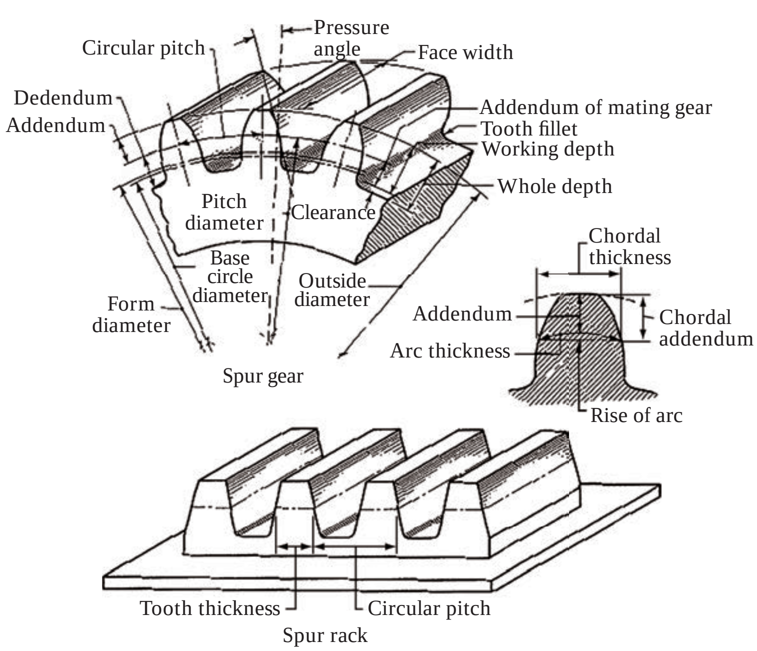


FIGURE 1.18 Spur-gear and rack terminology.

Single-helical gears impose both thrust and radial loads on their bearings. Double-helical gears develop equal and opposite thrust reactions, which have the effect of canceling out the thrust load. Usually double-helical gears have a gap between helices to permit a runout clearance for the hob, grinding wheel, or other cutting tool. One kind of gear shaper has been developed that permits double-helical teeth to be made *continuous* (no gap between helices).

Helical-gear teeth are usually made with an involute pro le in the *transverse* section (the transverse section is a cross section perpendicular to the gear axis). Small changes in center distance do not affect the action of helical gears.

Helical-gear teeth may be made by hobbing, shaping, milling, or casting. Sintering has been used with limited success. Helical teeth may be nished by grinding, shaving, rolling, lapping, or burnishing.

The size of helical-gear teeth is speci ed by module for the metric system and by diametral pitch for the English system. The helical tooth will frequently have some of its dimensions given in the *normal* section and others are given in transverse section. Thus, standard cutting tools could be speci ed for either section—but not for *both* sections. If the helical gear is small (less than 1 m pitch diameter), most designers will use the same pressure angle and standard tooth size in the normal section of the helical gear as they would use for spur gears. This makes it possible to hob helical gears with standard spur gear hob. (It is not possible, though, to cut helical gears with standard spur-gear shaper-cutters.)

Helical gears often use 20° as the standard pressure angle in the normal section. However, higher pressure angles, such as 22.5° or 25°, may be used to get extra load-carrying capacity.

Figure 1.19 shows the terminology for a helical gear and a helical rack. On the transverse plane, the elements of a helical gear are the same as those of a spur gear. Equations 1.1 through 1.10 apply just as well to the transverse plane of a helical gear as they do to a spur gear. Additional general formulas for helical gears are the following:

$$\text{Normal circular pitch} = \text{circular pitch} \times \text{cosine helix angle}, \tag{1.11}$$

$$\text{Normal module} = \text{transverse module} \times \text{cosine helix angle}, \tag{1.12}$$

$$\text{Normal diameter pitch} = \text{transverse diametral pitch} \times \text{cosine helix angle}, \tag{1.13}$$

$$\text{Axial pitch} = \text{circular pitch} \div \text{tangent helix angle} \tag{1.14}$$

$$= \text{normal circular pitch} \div \text{sine helix angle}. \tag{1.15}$$

1.2.3 Internal Gears

Two internal gears will not mesh with each other, but an external gear may be meshed with an internal gear. The external gear must not be larger than about two-thirds the pitch diameter of the internal gear when full-depth 20° pressure angle teeth are used. The axes on which the gears are mounted must be parallel.

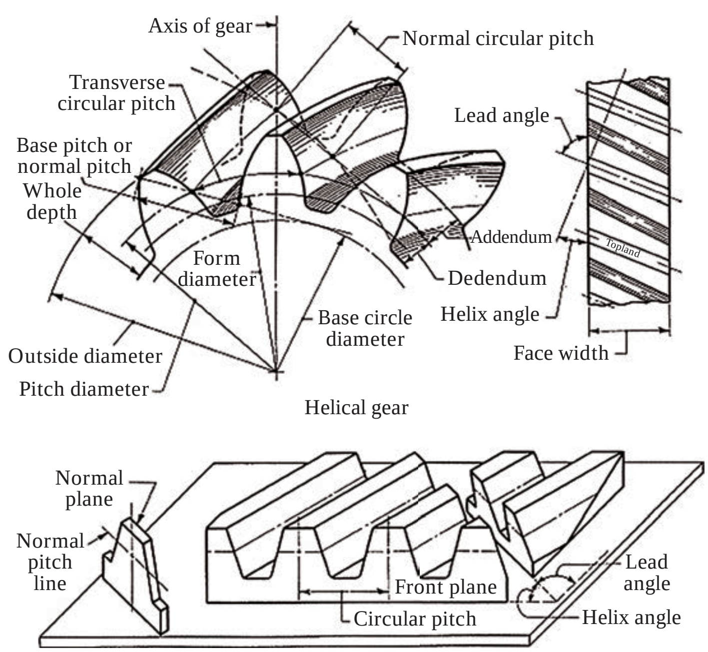


FIGURE 1.19 Helical-gear and rack terminology.

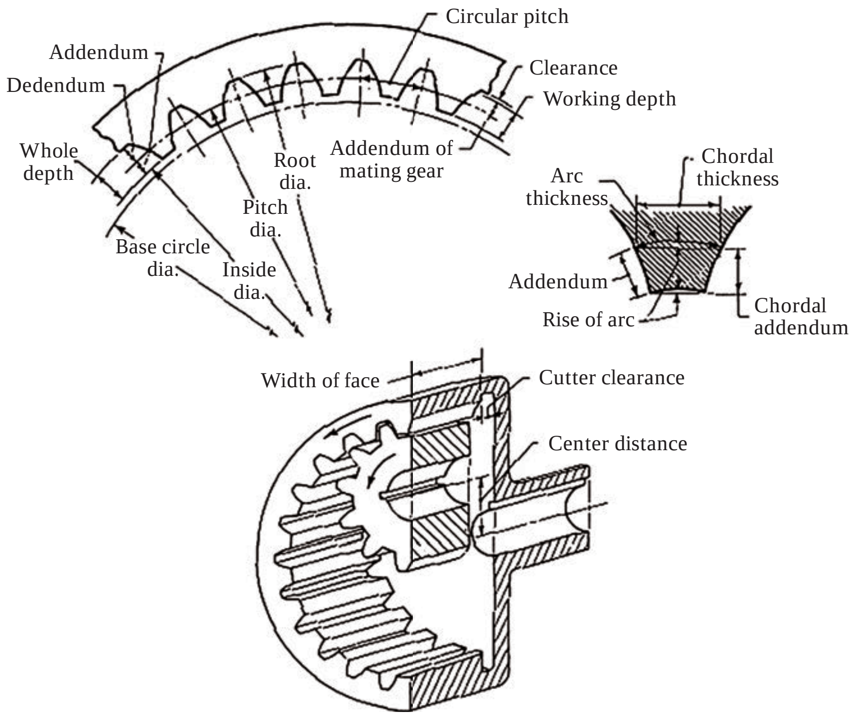


FIGURE 1.20 Internal-gear terminology. dia.: diameter.

Internal gears may be either spur or helical. Even double-helical internal gears are used occasionally.

An internal gear is a necessary in an epicyclic type of gear arrangement. The short center distance of an internal gearset makes it desirable in some applications where space is very limited. The shape of an internal gear forms are natural guard over the meshing gear teeth. This is very advantageous for some types of machinery.

Internal gears have the disadvantage that fewer types of machine tools can produce them. Internal gears cannot be hobbed.* They can be shaped, milled, or cast. In small sizes they can be broached. Both helical and spur internals can be nished by shaving, grinding, lapping, or burnishing.

An internal gear has the same helix angle in degrees and the same hand as its mating pinion (an RH pinion meshes with an RH gear and vice versa).

Figure 1.20 shows the terminology used for a spur internal gear. All the previously given formulas apply to internal gearing except those involving center distance (Equations 1.7 through 1.9 do not hold for internals). The formulas for internal-gear center distance are as follows:

$$\text{Center distance} = \frac{\text{pitch diameter of gear} - \text{pitch diameter of pinion}}{2}, \quad (1.16)$$

$$\text{Pitch diameter (operating) of pinion} = \frac{2 \times \text{operating center distance}}{\text{ratio} - 1}, \quad (1.17)$$

$$\text{Pitch diameter (operating) of gear} = \frac{2 \times \text{operating center distance} \times \text{ratio}}{\text{ratio} - 1}. \quad (1.18)$$

1.2.4 Straight Bevel Gears

Bevel-gear blanks are conical. The teeth are tapered in both tooth thickness and tooth height. At one end the tooth is larger, while at the other end it is small. The tooth dimensions are usually speci ed for the *large* end of the tooth. However, in calculating bearing loads, the central-section dimensions and forces are used.

The simplest type of bevel gear is the *straight* bevel gear. These gears are commonly used for transmitting power between intersecting shafts. Usually the shaft angle is 90°, but it may be almost any angle. The gears impose both radial and thrust load on their bearing.

Bevel gears must be mounted on axes whose shaft angle is almost exactly the same as the design shaft angle. Also, the axes on which they are mounted must either intersect or come very close to intersecting. In addition to the accuracy required of the axes, bevel gears must be mounted at the right distance from the cone center. The complications involved in mounting bevel gears make it dif cult to use sleeve bearings with large clearances (which is often done on high-speed, high-power spur and helical gears). Ball and roller bearings are the kinds

* Some very special hobs and hobbing machines have been used—to a rather limited extent—to hob internal gears.

commonly used for bevel gears. The limitations of these bearings' speed and load-carrying capacity indirectly limit the capacity of bevel gears in some high-speed applications.

Straight bevel teeth are usually cut on bevel-gear generators. In some cases, where accuracy is not too important, bevel-gear teeth are milled. Bevel teeth may also be cast. Lapping is the process often used to nish straight bevel teeth. Shaving is not practical for straight bevel gears, but straight bevels may be ground.

The size of bevel-gear teeth is de ned in module for the metric system and in diametral pitch for the English system. The speci ed size dimensions are given for the large end of the tooth. A bevel-gear tooth which is 12 module at the large end may be only around 10 module at the small end. The commonly used modules (or diametral pitches) are the same as those used for spur gears (see Section 3.2.1). There is no particular advantage to using standard tooth sizes for bevel gears. A set of cutting tools will cut more than a single pitch.

The two views of bevel gears in Figure 1.21 show bevel-gear terminology. Bevel-gear teeth have pro les which closely resemble an involute curve. The shape of a straight bevel-gear tooth (in a section normal to the tooth) closely approximates

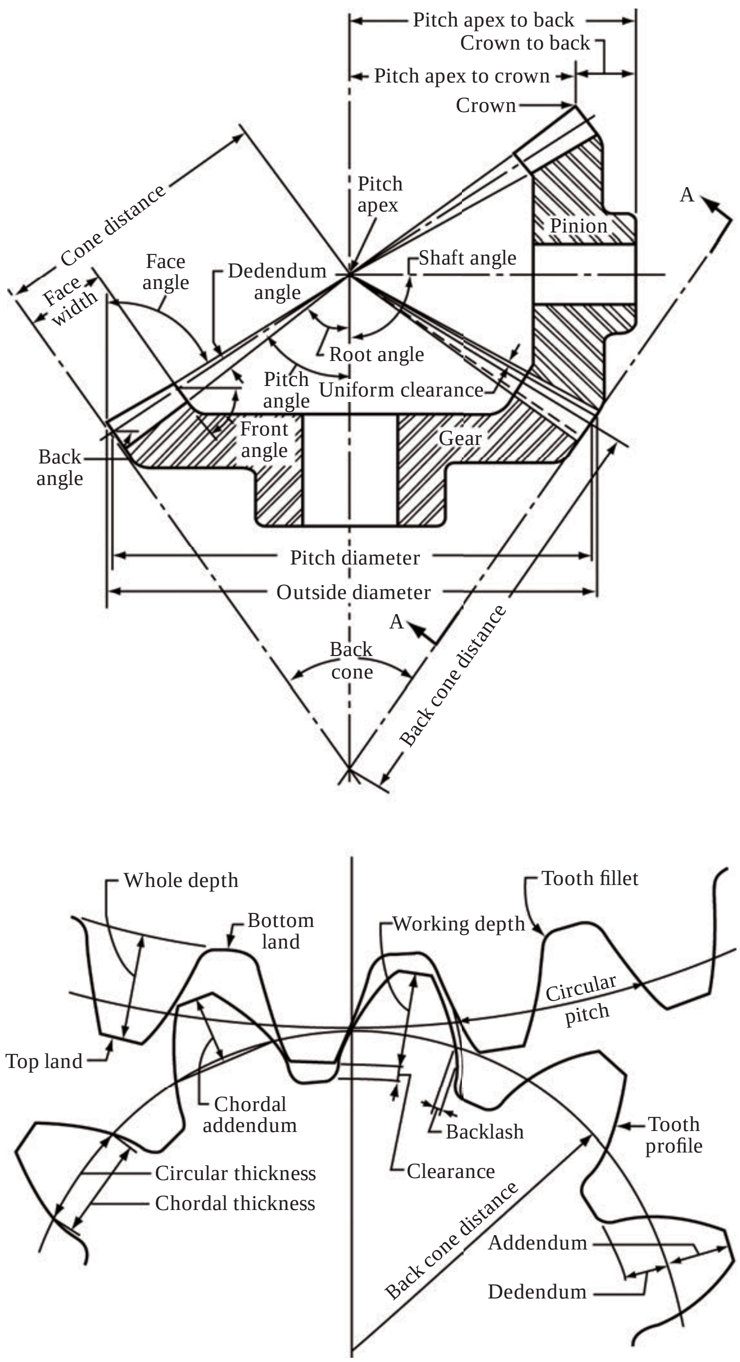


FIGURE 1.21 Bevel-gear terminology.

that of an involute spur gear with a larger number of teeth. This larger number of teeth, called the virtual number of teeth, is equal to the actual number of teeth divided by the cosine of the pitch angle.

Straight bevel-gear teeth have been commonly made with 14.5°, 17.5°, and 20° pressure angles. The 20° design is the most popular.

The pitch angle of a bevel gear is the angle of the pitch cone. It is a measure of the amount of taper in the gear. For instance, as the taper is reduced, the pitch angle approaches zero and the bevel gear approaches a spur gear.

The pitch angles in a set of bevel gears are defined by lines meeting at the cone center. The root and face angles are defined by lines which *do not* hit the cone center (or apex). In old-style designs these angles did meet at the apex, but modern designs make the outside cone of one gear parallel to the root cone of its mate. This gives a constant clearance and permits a better cutting-tool design and gear-tooth design than the old-style design with its tapering clearance.

The circular pitch and the pitch diameters of bevel gears are calculated in the same way as that for spur gears. (See Equations 1.1 through 1.3.) The pitch-cone angles may be calculated by one of the following sets of equations:

$$\tan(\text{pitch angle, pinion}) = \frac{\text{number of teeth in pinion}}{\text{number of teeth in gear}}, \quad (1.19)$$

$$\tan(\text{pitch angle, gear}) = \frac{\text{number of teeth in gear}}{\text{number of teeth in pinion}}. \quad (1.20)$$

When the shaft angle is less than 90°,

$$\tan(\text{pitch angle, pinion}) = \frac{\sin(\text{shaft angle})}{\text{ratio} + \cos \text{shaft angle}}, \quad (1.21)$$

$$\tan(\text{pitch angle, gear}) = \frac{\sin(\text{shaft angle})}{1/\text{ratio} + \cos(\text{shaft angle})}. \quad (1.22)$$

When the shaft angle is over than 90°,

$$\tan(\text{pitch angle, pinion}) = \frac{\sin(180° - \text{shaft angle})}{\text{ratio} - \cos(180° - \text{shaft angle})}, \quad (1.23)$$

$$\tan(\text{pitch angle, gear}) = \frac{\sin\left(180° - \text{shaft angle}\right)}{1/\text{ratio} - \cos\left(180° - \text{shaft angle}\right)}. \quad (1.24)$$

In all the preceding cases,

$$\text{Pitch angle, pinion} + \text{pitch angle, gear} = \text{shaft angle}. \quad (1.25)$$

FIGURE 1.22 A pair of Zerol bevel gears. (Courtesy of the Gleason Works, Rochester, New York.)

1.2.5 Zerol Bevel Gears

Zerol* bevel gears are similar to straight bevel gears except that they have a curved tooth in the lengthwise direction; see Figure 1.22. Zerol bevel gears have 0° spiral angle. They are made in a kind of machine different from that used to make straight bevel gears. The straight bevel gear–generating machine has a cutting tool which moves back and forth in a straight line. Zerol is generated by a rotary cutter that is like a face mill. It is the curvature of this cutter that makes the lengthwise curvature in the Zerol tooth.

A Zerol gear has a profile which somewhat resembles an involute curve. The pressure angle of the tooth varies slightly in going across the face width. This is caused by the lengthwise curvature of the tooth.

Zerol gear teeth may be finished by grinding or lapping. Since a Zerol gear can be ground, it is favored over straight bevel gears in applications in which both high accuracy and full hardness are required. Even in applications in which cut gears of machinable hardness can be used, Zerol may be the best choice if speeds are high. Because of its lengthwise curvature, a Zerol tooth has a slight overlapping action. This tends to make it run more smoothly than the straight-bevel-gear tooth. A cut Zerol bevel gear is usually more accurate than a straight bevel gear.

In making a set of Zerol gear, one member is made first, using theoretical machine settings. Then a second gear is finished in such a way that its profile and lengthwise curvature will give satisfactory contact with the first gear. Several trial cuts and adjustments to machine settings may be required to develop a set of gears which will conjugate properly. If a number of identical sets of gears are required, a matching set of test gears is made. Then each production gear is machined so that it will mesh satisfactorily with one or the other of the test gears. In this way a number of sets of interchangeable gears may be made.

Zerol gears are usually made to a 20° pressure angle. In a few ratios where pinion and gear have small numbers of teeth, 22.5° or 25° is used.

* Zerol is a registered trademark of Gleason Works, Rochester, New York, United States.

The calculations for pitch diameter and pitch-cone angle are the same for Zerol bevel gears as for straight bevel gears.

1.2.6 Spiral Bevel Gears

Spiral bevel gears have a lengthwise curvature like Zerol gears. However, they differ from Zerol gears in that they have an appreciable angle with the axis of the gear; see Figure 1.23. Although spiral bevel teeth do not have a true helical spiral, a spiral bevel gear looks somewhat like a helical bevel gear.

Spiral bevel gears are generated by the same machines that cut or grind Zerol gears. The only difference is that the cutting tool is set at an angle to the axis of the gear instead of being set essentially parallel to the gear axis.

Spiral bevel gears are made in matched sets like Zerol bevel gears. Different sets of the same design are not interchangeable unless they have been purposely built to match a common set of test gears.

Generating types of machines are ordinarily used to cut or grind spiral bevel gear teeth. In some high-production jobs, a special kind of machine is used which cuts the teeth without going through a generating motion. Spiral bevel gear teeth are frequently given a lapping operation to finish the teeth and obtain the desired tooth bearing.

In high-speed gear work, the spiral bevel is preferred over the Zerol bevel because its spiral angle tends to give the teeth a considerable amount of overlap. This makes the gear run more smoothly, and the load is distributed over more tooth surface. However, the spiral bevel gear imposes much more thrust load on its bearings than does a Zerol bevel gear.

Spiral bevel gears are commonly made of 16°, 17.5°, 20°, and 22.5° pressure angles. The 20° angle has become the most popular. It is the only angle used on aircraft and instrument gears. The most common spiral angle is 35°.

1.2.7 Hypoid Gears

Hypoid gears resemble bevel gears in some respects. They are used on crossed-axis shafts, and there is a tendency for the parts to taper as do bevel gears. They differ from true bevel gears in that their axes do not intersect. The distance between a hypoid pinion axis and the axis of a hypoid gear is called *offset*. This distance is measured along the perpendicular common to the two axes. If a set of hypoid gears had *no offset*, they would simply be spiral bevel gears. See Figure 1.24 for *offset* and other terms.

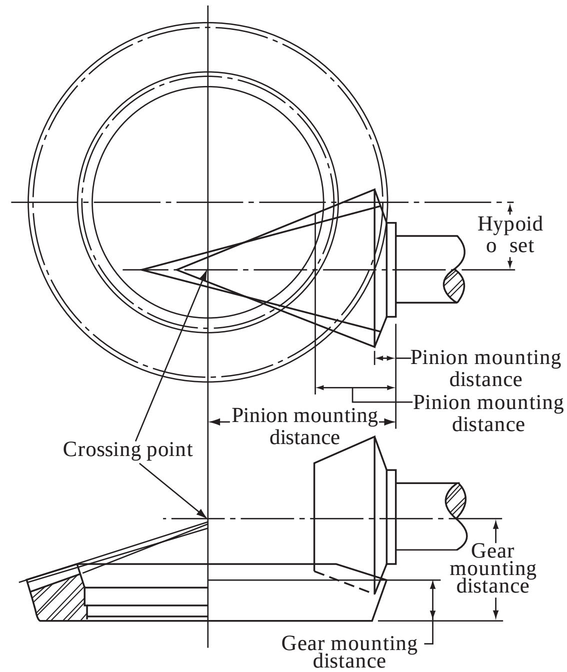


FIGURE 1.24 Hypoid-gear arrangement.

Hypoid pinions may have as few as five teeth in a high ratio. Since the various kinds of bevel gears do not often go below 100 teeth in a pinion, it can be seen that it is easier to get high ratios with hypoid gears.

Contrary to the general rule with spur, helical, and bevel gears, hypoid pinions and gears *do not* have pitch diameters which are in proportion to their numbers of teeth. This makes it possible to use a large and strong pinion even with a high ratio and only a few pinion teeth. See Figure 1.25.

Hypoid teeth have unequal pressure angles and unequal profile curvatures on the two sides of the teeth. This results from the unusual geometry of the hypoid gear rather than from a nonsymmetrical cutting tool.

Hypoid gears are matched to run together, just as Zerol or spiral gearsets are matched. Interchangeability is obtained by making production gears fit with test masters.

FIGURE 1.23 A pair of spiral bevel gears. (Courtesy of Gleason Works, Rochester, New York.)

FIGURE 1.25 A pair of hypoid gears. (Courtesy of Gleason Works, Rochester, New York.)

Hypoid gears and pinions are usually cut on a generating type of machine. They may be nished by either grinding or lapping.

The hypoid gears and pinions are usually cut on a generating type of machine. They may be nished by either grinding or lapping.

The hypoid gears for passenger cars and for industrial drives usually have a basic pressure angle of 21°15′. For tractors and trucks the average pressure angle is 22°30′. Pinions are frequently made with a spiral angle of 45° or 50°.

In hypoid gearing, module and diametral pitch are used for the gear *only*. Likewise, the pitch diameter and the pitch angle are gured for the gear only. If a pitch were used for the pinion, it would be smaller than that of the gear. The size of a hypoid pinion is established by its outside diameter and its number of teeth. The geometry of hypoid teeth is de ned by the various dimensions used to set up the machines to cut the teeth.

1.2.8 Face Gears

Face gears have teeth cut on the end face of a gear, just as the term *face* implies. They are not ordinarily thought of as bevel gears, but functionally they are more akin to bevel gears than to any other kind.

A spur pinion and a face gear are mounted—like bevel gears—on shafts that intersect and have a shaft angle (usually 90°). The pinion bearing carries mostly radial load, while the gear bearing has both thrust and radial load. The mounting distance of the pinion from the pitch-cone apex is not critical, as it is in bevel or hypoid gears. Figure 1.26 shows the terminology used for face gears.

The pinion that goes with a face gear is usually made spur, but it can be made helical if necessary. The formulas for determining the dimensions of a pinion to run with a face gear are no different from those for the dimensions of a pinion to run with a mating gear on parallel axes. The pressure angles and pitches used are similar to spur-gear (or helical-gear) practice.

The pinion may be nished or cut by all the methods previously mentioned for spur and helical pinions. The gear, however, must be nished with a shaper-cutter with is almost the same size as the pinion. Equipment to grind face gears is not available. The teeth can be lapped, and they might be shaved without too much dif culty, although ordinarily they are not shaved.

The face-gear tooth changes shape from one end of the tooth to the other. The face width of the gear is limited at the outside end by the radius at which the tooth becomes pointed. At the inside end, the limit is the radius at which undercut becomes excessive. Practical considerations usually make it desirable to make the face width somewhat short of these limits.

The pinion to go with a face gear is usually made with 20° pressure angle.

1.2.9 Crossed-Helical Gears (Nonenveloping Worm Gears)

The word *spiral* is rather loosely used in the gear trade. The word may be applied to both helical and bevel gears. In this section we shall consider the special kind of worm gear that is often called a spiral gear. More correctly, though, it is a *crossed-helical* gear.

Crossed-helical gears are essentially nonenveloping worm gears. Both members are cylindrically shaped. (See Figure 1.27.) In comparison, the *single-enveloping* worm gearset has a cylindrical worm, but the gear is throated so that is tends to wrap around the worm. The *double-enveloping* worm gearset goes still further; both members are throated, and both members wrap around each other.

Crossed-helical gears are mounted on axes that do not intersect and that are at an angle to each other. Frequently the angle between the axes is 90°. The bearings for crossed-helical gears have both thrust and radial loads.

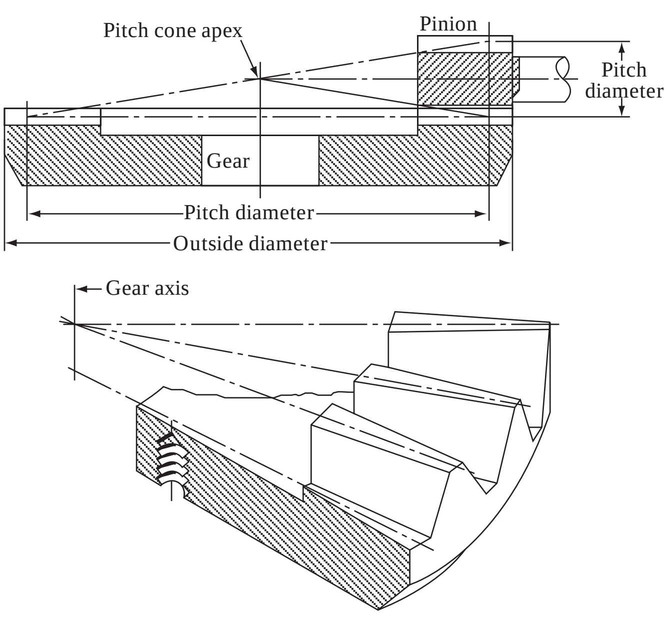


FIGURE 1.26 Face-gear terminology.

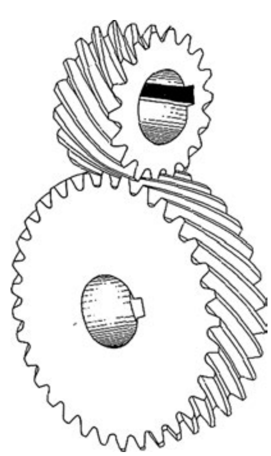


FIGURE 1.27 Crossed-helical-gear drive.

A *point contact* is made between two spiral gear teeth in mesh with each other. As the gears revolve, this point travels across the tooth in a sloping line. After the gears have worn in for some time, a shallow, sloping line of contact is worn into each member. This makes the original point contact increase to a line as long as the width of the sloping band of contact. The load-carrying capacity of crossed-helical gears is quite small when they are new, but if they are worn in carefully, it increases quite appreciably.

Crossed-helical gears are able to stand small changes in center distance and small changes in shaft angle without any impairment in the accuracy with which the set transmits motion. This fact, and the fact that shifting either member endwise makes no difference in the amount of contact obtained, makes this the easiest of all gears to mount. There is no need to get close accuracy in center distance, shaft alignment, or axial position—provided the teeth are cut with reasonably generous face width and backlash.

Crossed-helical gears may be made by any of the processes used to make single-helical gear of the same hand. Up to the point of mounting the gear in a gearbox, there is no difference between a crossed-helical gear and a helical gear.

Usually a crossed-helical gear of one hand is meshed with a crossed-helical gear of the same hand. It is not necessary, though, to do this. If the shaft angle is properly set, it is possible to mesh opposite hands together. Thus, the range of possibilities is as follows:

- RH drive with RH drive
- LH drive with LH drive
- RH drive with LH drive
- LH drive with RH drive

The pitch diameters of crossed-helical gears—like those of hypoid gears—are not in proportion to the tooth ratio. This makes the use of the word *pinion* for smaller member of the pair inappropriate. In a crossed-helical gearset, the small pinion might easily have more teeth than the gear!

The same helix angle in degrees does not have to be used for each member. Whenever different helix angles are used, the module (or diametral pitch) for two crossed-helical gears that mesh with each other is not the same. The thing that is the same in all cases is the normal module (and the normal circular pitch). This makes the normal module (or normal diametral pitch) the most appropriate measure of tooth size.

Designers of crossed-helical gears usually get the best results when there is a contact ratio in the normal section of at least 2. This means that in all positions of tooth engagement, the load will be shared by at least two pairs of teeth. To get this high contact ratio, a low normal pressure angle and a deep tooth depth are needed. When the helix is 45°, a normal pressure angle of 14.5° gives good results.

Some of the basic formulas for crossed-helical gears are as follows:

$$\text{Shaft angle} = \text{helix angle of driver} \pm \text{helix angle of driven}, \quad (1.26)$$

$$\text{Normal module} = \text{normal circular pitch} \div \pi, \quad (1.27)$$

$$\text{Normal diameter pitch} = \frac{\pi}{\text{normal circular pitch}}, \quad (1.28)$$

$$\text{Pitch diameter} = \frac{\text{number of teeth} \times \text{normal module}}{\text{cosine of helix angle}}, \quad (1.29)$$

$$\text{Center distance} = \frac{\text{pitch diameter driver} + \text{pitch diameter driven}}{2}, \quad (1.30)$$

$$\text{Cosine of helix angle} = \frac{\text{number of teeth} \times \text{normal circular pitch}}{\pi \times \text{pitch diameter}}. \quad (1.31)$$

1.2.10 Single-Enveloping Worm Gears

Figure 1.28 shows a single-enveloping worm gear. Worm gears are characterized by one member having a screw thread. Frequently the thread angle (lead angle) is only a few degrees. The worm in this case has the outward appearance of the thread on a bolt, greatly enlarged. When a worm has multiple threads and a lead angle approaching 45°, it may be (if it has an involute pro le) geometrically just the same as a helical pinion of the same lead angle. In this case the only difference between a worm and a helical pinion would be in their usage.

Worm gears are usually mounted on nonintersecting shafts which are at a 90° shaft angle. Worm bearings usually have a high thrust load. A worm-gear bearing has a high radial load and a low thrust load (unless the lead angle is high).

A single-enveloping worm gear has a line contact which extends either across the face width or across the part of the tooth that is in the zone of action. As the gear revolves, this line sweeps across the whole width and height of the tooth. The meshing action is quite similar to that of helical gears on parallel shafts, except that much higher sliding velocity is obtained for the same pitch-line velocity. In a helical gearset, the sliding velocity at the tooth tips is usually mot more than about one-fourth the pitch-line velocity. In a high-ratio

FIGURE 1.28 Single-enveloping worm gearset. (Courtesy of Delroyd Worm Gear Division, Transamerica Delaval, Trenton, New Jersey.)

worm gearset, the sliding velocity is greater than the pitch-line velocity of the worm.

Worm gearsets have considerably more load-carrying capacity than crossed-helical gearsets. This results from the fact that they have *line* contact instead of *point* contact. Worm-gearsets must be mounted on shafts that are very close to being correctly aligned and at the correct center distance. The axial position of a single-enveloping worm is not critical, but the worm gear must be in just the right axial position so that it can wrap around worm properly.

Several different kinds of worm-thread shapes are in common use. These are the following:

- Worm thread produced by straight-sided conical milling or grinding wheel
- Worm thread straight sided in the axial section
- Worm thread straight sided in the normal section
- Worm thread an involute helicoid shape

The shape of the worm thread de nes the worm-gear tooth shape. The worm gear is simply a gear element formed to be “conjugate” to a speci ed worm thread.

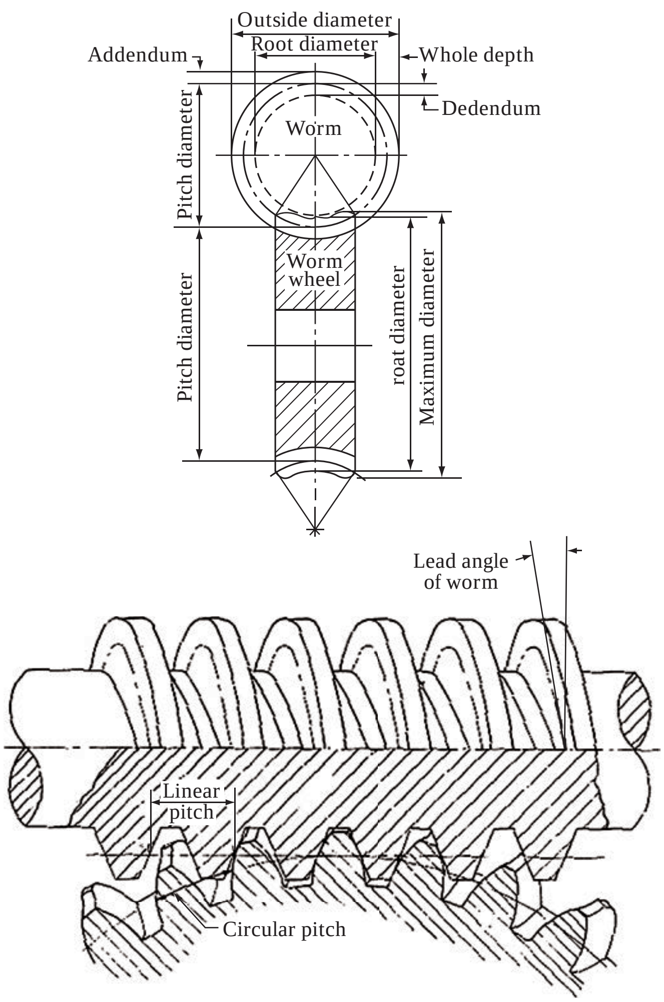


FIGURE 1.29 Worm-gear terminology (linear pitch = axial pitch).

A worm and a worm gear have the same hand of helix. An RH worm, for example, meshes with a RH gear. The helix angles are usually very different for a worm and a worm gear. Usually the worm has a more than 45° helix angle, and the worm gear has a less than 45° helix angle. Customarily the *lead angle*—which is the complement of the helix angle—is used to specify the angle of the worm thread. See Figure 1.29 for a diagram of the worm gear and its terminology. When the worm gearset has a 90° shaft angle, the worm lead angle is numerically equal to the worm-gear helix angle.

The *axial pitch* is the dimension that is used to specify the size of worm threads. It is the distance from thread to thread measured in an axial plane. When the shaft angle is 90°, the axial pitch of the worm is numerically equal to the worm-gear circular pitch. In the metric system, popular axial pitch values are 5, 7.5, 10, 15, 20, 30, and 40 mm. In the English system, the commonly used values have been 0.250, 0.375, 0.500, 0.750, 1.000, 1.250, and 1.500 in. Fine pitch worm gears are often designed to standard lead and pitch diameter values so as to obtain even lead-angle values.

Worm threads are usually milled or cut with a single-point lathe tool. In ne pitches, some designs can be formed by rolling. Grinding is often employed as a nishing process for high-hardness worms. In ne pitches, worm threads are sometimes ground from the solid.

Worm-gear teeth are usually hobbed. The cutting tool is essentially a duplicate of the mating worm in size and thread design. New worm designs should be based on available hobs wherever possible to avoid the need for procuring a special hob for each worm design.

A variety of pressure angles are used for worms. Single-thread worms used for indexing purposes frequently have low axial pressure angles, like 14.5°. Multiple-threaded worms with high lead angles such as 30° or 40° are often designed with about 30° axial pressure angles.

The following formulas apply to worm gears which are designed to run on 90° axes:

$$\text{Axial pitch of worm} = \text{circular pitch of worm gear}, \tag{1.32}$$

$$\text{Pitch diameter of gear} = \frac{\text{number of teeth} \times \text{circular pitch}}{\pi}, \tag{1.33}$$

$$\text{Pitch diameter of worm} = 2 \times \text{center distance} - \text{pitch diameter of gear}, \tag{1.34}$$

$$\text{Lead of worm} = \text{axial pitch} \times \text{number of threads}, \tag{1.35}$$

$$\tan \text{lead angle} = \frac{\text{lead of worm}}{\text{pitch diameter} \times \pi}, \tag{1.36}$$

$$\text{Lead angle of worm} = \text{helix angle of gear}. \tag{1.37}$$

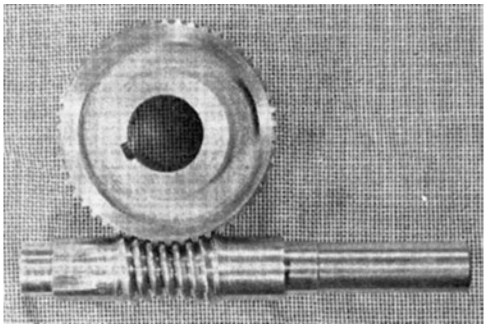


FIGURE 1.30 Double-enveloping worm gearset. (Courtesy of Cone Drive Division, Ex-Cell-O Corp., Traverse City, Michigan.)

1.2.11 Double-Enveloping Worm Gears

The double-enveloping worm gear is like the single-enveloping gear except that the worm envelopes the worm gear. Thus, both members are throated. See Figure 1.30.

Double-enveloping worm gears are used to transmit power between nonintersecting shafts, usually those at 90° angle. Double-enveloping worm gears load their bearings with thrust and radial loads the same as single-enveloping worm gears do.

Double-enveloping worm gears should be accurately located on all mounting dimensions. Shafts should be at the right shaft angle and at the right center distance.

The double-enveloping type of worm gear has more tooth surface in contact than a single-enveloping worm gear. Instead of line contact, it has *area* contact at any one instant. The larger contact area of the double-enveloping worm gear increases the load-carrying capacity. On most double-enveloping worm gearsets, the worm rubbing speed is below 10 m/s (2000 fpm). Above 10 m/s, it is possible to get good results with oil lubrication, using a circulating system and coolers. The lubrication system must be good enough to prevent scoring and overheating.

The only double-enveloping worm gear that is in widespread use today is the Cone Drive* design. Figure 1.31 shows the terminology used for Cone Drive worm gears.

A Cone Drive worm has a straight-sided profile in the axial section, but this profile changes its inclination as you move along the thread. At any one position, this slope is determined by a line which is tangent to the base cylinder of the gear. The base cylinder of a Cone Drive gear is like an involute base circle in that it is an imaginary circle used to define a profile. Geometrically, though, the base circle of a Cone Drive gear is not used in the same way as the base circle of an involute gear.

The size of Cone Drive gear teeth is measured by the circular pitch of the gear.

The normal pressure angle is ordinarily 20° to 22°. The Cone Drive worm and gear diameters are not in proportion to the ratio. With low ratios, it is possible (although not recommended) to have a worm which is larger than the gear!

* Cone Drive is a registered trademark of the Cone Drive Division, Ex-Cell-O Corp., Traverse City, Michigan, United States.

Equations 1.33 and 1.34 apply to Cone Drive gears as well as to regular worm gears. The other formulas for single-enveloping worm gears apply only to the center of the Cone Drive worm, since it does not have a fixed axial pitch and lead like a cylindrical worm.

In both single-enveloping worm gears and Cone Drive gears, it is generally recommended that the worm or pinion diameter be made a function of the center distance. Thus,

$$\text{Pitch diameter of worm} = \frac{(\text{center distance})^{0.875}}{2.2}. \quad (1.38)$$

Following the recommendation of Equation 1.38 is, of course, not necessary. This formula merely recommends a good proportion of worm to gear diameter for best power capacity. In instrumental and control work, the designer may not be interested in power transmission at all. In such cases, it may be desirable to depart considerably from Equation 1.38 in picking the size of a worm or pinion. In fact, the source shows a whole series of worm diameters for fine-pitch work which do not agree with Equation 1.38.

When a worm diameter is picked in accordance with Equation 1.38, the gear diameter and the circular pitch may be obtained by working backward through Equations 1.34, 1.33, and 1.32.

The helix angle of a worm gear or a Cone Drive gear may be obtained from the following general formula:

$$\tan(\text{center helix angle of gear}) = \frac{\text{pitch diameter of gear}}{\text{pitch diameter of worm} \times \text{ratio}}. \quad (1.39)$$

1.2.12 Spiroid Gears

The Spiroid family of gears operates on nonintersecting, nonparallel axes. The most famous family member is called Spiroid. It involves a tapered pinion that somewhat resembles a worm (see Figure 1.32). The gear member is a face gear with teeth curved in a lengthwise direction; the inclination to the tooth is like a helix angle—but not a true helical spiral.

Figure 1.33 shows the schematic relation on the Spiroid type of gear to worm gears, hypoid gears, and bevel gears.

The Spiroid family has Helicon† and Planoid types as well as the Spiroid type. A Helicon is essentially a Spiroid with no taper in the pinion. A Planoid is used for lower ratios than those used for a Spiroid, and its offset is lower—more in the range of the hypoid gear.

Spiroid pinions may be made by hobbing, milling, rolling, or thread chasing. Spiroid gears are typically made by hobbing, using a specially built (or modified) hobbing machine and special hobs. The gear may be made with molded or sintered gear teeth using tools (dies or punches) that have teeth

† Helicon and Planoid are registered trademarks of Illinois Tool Works Inc., Chicago, Illinois, United States, as is Spiroid.

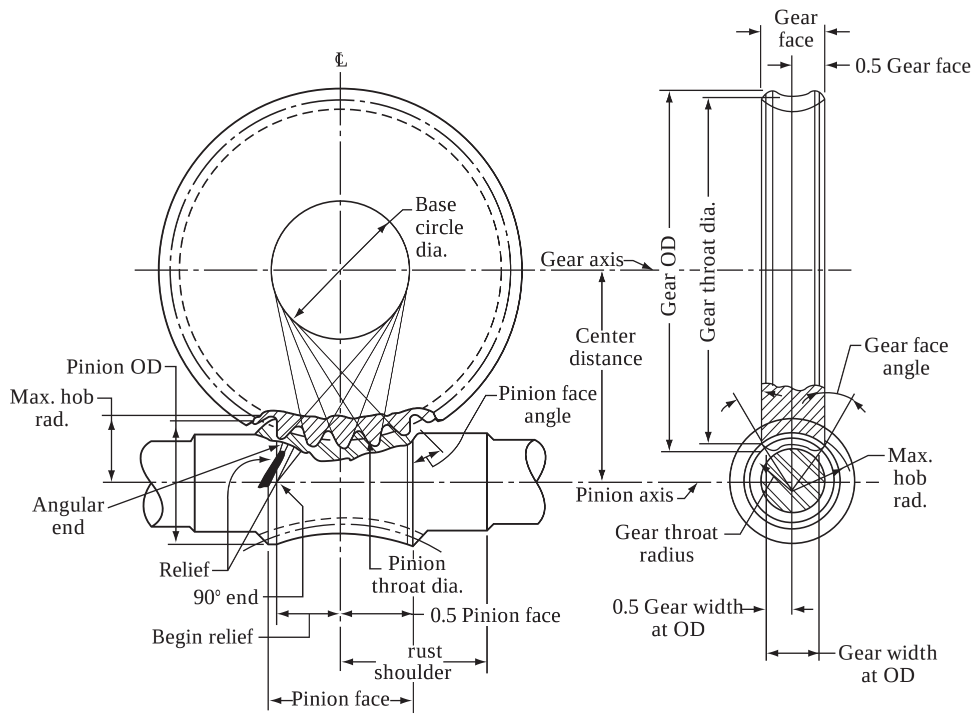


FIGURE 1.31 Terminology for Cone Drive worm gears. Max.: maximum; OD: outside diameter; rad.: radius.

FIGURE 1.32 Double-reduction Spiroid linear actuator unit. (Courtesy of Spiroid Division of Illinois Tool Works, Inc., Chicago, Illinois.)

resembling hobbed gears. Shaping and milling are not practical to use in making Spiroid gears.

Spiroid gears may be lapped as a nal nishing process. Special "shaving"-type hobs may also be used in a nishing operation.

Spiroid gears are used in a wide variety of applications, ranging from aerospace actuators to automotive and appliance use. The combination of a high ratio in compact arrangements, low cost when mass-produced, and good load-carrying capacity makes the Spiroid-type gear attractive in many situations. The fact that the gearing can be made with lower-cost machine tools and manufacturing processes is also an important consideration.

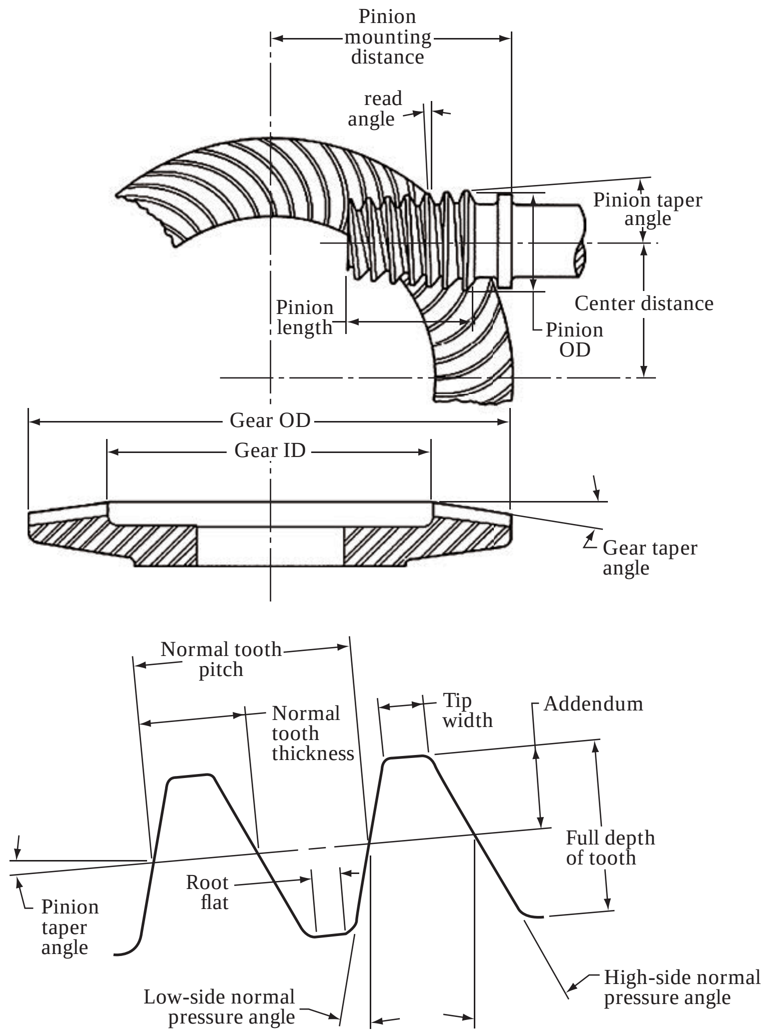


FIGURE 1.33 Spiroid gear terminology.

2 Gear Types and Nomenclature

Before we can discuss the different types of gears, we would do well to define just what we mean by a *gear*. Perhaps the most sufficient definition ever heard was provided by a loading dockworker who was asked where he should deliver the "wheels with notches in them." In more technically correct terms, a gear is a toothed wheel that is usually, but not necessarily, round. The teeth may have any of an almost infinity variety of profiles. The purpose of gearing is to transmit motion and/or power from one shaft to another. This motion transfer may or may not be uniform, and may also be accompanied by changes in direction, speed, and shaft torque.

Ultimately, even among gear specialists, there is some ambiguity in the terms used to describe gears and gear-related parameters. In our treatment herein, to the extent possible, we follow the conventions recommended by the AGMA.

2.1 TYPES OF GEARS

The first part of this chapter covers the types of gears in common use. The basic nomenclature and formulas for each type of gear are covered in the second part of this chapter.

The text material for each type of gear gives some comments on where the gears are used, how the gears may be made, the possible efficiency, and the occasional comments on lubrication of the gears. These comments should all be considered introductory. Much more specific information is given in the following chapters of this book. Also, the references given in this book show the reader where more comprehensive data can be obtained.

2.1.1 Classifications

In general, gears may be divided into several broad classifications based on the arrangement of the axes of the gear pair. The most general type of gearing consists of a gear pair whose axes are neither perpendicular nor parallel and do not intersect (that is, they do not lie on the same plane). All other types are special cases of this basic form. In the ensuing sections, we discuss most of the major gear types, generally in order of increasing complexity. We limit our discussion to specific gear types and do not include arrangements of these types.

Our discussions consider, qualitatively, the various aspects of many different types of gears. In order to provide an easy basis for comparing these many and varied gear types, Table 2.1 provides a broad-based comparison. It is by no means an exhaustive comparison, but it serves to provide a reasonable basis for preliminary design. In using this table, the reader must clearly keep in mind the fact that the data are typical or nominal. Surely, any experienced gear technologist can point out specific cases that substantially vary in virtually every category.

Within each type of gear classification, there exist many variations in actual tooth form. The involute- or involute-based form is, at least for parallel axis gears, the most common tooth form; however, many other special forms exist. We do not discuss tooth forms, per se, in this section.

2.1.2 Parallel Axis Gears

The simplest types of gears are those that connect parallel shafts. They are generally relatively easy to manufacture and are capable of transmitting large amounts of power with high efficiency. Parallel axis gears transmit power with greater efficiency than any other type or form of gearing.

2.1.2.1 Spur Gears

The spur gear has teeth on the outside of a cylinder and the teeth are parallel to the axis of the cylinder. This simple type of gear is the most common type. Its volume of usage is the largest of all types.

The shape of the tooth is that of an involute form. There are, however, some notable exceptions. Precision mechanical clocks very often use cycloidal teeth, since they have lower separating loads and generally operate more smoothly than involute gears and have fewer tendencies to bind. The cycloidal form is not used for power gearing because such gears are difficult to manufacture, sensitive to small changes in center distance, and not as strong or as durable as their involute brothers.

Figure 2.1 shows a close-up view of the teeth on a set of spur gears having about a 5:1 ratio. The teeth are 20° involute tooth form. The pinion is made with more than a standard addendum and the gear addendum is shorter than the standard. The whole depth is standard for high-strength gears. Note the large radius of curvature in the root fillet region. This reduces the bending stress due to a lower stress concentration factor. Also note that the pinion teeth look very sturdy. This is an effect of the long- and short-addendum designs just mentioned. (Details of specific design strategy are covered in later chapters of this book.) Figure 2.2 shows a set of spur gears in an accessory gearbox.

The most common pressure angles used for spur gears are 14.5°, 20°, and 25°. In general, 14.5° pressure angle is not used for new designs (and has, in fact, been withdrawn as an AGMA standard tooth form); however, it is used for special designs and for some replacement gears. Lower pressure angles have the advantage of smoother and quieter tooth action because of the large profile contact ratio. In addition, lower loads are imposed on the support bearings because of a decreased radial load component; however, the tangential load component remains unchanged with pressure angle. The problem of undercutting associated with small numbers

TABLE 2.1
General Comparison of Gear Types

Type of Gear	Approximate Range of Efficiency	Maximum[a] Width Generally Used[b]	Type of Load Imposed on Support Bearings	Nominal Range of Reduction Ratio[c]	Nominal Maximum Pitch-Line Velocity (fpm) (5.08×10^{-3} m/s): Aircraft and High Precision	Nominal Maximum Pitch-Line Velocity (fpm) (5.08×10^{-3} m/s): Commercial	Member of Pair	Methods of Manufacture	Methods of Refining
Parallel Axes									
External spur gears	97–99.5	$F = d$	Radial	1:1–5:1	20,000	4000	Both	Hob, shape, mill, broach, stamp, sinter	Grind, shave, lap (crossed axis only), hone
Internal spur gears	97–99.5	Directed by mating gear	Radial	1:5–7:1	20,000	4000		Shape, mill, broach, stamp, sinter	Grind, shave, lap (crossed axis only), hone
External helical gears	97–99.5	$F = d$	Radial and thrust	1:1–10:1	40,000	4000	Both	Hob, shape, mill	Grind, shave, lap, hone
Internal helical gears	97–99.5	Directed by mating gear	Radial and thrust	5:1–10:1	20,000	4000		Shape, mill	Shave, grind, lap, hone
Internal herringbone or double helical	97–99.5	Directed by mating gear	Radial	2:1–20:1	20,000	4000		Shape	Lap, shave, hone
External herringbone or double helical	97–99.5	$F = 2d$	Radial	1:1– 20:1	40,000	4000	Both	Hob, shape, mill	Grind, shave, lap, hone
Intersecting Axis									
Straight bevel gear	97–99.5	⅓ cone distance	Radial and thrust	1:1–8:1	10,000	1000	Both	Generating forming	Grind
Zerol bevel gear	97–99.5	28% of cone distance	Radial and thrust	1:1–8:1	10,000	1000	Both gear	Generating forming	Grind
Spiral bevel gear	97–99.5	⅓ cone	Radial and thrust	1:1–8:1	25,000	4000	Both gear	Generating forming	Grind, lap
Face gear	95–99.5	From 0.2d at low ratio to d at a high ratio	Radial and thrust	3:1–8:1	5000	4000	Pinion	Same as external spur gear	Lap, grind, shave, hone
							Gear	Shape	Shape, lap

(*Continued*)

TABLE 2.1 (CONTINUED)
General Comparison of Gear Types

Type of Gear	Approximate Range of Efficiency	Maximum[a] Width Generally Used[b]	Type of Load Imposed on Support Bearings	Nominal Range of Reduction Ratio[c]	Nominal Maximum Pitch-Line Velocity (fpm) (5.08×10^{-3} m/s): Aircraft and High Precision	Nominal Maximum Pitch-Line Velocity (fpm) (5.08×10^{-3} m/s): Commercial	Member of Pair	Methods of Manufacture	Methods of Refining
Intersecting Axis									
Beveloid	95–99.5	$5/p_n$	Radial and thrust	1:1–8:1	5000	4000	Both	Hob generating	Lap, grind
Crossed helical	50–95	$F_1 = 4p_n \sin \psi_1$ $F_2 = 4p_n \sin \psi_2$	Radial and thrust	1:1–100:1	10,000	4000	Both	Hob, shape, mill	Grind, lap, shave
Cylindrical worm	50–90	$F_w = 5\, p_n \cos \lambda$	Radial and thrust	3:1–100:1	10,000	5000	Pinion	Mill, hob	Grind
		$F_g = 0.67d$					Gear	Hob	Lap
Double-enveloping worm	50–98	$F_w = 0.9D$	Radial and thrust	3:1–100:1	10,000	4000	Worm	Shape, hob	Lap, grind
		$F_g = 0.9d$					Wheel	Hob, mill	
Hypoid	90–98	$F_g = ⅓$ cone distance	Radial and thrust	1:1–10:1	10,000	4000	Both	Generating Forming	Grind, lap
High-reduction hypoid	50–90	$F_g = 0.15D$	Radial and thrust	10:1–50:1	10,000	4000	Pinion	Generating	Grind, lap
							Gear	Forming	Grind, lap
Spiroid	50–97	$F_p = 0.24D$	Radial and thrust	9:1–100:1	10,000	6000	Pinion	Mill, hob	Grind, chase
		$F_g = 0.14D$					Gear	Hob, mold	
Planoid	90–98	$F = ⅓$ cone distance	Radial and thrust	1.5:1–10:1	10,000	4000	Pinion	Hob	Lap, grind
							Gear	Mill, broach	Grind
Helicon	50–98	$F_p = 0.21D$	Radial and thrust	3:1–100:1	10,000	6000	Pinion	Mill, hob	Grind, chase
		$F_g = 0.12D$					Gear	Hob, mold	
Face gear	95–99.5	From 0.2D at low ratio to D at a high ratio	Radial and thrust	3:1–8:1	10,000	4000	Pinion	Same as external spur	Same as external spur
							Gear	Shape	Lap
Beveloid	50–95	$5/p_n$	Radial and thrust	1:1–100:1	10,000	4000	Both	Generating, hob	Grind, lap

[a] Face width given is only a nominal maximum. Consult other parts of the *Gear Handbook* for details on the limitations on face width.
[b] d: pinion pitch diameter; D: gear pitch diameter; λ: lead angle; ψ: helix angle; p_n: normal circular pitch.
[c] The gear types showing an upper ratio of 100:1 can be built to 300:1 or higher. The ratio of 10:1 is shown as a normal maximum limit.

FIGURE 2.1 Sixteen-tooth spur pinion driving a gear with about 75 teeth. This is a special design of 20° involute teeth for high load-carrying capacity.

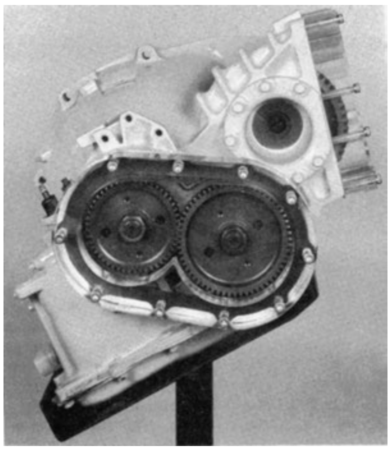


FIGURE 2.2 With the cover removed, a set of high-speed spur gears can be seen in an accessory box. (Courtesy of the Gleason Works, Rochester, New York.)

of pinion teeth is more severe with the lower pressure angle. Lower-pressure-angle gears also have lower bending strength and surface durability ratings and operate with higher sliding velocities (which contribute to their relatively poor scoring and wear performance characteristics) than their higher-pressure-angle counterparts.

Higher pressure angles have the advantage of better load-carrying capacity, with respect to both strength and durability, and lower sliding velocities (thus, better scoring and wear performance characteristics). In some cases, very high pressure angles, 28°, 30°, and, in a few cases, as high as 45°, are employed in some special slow-speed gears for very high load capacity where noise is not the predominant consideration.

A minimum of equipment is required to produce this type of gear; thus, it is usually the least expensive of all forms of gearing. While the most common tooth form for spur gears is the involute, other tooth forms are possible as long as they provide conjugate motion.

2.1.2.2 Helical Gears

When the gear teeth are cut on a spiral that wraps around a cylinder, they are designed as helical. Helical teeth progressively enter the meshing zone and, therefore, have a smoother action than spur gear teeth and tend to be quieter. In addition, the transmitted load may be somewhat larger, or the life of the gears may be greater for the same loading, than with that of an equivalent pair of spur gears. Conversely, in some cases, smaller helical gears (compared with spur gears) may be used to transmit the same loading. Helical gears produce an end thrust along the axis of the shafts in addition to separating and tangential (driving) loads of spur gears. Where suitable means can be provided to take this thrust, such as thrust collars or ball or tapered roller bearings, it is no great disadvantage. The efficiency of a helical gearset, which is dependent on the total normal tooth load (as well as the sliding velocity and the friction coefficient, and so forth), will usually be slightly lower than that of an equivalent spur gearset.

Conceptually, helical gears may be thought of as stepped spur gears in which the size of the step becomes infinitely smaller. For external parallel axis helical gears to mesh, they must have the same helix angle but be of different hand. Just the opposite is true for an internal helical mesh; that is, the external pinion and the internal gear must have the same hand of helix. For crossed axis helical gears (discussed later in Section 2.1.4.1), the helix angles on the pinion and the gear may or may not be of the same hand, depending on the direction of rotation desired and the magnitude of their individual helix angles.

Involute profiles are usually employed for helical gears and the same comments made earlier about spur gears hold for helical gears.

In order to provide significant improvement in noise over spur gears, the face overlap must be at least unity. That is, one end of the gear face must be advanced at least one circular pitch from the other. If the face overlap (also called face contact ratio) is less than unity, the helical gear, for most analytical purposes (that is, stress calculations), is treated as if it were a spur gear with no advantage taken of the fact that it is helical.

Helix angles from only a few degrees up to about 45° are practical. As the helix angle increases from zero, in general, the noise level is reduced and the load capacity is increased. At angles much above 15° to 20°, however, the tooth bending capacity generally begins to drop. This is due to the fact that the transverse tooth thickness rapidly decreases.

Double-helical or herringbone gears are frequently used to obtain the noise benefits of single-helical gears without the disadvantage of thrust loading. The common types of helical gears are shown in Figure 2.3, while Figure 2.4 shows a typical single-helical set. The terms *double helical* and *herringbone* are sometimes used interchangeably. Actually, herringbone is more correctly used to describe double-helical teeth that are continuously cut into a solid blank, as shown in Figure 2.3c. Double-helical gears may be cut integral with the blank either inline or staggered, or two separate parts may be assembled to form the double.

While double-helical or herringbone gears do eliminate the net thrust load on the shaft, it is important to note that the two halves of the gear must internally react to the full thrust load. This being the case, the design of the gear blank for one-piece gears, or the construction of the assembly, must

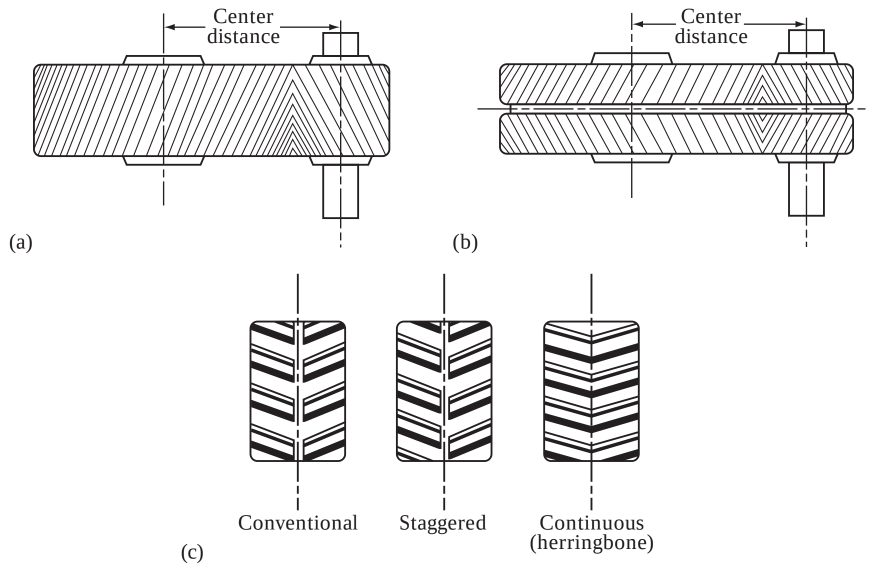


FIGURE 2.3 Types of helical gear teeth: (a) single helical and (b) double helical; and (c) types of double helical.

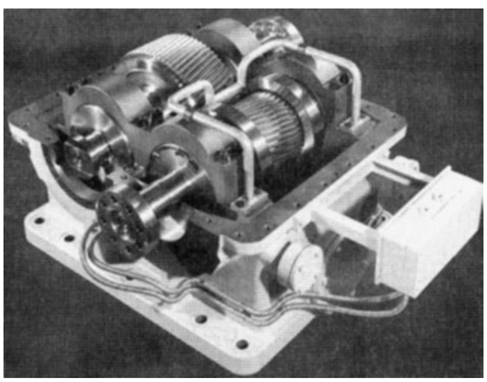


FIGURE 2.4 Single-helical gearset designed for high speed and high horsepower. (Courtesy of Maag Gear Wheel Company, Zurich, Switzerland.)

be carefully evaluated to ensure that it is strong enough to take the thrust loads and to provide enough rigidity so that the deflections are not excessive.

The efficiency of helical gears is quite good, although not quite equal to that of simple spurs. This is due to the fact that all other things being equal, the normal (or total) tooth load on a helical gear is higher than that of a spur for an equivalent tangential load.

2.1.2.3 Internal Gears

The internal gear has teeth on the inside of a cylinder. The teeth may be made either spur or helical. The teeth of an involute form internal gear have a concave shape rather than a convex shape. Internal gears are generally more efficient, since the sliding velocity along the profile is lower than that of an equivalent external set. Because of the concave nature of the internal tooth profile, its base is thicker than that of an equivalent external gear tooth (either spur or helical). The tooth strength of an internal gear is greater than that of an equivalent external gear.

The internal gear has other advantages. It operates at a closer center distance with its mating pinion than external gears of the same size do. This permits a more compact design. The internal gear eliminates the use of an idler gear when it is necessary to have two parallel shafts rotate in the same direction. The internal gear forms its own guard over the meshing gear teeth. This is highly desirable for preventing accidents in some kinds of machines.

Internal gears cannot be used where the number of teeth in the pinion is almost the same as that of the gear. When this occurs, the tips of the pinion teeth interfere with the tips of the gear teeth. While a good guide is to maintain a ratio of 2:1 between the number of teeth on the internal gear and its mating pinion, lower ratios may be practical in some cases, particularly if the tooth utilizes shifted involute design characteristics. For full-depth 20° pressure-angle teeth, for example, the internal gear pitch diameter must be at least one and one-half times that of the external gear; smaller ratios require considerable modification of the tooth shape to prevent interference as the teeth enter and leave the mating gear. Internal gears also have the disadvantage that few machine tools can produce them.

It may be more difficult to support a pinion in mesh with an internal gear because the pinion must usually be overhung mounted. Internal gears tend to find their greatest application in various forms of epicyclic gear systems and in large slewing-type drives, generally with integral bearings. In such configurations, the internal gear is frequently much larger than its mating external pinion, and thus, it is usually possible to build a carrier structure inside the internal gear so that the pinion can be straddle mounted.

A typical internal gear arrangement is shown in Figure 2.5, while Figure 2.6 shows a large internal spur gear. In the case of an internal helical gear, its hand of helix must be the same as its mating pinion; that is, both the external helical pinion and its mating helical internal gear must be either right or left handed.

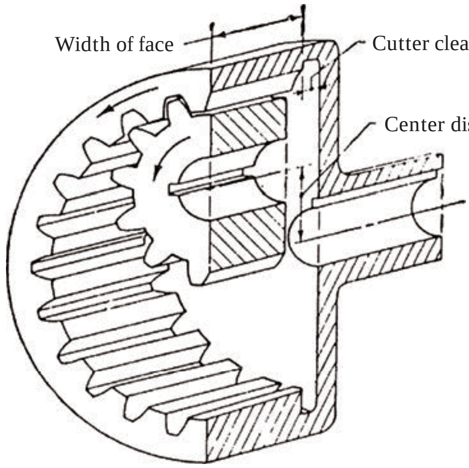


FIGURE 2.5 Typical internal-gear arrangement.

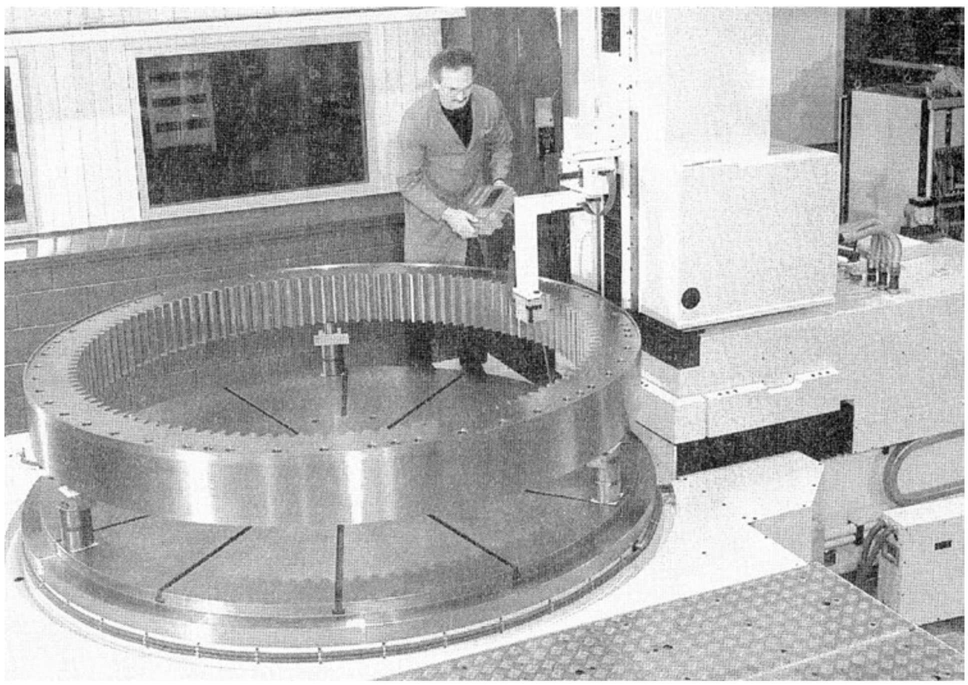


FIGURE 2.6 Ninety-inch diameter internal ring gear for an epicyclic speed reducer being measured on a 200-inch diameter Maag gear checker. (Courtesy of Philadelphia Gear Corp., King of Prussia, Pennsylvania.)

2.1.3 Nonparallel, Coplanar Gears (Intersecting Axes)

While the involute is the tooth form that is almost the universal choice for parallel axis gears, most gears that operate on nonparallel, coplanar axes do not employ involute pro les. A wide variety of gear types falls into this category. The most obvious use for gears of this type is the reduction of power ow around a corner, such as might be required, for example, when connecting a horizontally mounted turbine engine to the vertically mounted rotor shaft on a helicopter. In fact, while power ow is a frequent reason for choosing one of these gear types, other features unique to a particular type of gear, such as the ability of straight bevel gears to be used in a differential arrangement for a rear-wheel-drive automobile, might well be the determining factor in making the decision to use one of these gear types. Descriptions of the major, and some interesting minor, ones follow.

2.1.3.1 Bevel Gears

There are four basic types of bevel gears: straight, Zerol,* spiral, and skew tooth. In addition, there are three different manufacturing methods (face milling, face hobbing, and tapered hobbing) that are employed to produce true curved-tooth

* Zerol is a registered trademark of Gleason Works, Rochester, New York.

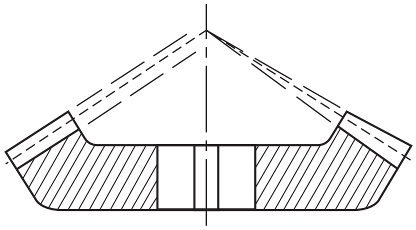


FIGURE 2.7 Basic bevel-gear configuration. Teeth may be tapered in depth or they may be made with a parallel depth, depending on manufacturing system used.

bevel gears, each of which produces different tooth geometry. In our discussion herein, we address the overall generic attributes of the different types of bevel gears without addressing the manufacturing variations.

All bevel gears impose both thrust and radial loads in addition to the transmitted tangential forces on their support bearings. The thrust is a result of the tapered nature of the gear blank regardless of the tooth form employed (that is, even straight bevel gears produce an axial thrust). The diametral pitch for all bevel gears is conventionally measured at the heel of the tooth, while the pressure and spiral angles are conventionally measured at the mean section (tooth midface). In all cases, the pitch apex of the pinion and that of its mating gear must intersect at the point at which the axes of the pinion and the gear shafts intersect. Figure 2.7 shows the basic bevel gear configuration that is common to all types of bevel gears. By convention, the direction of rotation of a bevel gear is specified as clockwise or counterclockwise as viewed from the heel end of the gear looking toward the pitch apex.

2.1.3.1.1 Straight

The straight form is the simplest form of the bevel gear. Except for the thrust load produced by the blank taper, straight bevel gear teeth are a three-dimensional analogy to spur gear teeth. One very common use for straight bevel gears is in a bevel gear differential. Extensions of the straight teeth intersect at the axis of the gear. The tooth profile in a section normal to the tooth approximates that of an involute spur gear having a number of teeth equal to the actual number of teeth in the bevel gear divided by the cosine of the pitch angle (equivalent number of teeth). The teeth are always tapered in thickness and may have either constant or tapering height. The outer or heel part of the tooth is larger than the inner part called the *toe*. See Figure 2.8. Also, straight bevel gears have instantaneous line contact; they are often manufactured to have localized contact to permit more tolerance in mounting. One method of achieving this type of contact is the Coniflex* system, employed by Gleason Works.

2.1.3.1.2 Zerol

An improvement over straight bevel gears in terms of contact conditions, noise level, and power capacity is the Zerol bevel gear (Figure 2.9). This gear is similar to a straight bevel gear except that the teeth are curved along their axis; however, the mean spiral angle is zero[†]; thus, the bearing reaction loads (particularly the thrust loads) are the same for straight bevels. Zerol gears are manufactured with the same type of cutters and on the same machines as spiral bevel gears. This is economically important as it eliminated the need for more than one type of bevel gear cutting equipment. The localized tooth bearing of Zerol gears is accomplished by lengthwise mismatch (face curvature mismatch) and profile modification. The face width of a Zerol bevel gear is restricted to specific values due to cutter limitations. Zerol bevel gears are very frequently used in turbine engine, helicopter, and other high-speed devices such as accessory drives.

FIGURE 2.8 Straight bevel gears. (Courtesy of Gleason Works, Rochester, New York.)

FIGURE 2.9 Zerol bevel gears. (Courtesy of Gleason Works, Rochester, New York.)

Zerol bevel gears may be compared to double-helical or herringbone parallel axis gear teeth in that they have no more thrust load than their straight counterparts but provide advantages related to the improved contact ratio due to their lengthwise curvature. A pressure angle of 20° is the most common; however, Zerol gears may be manufactured with 14.5°, 22.5°, or 25° as well. Because Zerol gears are not generally used in very high load applications, the use of high pressure angles, which tend to reduce contact ratio and thus increase noise, should be carefully considered.

2.1.3.1.3 Spiral

The most complex form of bevel gear is the spiral bevel. It is commonly used in applications that require high load capacity at higher operating speeds than are typically possible with

* Coniflex is a registered trademark of the Gleason Works, Rochester, New York.

† Actually, the term *Zerol* applies to any spiral bevel gear with a spiral angle of 10° or less, but most often Zerol bevels have a zero mean spiral angle.

either straight or Zerol bevel gears. The relationship between spiral and straight bevel gears is comparable to that between helical and spur gears. By convention, the hand of a spiral gear is defined as either left- or right-hand as determined by viewing the gear from the apex end looking toward the heel end.

The teeth of spiral bevel gears are curved and oblique (Figure 2.10), and as a result, they have a considerable amount of overlap. This increases more than one tooth in contact at all times and results in gradual engagement and disengagement with continuous multiple-tooth contact. This results in higher overlap contact ratios than are possible with either straight or Zerol gears. Because of this improved contact ratio and the resultant load sharing, spiral bevel gears have better load-carrying capacity and run more smoothly and quietly than either straight or Zerol bevel gears of the same size. The improved load capacity of spiral bevel gears also allows them to be smaller in size for a given load capacity than that of an equivalent straight or Zerol bevel gear.

Spiral bevel gears impose significantly more thrust load on their support bearings than either Zerol or straight bevel gears do. While plain thrust bearings have been successfully used, a rolling element thrust bearing is usually required with spiral bevel gears. The more common pressure angles for spiral bevel gears are 16° and 20°, with the latter now being almost the standard. The most common spiral angle in use is 35°. In more highly loaded applications, particularly those that operate at high speeds as well, such as aircraft or helicopter transmissions, a higher pressure angle (typically 20.5° to 25°) and lower spiral angles (typically 15° to 25°) are more often used.

The profile contact ratio on all bevel gears is usually substantially lower than that of equivalent spur gears; thus, they tend to be somewhat noisier. In addition, due to their higher total load for a given tangential load and the higher sliding that accompanies the localized contact, they are less efficient than either spur or helical gears. Still, the efficiency of a well-made set of bevel gears is generally better than 98%, and that of hardened and ground sets is usually above 99.2%.

Spiral bevel gears can be made by any of three generic methods. Each of these three methods produces a unique lengthwise tooth curvature and profile. In general, while gears within one system may be interchangeable, and gearsets made by each system are setwise interchangeable, pinions and gears are not interchangeable between systems, despite the fact that the basic configuration (that is, diametral pitch, spiral angle, pressure angle, etc.) may be identical.

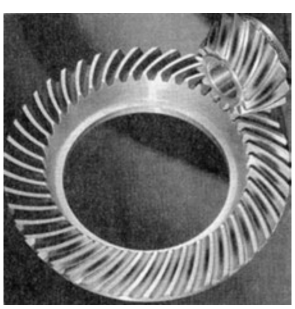


FIGURE 2.10 Spiral bevel gears. (Courtesy of Gleason Works, Rochester, New York.)

1. *Circular lengthwise tooth curvature*: For circular lengthwise tooth curvature systems, developed in the United States, face-milled cutters with multiple blades are used. While several methods of implementing this process exist, they are all somewhat similar in their basic motions. The blank executes a rolling motion relative to the cutter, which simulates one tooth of the imaginary crown gear. This is repeated for every tooth space, the cutter being withdrawn and turned back into starting position every time while the job is indexed for the next gap. This process is known as single indexing.

   All tooth spaces first have to undergo a roughing operation and are finished by a subsequent operation. In general, the root lines of the teeth are not parallel to the pitch line, but set at a specific root angle. The cutters must be set parallel to the root lines while producing the gears; that is, they must be tilted relative to the pitch line. This causes the normal pressure angle to change, so that the pressure angles of the cutters must be corrected accordingly. This correction depends on the root angle and on the spiral angle and is different for either tooth side. In operation, the concave tooth surfaces of one member are always in mesh with the convex tooth side of the other. This system is used by Gleason machines and is the only one for which grinding machines have been developed.
2. *Involute lengthwise tooth curvature*: The involute lengthwise tooth curvature system was originally developed in Germany. The tooth is a tapered hob, usually single thread, with a constant pitch determined in accordance with the normal diametral pitch. The machining process is by continuous indexing: The tool and the hob rotate continuously and uniformly, the relationship between the two rotary motions being established by change gears as a function of the number of teeth being cut. The hob is adjusted with its cone surface tangential to the surface of the imaginary flat gear.

   This system is utilized by some Klingelnberg machines but is rapidly being replaced by the more versatile face-milling and face-hobbing processes.
3. *Epicycloidal lengthwise tooth curvature*: The epicycloidal lengthwise tooth curvature system, originally evolved in Italy and England but still not free from some shortcomings, has been further developed in Switzerland and Germany. The machines available for producing this type of spiral bevel gear are also known as cyclo-palloid tooth form bevel gears and are similar to the one for the production of circular tooth length curves in that the tool is also a face-type milling cutter with inserted blades, which can

be of the inside-cutting, outside-cutting, and gashing types, alternatively. The indexing is now, however, not intermittent but continuous. The cutter and the blank rotate continuously and uniformly, so that not only one tooth space generated at a time, but also all tooth spaces consecutively. Accordingly, the blades are not evenly distributed over the periphery of the cutter, but combined in groups (that is, the cutter has multiple starts). One group generally comprises one roughing, one outside-cutting, and one inside-cutting blades, and allows simultaneous machining of the tooth bottom and both the tooth sides. The number of blade groups depends on size of the cutter and usually varies from 3 to 11. This basic system is utilized by both Oerlikon and Klingelnberg, with some variation on each.

2.1.3.1.4 Skew

The nal type of bevel gear is the skew tooth. This is similar to a spiral bevel gear and is also quite often (incorrectly) referred to as a spiral bevel gear. Actually, the skew tooth has no lengthwise curvature; rather, the teeth are simply cut straight but at an angle to the shaft centerline (as shown in Figure 2.11), so that some face overlap may occur. This provides an improvement in load capacity (Figure 2.12) when compared with a straight bevel gear, but not to the level possible with a true spiral (curved tooth) bevel gear. Skew tooth gears are primarily used in large (over 30 in. [762 mm] gear pitch diameter) sizes only. They are produced on planing generator machines.

Until the late 1980s, because of limitations in the size of the face-milling machines available, most spiral bevel gears over 30 to 40 in. (762 to 1016 mm) in diameter were planed; thus, they were actually skew tooth, not true spiral bevel gears. Since pinion and gear must be conjugate, the pinions that mate with these large gears must also be planed. Relatively recent advances in spiral bevel gear manufacturing have led to the development of large machines of both the face-milling and face-hobbing types that can cut true spiral bevel gear teeth on gears over 100 in. (2540 mm) in pitch diameter.

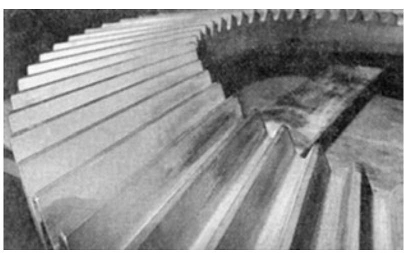


FIGURE 2.11 Skew bevel gear.

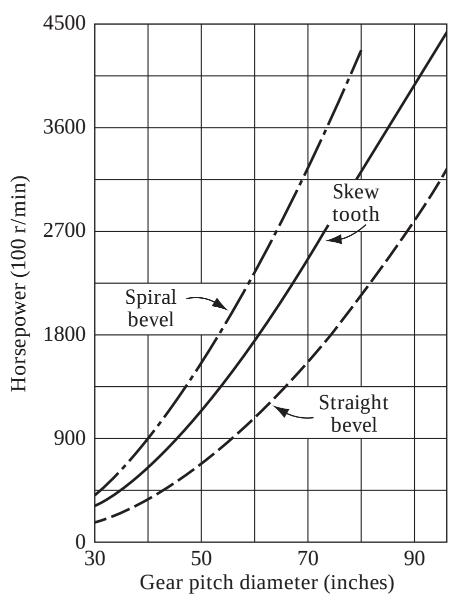


FIGURE 2.12 Comparison of large straight skew and spiral bevel gear capacities. (Data from Philadelphia Gear Corp., King of Prussia, Pennsylvania.)

2.1.3.2 Face Gears (On-Center)

A face gearset is actually composed of a spur or a helical pinion that is in mesh with a face gear. The pinions are really not any different from their parallel axis counterparts except for the fact that they are in mesh with a face gear. Face gears have teeth cut into the blank such that the axis of the teeth lies on a plane that is perpendicular to the shaft axis. The mating pinion is either a spur or a helical gear. The pinion and gear axes are coplanar for on-center face gears. The pinion and the face gear most often form a 90° shaft angle; however, small deviations from 90° are possible. In operation, this type of gear is similar to an equivalent set of straight bevel gears.

Face gear tooth changes shape from one end of the tooth to the other. The load capacity of face gears, compared with that of bevel gears, is rather small; thus, they are mostly used for motion transmission rather than as power gears. Face gears are, relative to bevel gears, easy to make and somewhat less expensive as well. A typical arrangement is shown in Figure 2.13. Toys and games make extensive use of face gears.

2.1.3.3 Conical Involute Gearing

Conical involute gearing is a completely generalized form of involute gear. It is an involute gear (as shown in Figure 2.14) with tapered tooth thickness, root, and outside diameter. Commonly known as *beveloid* gears, they are primarily useful for precision instrument drives where the combination of high precision and limited load-carrying ability ts the application. Like other involute gears, beveloid gears are relatively insensitive to positional errors, although shaft angles must

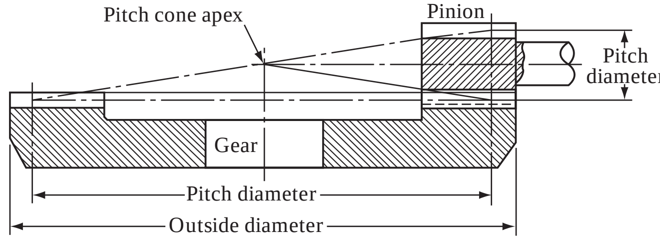


FIGURE 2.13 Face gear arrangement.

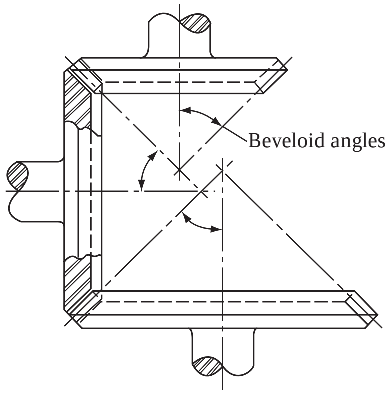


FIGURE 2.14 Beveloid gear arrangement.

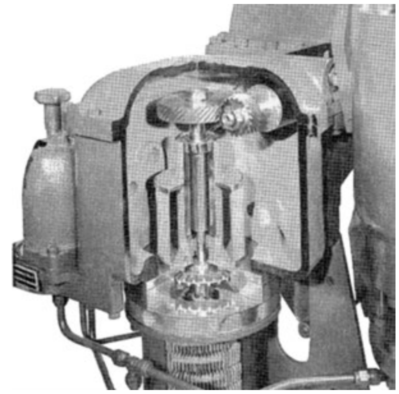


FIGURE 2.15 Crossed-axis-helical gearset, driving a pump and governor for a steam turbine application. (Courtesy of General Electric Co., Lynn, Massachusetts.)

be quite accurate. They will conjugately mate with all other involute gears (spur, worms, helical gears, and racks). Like crossed axis helical gears (see Section 2.1.4.1), however, the area of contact of these gears and the beveloid gears is limited; thus, the load capacity is low. For small tooth numbers and large cone angles, undercutting is quite severe.

This type of gear is not in widespread use at this time, but may be used to advantage in some circumstances, especially for precision motion-transmission applications in which the gear shafts must be located at raying orientations in space.

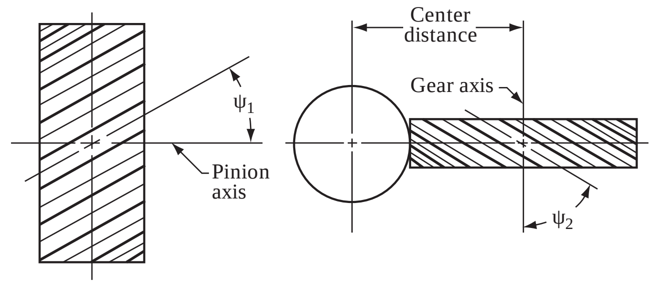


FIGURE 2.16 Crossed axis helical gear arrangement.

2.1.4 Nonparallel, Noncoplanar Gears (Nonintersecting Axes)

Gears in nonparallel, non-coplanar gear classi cation are generally the most complex, both in terms of geometry and manufacture. The simpler types, discussed rst in the following discussions are, however, quite easy to manufacture and are reasonably inexpensive, but they do not carry large loads. The more complex types are generally more expensive but provide better load capacity and other features that make them especially suited for a wide variety of special applications.

2.1.4.1 Crossed Axis Helical Gears

Crossed-helical gears are satisfactory for the normal range of ratios used for single reduction helical gears. They provide both speed reductions and extreme versatility of shaft positioning at a relatively low initial cost. At higher ratios or for anything above moderate loads, however, worm, Spiroid, or Helicon* gears are generally preferable. (See Figure 2.15 for typical crossed-helical gears.)

Crossed axis helical gears are usually cut in the same manner as conventional helical gears using identical tooling, since they are, until mounted on their crossed axes, actually nothing more than conventional helical gears! Since crossed-helical gears have a great deal of sliding action, special attention must be given to the selection of gear materials and their lubricants to reduce friction to a minimum and eliminate any possibility of seizing between mating gears. Experience has indicated that iron on iron satisfactorily functions. However, a hardened steel pinion driving a bronze gear will also work quite well.

Figure 2.16 shows some of the special relations that must exist for this type of gearing to properly function. It is interesting to note that while mating, parallel axis external helical gears must have opposite hands of the helix; crossed axis helical gears can have either the same or the opposite hands of the helix, depending on the shaft angle and the relative directions of rotation of the driver and the driven gears. Additionally, the reduction ratio between the pinion and the gear is a function of their relative numbers of teeth but not directly of their pitch diameters. This provides a great deal of exibility in choosing the ratio, the center distance, and the diameters of the gears.

Finally, because the relative contact between the mating gears is the theoretical intersection of two cylinders, the load capacity of the gears is not directly in uenced by the face width; that is, once the face width is extended such that it covers the full cylindrical intersection, extending either or both face widths farther is of no practical value, since the extra face width will not be in contact.

2.1.4.2 Cylindrical Worm Gearing

The most basic form of worm gearing is a straight cylindrical worm in mesh with a simple helical gear. In reality, this is really an extreme case of crossed axis helical gearing. Such gears can provide considerably higher reduction ratios than simple crossed axis gearsets, but their load capacity is low,

* Spiroid and Helicon are registered trademarks of Illinois Tool Works, Chicago, Illinois.

and the wear rate is high. For light loads, however, this configuration can be an economical alternative.

2.1.4.3 Single-Enveloping Worm Gearing

Better load capacity can be achieved if the simple helical gear is modified such that it is throated to allow the worm to fit farther down into the gear to achieve greater tooth contact area and thus smoother operation and improved load capacity. This common worm gearset is often referred to as *single enveloping*, since the gear envelopes the worm but the worm remains straight. While many variations of this configuration are used, in the most common case, the worm is essentially straight sided while the gear or the wheel is generally involute in form. The contact point is theoretically a line varying in length up to the full-face width of the gear with different tooth designs. Under load, this line becomes a thin elliptical band of contact. See Figure 2.17 for a two-stage worm gear drive.

Basic arrangement views are shown in Figure 2.18. The hand of the helix for both members is the same. While the worm is generally made of steel, the wheel is usually made of either cast iron or, more frequently, one of several types of bronze. The worm teeth may be through hardened, but they are often, especially in higher-load and higher-speed applications, case hardened and ground. In some cases, however, to improve accuracy and efficiency, the worm teeth are ground even though they are not hardened after cutting. The wheel, due to the high sliding conditions that exist in this type of gearing, is usually made of cast iron or bronze, as noted in the foregoing; however, in some high-temperature environments it is necessary to make the wheel from steel as well.

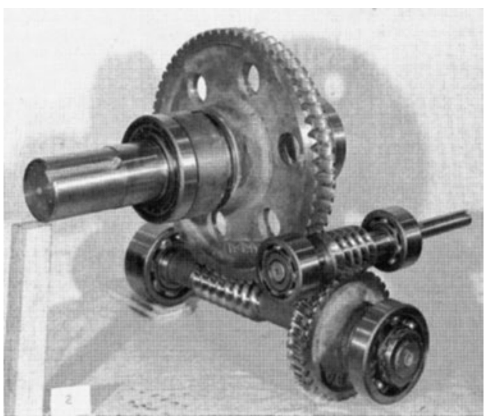


FIGURE 2.17 Two stages of cylindrical worm gears. (Courtesy of Hamilton Gear and Machine Co., Ltd., Hamilton, Ontario, Canada.)

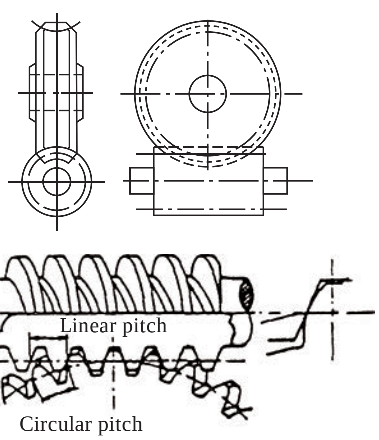


FIGURE 2.18 Schematic of cylindrical worm gears.

By virtue of their inherently high contact ratio, the mechanical power rating of worm gears is quite high; however, in practice, their actual continuous-duty rating is substantially lower. This is due to high heat generation that can raise the lubricant temperature to unacceptable levels when the box is continuously operated. Fan-cooled worm boxes are quite common, and higher–power capacity worm housings are almost always fanned to aid heat dissipation. This large difference between the thermal and mechanical ratings of the typical worm gear drives rises to a peculiar property associated with worm drives, that is, their apparent ability to sustain relatively high short-term overloads without experiencing any damage. In reality, worm gears do not have a particularly good overload capacity; rather, their thermal limitations cause them to be operated at loads below their mechanical limits. When operated for short periods at overload, they are actually operating above their continuous-duty thermal limits but below their mechanical (stressed-related) ratings; thus, since it takes an appreciable period for the temperature to rise, they sustain these short-term overloads quite well.

The advent of synthetic fluids has been a boon for worm drives of all types for two reasons. First, synthetic fluids have the ability to operate at higher bulk temperatures than the compounded mineral-based fluids that are commonly used for worm gears. Second, the friction coefficient associated with the use of synthetic fluids tends to be somewhat lower than that associated with compounded worm gear oils; thus, less heating is produced. These factors combine to decrease the margin between the thermal and mechanical limits of newer work gearsets designed and rated to run with synthetic fluids; thus, their apparent overload capability is reduced.

Worm gear efficiency is quite dependent on operating speed. The same set may show an efficiency of, say, 75% at a low speed and 85% at a higher speed. Ratio, material, accuracy, and geometric design all affect worm gear efficiency. Typical efficiencies run from 35% to 90%, with higher or lower values occurring in special cases.

A worm set can be used where irreversibility is desired, since, if the lead angle is less than the friction angle, the wheel cannot drive the worm. Usually, worms with lead angles less than 5° are self-locking. Care should be exercised when designing self-locking worms, since this feature is a static one. Vibration can cause the set to slip under dynamic conditions (that is, during a cutoff of power under load, the wheel, due to the inertia of the driving load, may overdrive the worm for a considerable time). Similarly, a worm set that, when stationary, cannot be driven through the gear shaft may well begin to rotate if the unit is subjected to vibrations. The property of self-locking or irreversibility is better thought of

as antibackdriving rather than positive irreversibility. In critical applications, a brake on the worm should also be provided to ensure that the system will positively not backdrive.

When the gears are assembled into their housing, the position of the gear along its axis must be adjusted such that an acceptable contact pattern is obtained on the bench. This contact pattern should favor the leaving side of the mesh so that a wedge is formed at the entering side of the mesh. This wedge at the entering side will cause lubricant to be drawn in the contact zone and thus minimizes wear.

2.1.4.4 Double-Enveloping Worm Gearing

The capacity of a single-enveloping worm gearset as described in the foregoing discussion is improved by allowing the gear or the wheel to envelop the worm. A further improvement in capacity may be achieved by allowing the worm to envelop the wheel as well. Such drives are known as *double enveloping.* Double-enveloping worm gear drives, by getting more teeth into contact, tend to provide higher load capacity than cylindrical or single-enveloping worm sets do. This is accomplished by changing the shape of the worm (as shown in Figure 2.19) from a cylinder to an hourglass.

All the comments made earlier pertaining to cylindrical worms also apply to double-enveloping worms. Because of the shape of the worm, clearly shown in Figure 2.20, this type of worm gearing is more expensive to produce; but where weight or size are considerations, the cost differential is relatively small.

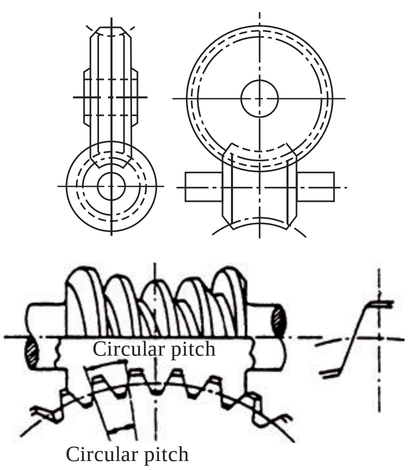


FIGURE 2.19 Schematic of double-enveloping worm gears.

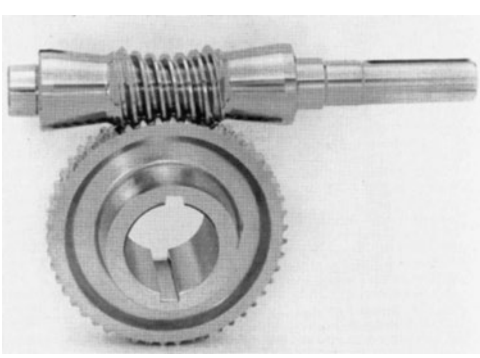


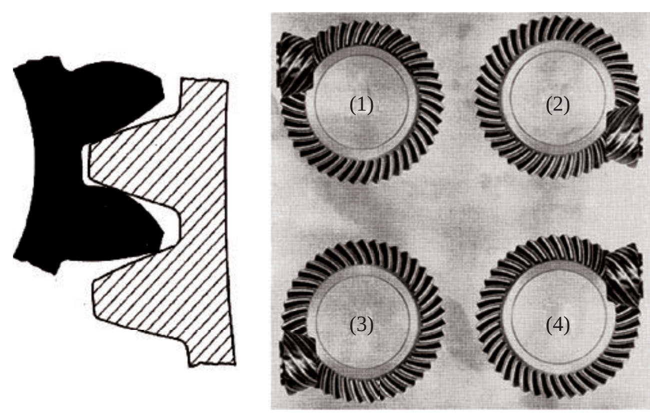
FIGURE 2.20 Double-enveloping worm gearset.

The forms of the worm and the gear in the double-enveloping drive design are regenerative; that is, the worm and gear tend to reproduce each other in use. This condition aids proper break-in and contact pattern development. Since the worm and wheel envelop each other, the assembly is not as straightforward with double-enveloping gearsets as it is with a single-enveloping set. In general, the worm and wheel must be assembled obliquely, and thus, provisions must be made in the design of the housing to accommodate this requirement. This can often be accomplished by splitting the housing such that the worm is one part and the wheel is the other half. In addition, the position of the worm along its axis and the position of the gear along its axis must be simultaneously adjusted such that an acceptable pattern is obtained. This bench patterning procedure is in contrast to the simpler procedure required of a single-enveloping set in which only the position of the gear along its axis needs be adjusted at assembly to obtain an acceptable contact pattern.

2.1.4.5 Hypoid Gears

Hypoid gears (Figure 2.21) resemble spiral bevel gears except that the teeth are asymmetrical; that is, the pressure angle on each side of the tooth is different. Many of the same machines used to manufacture spiral bevel gears can also be used to manufacture hypoid gears.

The pitch surfaces of hypoid gears are hyperboloids of revolution. The teeth in mesh have line contact; however, under load, these lines spread to become elliptical regions of contact inclined across the face width of the teeth. One condition that must exist if a hypoid gearset is to have conjugate action is that the normal pitch of both members must be the same. The number of teeth in a gear and pinion are not, however, directly



(a) (b)

FIGURE 2.21 Hypoid gear data. (a) Hypoid tooth profile showing unequal pressure angles and unequal profile curvatures on the two sides of the tooth. (b) Hypoid gears and pinions 1 and 2 are referenced to having an offset below center, while those in 3 and 4 have an offset above center. In determining the direction of the offset, it is customary to look at the gear with the pinion at the right. For a below-center offset, the pinion has an LH spiral, and for above-center offset the pinion has an RH spiral.

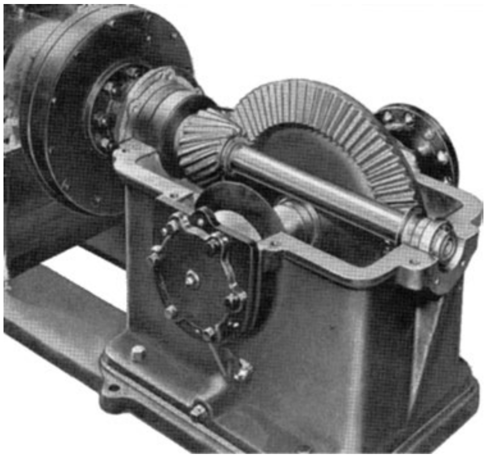


FIGURE 2.22 Hypoid gearset. Both pinion and gear are straddle mounted. (Courtesy of Gleason Works, Rochester, New York.)

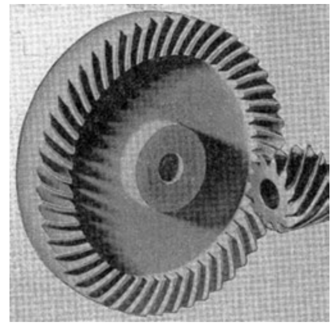


FIGURE 2.24 Helicon gearset. (Courtesy of Spiroid Division, Illinois Tool Works, Chicago, Illinois.)

proportional to the ratio of their pitch diameters. This makes it possible to make large pinions while minimizing the size of the driven gear. This one of the most attractive features of hypoid gearing. High reduction ratios with small offsets must usually utilize an overhung-mounted pinion. Frequently, however, the pinion must be straddle mounted (Figure 2.22) if suf cient offset exists. Aside from space considerations, the tradeoff usually centers on ef ciency, since ef ciency generally decreases with increasing offset.

In operation, hypoid gears are usually smoother and quieter than spiral bevel gears due to their inherently higher total contact ratio. However, as in all cases of nonintersecting gearsets, high sliding takes place across the face of the teeth. The ef ciency of hypoid gears is thus much less than that of a similar set of spiral bevel gears, typically 90% to 95% as compared with over 99% for many spiral bevel gears. Hypoid gears generally do, however, have greater tolerance to shock loading and can frequently be used at much higher single-stage ratios than spiral bevel gears.

2.1.4.6 Spiroid and Helicon Gearing

Like most other skew axis concepts, Spiroid and Helicon are primarily screw action gears while, by comparison, spur, helical, and bevel gears are primarily rolling action gears.

The gear member of either a Spiroid (Figure 2.23) or a Helicon (Figure 2.24) set resembles a high pitch–angle bevel gear, while the pinions are more akin to worms. Both forms have more teeth in contact than that of an equivalent-sized worm set. The primarily differences between Spiroid and Helicon gears are their minimum ratio capabilities (about 0:1 for Spiroid and 4:1 for Helicon [although lower ratios are practical if powdered metal fabrication is employed]) and the range of acceptable offsets. Spiroid pinions (Figure 2.23) are typically tapered by about 5° or 10° on a side, while Helicon pinions (Figure 2.24) are cylindrical. Pinions of both types, used in lower ratio designs (that is, less than 30:1), typically utilize multiple threads so that the number of teeth in the gear may be maintained at least at 30.

This type of gearing ts between bevel and worm gears (Figure 2.25) in terms of ratio, and generally provides performance that worm gearing does. They can, like worms, be designed to be antibackdriving, but again like worms, they cannot be designed to be entirely self-locking under all operating conditions, particularly where signi cant vibrations are present. They can also incorporate accurate control of backlash since the contact pattern is largely controlled by adjusting the pinion position, while the backlash is largely controlled by adjusting the position of the gear along its axis at assembly. These features make them a good choice for positioning drives such as radar or other antenna. These gear types are relatively insensitive to small position shifts; thus, in precision position-control applications, they are usually shimmed into intimate double- ank contact to eliminate backlash.

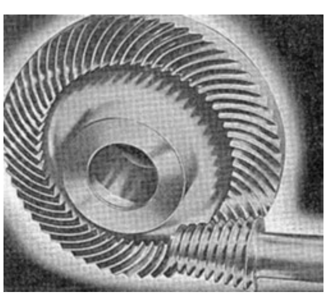


FIGURE 2.23 Spiroid gearset. (Courtesy of Spiroid Division, Illinois Tool Works, Chicago, Illinois.)

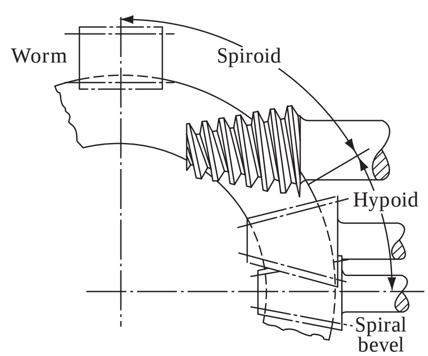


FIGURE 2.25 Schematic comparison of worm gears, Spiroid gears, Helicon gears, hypoid gears, and spiral bevel gears.

The contact conditions are line, again like worm and hypoid gears, but the contact line is essentially radial; thus, the ow of oil onto the contact zone is good, and the ef ciency is improved over worms—usually about as good as hypoid gearing. As with worms, the ef ciency of Spiroid or Helicon set varies with speed, but not to the large extent that occurs with worms.

One feature of Spiroid and Helicon gears that makes them both quiet and capable of great positional accuracy is the relatively high number of teeth in contact. The multiplicity of teeth in contact (for a typical design, perhaps 10% of the gear teeth are theoretically in contact) provides an error averaging function so that individual tooth errors are not signi cant in terms of gear position.

Although generally less ef cient than spur or helical gears, Spiroid or Helicon gears are usually superior to worm drives particularly at ratios less than 40:1, when compared on the basis of constant pinion size.

The offset required for a typical Spiroid gearset is about one-third the gear diameter, but small variations can also be accommodated. Similarly, the shaft angle is usually limited to 80° to 100°, with 90° being the standard in the vast majority of cases. In some special applications, shaft angle as low as 70° or as high as 120° are possible, but each case is a special design.

In general, the gears may be cut on standard hobbing machines. For a given con guration, a Spiroid gearset will have a greater load capacity than a Helicon set, but the Helicon position is somewhat easier to manufacture due to the cylindrical nature of its pinion as compared with the tapered shape of the Spiroid pinion.

2.1.4.7 Face Gears (Off-Center)

If the pinion of a face gearset is offset such that its axis and that of the gear do not intersect, it is termed *off center*. The discussion provided earlier for on-center face gears applies equally well here. Such gearsets are not ordinarily used to transmit signi cant amounts of power; rather, they are generally used in motion-transmission applications where uniformity of motion is not a critical factor.

2.1.5 Special Gear Types

Our discussion to this point would lead the reader to believe that all gears are in the form of surfaces of revolution (that is, cylinders and cones). Such is not the case. There are many applications in which gears are used to transmit motions that are intentionally nonuniform. There are few areas in which humans' mechanical ingenuity can be more effectively employed than in the design of special-motion gears. The variety is almost endless, limited only by application and imagination. In order to provide some insight into the subject, we discuss some representative cases.

2.1.5.1 Square or Rectangular Gears

Consider the case in which a constant input speed is to be converted into a varying output speed. Gears that are square or rectangular, depending on the actual output required, can be used to produce such motion (as shown in Figure 2.26). While the gears are in the position shown, the greatest radius of the driver mates with the smallest radius of the driven gear; the speed of the driven gear is then, therefore, in its maximum. As the gears revolve in the directions indicated by the arrows, the radius of the driver gradually decreases, and that of the driven gear gradually increases, until the points *b* and *b′* are in contact. The speed of the driven gear, therefore, gradually decreases during this eighth of a revolution. From the moment of contact between the points *b* and *b′*, the reverse action takes place, and the speed of the driven gear gradually increases until the points *c* and *c′* are in contact. Thus, during the entire revolution, the driven gear continues to alternate from a gradually decreasing to a gradually increasing speed, and vice versa.

For rectangular gears to properly work together, it is rst necessary that the pitch peripheries of the two gears be equal in length, and, second, that the sum of the radii of each pair of points (points that come into contact with each other) on the two pitch peripheries be equal to the distance between the centers of the gears.

2.1.5.2 Triangular Gears

Figure 2.27 represents a pair of triangular gears; the object of which is to obtain an alternating, varying speed from the uniformly rotating driver, as in rectangular gears. Triangular gears give fewer changers of speed per revolution than rectangular gears do. In Figure 2.27, the speed of the driven gear is at its minimum when the gears are in the position shown in the gure. Since, from the positions, the radius of the driver gradually increases, and that of the driven gear decreases,

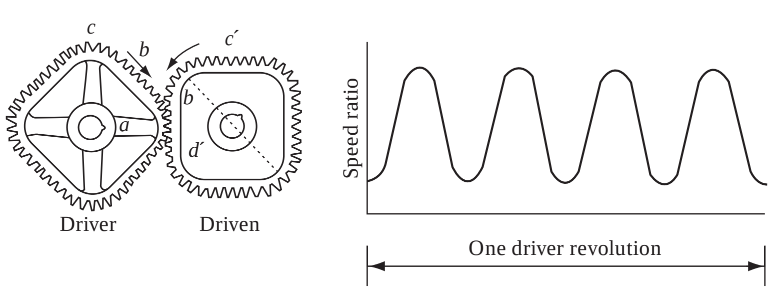


FIGURE 2.26 Square gear characteristics.

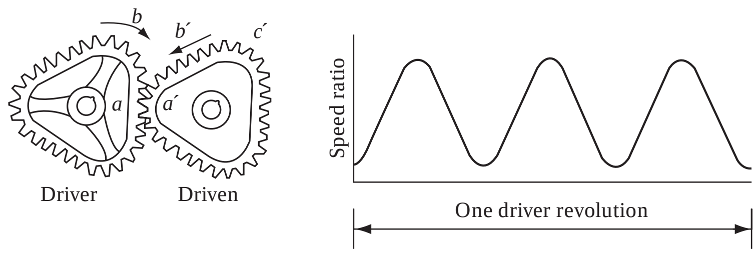


FIGURE 2.27 Triangular gear characteristics.

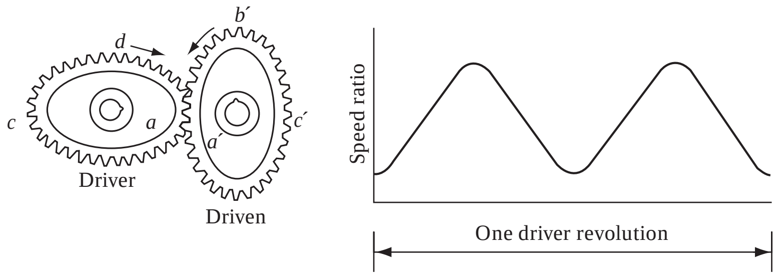


FIGURE 2.28 Elliptical gear characteristics.

as far as points *b* and *b*′, the speed of the driven gear will gradually increase until points *b* and *b*′ are in contact—or for one-sixth of an entire revolution. The reverse action will then take place until points *c* and *c*′ are in contact, and so on. Thus, while in rectangular gears, each gradually increasing or decreasing period takes place during one-eighth of revolution, in triangular gears, there are eight alternatively increasing and decreasing periods in one entire revolution of the driven gear, and in triangular gearing, there are but six.

2.1.5.3 Elliptical Gears

With elliptical gears (Figure 2.28), we have still another means of obtaining the same result, with the difference that in elliptical gears, each period of gradually increasing and decreasing speed takes place during one-fourth of a revolution. In other words, there are but four periods of increasing and decreasing speed during an entire revolution of the driver.

2.1.5.4 Scroll Gears

Figure 2.29 shows a pair of scroll gears. From the positions shown in the figure (in which the greatest radius of the driver meshes with the smallest radius of the driven gear), as the gears revolve in the directions indicated by the arrows, the radius of the driver gradually and uniformly decreases, while that of the driven gear gradually and uniformly increases. The speed of the driven gear is, therefore, at its maximum when the gears are in the positions shown, and gradually and uniformly decreases during the entire revolution. The moment before the positions shown in the figure are reached, the smallest radius of the driver gears with the greatest radius of the driven gear, the speed of the latter is at its minimum and suddenly (as the gears assume the positions in the figure) changes to its maximum.

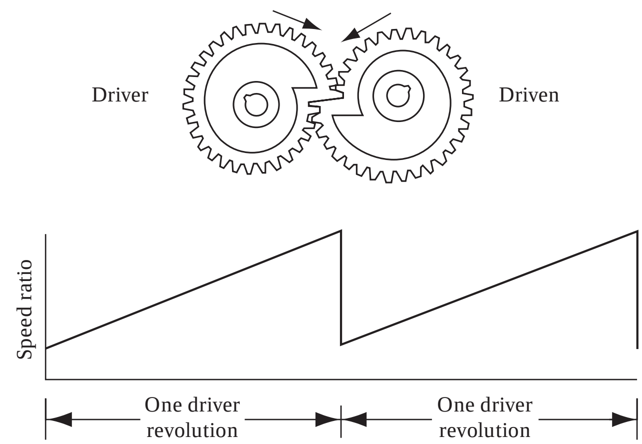


FIGURE 2.29 Scroll gear characteristics.

2.1.5.5 Multiple-Sector Gears

The mechanism represented in Figure 2.30 is a pair of multiple-sector gears, and the object is to obtain a series of discrete, different uniform speeds. In the figure, as long as the arcs *ab* and *a*′*b*′ are in mesh, the speed of the driven gear is the same. When the arcs *cd* and *c*′*d*′ come into mesh, the speed of the driven gear becomes slower, but remains the same throughout the meshing of these two arcs. Similarly, when the arcs *ef* and *e*′*f*′ come into mesh, the speed of the driven gear still becomes slower, but remains uniform during the meshing of these arcs. Thus, during each revolution, the driven gear has three periods of uniform speed, each differing from the others. For sector gears to properly work together, it is necessary that the arcs that mesh together be equal in length ($ab = a'b'$, $cd = c'd'$, etc.) and that the sum of the arc lengths on one gear be equal to the sum of the arc lengths on the other

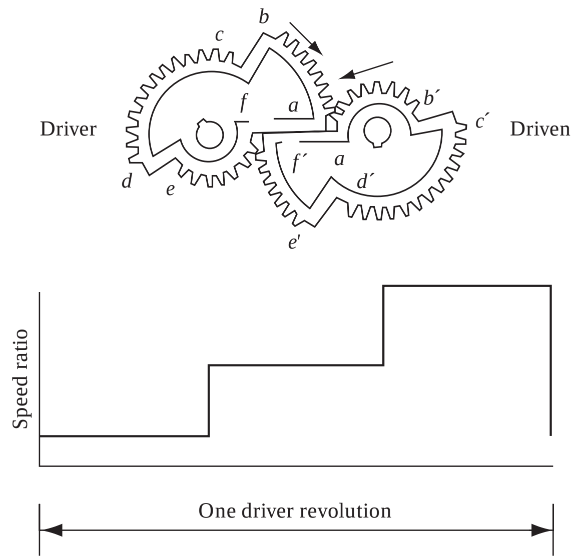


FIGURE 2.30 Multiple-sector gear characteristics.

($ab + cd + ef = a'b' + c'd' + e'f'$). Also, the sum of the radii of each two arcs together must be equal to the distance between the centers of the gears. Sector gears are somewhat dif cult to construct because considerable care must be taken to ensure that no two sectors of the driver gear mesh at the same time with the driven gear. To illustrate, suppose that the arcs *ab* and *a′b′* mesh together at the same time as the arcs *ef* and *e′f′* do; that is, that the last few teeth of *ab* mesh with the driven gear at the moment when the rst few teeth of *cf* do the same. The driven gear will then strive to drive the driven gear at its maximum and minimum speeds at the same time, an attempt that will obviously result in a fracture. In the gure, the arc *cf* ceases to mesh with the driven gear at the moment when the arc *ab* begins to gear. Thus, each arc of the driver must escape engagement just in time for its successor to begin engagement, and yet leave between these events no appreciable interval to disturb the uniformity of motion.

In actuality, a discontinuous condition will exist for all sector-type gears at the time when different sectors exchange control of the speed of the driven member. These seemingly instantaneous periods of acceleration and deceleration limit the application of such gears to very low speeds lest the acceleration becomes exceedingly large.

2.2 NOMENCLATURE OF GEARS

In this part of Chapter 2, the technical nomenclature of the various types of gears is given. Table 2.2 shows metric system symbols and English system symbols. The following tables (due to space limitations) show nomenclature tables in English symbols only.

2.2.1 Spur Gear Nomenclature and Basic Formulas

Figure 2.31 shows tooth elements that directly apply to spur gears. The same elements apply to other gear types.

TABLE 2.2
Gear Terms, Symbols, and Units Used in the Calculation of Gear Dimensional Data

	Metric		English	
Term	**Symbol**	**Units**	**Symbol**	**Units**
Number of teeth, pinion	z_1	–	N_P or n	–
Number of teeth, gear	z_2	–	N_G or N	–
Number of threads, worm	z_1	–	N_W	–
Number of crown teeth	z	–	N_c	–
Tooth ratio	u	–	m_G	–
Addendum, pinion	h_{a1}	mm	a_P	in.
Addendum, gear	h_{a2}	mm	a_G	in.
Addendum, chordal	h_a	mm	a_c	in.
Rise of arc	–	mm	–	in.
Dedendum	h_f	mm	b	in.
Working depth	h'	mm	h_k	in.
Whole depth	h	mm	h_t	in.
Clearance	c	mm	c	in.
Tooth thickness	s	mm	t	in.
Arc tooth thickness, pinion	s_1	mm	t_P	in.
Arc tooth thickness, gear	s_2	mm	t_G	in.
Tooth thickness chordal	$\bar{s}$	mm	t_c	in.
Backlash, transverse	j	mm	B	in.
Backlash, normal	j_n	mm	B_n	in.
Pitch diameter, pinion	d_{p1}	mm	d	in.
Pitch diameter, gear	d_{p2}	mm	d	in.
Pitch diameter, cutter	d_{p0}	mm	d_c	in.
Base diameter, pinion	d_{b1}	mm	d_b	in.
Base diameter, gear	d_{b2}	mm	D_b	in.
Outside diameter, pinion	d_{a1}	mm	d_o	in.
Outside diameter, gear	d_{a2}	mm	D_o	in.
Inside diameter, face gear	d_{i2}	mm	D_i	in.
Root diameter, pinion or worm	d_{f1}	mm	d_R	in.
Root diameter, gear	d_{f2}	mm	D_R	in.
Form diameter	d'_f	mm	d_f	in.
Limit diameter	d_l	mm	d_l	in.
Excess involute allowance	Δd_l	mm	Δd_l	in.
Ratio of diameters	ε	–	m	–
Center distance	a	mm	C	in.
Face width	b	mm	F	in.
Net face width	b'	mm	F_e	in.
Module, transverse	m or m_t	mm	–	–
Module, normal	m_n	mm	–	–
Diametral pitch, transverse	–	–	P_d or P_t	in.$^{-1}$
Diametral pitch, normal	–	–	P_n	in.$^{-1}$
Circular pitch	p	mm	p	in.
Circular pitch, transverse	p_t	mm	p_t	in.
Circular pitch, normal	p_n	mm	p_n	in.
Base pitch	p_b	mm	p_b	in.
Axial pitch	p_x	mm	p_x	in.
Lead (length)	p_z	mm	L	in.
Pressure angle	α or α_t	deg	ϕ or ϕ_t	deg
Pressure angle, normal	α_n	deg	ϕ_n	deg
Pressure angle, axial	α_x	deg	ϕ_x	deg
Pressure angle of cutter	α_0	deg	ϕ_c	deg
Helix angle	β	deg	ψ	deg

(*Continued*)

TABLE 2.2 (CONTINUED)
Gear Terms, Symbols, and Units Used in the Calculation of Gear Dimensional Data

	Metric		English	
Term	**Symbol**	**Units**	**Symbol**	**Units**
Lead angle	γ	deg	λ	deg
Shaft angle	Σ	deg	Σ	deg
Roll angle	θ_r	deg	ε_r	deg
Pitch angle, pinion	δ'_1	deg	γ	deg
Pitch angle, gear	δ'_2	deg	Γ	deg
Pi	π	–	π	–
Contact ratio	ε_a	–	m_p	–
Zone of action	g_a	mm	Z	in.
Edge radius, tool	r_{ao}	mm	r_T	in.
Radius of curvature, root llet	ρ_f	mm	ρ_f	in.
Circular thickness factor	k	–	k	–
Cone distance	R	mm	A	in.
Outer cone distance	R_a	mm	A_o	in.
Mean cone distance	R_m	mm	A_m	in.
Inner cone distance	R_f	mm	A_i	in.

Note: deg: degrees; in.: inches; mm: millimeters.

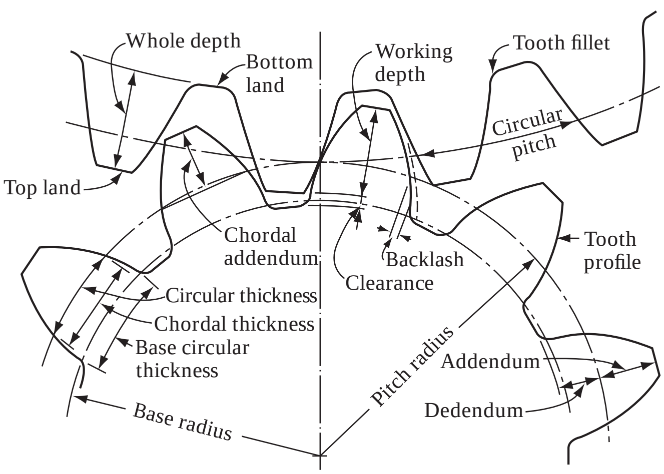


FIGURE 2.31 Spur gear nomenclature.

Table 2.3 shows spur gear formulas. Note that the value of π used may need to be 3.14159265 when very precise calculations are needed. Also, the determination of tooth thickness has complications of what backlash is needed and effects of long or short addendum. (These matters are covered later in this book.)

2.2.2 Helical Gear Nomenclature and Basic Formulas

Helical gear teeth spiral a base cylinder. The angle of this spiral with respect to the axis is the helix angle. The advance of the spiral in going 360° around the base cylinder is a xed quantity called the *lead*.

The spur gear tooth is parallel to the axis. This means that a spur gear can be thought of as a special case of a helical gear in which the numerical value of the lead is in nity!

Figure 2.32 shows the nomenclature of helical gear teeth. Figure 2.32a shows a section through the teeth that is perpendicular to the gear axis. This is known as the *transverse section*. Figure 2.32b shows how a normal section data can be taken through a tooth perpendicular to a point on the pitch helix.

Both the transverse section and the normal section data must be used in helical gear calculations. Table 2.4 shows the formulas needed to the calculate data for both of these sections.

A rack is a gear part in which the number of teeth in a 360° circle is in nite. Circular gears, of course, become straight when the radius of curvature is in nite.

Involute teeth become straight sided for the rack tooth form. Figure 2.33 shows rack tooth parts for spur, helical, and straight bevel teeth. The helical rack very clearly shows the difference between the normal plane and the front plane (which corresponds to the transverse plane of circular gears).

2.2.3 Internal Gear Nomenclature and Formulas

The internal gear has teeth on the inside of a ring rather than on the outside. An internal gear must mesh with a pinion having external teeth. (Two internal gears cannot mesh with each other.)

The nomenclature for the transverse section of an internal gear drive is shown in Figure 2.34. Note that the internal tooth tends to have concave curves rather than convex curves. Also note that the pips of the teeth almost tend to hit each other as they enter and leave the meshing zone.

The formulas for internal gear dimensions are given in Table 2.5. Note that some of these formulas are similar to the formulas in Table 2.3 except for minus signs.

Internal gears may be either spur or helical. If the teeth are helical, data for the teeth in the normal section may be determined by the formulas in Table 2.4.

2.2.4 Crossed-Helical Gear Nomenclature and Formulas

Since the axes of crossed-helical gears are not parallel to each other and do not intersect, the two meshing gears tend to have different helix angles. (If the numerical amount of each helix angle is the same, then the two gears cannot have opposite helix angles—because this would make the axes parallel instead of crossed.) Figure 2.35 shows standard nomenclature for the crossed axis situation with helical gears. The basic formulas are given in Table 2.6.

2.2.5 Bevel Gear Nomenclature and Formulas

Section 2.1.3 gives general information on gear types in the bevel gear family of gears. Nomenclature and formulas will now be given for three of the types that are rather common in usage.

TABLE 2.3
Spur Gear Formulas

To Find	Having	Formula
Diametral pitch	Number of teeth and pitch diameter	$P = \frac{N}{D}$
Diametral pitch	Circular pitch	$P = \frac{3.1416}{p}$*
Pitch diameter	Number of teeth and diametral pitch	$D = \frac{N}{P}$
Outside diameter	Pitch diameter and addendum	$D_o = D + 2a$
Root diameter	Outside diameter and whole depth	$D_R = D_o - 2h_t$
Root diameter	Pitch diameter and dedendum	$D_R = D - 2b$
Number of teeth	Pitch diameter and diametral pitch	$N = D \times P$
Base-circle diameter	Pitch diameter and pressure angle	$D_b = D \times \cos \phi$
Circular pitch	Pitch diameter and number of teeth	$p = \frac{3.1416D}{N}$
Circular pitch	Diametral pitch	$p = \frac{3.1416}{P}$
Center distance	Number of gear teeth and number of pinion teeth and diametral pitch	$C = \frac{N_G + N_P}{2P} = \frac{D_G + D_P}{2}$
Approximate thickness of tooth	Diametral pitch	$t_t \approx \frac{1.5708}{P}$

* Exact value for π is 3.14159265….

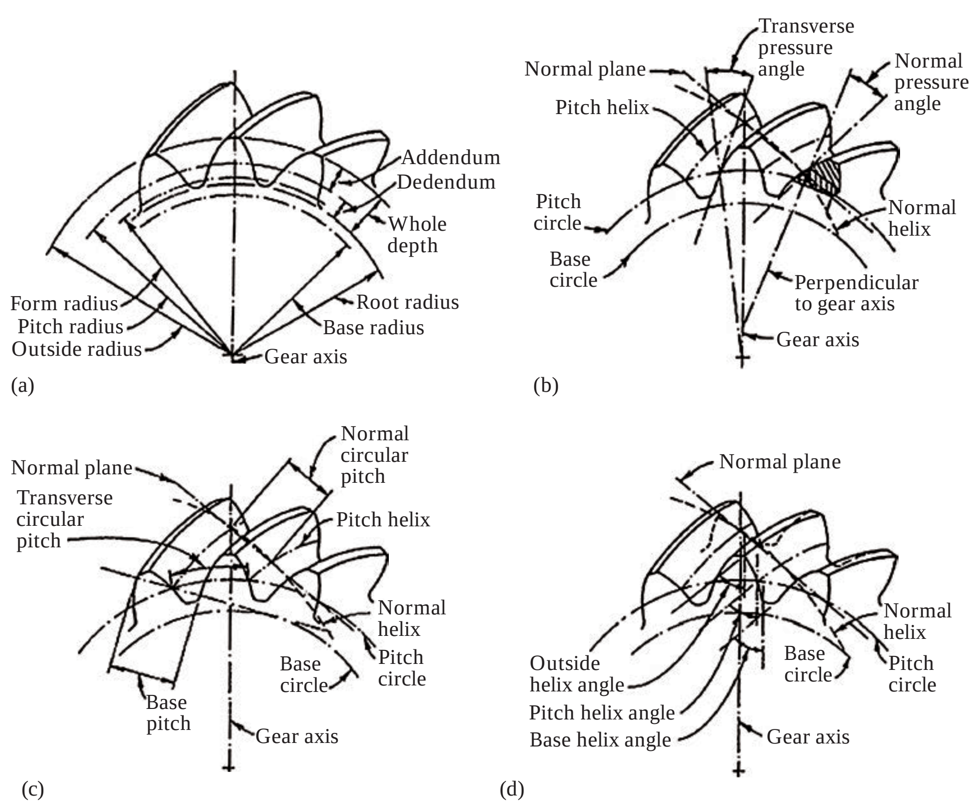


FIGURE 2.32 Helical gear nomenclature. (a) Radial dimensions of a gear, (b) pressure angles of a gear, (c) gear pitches, and (d) helix angles of a gear.

TABLE 2.4
Helical Gear Formulas

To Find	Having	Formula
Normal diametral pitch	Number of teeth, pitch diameter, and helix angle	$P_n = \dfrac{N}{D\cos\psi}$
Normal diametral pitch	Normal circular pitch	$P_n = \dfrac{3.1416}{p_n}$
Normal diametral pitch	Transverse diametral pitch and helix angle	$P_n = \dfrac{P}{\cos\psi}$
Normal circular pitch	Normal diametral pitch	$p_n = \dfrac{3.1416}{P_n}$
Normal circular pitch	Transverse circular pitch	$p_n = p\cos\psi$
Pitch diameter	Number of teeth, normal diametral pitch, and helix angle	$D = \dfrac{N}{P_n\cos\psi}$
Center distance	Pinion and gear pitch diameter	$C = \dfrac{D_P + D_G}{2}$
Outside diameter	Pitch diameter and addendum	$D_o = D + 2a$
Approximate normal tooth thickness	Normal diametral pitch	$t_n \approx \dfrac{1.571}{P_n}$
Transverse tooth thickness	Normal tooth thickness and helix angle	$t_t = \dfrac{t_n}{\cos\psi}$
Normal pressure angle	Transverse pressure angle and helix angle	$\tan\phi_n = \tan\phi\cos\psi$

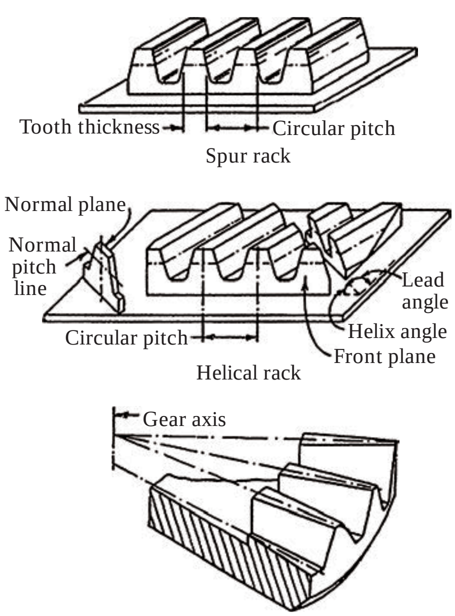


FIGURE 2.33 Spur, helical, and bevel gear racks.

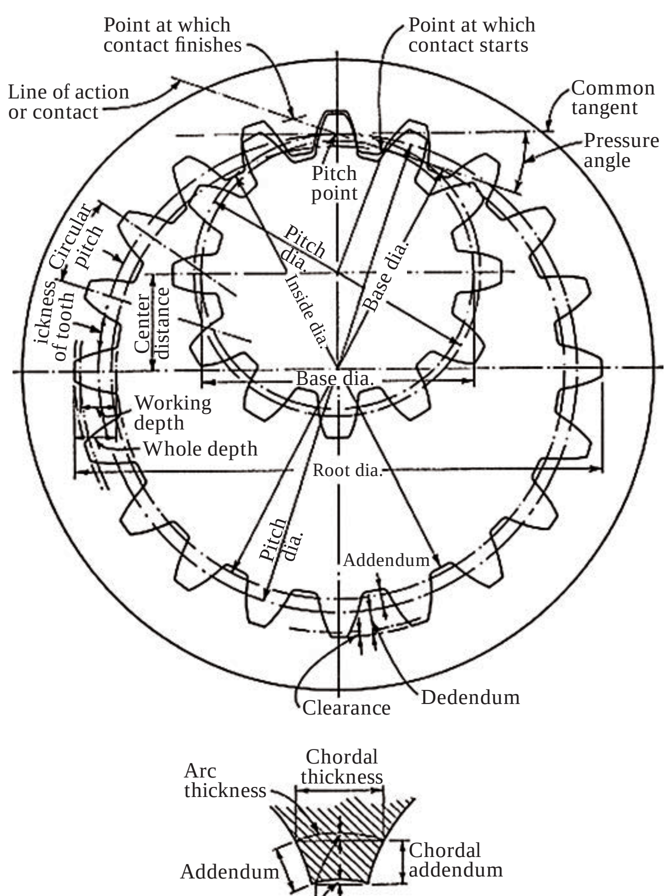


FIGURE 2.34 Internal gear and pinion nomenclature.

TABLE 2.5
Internal Spur Gear Formulas

To Find	Having	Formula
Pitch diameter of gear	Number of teeth in gear and diametral pitch	$D = \frac{N_G}{P}$
Pitch diameter of pinion	Number of teeth in pinion and diametral pitch	$d = \frac{N_P}{P}$
Internal diameter (gear)	Pitch diameter and addendum	$D_i = D - 2a$
Center distance	Number of teeth in gear and pinion; diametral pitch	$C = \frac{N_G - N_P}{2P}$
Center distance	Pitch diameter of gear and pinion	$C = \frac{D - d}{2}$

TABLE 2.6
Crossed-Helical Gear (Nonparallel Shaft, Helicals)

To Find	Having	Formula
Shaft angle	Helix angle of pinion and helix angle of gear	$\Sigma = \psi_P + \psi_G$
Pitch diameter of pinion	Number of teeth in pinion, normal diametral pitch, and helix angle of pinion	$D_P = \frac{N_P}{P_n \times \cos\psi_P}$
Pitch diameter of gear	Number of teeth in gear, normal diametral pitch, and helix angle of gear	$D_G = \frac{N_G}{P_n \times \cos\psi_G}$
Center distance	Pitch diameter of pinion, and pitch diameter of gear	$C = \frac{D_P + D_g}{2}$
Speed ratio	Number of teeth in gear, and number of teeth in pinion	$m_G = \frac{N_G}{N_P}$

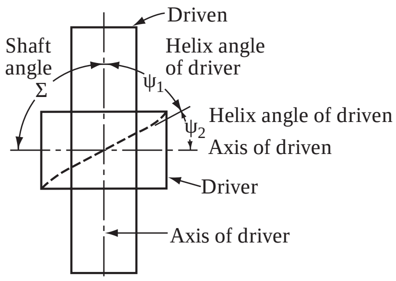


FIGURE 2.35 Crossed-helical shaft angle and helix.

2.2.5.1 Straight

Figure 2.36 shows the section of a pair of straight bevel gears in mesh. Note how the teeth are on the outside of cones. The dimensions are measured to a crown point that exists in space but may not exist in metal! (Usually the sharp corner is rounded in the process of actual manufacture.)

The formulas for the straight bevel are given in Table 2.7. By tradition, the pitch diameters are taken at the large end of the tooth. This end also goes by names such as the *back face*

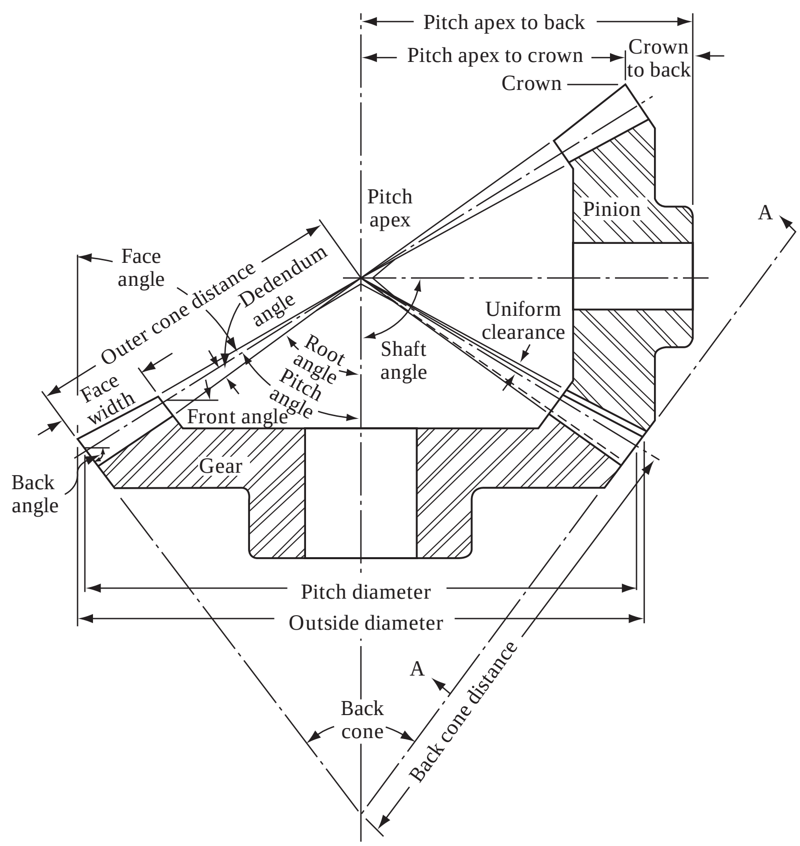


FIGURE 2.36 Bevel gear nomenclature.

TABLE 2.7
Straight Bevel Gear Formulas (20° Pressure Angle, 90° Shaft Angle)

To Find	Having	Formula
Pitch diameter of pinion	Number of pinion teeth and diametral pitch	$d = \frac{N_P}{P_d}$
Pitch diameter of gear	Number of gear teeth and diametral pitch	$D = \frac{N_G}{P_d}$
Pitch angle of pinion	Number of pinion teeth and number of gear teeth	$\gamma = \tan^{-1}\left(\frac{N_P}{N_G}\right)$
Pitch angle of gear	Pitch angle of pinion	$\Gamma = 90° - \gamma$
Outer cone distance of pinion and gear	Gear pitch diameter and pitch angle of gear	$A_o = \frac{D}{2\sin\Gamma}$
Circular pitch of pinion and gear	Diametral pitch	$p = \frac{3.1416}{P_d}$
Dedendum angle of pinion	Dedendum of pinion and outer cone distance	$\delta_P = \tan^{-1}\left(\frac{b_{oP}}{A_o}\right)$
Dedendum angle of gear	Dedendum of gear and outer cone distance	$\delta_G = \tan^{-1}\left(\frac{b_{oG}}{A_o}\right)$
Face angle of pinion blank	Pinion pitch angle and dedendum angle of gear	$\gamma_o = \gamma + \delta_G$
Face angle of gear blank	Gear pitch angle and dedendum angle of pinion	$\Gamma_o = \Gamma + \delta_P$
Root angle of pinion	Pitch angle of pinion and dedendum angle of pinion	$\gamma_R = \gamma - \delta_P$
Root angle of gear	Pitch angle of gear and dedendum angle of gear	$\Gamma_R = \Gamma - \delta_G$
Outside diameter of pinion	Pinion pitch diameter, pinion addendum, and pitch angle of pinion	$d_o = d + 2\,a_{oP}\cos\gamma$
Outside diameter of gear	Pitch diameter of gear, gear addendum, and pitch angle of gear	$D_o = D + 2\,a_{oG}\cos\Gamma$
Pitch apex to crown of pinion	Pitch diameter of gear, addendum, and pitch angle of pinion	$x_o = \frac{D}{2} - a_{oP}\sin\gamma$
Pitch apex to crown of gear	Pitch diameter of pinion, addendum, and pitch angle of gear	$X_o = \frac{d}{2} - a_{oG}\sin\Gamma$
Circular tooth thickness of pinion	Circular pitch and gear circular tooth thickness	$t = p - T$
Chordal thickness of pinion	Circular tooth thickness, pitch diameter of pinion, and backlash	$t_c = t - \frac{t^3}{6d^2} - \frac{B}{2}$
Chordal thickness of gear	Circular tooth thickness, pitch diameter of gear, and backlash	$T_c = T - \frac{T^3}{6D^2} - \frac{B}{2}$
Chordal addendum of pinion	Addendum angle, circular tooth thickness, pitch diameter, and pitch angle of pinion	$a_{cP} = a_{oP} + \frac{t^2\cos\gamma}{4d}$
Chordal addendum of gear	Addendum angle, circular tooth thickness, pitch diameter, and pitch angle of gear	$a_{cG} = a_{oG} + \frac{T^2\cos\Gamma}{4D}$
Tooth angle of pinion	Outer cone distance, tooth thickness, dedendum of pinion, and pressure angle	$\frac{3.438}{A_o}\left(\frac{t}{2} + b_{oP}\tan\phi\right)$, minutes
Tooth angle of gear	Outer cone distance, tooth thickness, dedendum of gear, and pressure angle	$\frac{3.438}{A_o}\left(\frac{T}{2} + b_{oG}\tan\phi\right)$, minutes

Note: $\tan^{-1}$ means "the angle whose tangent is."

or the *heel* end. (The other end of the tooth is termed the *front face* or *toe* end.)

2.2.5.2 Spiral

The spiral bevel gear has a curved shape (lengthwise). This curved shape is positioned at an angle to a pitch cone element, the angle at the center of the face width (not the large end of the tooth where the pitch diameter is speci ed). Figure 2.37 shows the spiral angle.

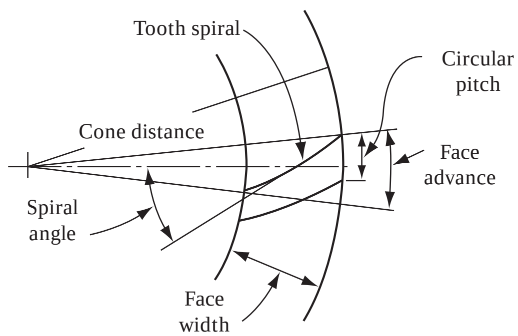


FIGURE 2.37 Spiral bevel gear nomenclature.

The formulas for conventional spiral bevel gears are given in Table 2.8. Spiral bevel gears are often made to pressure angles other than 20°. They can also be made for shaft angles other than 90°. Beyond this, there is more than one system for the design and the manufacture of bevel gears. Considerably more bevel gear data can be found out in later chapters of the book.

2.2.5.3 Zerol

The Zerol bevel gear tooth has lengthwise curvature like the spiral bevel gear tooth at the center of the face width, although the spiral angle is 0°.

Figure 2.38 shows how the hand of Zerol bevel gear is designated. Note that an RH spiral bevel pinion has to have an LH spiral bevel gear to mesh with it, and vice versa. This same rule applies to spiral bevel gears.

Table 2.9 shows formulas for Zerol bevel gears. Both Zerol and spiral bevel gears have the pitch diameters speci ed at the large end of the tooth.

TABLE 2.8
Spiral Bevel Gear Formulas (20° Pressure Angle, 90° Shaft Angle)

To Find	Having	Formula
Pitch diameter of pinion	Number of pinion teeth and diametral pitch	$d = \dfrac{N_P}{P_d}$
Pitch diameter of gear	Number of gear teeth and diametral pitch	$D = \dfrac{N_G}{P_d}$
Pitch angle of pinion	Number of pinion teeth and number of gear teeth	$\gamma = \tan^{-1}\left(\dfrac{N_P}{N_G}\right)$
Pitch angle of gear	Pitch angle of pinion	$\Gamma = 90° - \gamma$
Outer cone distance of pinion and gear	Pitch diameter of gear and pitch angle of gear	$A_o = \dfrac{D}{2\sin\Gamma}$
Circular pitch of pinion and gear	Diametral pitch	$p = \dfrac{3.1416}{P_d}$
Dedendum angle of pinion	Dedendum of pinion and outer cone distance	$\delta_P = \tan^{-1}\left(\dfrac{b_{oP}}{A_o}\right)$
Dedendum angle of gear	Dedendum of gear and outer cone distance	$\delta_G = \tan^{-1}\left(\dfrac{b_{oG}}{A_o}\right)$
Face angle of pinion blank	Pitch angle of pinion and dedendum angle of gear	$\gamma_o = \gamma + \delta_G$
Face angle of gear blank	Pitch angle of gear and dedendum angle of pinion	$\Gamma_o = \Gamma + \delta_P$
Root angle of pinion	Pitch angle of pinion and dedendum angle of pinion	$\gamma_R = \gamma - \delta_P$
Root angle of gear	Pitch angle of gear and dedendum angle of gear	$\Gamma_R = \Gamma - \delta_G$
Outside diameter of pinion	Pitch diameter, addendum, and pitch angle of pinion	$d_o = d + 2\, a_{oP} \cos\gamma$
Outside diameter of gear	Pitch diameter, addendum, and pitch angle of gear	$D_o = D + 2\, a_{oG} \cos\Gamma$
Pitch apex to crown of pinion	Pitch diameter of gear, pitch angle, and addendum of pinion	$x_o = \dfrac{D}{2} - a_{oP}\sin\gamma$
Pitch apex to crown of gear	Pitch diameter of gear, pitch angle, and addendum of gear	$X_o = \dfrac{d}{2} - a_{oG}\sin\Gamma$
Circular tooth thickness of pinion	Circular pitch of pinion and circular pitch of gear	$t = p - T$

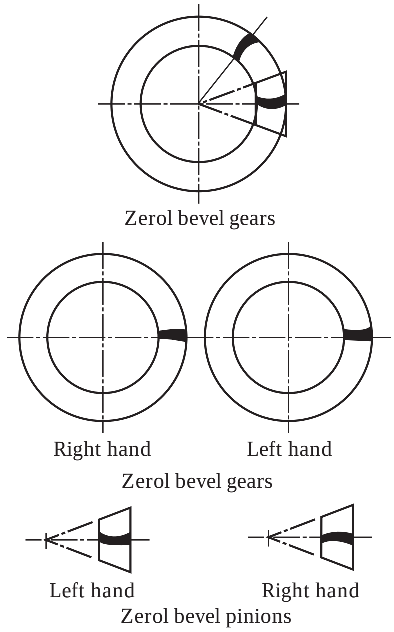


FIGURE 2.38 Zerol bevel gear nomenclature.

2.2.6 Worm Gear Nomenclature and Formulas

When the word *worm* is used in connection with gear types, the implication is that there is some enveloping. A single-enveloping set of worm gears has a cylindrical worm in mesh with a gear that is throated to tend to wrap around the worm. Figure 2.39 shows a typical arrangement.

When both the worm and the gear wrap around each other, the combination is designated as double enveloping. Figure 2.40 shows a typical arrangement of this kind.

2.2.6.1 Cylindrical Worm Gears

The worm thread (of tooth) may be dimensioned in both a normal section and an axial section. The traverse section is normally not used. Figure 2.39 shows the nomenclature that is used.

The special formulas for worm gearing are given in Table 2.10. The pitch diameter of the throated gear changes as you go across the face width. The standard practice is to take the pitch diameter at the center of the throat. Note the details of how this is done in Figure 2.39.

TABLE 2.9
Zero Bevel Gear Formulas (20° Pressure Angle, 90° Shaft Angle)

To Find	Having	Formula
Pitch diameter of pinion	Number of pinion teeth and diametral pitch	$d = \frac{N_P}{P_d}$
Pitch diameter of gear	Number of gear teeth and diametral pitch	$D = \frac{N_G}{P_d}$
Pitch angle of pinion	Number of pinion teeth and number of gear teeth	$\gamma = \tan^{-1}\left(\frac{N_P}{N_G}\right)$
Pitch angle of gear	Pitch angle of pinion	$\Gamma = 90° - \gamma$
Outer cone distance of pinion and gear	Pitch diameter of gear and pitch angle of gear	$A_o = \frac{D}{2\sin\Gamma}$
Circular pitch of pinion and gear	Diametral pitch	$p = \frac{3.1416}{P_d}$
Face angle of pinion blank	Pitch angle of pinion and dedendum angle of gear	$\gamma_o = \gamma + \delta_G$
Face angle of gear blank	Pitch angle of gear and dedendum angle of pinion	$\Gamma_o = \Gamma + \delta_P$
Root angle of pinion	Pitch angle of pinion and dedendum angle of pinion	$\gamma_R = \gamma - \delta_P$
Root angle of gear	Pitch angle of gear and dedendum angle of gear	$\Gamma_R = \Gamma - \delta_G$
Outside diameter of pinion	Pitch diameter, pitch angle, and dedendum of pinion	$d_o = d + 2a_P \cos\gamma$
Outside diameter of gear	Pitch diameter, pitch angle, and dedendum of gear	$D_o = D + 2a_G \cos\Gamma$
Pitch apex to crown of pinion	Pitch diameter of gear, pitch angle, and addendum of pinion	$x_o = \frac{D}{2} - a_P \sin\gamma$
Pitch apex to crown of gear	Pitch diameter of pinion, pitch angle, and addendum of gear	$X_o = \frac{d}{2} - a_G \sin\Gamma$
Circular tooth thickness of pinion	Circular pitch of pinion and circular thickness of gear	$t = p - T$
Circular tooth thickness of gear	Circular pitch, pressure angle, and addendum of pinion and gear	$T = \frac{p}{2} - (a_P - a_G)\tan\phi$

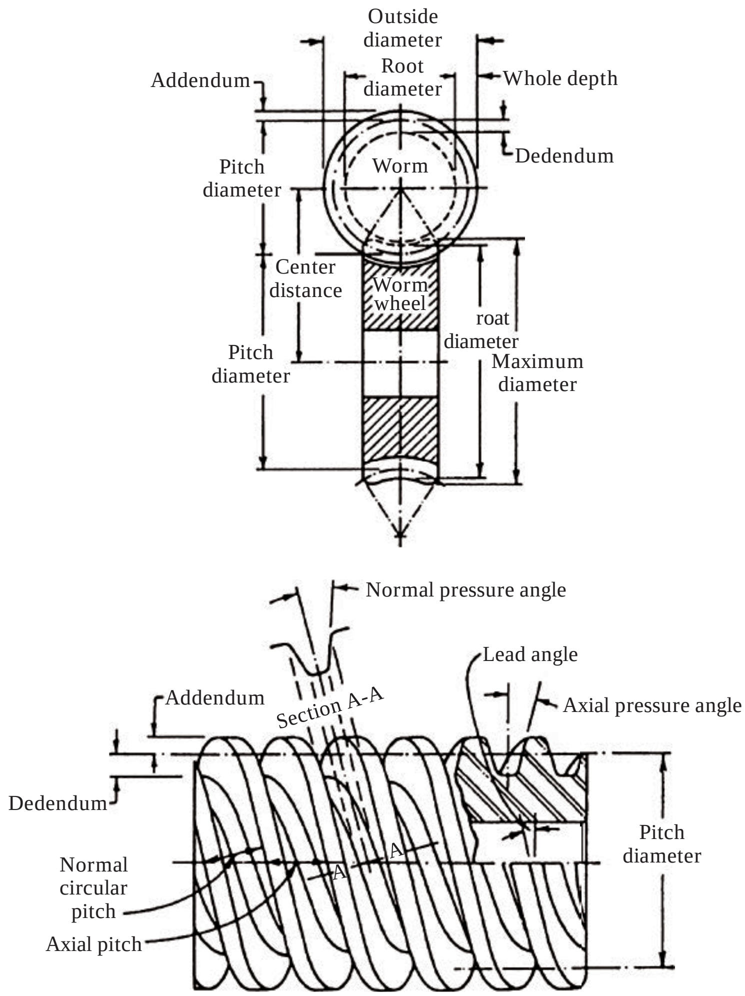


FIGURE 2.39 Cylindrical worm gear nomenclature. OD: outside diameter.

2.2.6.2 Double-Enveloping Worm Gears

The typical style of design and the normal nomenclature for this kind of gearing is shown in Figure 2.40. The pitch diameters are speci ed at the center of the throat on the worm and the center of the throat of the gear.

The formulas for double-enveloping worm gears are given in Table 2.11.

2.2.7 Face Gears

The gear member of a face gearset has teeth cut on the end of a cylindrical-shaped blank. Figure 2.41 shows a typical face gear arrangement and the nomenclature used.

The formulas for an on-center 90° shaft-angle set of face gears are given in Table 2.12. Additional design data (for a standard design) are given in Tables 2.13 and 2.14.

There are other possible face gear designs with shaft angles that are not 90° or conditions of not being on center. These can be thought of as special designs beyond the scope of this book.

2.2.8 Spiroid Gear Nomenclature and Formulas

There is a family of gears that is generally described by the name Spiroid and Helicon. The gear member of each of these kinds of gearsets has teeth cut on the end of a blank. The pinion member does not mesh on center but instead meshes with a considerable offset from the central position. Figure 2.42 shows a standard Spiroid arrangement. Column A in Table 2.15 is the offset.

The Spiroid pinion is tapered. The teeth spiral around the pinion somewhat like threads on a worm, but they are on the surface of a cone rather than on that of a cylinder. The Helicon pinion differs from the Spiroid pinion by being cylindrical. This makes it quite comparable to a cylindrical worm.

Table 2.16 gives some formulas for Spiroid gearsets. Other dimensions needed to make a standard design are given as part of Figure 2.42. Speci c data for Helicon gearsets are not given. The Spiroid Division of the Illinois Tool Works in Chicago, Illinois should be consulted for additional data.

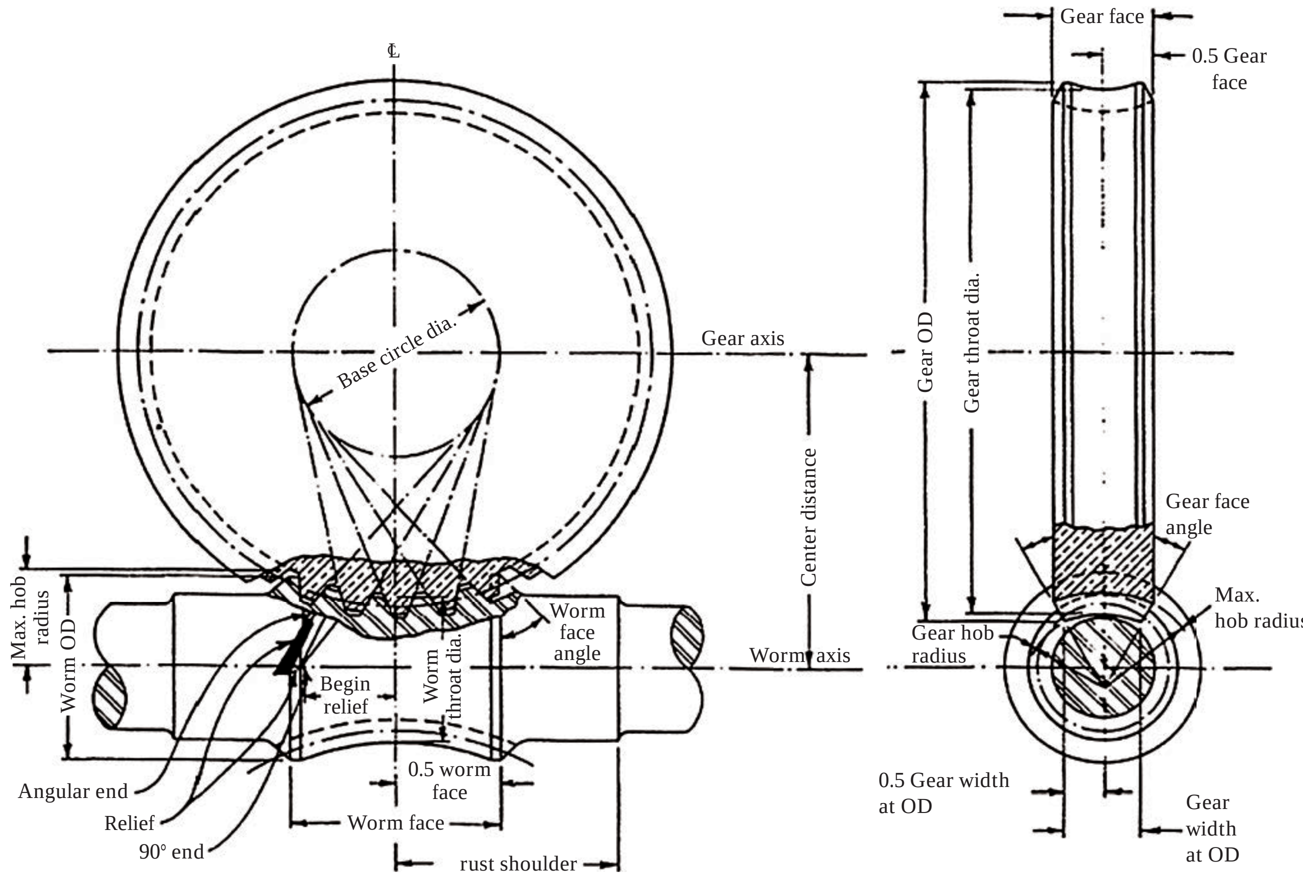


FIGURE 2.40 Double-enveloping worm gear nomenclature.

TABLE 2.10
Worm Gear Formulas

To Find	Having	Formula
	Worm	
Lead	Number of threads in worm and axial pitch	$L = N_W \times p_x$
Pitch diameter	Center distance	$d = \dfrac{C^{0.875}}{1.7}$ to $\dfrac{C^{0.875}}{3}$
Rood diameter	Outside diameter of pinion and whole depth of tooth	$d_R = d_o - 2h_t$
Outside diameter	Pitch diameter and addendum	$d_o = d + 2a$
Minimum face	Throat diameter, pitch diameter, and addendum	$f = 2\sqrt{\left(\dfrac{D_t}{2}\right)^2 - \left(\dfrac{D}{2} - a\right)^2}$
Lead angle	Lead and pitch diameter	$\tan\lambda = \dfrac{L}{3.1416 \times d}$
Normal pitch	Axial pitch and lead angle	$p_n = p \times \cos\lambda$
	Gear	
Nominal pitch diameter	Number of teeth in gear and axial pitch	$D = \dfrac{N_G \times p_x}{3.1416}$
Throat diameter	Nominal pitch diameter and addendum	$D_t = D + 2a$
Effective face	Pitch diameter and working depth	$F_e = \sqrt{(d + h_k)^2 - d^2}$
Center distance	Pitch diameter of gear and pitch diameter of worm	$C = \dfrac{D + d}{2}$

TABLE 2.11
Double-Enveloping Worm Gear Formulas

To Find	Having	Formula
Worm root diameter[a]	Pitch diameter of pinion and dedendum of gear	$d_R = d - 2b_G$
Worm root diameter[a]	Center distance	$d_R = \frac{C^{0.875}}{3}$ (approximate)
Worm pitch diameter[a]	Center distance	$d = \frac{C^{0.875}}{2.2}$ (approximate)
Gear pitch diameter	Center distance and worm pitch diameter	$D = C - 2d$
Axial circle pitch	Gear pitch diameter and number of teeth in gear	$p_x = \frac{\pi D}{N_G}$
Normal circular pitch	Axial circular pitch and pitch cone angle of pinion	$p_n = p_x \cos \lambda$
Whole depth of tooth[a]	Normal circular pitch	$h_t = \frac{p_n}{2}$
Working depth of tooth[a]	Whole depth of tooth	$h_k = 0.9h_t$
Dedendum[a]	Working depth of tooth	$b_G = 0.611h_k$
Normal pressure angle[a]	–	$\phi_n = 20°$
Axial pressure angle	Normal pressure angle and lead angle at center of worm	$\phi_x = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\lambda_c}\right)$
Lead angle at center of worm	Pitch diameter, gear ratio, and worm pitch diameter	$\lambda_c = \tan^{-1}\left(\frac{D}{m_g d}\right)$
Lead angle average[a]	Pitch diameter, gear ratio, and worm pitch diameter	$\lambda = \tan^{-1}\left(\frac{0.87D}{m_g d}\right)$

[a] Proportions given in these formulas represent recommendations of Cone Drive gears. Cone Drive is a registered trademark of Cone Drive Textron, Traverse City, Michigan.

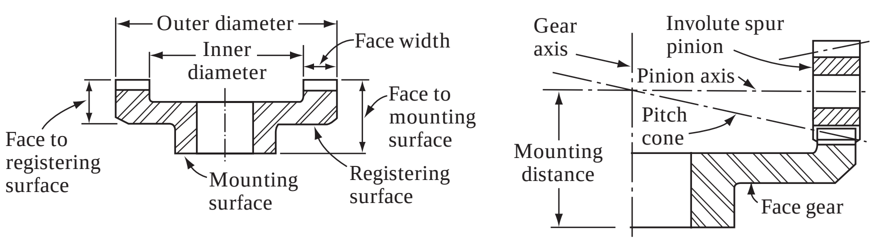


FIGURE 2.41 On-center face gear nomenclature.

TABLE 2.12
Face Gears—On-Center 90° Shaft Angle

To Find	Having	Formula
Gear ratio	Number of gear and pinion teeth	$m_G = \frac{N_G}{N_P}$ (from 1.5 to 12.5)
Inside diameter	Diametral pitch, number of gear and pinion teeth, pressure angle	$D_i = \frac{1}{P}\sqrt{8N_P + N_G \cos^2 \phi}$
Outside diameter (pointed teeth)	Diametral pitch, number of gear teeth, pressure angle, and tangency pressure angle	$D_{oG} = \frac{N_G \cos\phi \sec\phi_o}{P}$ (see Table 2.13)
Maximal face width	Face gear inside and outside diameters	$F_g = \frac{D_o - D_i}{2}$
Face to pinion axis	Pinion outside diameter and working depth of tooth	Face to pinion axis $= \frac{d_o}{2} - h_k$
Face to mounting surface	Mounting distance and face to pinion axis	Face to mounting surface = mounting distance − face to opinion axis
Minimal number of teeth	–	(See Table 2.14)

TABLE 2.13
Tangency Pressure Angle at Face Gear Outside Diameter

N_P	ϕ_o	$\sec\phi_o$
12	40.765	1.32–1.31
13	39.669	1.29913
14	38.692	1.28120
15	37.815	1.26583
16	37.022	1.25249
17	36.300	1.24081
18	36.012	1.23626
19	35.778	1.23261
20	35.557	1.22919
21	35.346	1.22598
22	35.145	1.22295
23	34.954	1.22009
24	34.771	1.21738
25	34.596	1.21481
26	34.429	1.21237
27	34.268	1.21005
28	34.113	1.20783
29	33.965	1.20572
30	33.821	1.20369
31	33.683	1.20176
32	33.550	1.19990
33	33.421	1.19812
34	33.297	1.19640
35	33.176	1.19475
36	33.059	1.19317
37	32.946	1.19164
38	32.837	1.19016
39	32.730	1.18874
40	32.627	1.18736
41	32.525	1.18602
42	32.427	1.18473
43	32.332	1.18352
44	32.240	1.18228
45	32.150	1.18111
46	32.061	1.17996
47	31.976	1.17887
48	31.893	1.17781
49	31.810	1.17675
50	31.734	1.17578

2.2.9 Beveloid Gears

This special type of gear is patented by Vinco Corporation. Vinco Corporation went out of business about 50 years ago, but this type of gear is still in use. The Invincible Gear Company in Livonia, Michigan, now holds the patent rights for this gear type and still manufactures beveloid gears.

Figure 2.24 shows a beveloid gear and its features. Beveloid gears can be engaged with spur gears, helical gears, cylindrical worms, racks, and other beveloid gears. They can be used with intersecting, parallel, or skew shafts. They are relatively insensitive to mounting errors.

TABLE 2.14
Minimum Numbers of Teeth in Pinion and Face Gear

	Minimum Number of Teeth	
Diametral Pitch Range	**Pinion**	**Gear**
20–48	12	18
49–52	13	20
53–56	14	21
57–60	15	23
61–64	16	24
65–68	17	26
69–72	18	27
73–76	19	29
77–80	20	30
81–83	21	32
85–88	22	33
89–92	23	35
93–96	24	36
97–100	25	38

2.2.10 An Advanced Set of Terms and Definitions for Design Parameters in Gearing*

Almost all necessary design parameters in a gear pair are covered by a conventional set of terms and de nitions known from many sources, that is, from national as well as international standards on gearing, handbooks, manuals, and so forth. As the theory of gearing evolves, new design parameters and de nitions are introduced targeting more in-depth description of the gear kinematics, tooth ank geometry, operating, and others.

The following is short list of newly introduced along with some known terms and de nitions that pertain to gearing. The terms and de nitions are listed in the order starting from the most fundamental.

Axis of rotation—The axis of rotation is a straight line associated with a gear, O_g (with a pinion, O_p) that remains motionless when the gear (the pinion) rotates (Figure 2.43).

Rotation of a gear—The rotation of a gear is an angular velocity ω_g of a rotated gear. The similar is valid with respect to a rotated pinion, ω_p.

Rotation vector of a gear—The rotation vector of a gear is a vector† ω_g along the axis of rotation of the gear O_g. The magnitude of the vector ω_g is equal to the angular velocity ω_g of a rotated gear. The direction of the rotation vector ω_g is speci ed by the right-hand rule. The rotation vector ω_g is a type of sliding vectors. This means that the vector ω_g can be applied at any point within the axis of rotation of the gear O_g.

* The beginning of wisdom is to call things by their right names—an old Chinese proverb.

† It should be stressed here that a rotation is not a vector in nature. However, rotations can be treated as vectors if special care is taken; that is, when coordinate system transformations are applied, the order of multipliers becomes critical and it cannot be altered.

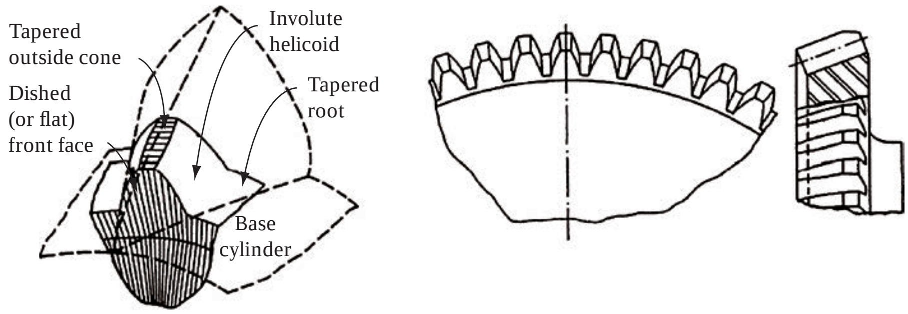
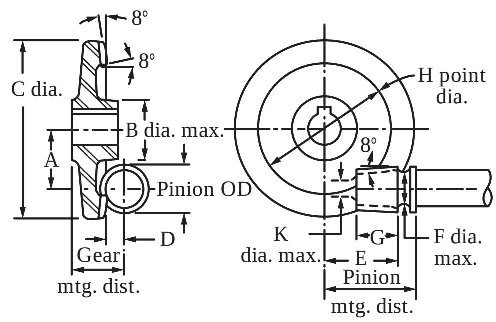


FIGURE 2.42 Beveloid tooth and gear nomenclature.

TABLE 2.15
Spiroid Gear Dimensions



A	B	C	D	E	F	G	H	K
0.500	0.625	1.500	0.129	0.596	0.2969	0.365	1.101	0.2031
0.750	1.000	2.250	0.176	0.894	0.4219	0.548	1.651	0.2812
1.000	1.375	3.000	0.248	1.192	0.5938	−0.731	2.202	0.3750
1.250	1.625	3.750	0.295	1.490	0.7031	0.914	2.752	0.4531
1.500	2.000	4.500	0.338	1.788	0.8281	1.096	3.303	0.5312
1.875	2.625	5.625	0.402	2.236	1.0156	1.370	4.129	0.6406
2.250	3.187	6.750	0.461	2.683	1.1562	1.644	4.954	0.7500
2.750	4.000	8.250	0.536	3.279	1.3594	2.010	6.055	0.8750
3.250	4.750	9.750	0.608	3.875	1.5625	2.375	7.156	1.0156
3.750	5.625	11.250	0.677	4.471	1.7656	2.741	8.257	1.1406
4.375	6.750	13.125	0.757	5.216	2.000	3.197	9.633	1.2969
5.125	8.000	15.375	0.851	6.111	2.2812	3.745	11.285	1.4844

TABLE 2.16
Spiroid Gear Formulas

To Find	Having	Formula
Ratio	Number of teeth in gear, number of threads in pinion	$m_G = \frac{N_G}{N_P}$
Pinion spiral angle	Theoretical lead, pinion *OD* cone angle, pinion pitch radius	$\tan\alpha_P = \frac{L\sec\tau}{2\pi r}$
Gear spiral angle	Ratio, pinion pitch radius, gear pitch radius, pinion spiral angle	$\sin\alpha_G = m_G\left(\frac{r}{R_G}\right)\sin\alpha_P$
Pinion pitch point	Length of pinion primary pitch cone	= ⅓ axial length primary pitch cone from small end

FIGURE 2.43 Axis of rotation of a gear.

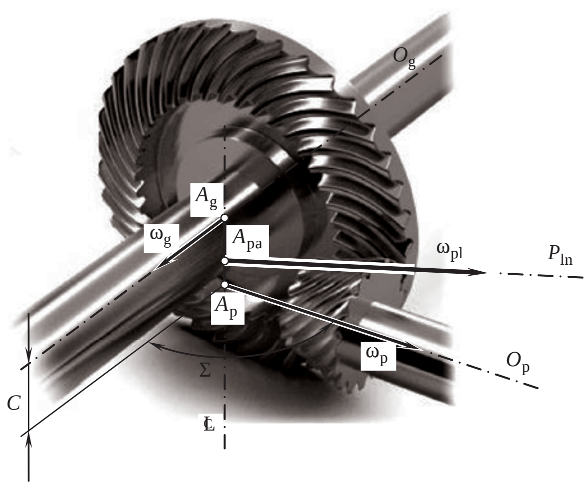


FIGURE 2.44 Elements of the kinematics of a gear pair.

The similar is valid with respect to a rotation vector of a pinion, ω_p.

Center distance—The center distance is the closest distance of approach C between the axes of rotation of the gear O_g and the pinion O_p (Figure 2.44).

Centerline—The centerline is the straight line along which the center distance C is measured.

Crossed axis angle (also the shaft angle)—The crossed axis angle is the angle Σ formed by the rotation vectors of the gear ω_g and the pinion ω_p, that is, $\Sigma = (\omega_g, \omega_p)$. Often, the crossed axis angle is speci ed as $\Sigma = 180° - (\omega_g, \omega_p)$ that is incorrect.

Gear apex—The gear apex is the point of intersection A_g of the gear axis of rotation O_g by the centerline .

Pinion apex—The pinion apex is the point of intersection A_p of the pinion axis of rotation O_p by the centerline .

Vector of instant rotation—The vector of instant rotation is a vector ω_{pl} of the instant rotation of a pinion in relation to the mating gear. Commonly, this vector is speci ed as $\omega_{pl} = \omega_p - \omega_g$.

Gear cone angle—The gear cone angle is the angle Σ_g that the rotation vector of the gear ω_g forms with the vector of instant rotation ω_{pl}, that is, $\omega_g = (\omega_{pl}, \omega_g)$. The gear cone angle ω_g is a signed value.

Pinion cone angle—The pinion cone angle is the angle Σ_p that the rotation vector of the pinion ω_p forms with the vector of instant rotation ω_{pl}, that is, $\omega_p = (\omega_{pl}\omega_p)$. The pinion cone angle ω_p is a signed value.

Axis of instant rotation (also the pitch line)—The axis of instant rotation is a straight line, P_{ln}, along the vector ω_{pl} of instant rotation of a pinion in relation to the mating gear.

Plane of action apex—The plane of action apex is the point of intersection A_{pa} of the pinion axis of instant rotation P_{ln} by the centerline .

Pitch-line plane—The pitch-line plane is the plane through the pitch line P_{ln} and the center line of the gear pair (Figure 2.45). In the case of I_a gearing, the P_{ln} plane is

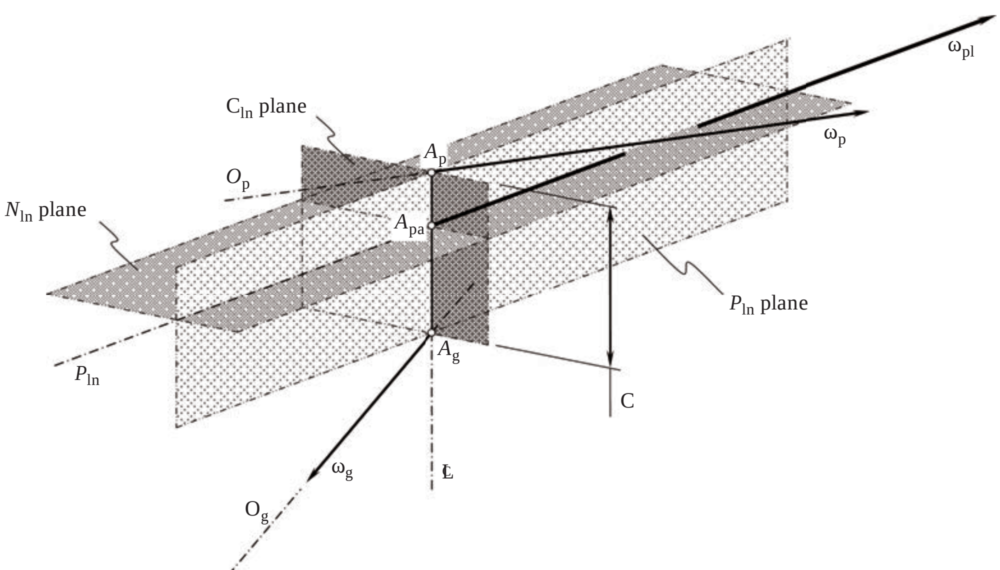


FIGURE 2.45 Principal planes: the pitch line plane (P_{ln} plane), the centerline plane (C_{ln} plane), and the normal plane (N_{ln} plane) associated with a gear pair.

the plane through the gear and pinion axes of rotation O_g and O_p, respectively. In the case of P_a gearing, the P_{ln} plane is the plane through the pitch line P_{ln}, and the centerline .

Centerline plane—The centerline plane is the plane through the center line of the gear pair perpendicular to the pitch line P_{ln}. It is revealed later that the centerline plane is congruent with the pitch plane of the gear pair. In the case of I_a gearing, the C_{ln} plane is the plane through plane-of-action apex A_{pa} perpendicular to the pitch line P_{ln}. In the case of P_a gearing, the C_{ln} plane is a plane perpendicular to the gear and pinion axes of rotation O_g and O_p, respectively.

Normal plane—The normal plane is the plane through the plane-of-action apex A_{pa} perpendicular to the centerline of the gear pair. In the case of I_a gearing, the N_{ln} plane is a plane perpendicular to the gear and pinion axes of rotation O_g and O_p, respectively. In the case of P_a gearing, the N_{ln} plane is a plane perpendicular to the gear and pinion axes of rotation O_g and O_p, respectively.

Main reference system—The origin of the main reference system $X_{ln}Y_{ln}Z_{ln}$ is coincident with the plane-of-action apex A_{pa}. The axes X_{ln}, Y_{ln}, and Z_{ln} are along the lines of intersection of the principal planes as illustrated in Figure 2.46: The axis Z_{ln} is along the vector of instant rotation ω_{pl}; the axis Y_{ln} is along the centerline ; and the X_{ln} axis complements the axes Y_{ln} and Z_{ln} to a left hand–oriented Cartesian coordinate system $X_{ln}Y_{ln}Z_{ln}$.

Motionless gear reference system—The origin of the motionless gear reference system $X_{g.m}Y_{g.m}Z_{g.m}$ is coincident with the gear apex A_g as shown in Figure 2.47a. The axis $Z_{g.m}$ is along the rotation vector of the gear ω_g; the axis $Y_{g.m}$ is along the centerline ; and the $X_{g.m}$ axis complements the axes $Y_{g.m}$ and $Z_{g.m}$ to a left hand–oriented Cartesian coordinate system $X_{g.m}Y_{g.m}Z_{g.m}$. Actually, the motionless gear reference system $X_{g.m}Y_{g.m}Z_{g.m}$ is turned through the gear cone angle Σ_g about Z_{ln} axis of the reference system $X_{ln}Y_{ln}Z_{ln}$ associated with a gear pair as illustrated in Figure 2.47b.

Gear reference system—The gear reference system $X_gY_gZ_g$ is rigidly associated with the gear, and is rotated together with the gear about its axis of rotation O_g. As illustrated in Figure 2.47c, the reference systems $X_gY_gZ_g$ and $X_{g.m}Y_{g.m}Z_{g.m}$ share the common axis $Z_g \equiv Z_{g.m}$, and is turned about the $Z_{m.g}$ axis through a gear rotation angle φ_g. A gear rotation angle φ_g and a pinion rotation angle φ_p correlate to one another in the following manner: $\varphi_p = u\varphi_g$, where u is the gear ratio of the gear pair.

Motionless pinion reference system—The origin of the motionless pinion reference system $X_{p.m}Y_{p.m}Z_{p.m}$ is coincident with the pinion apex A_p as shown in Figure 2.47d. The axis $Z_{p.m}$ is along the rotation vector of the pinion ω_p; the axis $Y_{p.m}$ is along the centerline ; and the $X_{p.m}$ axis complements the axes $Y_{p.m}$ and $Z_{p.m}$ to a left hand–oriented Cartesian coordinate system $X_{p.m}Y_{p.m}Z_{p.m}$. Actually, the motionless gear reference system $X_{p.m}Y_{p.m}Z_{p.m}$ is turned through the pinion cone angle Σ_p about Z_{ln} axis of the principal reference system $X_{ln}Y_{ln}Z_{ln}$ associated with a gear pair as illustrated in Figure 2.47e.

Pinion reference system—The pinion reference system $X_pY_pZ_p$ is rigidly associated with the pinion, and is rotated together with the pinion about its axis of rotation O_p. As illustrated in Figure 2.47f, the reference systems $X_pY_pZ_p$ and $X_{p.m}Y_{p.m}Z_{p.m}$ share the common axis $Z_p \equiv Z_{p.m}$, and is turned about $Z_{g.m}$ axis through a pinion rotation angle φ_p.

Principal reference systems—The reference systems $X_{ln}Y_{ln}Z_{ln}$, $X_{g.m}Y_{g.m}Z_{g.m}$, $X_gY_gZ_g$, $X_{p.m}Y_{p.m}Z_{p.m}$, and $X_pY_pZ_p$ are commonly referred to as the *principal reference systems* associated with a gear pair.

Plane of action—The plane of action is the plane PA within which the tooth anks of the gear and the pinion □ and □, respectively, interact with one another. In a case of crossed axis gearing (C_a *gearing*, for simplicity), the plane of action PA is a plane through the pitch line P_{ln} and the centerline , of the gear pair (Figure 2.45). This de nition is identical to that given earlier to the pitch-line plane P_{ln}. Therefore, in a case of gearing, the plane of action is congruent to the P_{ln} plane.

In a case of intersected axis gearing (I_a *gearing*, for simplicity), the plane of action PA is a plane through the pitch line P_{ln} that forms the transverse pressure angle $\phi_{t.\omega}$ that is perpendicular to the plane through the axes of rotation of the gear and the pinion.

In a case of parallel axis gearing (P_a *gearing*, for simplicity), the plane of action PA is a plane through the pitch line P_{ln} that forms the transverse pressure angle $\phi_{t.\omega}$ that is perpendicular to the plane through the axes of rotation of the gear and the pinion.

Transverse pressure angle—The transverse pressure angle is the angle $\phi_{t.\omega}$ formed by the pitch plane PP and the plane of action PA as shown in Figure 2.48.

Base cone (gear)—The gear base cone is a cone with an apex that is coincident with the gear apex A_g axis of the base cone that aligns with the gear axis of rotation O_g, and the gear base cone is tangent to the plane of action PA.

Base con (pinion)—The pinion base cone is a cone with an apex that is coincident with the pinion apex A_p axis of the base cone that aligns with the pinion axis of rotation O_p, and the pinion base cone is tangent to the plane of action PA.

Operating base pitch—The operating base pitch of a gear pair $\varphi_{b.op}$ is the angular distance between corresponding

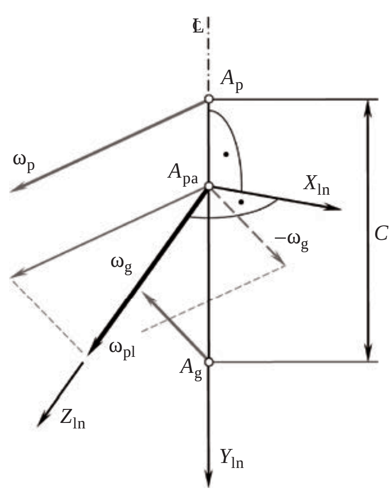


FIGURE 2.46 Principal reference system $X_{ln}Y_{ln}Z_{ln}$ associated with a gear pair.

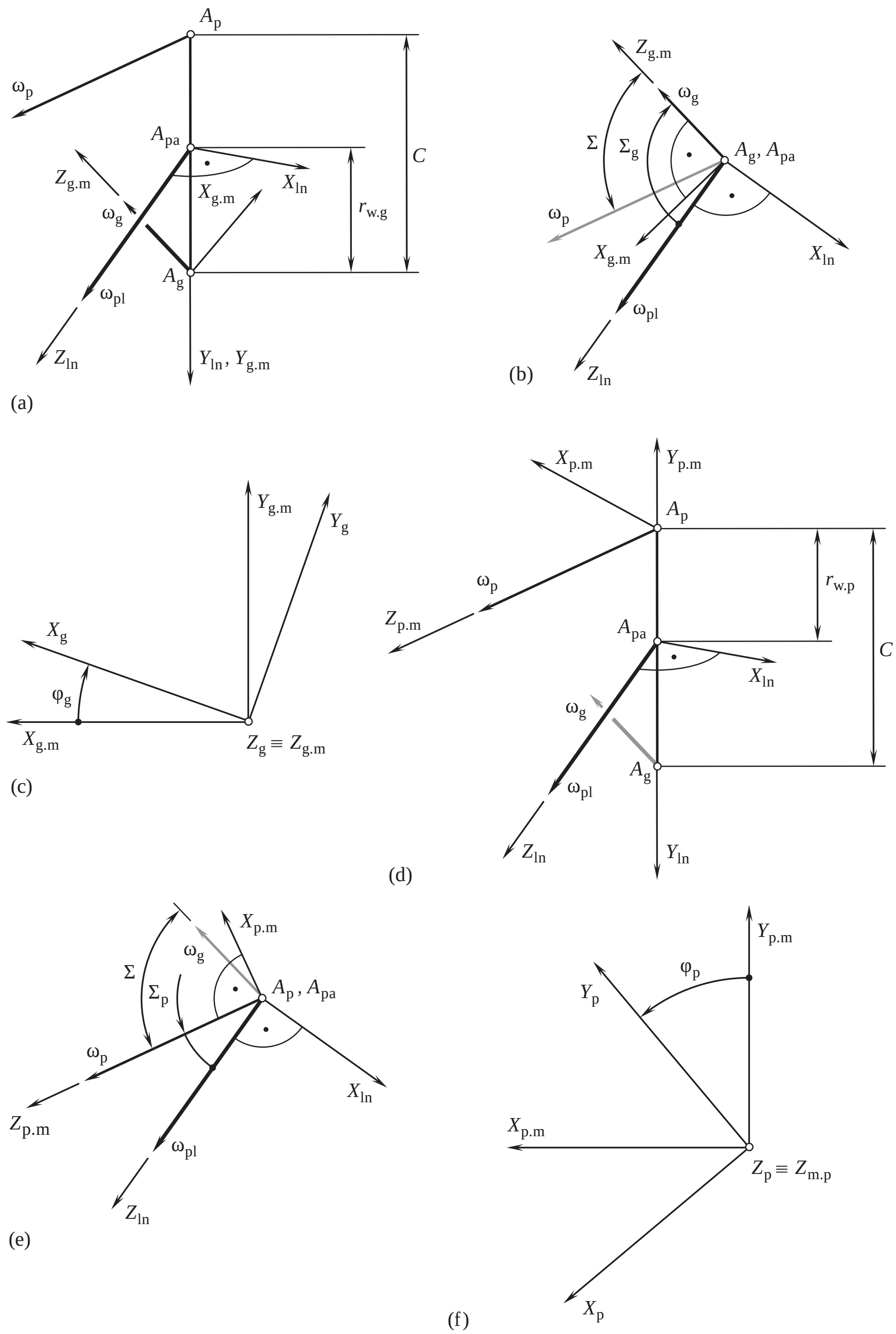


FIGURE 2.47 Rest of the principal reference systems. (a) Reference systems associated with the gear and the gear pair, (b) view along the center line, ·, onto the reference systems associated with the gear, (c) con guration of the gear reference systems, (d) reference systems associated with the pinion and the gear pair, (e) view along the center line, ·, onto the reference systems associated with the pinion, and (f) con guration of the pinion reference systems.

points within each two adjacent desired lines of contact LC^{i}_{des} and $LC^{i\pm1}_{\text{des}}$ (Figure 2.49). The distance $\varphi_{\text{b.op}}$ can be measured on a circular arc of an arbitrary radius, $r_{\text{y.pa}}$. The operating base pitch $\varphi_{\text{b.op}}$ is a calculated parameter; that is, it cannot be directly measured in a gear pair. The operating base pitch $\varphi_{\text{b.op}}$ is speci ed within the plane of action *PA*. The vertex of the operating base pitch angle $\varphi_{\text{b.op}}$ is coincident with the plane of action apex A_{pa}. The accuracy of the assembly of a gear and a mating pinion in the housing can be expressed in terms of the base pitch of the gear pair $\varphi_{\text{b.op}}$, that is, in terms of the deviation $\Delta\varphi_{\text{b.op}}$ of the actual operating base pitch of the gear pair $\varphi^{\text{ac}}_{\text{b.op}}$ from its nominal value $\varphi_{\text{b.op}}$. The deviation $\Delta\varphi_{\text{b.op}}$ is a signed value, and is equal to $\Delta\varphi_{\text{b.op}} = \varphi_{\text{b.op}} - \varphi^{\text{ac}}_{\text{b.op}}$.

Base pitch (gear)—The base pitch of a gear $\varphi_{\text{b.g}}$ is the angular distance between corresponding points within each two adjacent lines of intersection of the gear tooth anks G^{i}

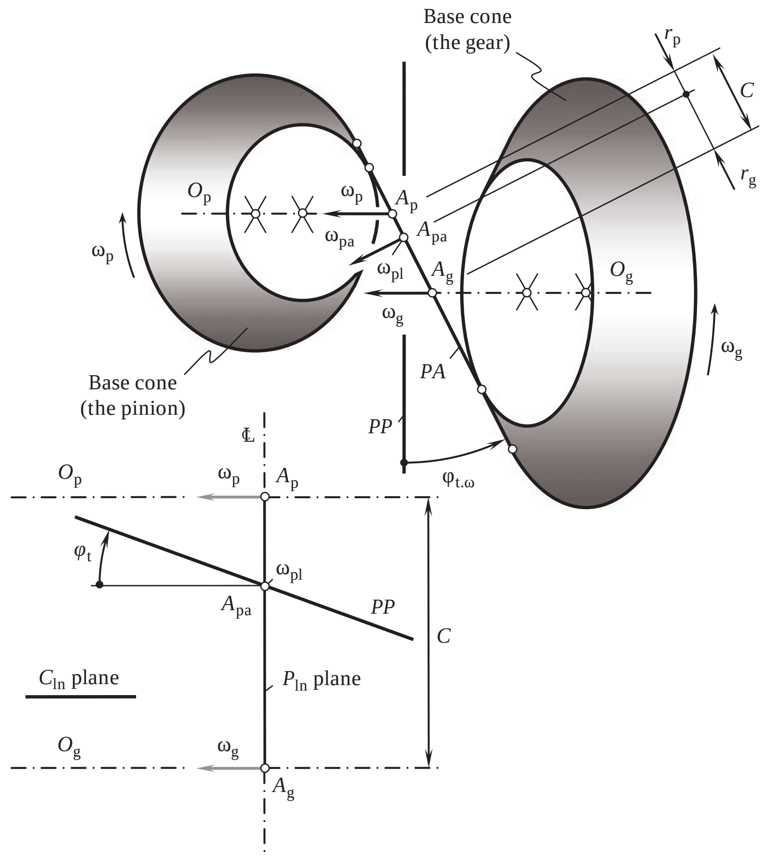


FIGURE 2.48 Plane of action *PA* and base cones associated with a gear pair shown in the centerline plane (C_{ln} plane.)

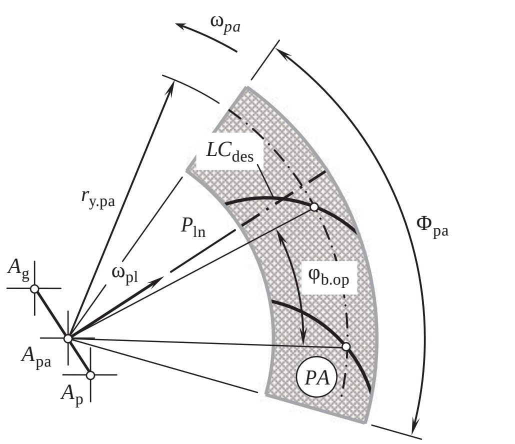


FIGURE 2.49 Operating base pitch $\varphi_{b.op}$ of a gear pair.

and $G^{i\pm1}$ by the plane of action *PA*. The gear base pitch $\varphi_{b.g}$ can be directly measured in a gear. The gear base pitch $\varphi_{b.g}$ is measured within the plane of action *PA* at an arbitrary angular configuration of the gear in relation to the pinion. The vertex of the gear base pitch angle $\varphi_{b.g}$ is coincident with the plane of action apex A_{pa}. The accuracy of the machined gear can be expressed in terms of the base pitch of the gear $\varphi_{b.g}$, that is, in terms of the deviation $\Delta\varphi_{b.g}$ of the actual base pitch of the gear $\varphi^{ac}_{b.g}$ from its nominal value $\varphi_{b.g}$. The deviation $\Delta\varphi_{b.g}$ is a signed value and is equal to $\Delta\varphi_{b.g} = \varphi_{b.g} - \varphi^{ac}_{b.g}$.

Base pitch (pinion)—The base pitch of a pinion $\varphi_{b.p}$ is the angular distance between corresponding points within each two adjacent lines of intersection of the pinion tooth flanks P^i and $P^{i\pm1}$ by the plane of action *PA*. The pinion base pitch $\varphi_{b.p}$ can be directly measured in a pinion. The pinion base pitch $\varphi_{b.p}$ is measured within the plane of action *PA* at an arbitrary angular configuration of the pinion in relation to the gear. The vertex of the pinion base pitch angle $\varphi_{b.p}$ is coincident with the plane of action apex A_{pa}. The accuracy of the machined pinion can be expressed in terms of the base pitch of the pinion $\varphi_{b.p}$, that is, in terms of the deviation $\Delta\varphi_{b.p}$ of the actual base pitch of the pinion $\varphi^{ac}_{b.p}$ from its nominal value $\varphi_{b.p}$. The deviation $\Delta\varphi_{b.p}$ is a signed value and is equal to $\varphi_{b.p} = \varphi_{b.p} - \varphi^{ac}_{b.p}$.

Fundamental law of gearing—The fundamental law of gearing states three conditions for a gear pair to be capable of transmitting a rotation smoothly:

- *The first condition*—The first condition is the so-called *condition of contact*. The condition of contact can be analytically described by the Shishkov's equation of contact: At every point of contact *K* of the tooth flanks of the gear □ and the pinion □, the vector of the relative motion $\mathbf{V}_\Sigma$ of the tooth flanks is always perpendicular to the unit vector of the common perpendicular $\mathbf{n}$ to the contacting surfaces □ and □, that is, $\mathbf{n} \cdot \mathbf{V}_\Sigma = 0$.

- *The second condition*—The second condition is the *condition of conjugacy* of the interacting tooth flanks □ and □ of the gear and the pinion, respectively (Radzevich, 2008). According to this condition, the instant line of action LA_{inst}, that is, a straight line along the common perpendicular **n** is always located within the plane of action *PA* and intersects the pitch line P_{ln}. Therefore, the path of contact P_c is a planar curve that is entirely located within the plane of action *PA*.

  The condition of conjugacy of the interacting tooth flanks □ and □ of the gear and the pinion, respectively, is more robust than the condition of their contact. This means that if the condition of conjugacy is met, then there is no necessity to verify whether or not the condition of contact is met: The condition of contact is met for sure.
- *The third condition*—The third condition requires equality of the base pitches (Radzevich, 2008). In order to smoothly transmit a rotation, first, the base pitch of the gear $\varphi_{b.g}$ must be equal to the operating base pitch $\varphi_{b.op}$ of the gear pair; that is, the equality $\varphi_{b.g} = \varphi_{b.op}$ must be observed; second, base pitch of the pinion $\varphi_{b.p}$ must be equal to the operating base pitch $\varphi_{b.op}$ of the gear pair, that is, the equality $\varphi_{b.p} = \varphi_{b.op}$ must be observed. Two equalities $\varphi_{b.g} = \varphi_{b.op}$ and $\varphi_{b.p} = \varphi_{b.op}$ can be combined into a generalized equality: $\varphi_{b.g} = \varphi_{b.p} = \varphi_{b.op}$.

It should be stressed here that the two equalities $\varphi_{b.g} = \varphi_{b.op}$ and $\varphi_{b.p} = \varphi_{b.op}$, as well as the combined equality $\varphi_{b.g} = \varphi_{b.p} = \varphi_{b.op}$, are more robust compared to the condition of conjugacy of the tooth flanks □ and □ of the gear and the pinion, respectively. This means that in a case when base pitches of the gear and the pinion are equal to the operating base pitch of the gear pair, there is no necessity to verify whether or not the condition of conjugacy, as well as the condition of contact of the tooth flanks □ and □ are satisfied: In this case, both the conditions are satisfied for sure.

Ultimately, one can conclude that the condition of equality of the base pitches of the gear and the mating pinion to the operating base pitch of the gear pair ($\varphi_{b.g} = \varphi_{b.p} = \varphi_{b.op}$) is the most robust condition the perfect gear pairs must met. Because of this, the conditions $\varphi_{b.g} = \varphi_{b.op}$ and $\varphi_{b.p} = \varphi_{b.op}$ (or the condition $\varphi_{b.g} = \varphi_{b.p} = \varphi_{b.op}$) are referred to as the *fundamental law of gearing** (Radzevich, 2008). If the conditions $\varphi_{b.g} = \varphi_{b.op}$ and $\varphi_{b.p} = \varphi_{b.op}$ (or the condition $\varphi_{b.g} = \varphi_{b.p} = \varphi_{b.op}$) are not met, then the gear pair is not capable of smoothly transmitting a rotation, and excessive vibration generation and noise excitation become inevitable.

* In a particular case of geometrically accurate gearing (ideal or perfect P_a gearing), that is, in a case of P_a gearing with zero axis misalignment, the angular base pitches $\varphi_{b.g}$, $\varphi_{b.p}$, and $\varphi_{b.op}$ reduce to the corresponding linear base pitches $p_{b.g}$, $p_{b.p}$, and $p_{b.op}$ correspondingly. In a reduced case, the fundamental law of gearing is represented either by two equations, $p_{b.g} = p_{b.op}$ and $p_{b.p} = p_{b.op}$, or by a generalized equation $p_{b.g} = p_{b.p} = p_{b.op}$.

Hand of helix—The hand of helix in helical gearing can be determined by means of the rule illustrated in Figure 2.50. This approach can be enhanced to the cases of gears for the I_a axis, as well as C_a axis gearings.

Conventional design parameters and terms used in gearing are summarized and depicted in Figures 2.51 and 2.52.

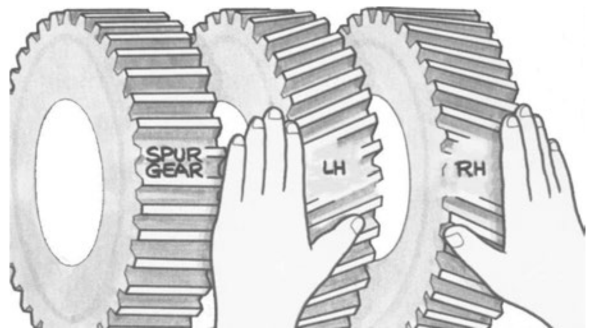


FIGURE 2.50 Hand of a helical gear.

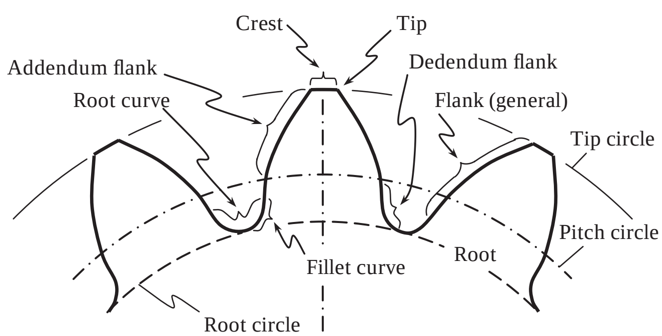


FIGURE 2.51 Tooth profile elements.

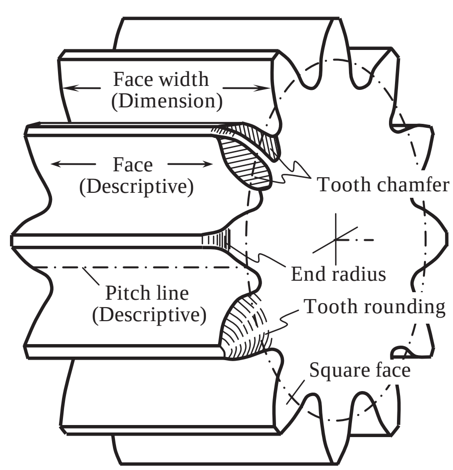


FIGURE 2.52 Lengthwise tooth elements of a spur gear.

3 Gear Tooth Design

In this chapter, the subject of the design of the gear tooth shape is covered. To the casual observer, some gear teeth appear tall and slim while others appear short and fat. Gear specialists talk about things like the pressure angle, long and short addenda, root fillet design, and the like. Obviously, there is some well-developed logic in the gear trade with regard to how to choose and specify factors related to the gear tooth shape. This chapter presents the basic data that are needed to exercise good judgment in gear tooth design.

3.1 BASIC REQUIREMENTS OF GEAR TEETH

The gear teeth mesh with each other and thereby transmit nonslip motion from one shaft to another. Those making and using gear teeth may expect the teeth to conform to some standard design system. If this is the case, the gear maker may be able to use some standard cutting tools that are already on hand. If a standard design that is already familiar to the gear user is used, the functional characteristics of the gears, such as relative load-carrying capacity, efficiency, quietness of operation, and the like, can be expected to be similar for new gear drives to those of gear drives already in service.

It should be kept in mind that the gear art has progressed to a point where there is much more versatility than before. Machine tools are computer controlled and can be programmed to cover much more variations than were possible in the past. In the gear design office, computers can find what may be believed to be the true optimum design for some important application. Frequently, the design chosen does not agree with a design that might have been considered standard in earlier years.

What this means is that a great variety of gear tooth designs are being used. This trend in gear engineering can be a good one—provided the gear designer has an in-depth knowledge of all the things that need to be considered. Serious mistakes, though, can be made when a decision is made by computer data and the computer program failed to consider some critical constraints in the application.

3.1.1 Definition of Gear Tooth Elements

Many features of the gear teeth need to be recognized and specified with appropriate dimensions (either directly or indirectly).

Spur and helical gears are usually made with an involute tooth form. If a section through the gear tooth is taken perpendicular to the axis of the part, the features shown in Figures 3.1 and 3.2 are revealed. Note the nomenclature used.

The *outside diameter* (dashed line in Figure 3.1) is the maximum diameter of the gear blank for spur, helical, worms, or worm gears. All tooth elements lie inside this circle. A tolerance on this diameter should always be negative.

The *modification diameter* (dashed line in Figure 3.1) is the diameter at which any tip modification is to begin. It is a reference dimension and may be given in terms of degrees of roll.

The *pitch diameter* (dashed line in Figures 3.1 and 3.2) is the theoretical diameter established by dividing the number of teeth in the gear by the diametral pitch of the cutter to be used to produce the gear. This diameter can have no tolerance.

The *limit diameter* (dashed line in Figure 3.1) is the lowest portion of a tooth that can actually come in contact with the teeth of a mating gear. It is a calculated value and is not to be confused with form diameter. It is the boundary between the active profile and the fillet area of the tooth.

The *form diameter* (dashed lines in Figure 3.1) is a specified diameter on the gear above which the transverse profile is to be in accordance with the drawing specification on profile. It is an inspection dimension and should be placed at a somewhat smaller radius than the limit diameter to allow for shop tolerances.

The *undercut diameter* (dashed lines in Figure 3.1) is the diameter at which the trochoid-producing undercut in a gear tooth intersects the involute profile.

The *base-circle diameter* (dashed lines in Figures 3.1 and 3.2) is the diameter established by multiplying the pitch diameter (mentioned earlier) by the cosine of the pressure angle of the cutter to be used to cut the gear. It is a basic dimension of the gear.

The *root diameter* (dashed lines in Figures 3.1 and 3.2) is the diameter of the circle that establishes the root lands of the teeth. All tooth elements should lie outside this circle. The tolerance should be negative.

Figure 3.3 shows a side view of a spur gear tooth to further depict the features and the nomenclature of gear teeth.

The *active profile* (shaded area in Figure 3.3a) is a surface and is the portion of the surface of the gear tooth which at some phase of the meshing cycle contacts the active profile of the mating gear tooth. It extends from the limit diameter (see Figure 3.1) near the root of the tooth to the tip round (see Figure 3.2) at the tip of the tooth and, unless the mating gear is narrower, then it extends from one side of the gear or the edge round (see Figure 3.3k) at one end of the tooth to the other side of the gear or the edge round.

The *top land* (shaded area in Figure 3.3b) is a surface bounded by the sides of the gear (see Figure 3.3d) and the active profiles, or if the tooth has been given end and tip rounds (see Figure 3.3h and 3.3i), the top land is bounded by these curved surfaces. The top land forms the outside diameter of the gear.

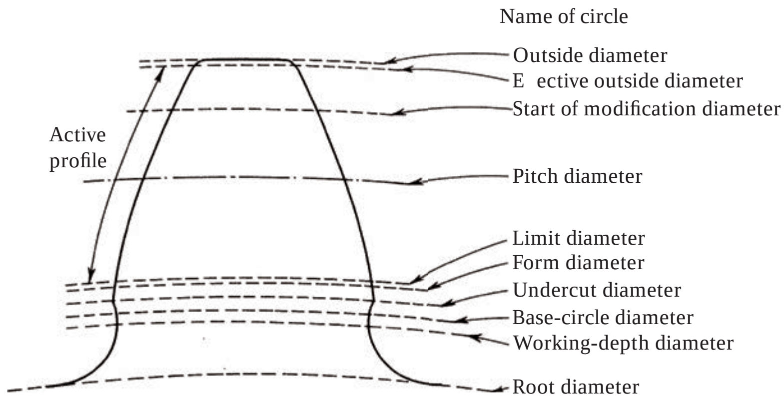


FIGURE 3.1 Nomenclature of gear circles.

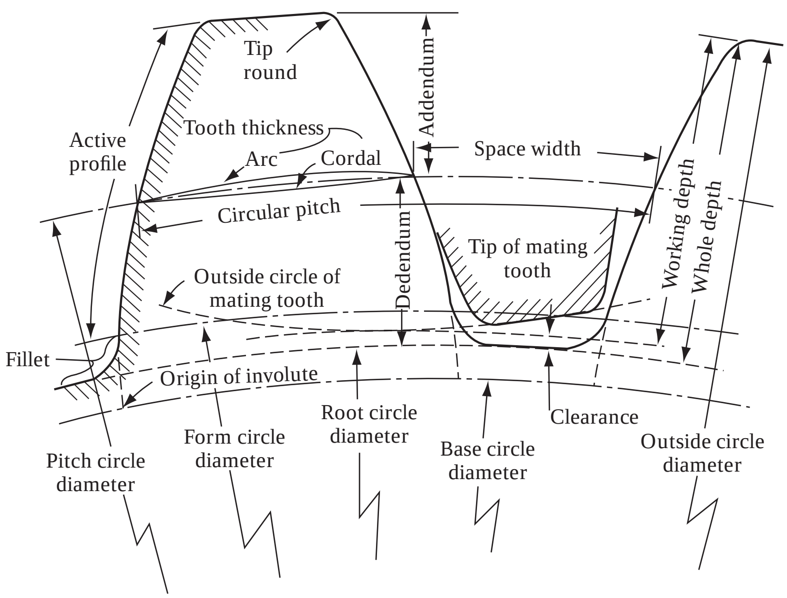


FIGURE 3.2 Gear tooth nomenclature.

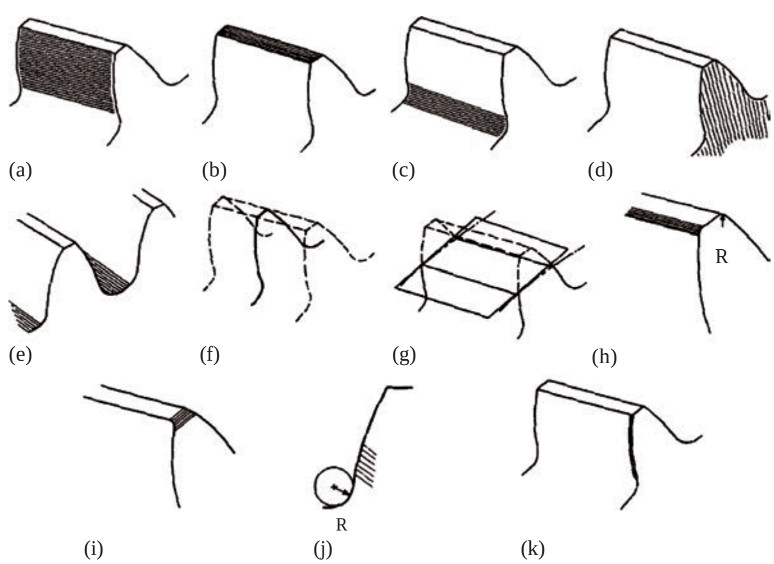


FIGURE 3.3 Nomenclature of gear tooth details: (a) active pro le, (b) top land, (c) llet, (d) sides of gear, (e) root land, (f) transverse pro- le, (g) normal section, (h) tip round, (i) end round, (j) llet radius, and (k) edge round.

The llet of a tooth (shaded area in Figure 3.3c) is a surface that is bounded by the form diameter (see Figure 3.1) and the root land (if present) (see Figure 3.3e) and by the ends of the teeth. In full- llet teeth, the llet of one tooth is considered to extend from the centerline of the space to the form diameter.

The *sides of gear* (shaded area in Figure 3.3d) are surfaces and are the ends of the teeth in spur and helical gears.

The *root land* (also known as bottom land) (shaded area in Figure 3.3e) is a surface bounded by llets (see Figure 3.3c) of the adjacent teeth and the sides of the gear blank.

The *transverse pro le* (heavy line in Figure 3.3f) is the shape of the gear tooth as seen in a plane perpendicular to the axis of rotation of the gear.

The *axial pro le* (heavy line in Figure 3.3g) is the shape of the gear tooth as seen in a plane tangent to the pitch cylinder at the surface of the tooth. In the case of helical gears, it is the shape of a tooth as seen on a pitch cylinder and may be developed to be shown in a plane.

The *tip round* (shaded area in Figure 3.3h) is a surface that separates the active pro le and the top land. It is sometimes applied to gear teeth either to remove the burrs or to lessen the chance of chipping, particularly in the case of hardened teeth. It may also be added as a very mild and crude form of pro le modi cation.

The *end round* (shaded area in Figure 3.3i) is a surface that separates the sides and the top land of the tooth. It is sometimes applied to gear teeth to reduce the chance of chipping, particularly in the case of hardened teeth.

The *llet radius* (shaded area in Figure 3.3j) is the minimum radius that a gear tooth may have.

The *end round* (shaded area in Figure 3.3k) is the surface that separates the active pro les of the teeth from the sides of the gear. These edges are of importance in the cases of helical gears, spiral bevel gears, and worms, since they become very sharp on the leading edge.

3.1.2 Basic Considerations for Gear Tooth Design

Gear teeth are a series of cam surfaces that contact similar surfaces on a mating gear in an orderly fashion. In order to drive in a given direction and to smoothly transmit power or motion and with a minimum loss of energy; the contacting cam surface on the mating gears must have the following properties:

- The height and the lengthwise shape of the active pro les of the teeth (cam surfaces) must be such that, before one pair of teeth goes out of contact during mesh, a second pair will have picked up its share of the load. This is called *continuity of action*.
- The shape of the contacting surfaces of the teeth (active pro les) must be such that the angular velocity of the driving member of the pair is smoothly imparted to the driven member in the proper ratio. The most widely used shape for active pro les of spur gears and helical gears that meets these requirements is the involute curve. There are many other specialized curves, each with speci c advantages in certain applications. This subject is developed further in Section 3.1.2.2.
- The spacing between the successive teeth must be such that a second pair of tooth-contacting surfaces (active pro les) is in the proper position to receive the load before the rst leave the mesh.

The continuity of action and the conjugate action are achieved by proper selection of the gear tooth proportions. Manufacturing tolerances on the gears govern the spacing accuracies of the teeth. Thus, to achieve a satisfactory design, it is necessary to specify the correct tooth proportions, and, in addition, the tolerances on the tooth elements must be properly speci ed.

As a general rule, gearing designed in accordance with the standard systems will not have problems of continuity of action or conjugate action. In those cases where it is necessary to depart from the tooth proportions given in the standard systems, the designer should check both the continuity of action and the conjugate action of the resulting gear design.

3.1.2.1 Continuity of Action

As discussed earlier, all gear tooth contact must take place along the *line of action*. The shape of this line of action is controlled by the shape of the active pro le of the gear teeth, and the length of the lines of action is controlled by the outside diameters of the gears (Figure 3.4). In order to provide a smooth continuous ow of power, at least one pair of teeth must be in contact at all times. This means that during a part of the meshing cycle, two pairs of teeth will be sharing the load. The second pair of teeth must be designed such that they will pick up their share of the load and be prepared to assume the full load before the rst pair of teeth goes out of action.

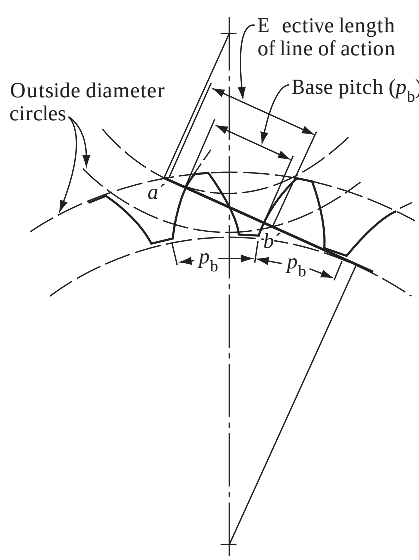


FIGURE 3.4 Zone of tooth action.

The control of the continuity of action is achieved in spur by varying the following:

- The slope of the line of action (in the case of involute gears, the operating pressure angle).
- The outside diameters of the pinion and the gear.
- The shape of the active profile.
- The relative sizes of the limit-diameter and undercut-diameter circles. (Limit circle must be larger.)

The control of the continuity of action in internal-type spur and helical gears is achieved by varying the following:

- The slope of the line of action (in the case of involute gears, the operating pressure angle).
- The inside diameter of the internal gear and the outside diameter of the pinion.
- The shape of the active profile.
- The relative sizes of the limit-diameter and undercut-diameter circles. The internal gear will not be undercut, but the external member may. Thus, the limit diameter of the pinion must be larger than the undercut diameter.

The control of the continuity of action in helical gears is achieved by varying the following:

- The slope of the line of action (in the case of involute gears, the operating pressure angle).
- The outside diameters of the pinion and the gear.
- The shape of the active profile.
- The relative sizes of the limit-diameter and the undercut-diameter circles. (Limit circles must be larger.) All the foregoing elements control the continuity of action in the transverse plane.
- The lead of the tooth.
- The length of the tooth. These two elements control the continuity of action in the axial plane. In order to assure continuity of action, the portion of the line of action bounded by the outside-diameter circles (the straight line segment *ab*) must be somewhat longer than the base pitch (see Figure 3.4).

The base pitch p_b is defined as follows:

$$p_b = p\cos\phi = \frac{\pi\cos\phi}{P_d}, \tag{3.1*}$$

where
p—circular pitch of gear
ϕ—pressure angle of gear
P_d—diametral pitch

Thus, either the outside-diameter circles, the operating pressure angle, or the base pitch must be adjusted so that *ab* exceeds the base pitch p_b by 20% to 40%.

The most general way of checking the continuity of action is by calculating the contact ratio.

A numerical index of the existence and degree of continuity of action is obtained by dividing the length of the line of action by the base pitch of the teeth (see Figure 3.4). This is called *contact ratio* m_p.

The AGMA recommends that the contact ratio for spur gears to not be less than 1.2:

$$m_p = \frac{L_a}{p_b} \geq 1.2. \tag{3.2}$$

A spur gear mesh has only a transverse contact ratio, m_p, whereas a helical gear mesh has a transverse contact ratio m_p, an axial contact ratio m_F (face contact ratio), and a total contact ratio m_t.

Equations for contact ratio are as follows:

- Spur gears and helical gears

$$m_p = \frac{\sqrt{\left(\frac{d'_o}{2}\right)^2 - \left(\frac{d_b}{2}\right)^2} + \sqrt{\left(\frac{D'_o}{2}\right)^2 - \left(\frac{D_b}{2}\right)^2} - C'\sin\phi'}{p_b} \tag{3.3}$$

- Internal, spur, and helical gears

$$m_p = \frac{\sqrt{\left(\frac{d'_o}{2}\right)^2 - \left(\frac{d_b}{2}\right)^2} + \sqrt{\left(\frac{D'_i}{2}\right)^2 - \left(\frac{D_{bi}}{2}\right)^2} - C'\sin\phi'}{p_b} \tag{3.4}$$

- Helical gears, axial contact ratio

$$m_F = \frac{F\tan\psi}{p} \tag{3.5}$$

- Helical gears, total contact ratio

$$m_t = m_p + m_F, \tag{3.6}$$

where
d'_o—outside diameter (effective) of pinion
D'_o—effective outside diameter of gear (diameter to intersection of tip round and active profile) (see the following discussion)
D'_i—inside diameter (effective), internal gear
d_b—base diameter of pinion
D_b—base diameter of gear
D_{bi}—base diameter of internal gear
C'—operating center distance of pair
ϕ'—operating pressure angle

* Equations 3.1 through 3.6 are given dimensionless. For English system calculations, use inches for all dimensions. For metric system calculations, use millimeters for all dimensions.

p—circular pitch (in plane of rotation)
p_b—base pitch
F—length of tooth, axial, face width of the gear
ψ—helix angle of helical gears

Note: To achieve correct answers on contact ratio, the following points should be observed:

- The *effective outside diameter* d_o' or D_o' is actually the diameter to the beginning of the tip round, usually $D_o' = D_o - 2$ (edge round specification) (maximum). This value rather than the drawing outside diameter should be used, since in many cases, manufacturing practices for removing burrs produce a large radius, particularly in fine-pitch gears. Thus, a considerable percentage of the addendum may not be effective. If the teeth are given a very heavy profile modification, consideration should be given to performing the calculation under (a) full load, assuming contact to the tip of the active profile, and (b) light load, assuming contact near the start of modification. In this case d_o' or D_o' is selected to have a value close to the diameter at the start of modification. This will give an index to the smoothness of operation at these conditions.
- It is also assumed that the form diameter is a larger value than the undercut diameter. If not, use the value of undercut diameter.
- On occasion, the outside diameter of one or both members is so large relative to the center distance and operating pressure angle that the tip extends below the base circle when it is tangent to the line of action. Since no involute action can take place below the base circle, the value $C \sin \phi$ (the total length of the line of action) should be substituted in the equation in place of the value $\sqrt{\left(d_o'/2\right)^2 - (d_b/2)^2}$ or $\sqrt{\left(D_o'/2\right)^2 - (D_b/2)^2}$ in the case that one or both become larger than the value of $C \sin \phi$.

3.1.2.2 Conjugate Action

Gear teeth are a series of cam surface that act on similar surfaces of the mating gear to impart a driving motion. Many different shapes of surfaces can be used on the teeth to produce uniform transmission of motion. Curves that act on each other with a resulting smooth driving action and with a constant driving ratio are called *conjugate curves*. The fundamental requirements governing the shapes that any pair of these curves must have are summarized in Willis's (1841) basic law of gearing, which state:

Basic law of gearing: Normals to the profiles of mating teeth must, at all points of contact, pass through a fixed point located on the line of centers (Willis, 1841).

In the case of spur- and helical-type gears, the curves used almost exclusively are those of the involute family. In this type of curves, the fixed point mentioned in the basic law is the pitch point. Since all contact takes place along the line of action, since the line of action is normal to both the driving and the driven involutes at all possible points of contact, and, lastly, since the line of action passes through the pitch point, it can be seen that the involute satisfies all the requirements of the basic law of gearing. Many years ago, the cycloidal family of curves was in common use; however, clockwork gears are about the only application of this type of tooth today. The involute curves has supplanted the cycloid because of its greater ease of design and because it is far less sensitive to manufacturing and mounting errors. In addition, the tools used to generate the cycloid are more difficult to make with the same degree of accuracy as those for the involute.

The use of the involute has become so universal that, with few exceptions, it is hardly necessary to specify on spur and helical gear drawings that the involute form is desired. It is only when a designer wishes to use a noninvolute tooth for a special application that it becomes apparent how much work must be done to properly design a satisfactory profile. The results of standardization and the deep understanding of gears on the part of competent manufacturers are appreciated mostly by those who require quality designs of a nonstandard nature.

Worm gearing, like bevel gearing, is noninvolute. The tooth form of worm gearing is usually based on the shape of the worm; that is, the teeth of the worm gear are made conjugate to the worm. In general, worms can be chased, as on a lathe, or cut by such process as milling or hobbing, or can be ground. Each process, however, produces a different shape of worm thread and generally requires a different shape of worm gear tooth in order to properly run.

In the case of face gearing, the pinion member is a spur or a helical gear of involute form, but the gear tooth is a special profile conjugate only to the specific pinion. Thus, pinions having a number of teeth larger or smaller than the number for which the face gear was designed will not properly run with the face gear.

3.1.2.3 Pitch Diameter

Although the pitch circles are not the fundamental circles on gears, they are traditionally the starting point on most tooth designs. The pitch circle is related to the base circle, which is the fundamental circle, by the relationships that follow. Some authorities list as many as nine distinct definitions of pitch circles. The following definitions cover the pitch circles considered in this chapter.

The *standard pitch diameter* is the diameter of the circle on a gear determined by dividing the number of teeth in the gear by the diametral pitch.

The diametral pitch is that of the basic rack defining the pitch and the pressure angle of the gear.

The AGMA gives the following equation for standard pitch diameter in the fine-pitch standards:

$$D = \frac{N}{P_d}, \tag{3.7}$$

where

D—diameter of standard pitch circle
P_d—diametral pitch of basic rack
N—number of teeth in gear

The *operating pitch diameter* is the diameter of the circle on a gear, which is proportional to the gear ratio, and to the actual center distance, at which the gear pair will operate.

A gear does not have an operating pitch diameter until it is meshed with a mating gear. The equations for operating pitch diameter are as follows:

- External spur and helical gearing

$$d' = \frac{2C'}{m_g + 1} \quad \text{for pinion member,} \tag{3.8*}$$

$$D' = \frac{2C'm_g}{m_g + 1} \quad \text{for gear member,} \tag{3.9}$$

- Internal spur and helical gearing

$$d' = \frac{2C'}{m_g - 1} \quad \text{for pinion member,} \tag{3.10}$$

$$D' = \frac{2C'm_g}{m_g - 1} \quad \text{for gear member,} \tag{3.11}$$

where

d'—operating pitch diameter of pinion
D—operating pitch diameter of gear
C'—operating center distance of mesh
m_g—gear ratio $= \dfrac{\text{number of teeth in gear}}{\text{number of teeth in pinion}}$

The operating and standard pitch circles will be the same for gears operated on center distances that are *exactly* standard. The distinction to be made usually involves tolerances on the gear center distance. Most practical gear designs involve center-distance tolerances that are the accumulated effects of machining tolerances on the center bores and the tolerances in bearings (clearances, runout of outer races, and so forth). Thus, all gears operate with maximum and minimum operating pitch diameters. The most important application occurs on gears operated on nonstandard center distances.

In Figure 3.5, a pair of gears designed to operate on enlarged center distances is shown. Both the standard and operating pitch circles are shown. It will be noted that the pitch circles are related to the base circle as follows:

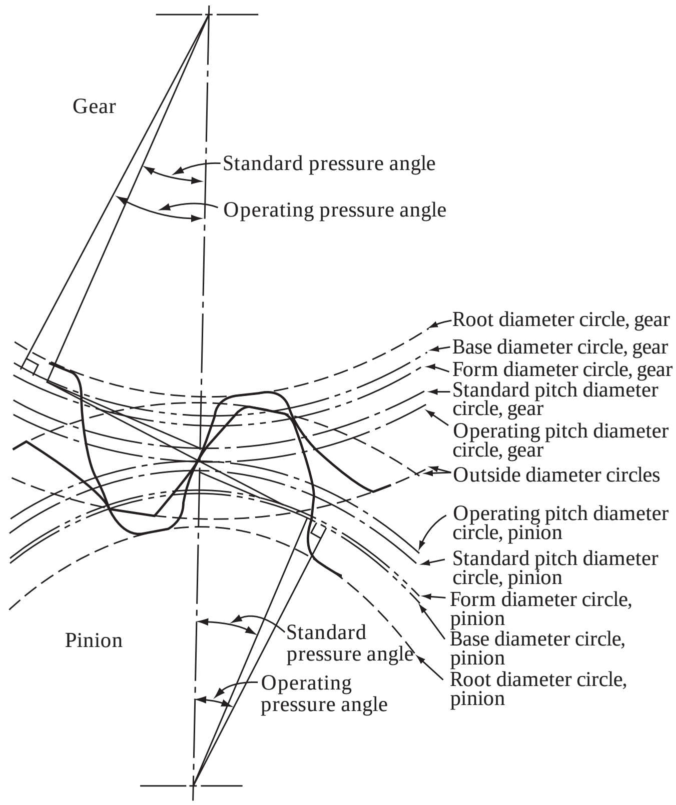


FIGURE 3.5 Nomenclature for gear cut to standard pressure angle and operated at an increased center distance.

$$D = \frac{D_b}{\cos\phi} \quad \text{standard pitch diameter,} \tag{3.12}$$

$$D' = \frac{D_b}{\cos\phi'} \quad \text{operating pitch diameter.} \tag{3.13}$$

It will also be noted that the standard pitch circles do not contact each other by the amount that the center distance has been increased from the standard.

In worm gearing, it is convenient to use pitch circles. In this case, however, it is common practice to make the pitch circle of the gear go through the teeth at a diameter at which the tooth thickness is equal to the space width. In the case of the worm, the pitch circle also de nes a cylinder at which the width of the threads and spaces are equal. It is also a good practice to slightly modify the worm teeth to achieve the required backlash. When this is done, the space widths are greater than the thread thicknesses, as measured on the standard pitch cylinder, by the amount of backlash introduced. The pitch cylinder also de nes the diameter at which the lead angle, as well as the pressure angle, is to be measured.

3.1.2.4 Zones in Which Involute Gear Teeth Exist

Although many gear designs utilize *standard*- or *equal-dedendum* tooth proportions, it is not always necessary or even desirable to use these proportions. One of the outstanding

* Equations 3.8 through 3.13 should use inches for English calculations, and millimeters for metric calculations.

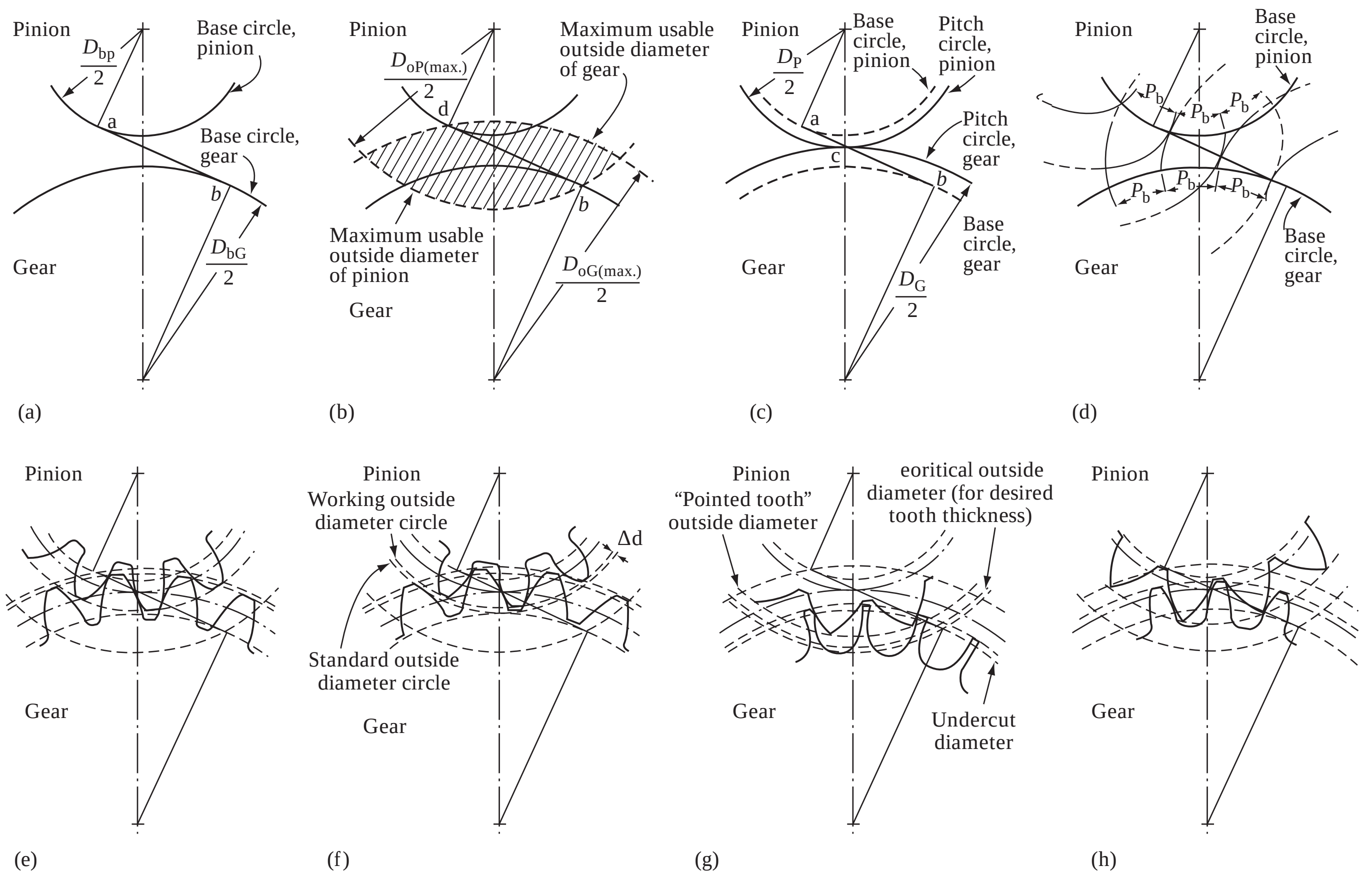


FIGURE 3.6 Study of involute tooth development. (a) Maximum usable length of line of action; (b) maximum zone in which conjugate action can take place; (c) operating pitch circles; (d) development of involute pro les; (e) standard-addendum tooth proportions; (f) short (pinion)–addendum tooth proportions; (g) long (pinion)–addendum (abnormal) tooth proportions; and (h) long (pinion)–addendum (normal) tooth proportions.

features of the involute tooth pro le is the opportunity it affords for the use of different amounts of addendum and tooth thicknesses on gears of any given pitch and numbers of teeth. These variations can be produced with standard gear tooth generating tools. It is not necessary to buy different cutting or checking tooling for each new value of tooth thicknesses or addendum if the proper tooth thicknesses–addendum relationship is maintained in the design.

As discussed in other chapters of this handbook, the limits of true involute action are established by the length of the line of action. It has been shown that the end limits de ning the maximum usable portion of the line of action are xed by the points at which this line becomes tangent to the basic circles. The limits *a* and *b* are shown in Figure 3.6a. The largest pinion or gear that will have correct gear tooth action is de ned by circles that pass through points *a* and *b* (Figure 3.6b). Therefore, any gear teeth that fully lie within the cross-hatched area on Figure 3.6b will have correct involute action on any portions of the teeth that are not undercut. Undercut limitations are more fully discussed in Section 3.1.2.6. One should not infer that the largest usable outside diameters will be simultaneously used on both members on a given gear design.

The equations for maximum usable outside diameter are as follows:

$$D_{oG} = 2\sqrt{(C'\sin\phi')^2 + \left(\frac{D_G}{2}\cos\phi\right)^2} \qquad \text{max}, \quad (3.14)^*$$

$$D_{oP} = 2\sqrt{(C'\sin\phi')^2 + \left(\frac{D_P}{2}\cos\phi\right)^2} \qquad \text{min}, \quad (3.15)$$

where

C'—operating center distance
ϕ—cutting or standard pressure angle
ϕ'—operating pressure angle
D_G—pitch diameter of gear
D_P—pitch diameter of pinion

Operating pitch circles for the gear ratio (2:1) and the center distance chosen for this illustration are shown in place in

* Equations 3.14 through 3.18 should use inches for English calculations, and millimeters for metric calculations.

Figure 3.6c. If this center distance is the standard for the number of teeth and pitch, the operating pitch circles shown are also the standard pitch circles.

To transmit uniform angular motion, a series of equally spaced involute curves are arranged to act on each other. These are shown in place in Figure 3.6d. On both the pinion and the gear member, the involute curves originate at the base circles and can theoretically go on forever. The more practical lengths are suggested by solid lines. The spacing of the involutes of both members measured on the base circles must be equal, and the chosen interval is called the *base pitch*. The base pitch of the gear or the pinion times the number of teeth in the member must exactly be equal to the circumference of the base circles.

If similar involute curves of opposite hands are drawn for both base circles, the familiar gear teeth are achieved. It is customary to measure the distance from one involute curve to the next along the standard pitch circle. This distance is called the *circular pitch*. It is also customary to make standard tooth proportions with tooth thicknesses equal to one-half the circular pitch. The standard addendum used for the gearing system shown in this chapter is equal to the circular pitch divided by π.

Figure 3.6e shows standard teeth developed on the base circles of Figure 3.6a through d. It will be noted that the addendum of the 12-tooth pinion is equal to that of the gear, and that the outside diameters thus established do not reach out to the maximum values established by the line-of-action limits.

In modern gear-cutting practice, the tooth thickness of gears as measured on the standard pitch circle is established by the depth to which the generating-type cutter (usually hob or shaper cutter) is fed, relative to the standard pitch diameter. In order to obtain a correct whole depth of tooth, the outside diameters of the gear blanks are made larger or smaller than the standard by twice the amount that the cutter will be fed in or held out relative to the normal or the standard amount that would be used for standard gears. If the cutter is to be held out a distance Δc, the outside diameter of the blank is made $2\Delta c$ larger than the standard and the resultant gear is said to be long addendum. Figure 3.6f shows an example of a gear in which the cutter was sufficiently held out so that the outside diameter of the gear was equal to the diameter of the maximum usable outside-diameter circle (see also Figure 3.6b). This is called a long-addendum gear. The cutter was fed into the pinion of an equal amount, making it short addendum. The reasons for making pinions short addendum are discussed in Section 3.1.3.2, and the effects of the undercutting produced are discussed in Section 3.1.2.6.

In order to avoid undercut, or to achieve a more equal balance in tooth strength, it is customary to make the pinion addendum long and that of the gear short. If an attempt is made to design a pinion with the maximum usable outside diameter as established by the line of action (see Figure 3.6b), difficulties may arise. In the example shown having 12 and 24 teeth, it is not possible to generate a pinion having such an outside diameter.

Figure 3.6g shows the tooth form resulting from such an attempt. The pinion blank was turned to the maximum usable outside diameter, and the cutter was held out to an equivalent distance Δc. As a result of the generating action, the sides of the teeth are involute curves that crossed over at a diameter smaller than the desired outside diameter. Thus, the teeth are pointed at the outside diameter and also the whole depth is less than anticipated. This effect is less serious in gears having a large number of teeth.

The gear tooth that results from this extreme modification is badly undercut. This effect would not have been so great if the gear had more teeth.

The amount of long and short addenda that may be applied to each member of a gear mesh is limited by the three following considerations:

- The length of the usable portion of the line of action will form a maximum limit on the outside diameter of a gear. Diameters in excess of this will not provide additional tooth contact area, since there is no involute portion on the mating gear that can contact this area.
- The diameter at which the teeth become pointed limits the actual or the effective outside diameter.
- Undercutting may limit the short-addendum gear. The undercut diameter should always be less than the form diameter.

These three considerations show the extreme limits that bound gear tooth modifications. The designer should not infer that it is necessary to approach these limits in any given gear design. In the treatment of gear tooth modifications, some reasons for making the given tooth modifications are considered. In each case, however, the designer must be sure that the amount actually used does not exceed the limits discussed here.

3.1.2.5 Pointed Teeth

The previous sections have shown how gear teeth generated by a specific basic rack can have different tooth thicknesses, and that the outside diameters of such gears are altered from standard as a function of the change in tooth thickness. In practice, these tooth profiles are achieved by feeding in or holding out the gear-generating tools. It is customary to feed a specific cutter to a definite depth into the gear blank which has a greater-than-standard outside diameter. The cutter, when working at its full cutting depth, will still be held out from the standard N/P_d pitch circle.

The generated teeth that are thicker than the standard will have tips of a width that is less than standard, since the cutter must be held out. For any given number of teeth, the tooth thickness can be increased such that the tip will become pointed at the outside-diameter circle. In Figure 3.7, four gears all having the same number of teeth, diametral pitch, and pressure angle are shown. In Figure 3.7a, the cutter shown by a rack has been fed to the standard depth. The outside diameter of this gear is standard $(N + 2)P_d$. Note the tooth

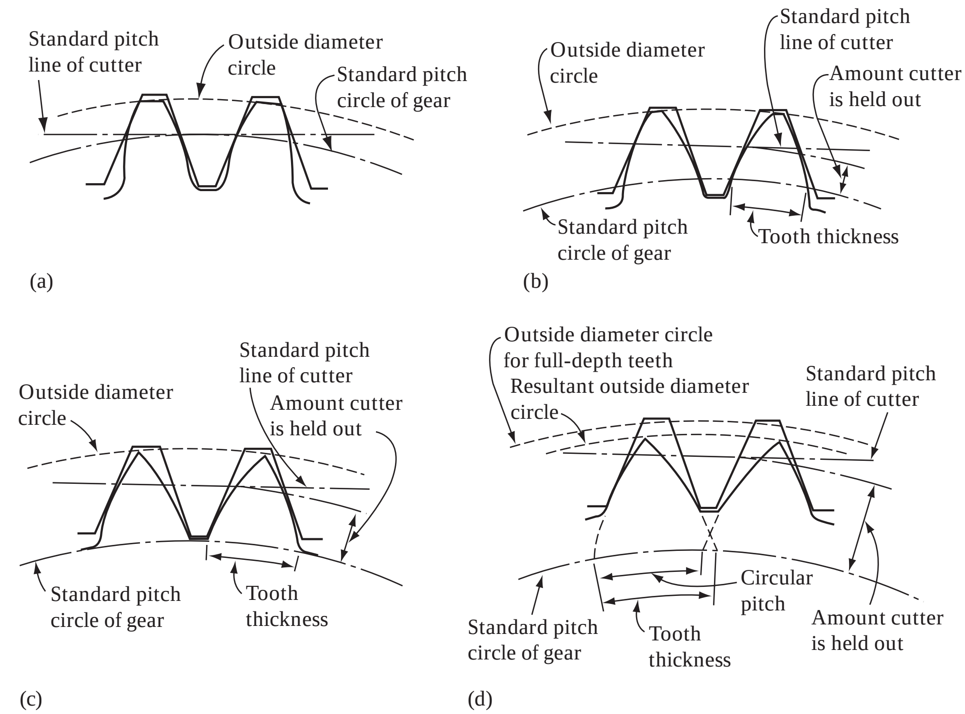


FIGURE 3.7 Cutting long-addendum teeth. (a) Standard tooth; (b) long-addendum tooth; (c) long-addendum (pointed) tooth; and (d) long-addendum (over-enlarged) tooth.

thickness at the tip. In Figure 3.7b, the outside diameter was somewhat enlarged and the cutter fed in the standard whole-depth distance starting from the enlarged outside diameter. This results in the thicker tooth (which would correctly operate with a standard gear on an enlarged center distance) and a somewhat thinner tip. In Figure 3.7c, the outside diameter has been established at a maximum value at which it is still possible to achieve a standard whole-depth tooth; the tooth thus generated is pointed. Figure 3.7d shows what happens if the maximum outside diameter and tooth thickness are exceeded. The resulting tooth does not have the correct whole depth because the involute curves cross over below the expected outside diameter. This tooth is similar to the one shown on the pinion member in Figure 3.6g.

The amount that the outside diameter of a gear is to be modi ed is usually a function of the desired tooth thickness. Equation 3.18 gives the relationship usually employed.

The maximum amount that the tooth thickness of a gear can be increased over the standard to just achieve a pointed tooth can be calculated from the simultaneous* solution of Equations 3.16 and 3.17.

$$\Delta D_o^{max} = N\left(\frac{\cos\phi}{\cos\phi''} - 1\right) - 2, \tag{3.16}$$

$$\Delta D_o^{max} = \frac{N(\text{inv}\,\phi'' - \text{inv}\,\phi) - \dfrac{\pi}{2}}{\tan\phi}. \tag{3.17}$$

The equivalent amount that the tooth thickness must be increased above the standard to achieve this increase in the outside diameter is given by

$$\Delta T = d(\text{inv}\ \phi'' - \text{inv}\ \phi) - T, \tag{3.18}$$

where D_o^{max} is the maximum outside diameter at which a tooth having full working depth will come to a point, as follows:

$$D_o^{max} = \frac{N+2}{P} + \Delta D_o, \tag{3.19}$$

where

T—tooth thickness at standard pitch diameter
ϕ—standard pressure angle of hob or rack
ϕ''—pressure angle at tip of tooth

3.1.2.6 Undercut

An undercut tooth is one in which a portion of the pro le in the active zone has been removed by secondary cutting

* Equations 3.16 and 3.17 are solved by assuming a series of values for ϕ''. The curves are plotted for ΔD_o^{max} versus ϕ''. The point where the curves cross indicate a *simultaneous* solution for the two equations. For instance, a 20-tooth pinion of a pitch having a 20° pressure angle and cut with a standard cutter has a crossing point for ϕ'' of 39.75° and ΔD_o^{max} of 2.44 in. It should be kept in mind that these equations solve problems for one pitch only (divided by the pitch to adjust the answer for other pitches) and make no allowance for backlash. In the metric system, the solution is in millimeters and for one module. (For other modules, multiply the answer by the module.)

action. Under certain conditions, the path swept out by the tip of a generating-type cutter will intersect the involute active profile at a diameter greater than the limit diameter. Such a tooth is said to be undercut, since no contact with the mating gear can take place between the undercut and the limit circles.

The amount of undercut will depend on the type of tool used to cut the gears; in general, a hob will undercut a gear to a greater degree than a circular, shaper-type cutter will. Form-type cutters do not normally produce undercut. In calculating the undercut diameter, it is customary to use an equation based on the type of tooling that will produce the greatest undercut, usually a hob. This is done in order to give the shop the greatest freedom of choice in selecting tools. If the most adverse choice will produce a satisfactory tooth profile, then all other types of cutter that might be chosen by the shop would prove to be satisfactory, except in those cases where undercut is intended to provide clearance for the tips of the mating teeth.

In general, spur gears of 20° pressure angle having 18 or more teeth and made to standard- or long-addendum tooth proportions will not be undercut. In each of the standard systems for different types of gear, the minimum number of teeth recommended for each pressure angle is usually based on the undercut.

Helical gears can usually be made with fewer teeth than can spur gears without getting into problems of undercut.

Figure 3.8 shows three gears having the same number of teeth, each produced by generating-type cutters. Figure 3.8a is the tooth form of a gear having standard addendum. Figure 3.8b shows the same shape of the tooth that results when the tip of the rack-type cutter is operating at a depth exactly passing through the point of intersection of the line of action and the base circle. Figure 3.8c shows the shape of a tooth that results when the cutter works at a depth somewhat below the intersection of the line of action and the base circle.

Undercutting is the product of generating-type cutters. Gears cut by forming-type cutters do not usually have undercut. In certain cases, the undercut may ensure tip clearance in mating gears. Racks operating with gears having a small number of teeth may show tip interference unless the pinions were generated with special shaper-type cutters or hobs.

3.1.3 Long- and Short-Addendum Gear Design

The previous section shows the geometric limitations on the amount that gears can be made long or short addendum. This section indicates cases in which long and short addendum should be considered.

The modification of the addendum of the pinion, and in most cases the gear number, is recommended for gears serving the following applications:

- Meshes in which the pinion has a small number of teeth.
- Meshes operating on nonstandard center distances because of limitations on ratio or center distances.
- Meshes of speed-increasing drives.
- Meshes designed to carry maximum power for the given weight allowance. (This type of gearing is usually designed to achieve the best balance in strength, wear, specific sliding, pitting, or scoring.)
- Meshes in which an absolute minimum of energy loss through friction is to be achieved.

3.1.3.1 Addendum Modification for Gears Having Small Number of Teeth

Undercutting is one of the most serious problems occurring in gearing having small numbers of teeth. The amount that gears with small numbers of teeth should be enlarged (made long addendum) to avoid undercut has been standardized by the AGMA. The values of modification are based on the use

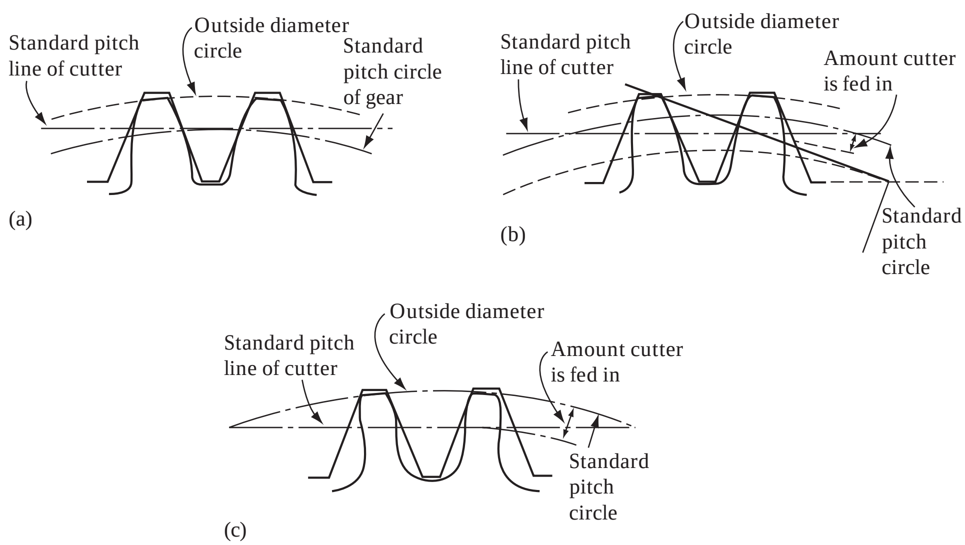


FIGURE 3.8 Cutting short-addendum teeth: (a) standard tooth; (b) short-addendum tooth; and (c) short-addendum (abnormal) tooth.

of a hob or a rack-type cutter and as a result are more than adequate for gears cut with circular, shaper-type cutters. The values of addendum modification recommended for each number of teeth are shown in Table 3.1.

When two gears, each containing a small number of teeth, must be operated together, it may be necessary to make both members long addendum to avoid undercut. Such teeth will have a tooth thickness which is larger than standard and which will necessitate the use of greater-than-standard center distance for the pair.

The modifications to avoid undercut fall into three categories.

- If both members have fewer teeth than the number critical to avoid undercut, increase the center distance so that the operating pressure angle is increased. Then get appropriate values of the addendum and the whole depth for the pinion and for the gear.
- If the pinion member has fewer teeth than the critical number, and the mating gear has considerably more, the usual practice is to decrease the addendum of the gear by the amount proportional to the amount that the pinion is increased. This results in a pair of gears free from undercut that will operate on a standard center distance.
- If the pinion member has fewer teeth than the critical number, and the gear just slightly more, a combination of the first two practices may be employed.

An alternate is to increase the pinion addendum by the required amount and increase the center distance by an equivalent amount to make it possible to use a standard gear.

3.1.3.2 Speed-Increasing Drives

Most gear trains are speed reducing (torque increasing), and most data on gear tooth proportions are based on the requirements of this type of gear application. The kinematics of speed-increasing drives is somewhat different, and as a result, special tooth proportions should be considered for this type of gear application. As in the case of conventional drives, the problems to be discussed here are most serious in meshes involving small number of teeth.

The first of these problems involves the tendency of the tip edge of the pinion tooth to gouge into the flank of the driving gear tooth. This gouging can come about as a result of spacing errors in the teeth of either member which allows the flank of the gear tooth to arrive at the theoretical contact point on the line of action before the pinion does. The pinion tooth has to deflect to get into the right position or it will gouge off a sliver of gear tooth side. If the gears are highly loaded, the unloaded pinion tooth entering the mesh will be out of position (lagging), since it is not deflected. The gear tooth is in effect slightly ahead of where it should be. The result is the same as if the gear tooth had an angular position error. The bearing and lubricating problems at the beginning point of contact are particularly bad. The edge of the pinion tooth tends to act as a scrapper and removes any lubricating film that may be present, for some distance along the flank of the tooth.

One possible solution to this problem is to give the tip of the pinion tooth a moderate amount of tip relief. This provides a sort of sled-runner condition which is easier to lubricate and which helps the pinion tooth find the proper position relative to the gear tooth with less impact.

TABLE 3.1
Values of Addendum

	Coarse-Pitch Teeth (1–19 P_d) (20°)			Coarse-Pitch Teeth (1–19 P_d) (25°)			Fine-Pitch Teeth (20 P_d and Finer) (20°)		
No. of Teeth in Pinion	α_P Pinion	α_G Gear	Recommended Min. No. of Teeth	α_P Pinion	α_G Gear	Recommended Min. No. of Teeth	α_P Pinion	α_G Gear	Recommended Min. No. of Teeth
7	–	–	–	–	–	–	1.4143[a]	0.4094	42
8	–	–	–	–	–	–	1.4369[a]	0.4679	39
9	–	–	–	–	–	–	1.4190[a]	0.5264	36
10	1.468	0.532	25	1.184	0.816	15	1.4151	0.5849	33
11	1.409	0.591	24	1.095	0.905	14	1.3566	0.6434	30
12	1.351	0.649	23	1.000	1.000	12	1.2982	0.7019	27
13	1.292	0.708	22	1.000	1.000	12	1.2397	0.7604	25
14	1.234	0.766	21	1.000	1.000	12	1.1812	0.8189	23
15	1.175	0.825	20	1.000	1.000	12	1.1227	0.8774	21
16	1.117	0.883	19	1.000	1.000	12	1.0642	0.9358	19
17	1.058	0.942	18	1.000	1.000	12	1.0057	0.9943	18
18	1.000	1.000	–	1.000	1.000	12	1.0000	1.0000	–

Note: The values in this table are for gears of 1 diametral pitch. For other sizes, divide by the required diametral pitch. The values of the addendum shown are the minimum increase necessary to avoid undercut. Additional addendum can be provided for special applications to balance strength. See Table 3.2.

[a] These values are less than the proportional amount that the tooth thickness is increased (see Table 3.14) in order to provide a reasonable top land.

A better solution, which may also be combined with the tip modi cation, is to modify the tooth thickness and the addenda so as to get as much of the gear tooth contact zone in the arc of recess as possible.

The action of gear teeth in the arc of approach and recess may be compared to a boy pushing or dragging a stick down the street. A gear tooth driving a pinion in the arc of approach is like the case where the boy pushes the stick along ahead of him. It tends to gouge into the ground. The gear tooth action in the arc of recess is like the case where the boy drags the stick along behind him. It has no tendency to gouge in; it rides up over bumps and is easier to pull. The relative gear ef cien-cies in each case are discussed in Section 3.1.3.4.

3.1.3.3 Power Drives (Optimal Design)

As shown in other chapters, gears fail in one or more of the following ways: actual breaking of the teeth, pitting, scoring, or wearing. In the case of drives with gears of standard tooth proportions and similar metallurgy, the weakest member is the pinion, and if tooth breakage does occur, it is generally in the pinion. This is a result of the weaker shape of the pinion tooth, as well as the larger number of fatigue cycles that it accumulates. This problem can be relieved to a considerable degree by making the pinion somewhat longer and, in so doing, increasing the thickness of the teeth and also improving their shapes. If a standard center distance is to be maintained, the gear addendum is reduced to a proportional amount. If the proper values are chosen, the pinion tooth strength will be increased and the gear tooth strength somewhat reduced, which will result in almost equal gear and pinion tooth strength. This will result in an overall increase in the strength of the gear pair.

Several authorities have suggested addendum modi cations which will balance scoring, speci c sliding, and tooth strength. Unfortunately, each balance results in different tooth proportions so that the designer has to use proportions that will balance only one feature or else proportions that are a compromise. Table 3.2 gives values that are such a compromise.

Experimental data seem to indicate that a pair that is corrected to the degree that seems to be indicated by tooth layouts or by calculation for balanced tooth strength will usually result in an overcorrection of the pinion member. The gear is not as strong as the form factors seem to indicate. Notch sensitivity in higher hardness ranges seems to be a problem, especially if the gear is to experience a great number of cycles of loading.

In small numbers of teeth, the correction required to avoid undercut on gears operated on standard center distances is excessive, in many cases, in respect to equal tooth strength. An overcorrection in pinion tooth thickness can lead to an

TABLE 3.2
Values of Addendum for Balance Strength

Gear Ratio, m_G (N_G/N_P)		Addendum, a		Gear Ratio, m_G (N_G/N_P)		Addendum, a	
From	To	Pinion, a_P	Gear, a_G	From	To	Pinion, a_P	Gear, a_G
1.000	1.000	1.000	1.000	1.421	1.450	1.240	0.760
1.001	1.020	1.010	0.990	1.451	1.480	1.250	0.750
1.021	1.030	1.020	0.980	1.481	1.520	1.260	0.0740
1.031	1.040	1.030	0.970	1.521	1.560	1.270	0.730
1.041	1.050	1.040	0.960	1.561	1.600	1.280	0.720
1.051	1.060	1.050	0.950	1.601	1.650	1.290	0.710
1.061	1.080	1.060	0.940	1.651	1.700	1.300	0.700
1.081	1.090	1.070	0.930	1.701	1.760	1.310	0.690
1.091	1.110	1.080	0.920	1.761	1.820	1.320	0.680
1.111	1.120	1.090	0.910	1.821	1.890	1.330	0.670
1.121	1.140	1.100	0.900	1.891	1.970	1.340	0.660
1.141	1.150	1.110	0.890	1.971	2.060	1.350	0.650
1.150	1.170	1.120	0.880	2.061	2.160	1.360	0.640
1.170	1.190	1.130	0.870	2.161	2.270	1.370	0.630
1.190	1.210	1.140	0.860	2.271	2.410	1.380	0.620
1.210	1.230	1.150	0.850	2.411	2.580	1.390	0.610
1.231	1.250	1.160	0.840	2.581	2.780	1.400	0.600
1.251	1.270	1.170	0.830	2.781	3.050	1.410	0.590
1.271	1.290	1.180	0.820	3.051	3.410	1.420	0.580
1.291	1.310	1.190	0.810	3.411	3.940	1.430	0.570
1.311	1.330	1.200	0.800	3.941	4.820	1.440	0.560
1.331	1.360	1.210	0.790	4.821	6.810	1.450	0.550
1.361	1.390	1.220	0.780	6.811	∞	1.460	0.540
1.391	1.420	1.230	0.770	–	–	–	–

Note: Do not select values from this table for the pinion members that are smaller than those given in Table 3.1.

excessive tendency to score. As a result, the values of addendum recommended in Table 3.2 represent a compromise among balanced strength, sliding, and scoring.

Gears with teeth ner than about (generally) 20 diametral pitches cannot score, since the tooth is not strong enough to support a scoring load; therefore, the values for addendum increase in ne-pitch gears are somewhat larger than the values for coarse-pitch power gearing.

3.1.3.4 Low-Friction Gearing

In cases where a speed-increasing gear train is to transmit power or motion with the least possible loss of energy, the selection of the tooth proportions is of considerable importance. The sliding should be kept as low as possible, and as much of the tooth action should be put into the arc of recess as possible.

Figure 3.9 shows two involute curves (tooth pro les) in contact at two different points along the line of action. The direction in which the driven pinion tooth slides along the driving gear tooth is shown by the arrows. This example is a speed-increasing drive which is the most sensitive to friction between the teeth. At the pitch point (where the line of centers crosses the line of action) there is no sliding, and it is at this point that the direction of relative sliding of one tooth on the other changes.

The forces shown in Figure 3.9 are those acting on the driven pinion. The subscripts a are the values considered in the arc of approach and r are those considered in the arc of recess. The normal driving force W_N is the force that occurs at the pitch point; if there were no friction at the point of gear tooth contact, it would be the force at all other points of contact along the line of action. Since there is friction, the friction vectors f_a and f_r oppose the sliding of the gear teeth in the arcs of approach and recess. Note the change in direction due to the change in direction of sliding. The angle of friction is Φ and is assumed to be the same in both cases. The torque exerted by the shaft driving the driving gear T_{DR} manifests itself in arc of approach as

$$T_{DR_a} = W_N \times R_{NG} \qquad \text{if no friction}, \tag{3.20}$$

$$T_{DR_a} = W_{f_a} \times R_{fG_a} \qquad \text{if friction is assumed}, \tag{3.21}$$

and in the arc of approach as

$$T_{DR_a} = W_N \times R_{NG} \qquad \text{if no friction}, \tag{3.22}$$

$$T_{DR_r} = W_{f_r} \times R_{fG_r} \qquad \text{if friction is assumed}. \tag{3.23}$$

The resisting moments are, in the arc of approach,

$$T_{DN_a} = W_N \times R_{NP} \qquad \text{if no friction}, \tag{3.24}$$

$$T_{DN_a} = W_{f_a} \times R_{fP_a} \qquad \text{if friction is assumed}, \tag{3.25}$$

and the corresponding moments are, in the arc of recess,

$$T_{DN_r} = W_N \times R_{NP} \qquad \text{if no friction}, \tag{3.26}$$

$$T_{DN_r} = W_{f_r} \times R_{fP_r} \qquad \text{if friction is assumed}. \tag{3.27}$$

Note that, in all the case above, single tooth contact is assumed.

Ef ciency is output divided by input and in this case is the torque that would appear on the driven shaft when friction losses are considered, compared with the torque that would result if no losses occurred.

Equation 3.28 shows the ef ciency of the mesh (single tooth contact) for the contacts occurring in the arc of approach, and Equation 3.29 shows the ef ciency in the arc of recess:

$$E_{approach} = \frac{R_{fP_a}}{R_{fG_a}} \times \frac{R_{NG}}{R_{NP}} \tag{3.28*}$$

$$E_{recess} = \frac{R_{fP_r}}{R_{fG_r}} \times \frac{R_{NG}}{R_{NP}}. \tag{3.29}$$

In the case of speed-increasing drives, the increased ef ciency can have considerable signi cance; cases have occurred in which, for every high ratios and poor lubrication, the speed increase actually became self-locking.

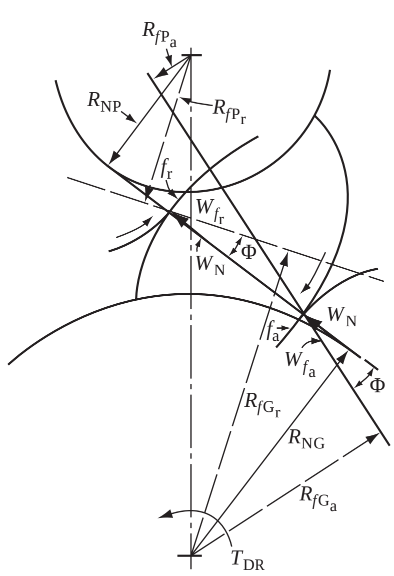


FIGURE 3.9 Effect of friction on tooth reactions.

* Equations 3.28 and 3.29 can be used in the metric system by using newtons for force and millimeters for distance. This makes the units of torque N·mm.

3.1.4 Special Design Considerations

Several special considerations should be kept in mind during the evolution of a gear design.

3.1.4.1 Interchangeability

Two types of interchangeability are related to gearing.

The first, and most generally recognized, is part interchangeability. This means that, if a part made to a specific drawing is damaged, a similar part made to the same drawing can be put in its place and can be expected to perform exactly the same quality of service. In order to achieve this kind of interchangeability, all parts must be held to carefully selected tolerances during manufacture, and the geometry of the parts must be clearly defined.

The second kind is engineering interchangeability. This type provides the means whereby a large variety of different sizes or number of teeth can be produced with a very limited number of standard tools. Parts so made may or may not be designed to have part interchangeability. The several systems of gear tooth proportions are based on engineering interchangeability. By careful design and skill in the application of each system, it is possible to design gears that will perform in almost any application and that can be made with standard tools. Only in the most exceptional cases will special tools be required. The advantages of gearing designed with engineering interchangeability are the lower tool costs and the ease with which replacement or alternate gears can be designed.

Gears procured through catalog source are good examples of gears having both part and engineering interchangeabilities.

3.1.4.2 Tooth Thickness

The thickness of the gear teeth determines the center distance at which they will operate, the backlash that they will have, and, as discussed in previous sections, their basic shape. One of the important calculations made during the design of gear teeth is the establishment of tooth thickness.

It is essential to specify the distance from the gear axis at which the desired tooth thickness is to exist. The usual convention is to use the distance that is established by the theoretical pitch circle *N*/*P*. Thus, if no other distance is shown, the specified tooth thickness is assumed to lie on the standard pitch circle. In certain cases, the designer may wish to specify a thickness, such as the chordal tooth thickness, at a diameter other than the standard pitch diameter. This specified diameter should be clearly defined on the gear drawings.

The actual calculation of tooth thickness is usually accomplished by the following procedure:

1. Theoretical tooth thickness is established.
   a. If the gears are of conventional design and are to be operated on standard center distance, the tooth thickness used is one-half the circular pitch.
   b. If special center distances are to be accommodated, Equations 3.30 and 3.31 may be used.

$$\cos\phi_2 = \frac{C}{C_2}\cos\phi, \tag{3.30}$$

where
ϕ_2—operating pressure angle
C—standard center distance
ϕ—standard pressure angle
C_2—operating center distance

$$T_P + T_G = \pi + (\text{inv}\,\phi_2 - \text{inv}\,\phi)(N_P + N_G), \qquad (3.31)^*$$

where
T_P—tooth thickness of pinion member (at 1 diametral pitch)
T_G—tooth thickness of gear member (at 1 diametral pitch)
N_P—number of teeth in pinion
N_G—number of teeth in gear

   c. If special tooth thicknesses for tooth strength considerations are required, the total thickness (sum of pinion and gear) must satisfy Equation 3.31. If standard centers are used, the sum of the modified thickness must be equal to 1 circular pitch.
2. After the theoretical tooth thickness is established, the allowance for backlash is made.
   a. *Backlash* allowance may be equally shared by pinion and gear. In this case, the theoretical tooth thickness of each member is reduced by one-half the backlash allowance.
   b. In case of pinions with small numbers of teeth which have been enlarged to avoid problems of undercut, it is customary to take all the backlash allowance on the gear member. This avoids the absurdity of increasing the tooth thickness to avoid undercut and then thinning the teeth to introduce backlash.

   In case a:

$$T_{\text{actual}} = T_{\text{theoretical}} - \frac{B}{2}, \tag{3.32}$$

where B is the backlash allowance.
In case b:

$$T_{\text{actual, pinion}} = T_{\text{theoretical}} \tag{3.33}$$

$$T_{\text{actual, gear}} = T_{\text{theoretical}} - B. \tag{3.34}$$

3. The teeth are lastly given an allowance for machining tolerance. This tolerance gives the machine

* The best way to use this equation is to start with C being the center distance for 1 diametral pitch (or for 1 module in metric dimensions). The C_2 is computed by Equation 3.30. The tooth thicknesses determined are not at the operating pitch diameters but are at the 1 diametral pitch center distance of C.

operator a size or a processing tolerance. Usually, this is a unilateral tolerance.

It is beyond the scope of this general-purpose book on gear technology to go deep into gear design details. Chapter 10 has the following sample problems worked out:

Problem 1: Drop-tooth design, fixed center distance
Problem 2: Fixed distance, standard tools
Problem 3: Fixed distance, adjust helix angle standard tools
Problem 4: Fixed design, standard tools, adjust center distance for tooth proportions

3.1.4.3 Tooth Profile Modifications

Errors of manufacture, deflections of mountings under load, and deflections of the teeth under load all combine to prevent the attainment of true involute contact in gear meshes. As a result, the teeth do not perform as they are assumed to by theory. Premature contact at the tips or excessive contact pressures at the ends of the teeth give rise to noise and/or gear failures. In order to reduce the causes of excessive tooth loads, profile modification is a usual practice. Remember: Only base pitch preserving modification of the tooth profile is allowed.

3.1.4.4 Transverse of Profile Modification

It is customary to consider the shape of the individual tooth profile. Actually, an operating gear presents to the mating gear tooth profiles whose shapes are distorted by errors in the profile shape (involute) and in tooth spacing, as well as by deflections due to contact loads acting at various places along the tooth. Ideally, the teeth under load should appear to the mating gear to have perfect involute profiles and to have perfect spacing. This would require a tooth profile when subjected to a load. Since loads vary, this is not practical. The usual practice is to give tip or root profile modifications to otherwise correct involute profiles that will be distorted by contact loads.

Tip modification usually takes the form of a sliding thinning of the tip of the tooth, starting at a point about halfway up the addendum. The amount of this modification is based on the probable accumulated effects of the allowances for errors of gear manufacture and allowances for deflection under load.

3.1.4.5 Allowances for Errors of Gear Manufacture

The correct involute profile may not be achieved as a result of manufacturing tolerances in the cutter used to cut the gear teeth or in the machine guiding the cutter. Errors in spacing may also be introduced by the cutter or the machine. The result is that the tips of the teeth will attempt to contact the mating gear too soon or too late. Tip modification produces a sort of sled-runner shape to help guide the teeth into full contact with the least impact.

3.1.4.6 Allowances for Deflection under Load

Although the teeth of a gear may have a correct profile under static conditions, the imposed loads may deflect the teeth in engagement to such a degree that the teeth that are just entering the mesh will not be in relative positions that are correct for smooth engagement. These tooth deflections cause two errors: The actual tooth profiles are not truly conjugate under load and therefore do not transmit uniform angular motion, and the spacing or the relative angular placement of the driving teeth relative to the driven teeth is such that smooth tooth engagement cannot take place. High dynamic loads are the result.

Several methods may be used to compensate for these effects. The most usual is to provide tip relief as discussed earlier. The flanks of the teeth may also be relieved. Tip and flank reliefs assume that the normal tooth would tend to contact too soon. If the tips of the driven gear are made slightly thin, the tooth will be able to get into a position on the line of contact before contact between that tooth and its driving mate actually occurs. The sled-runner effect of the tip modification will allow the tooth to gradually assume full contact load.

In the case of spur gearing, the amount of tip relief should be based on the sum of the allowance for probable tooth-to-tooth spacing errors and for assumed deflection of the teeth already in mesh. Equations 3.35 and 3.36 give good first approximations of the amount of tip relief required. The values obtained by these equations should be modified by experience for the best overall performance.

Modification at first point of contact:

$$\text{Modification} = \frac{\text{driving load (lb)} \times 3.5 \times 10^{-7}}{\text{face width (in.)}}. \qquad (3.35)$$

Remove stock from tip of driven gear.

Modification at last point of contact:

$$\text{Modification} = \frac{\text{driving load (lb)} \times 2.0 \times 10^{-7}}{\text{face width (in.)}}. \qquad (3.36)$$

Remove stock from tip of driven gear.

In general, gear teeth that carry a load in excess of 2000 lb per inch of face width for more than 1,000,000 cycles should have modification. Those under 1000 lb per inch of face width do not generally require modification.

Bevel gear teeth are often modified to accommodate mounting misalignments, tooth errors, and deflections due to load. The geometry of bevel gear teeth allows profile modifications both along the length of the teeth and from root to tip. In the case spiral and Zerol* bevel gears cut with face mill-type cutters, the contacting faces of the gear and pinion teeth can be easily made to a slightly different radius of curvature on each member. This is equivalent to the crowning of spur gears and is called *mismatch*.

Tips of the teeth may be given relief, and in some cases, the root of the flank is relieved (undercut). This is most commonly done on spiral bevel gears and hypoid gears that are to be lapped. It is done by cutters having a special protuberance called Toprem† cutters.

Mismatch is calculated into the machine settings and is therefore beyond the scope of this chapter.

* Zerol is a trademark registered by Gleason Works, Rochester, New York.
† Toprem is a trademark registered by Gleason Works, Rochester, New York.

3.1.4.7 Axial Modifications

In general, it is expected that a gear tooth (of a spur gear) will carry its driving load across the full face width.

Because of deflections in shafts, bearings, or mountings, when under load, or because of errors in the manufacturing of these parts or in the gears, the teeth may not be quite parallel, and end loading may result. Since a heavy load on the end of a gear tooth will often cause it to break off, attempts to avoid loading by relieving the ends of the teeth (crowning) are often made. A gear tooth that has been crowned is slightly thicker at the center section of the tooth than at the ends when measured on the pitch cylinder.

In effect, crowning allows a rocking chair–like action between the teeth when the shafts deflect into increasingly nonparallel positions. Heavy concentrations of load at the ends of the teeth are avoided. The fact that the whole face cannot act when the shafts are parallel requires that the load imposed upon a set of crowned gears be less than what could be carried on a similar noncrowned pair if parallel teeth could be maintained.

In general, the ends of the crowned gears are made to be 0.0005 to 0.002 in. thinner at the ends as compared with the middle, when measured on the circular-arc tooth thickness.

Spur and helical gearings are crowned by means of special attachments on the gear tooth finishing machines.

Bevel gearing is crowned, in effect, by the shape of the cutting tools and the way in which they are driven in relation to the teeth. Coniflex* teeth are straight bevel teeth cut on special generating machines which produce lengthwise crowning. Spiral and Zerol bevel gears and hypoid gears are given the effect of crowing by the use of a different radius of lengthwise curvature on the convex and the concave sides of the teeth.

Throated worm gears can be given the effect of crowning by the use of slightly oversized hobs.

Face gears are given the effect of crowning by the selection of a cutter having one or more teeth than the mating gear.

3.1.4.8 Root Fillets

The shape and minimum radius of curvature that the root fillet of a tooth will have depends on the type and the design of the cutting tool used to produce the gear tooth. The shape of the root fillet, as well as its radius, and the smoothness with which it blends into the root land and the active profile of a gear tooth can have a profound effect on the fatigue strength of the finished gear. The radius of the fillet at the critical cross section of the tooth is controlled on drawings by specifying the minimum acceptable fillet radius. The point of minimum radius occurs almost adjacent to the root land in the case of gears cut by hobs. An equation giving a reasonable evaluation or the minimum radius for teeth cut by hobs is

$$r_\mathrm{f} = 0.7\left[r_\mathrm{T} + \frac{(h_\mathrm{t} - a - r_\mathrm{T})^2}{\dfrac{d}{2\cos^2\psi} + h_\mathrm{t} - (a + r_\mathrm{T})}\right], \qquad (3.37)^\dagger$$

where

r_f—minimum calculated fillet radius produced by hobbing or generating grinding
r_T—edge radius of the generating rack, hob, or grinding wheel
a—addendum of gear
h_t—whole depth of gear
d—pitch diameter of gear
ψ—helix angle (use 0° for spur gears)

The edge radius specified for the generating tool will, in general, depend on the service the gear is to perform or on special manufacturing considerations. Table 3.3 shows suggested values of edge radius for various gear applications.

Other types of manufacturing tools can be designed to produce the minimum fillet radius as obtained from Equation 3.37. The shape of the fillet will be somewhat different, however.

A common method of checking the minimum radius of the coarser pitch gears is to lay a pin on the fillet zone and note that the contact is along a single line.

The constant 0.7 is to allow a reasonable working tolerance to the manufacturer of the tools. The edge breakdown of hobs and cutters will tend to increase the radius produced. This is particularly true of hobs and cutters for very fine–pitch gears.

3.1.4.9 Effective Outside Diameter

It is customary to consider the outside diameter of a gear as the outer boundary of the active profile of the tooth. In several cases, this approximation is not good enough.

- In very fine–pitch gears that have been burr brushed, the tip round may be quite large in proportion to the size of the teeth even though it is only a few thousands in radius by actual measurement. Since no part of this radius can properly contact the mating tooth, the outside diameter is, from the standpoint of conjugate action, limited to the diameter where the tip radius starts. Effective outside diameter should be used instead of outside diameter in calculations of contact ratio.
- In some gear meshes in which the pinion member contains a small number of teeth, the tips of the gear teeth may be found to be extending into the pinion spaces to a depth greater than that bounded by the line of action. In Figure 3.6b, the dimension $D_\mathrm{oP}/2$ is the maximum effective diameter to the pinion. The actual outside diameter of the pinion may exceed this value, however. In calculating contact ratio, the effective diameter as limited by the pinion base circle and center distance should be used.

3.1.4.10 Width of the Tip of the Tooth

The tooth thickness at the tip of the tooth is a convenient index of the quality of a gear design. For most power gearing applications, the thickness of the gear tooth should not be more than 1½ to 2 times that of the pinion tooth at the tip.

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* Coniflex is a trademark registered by Gleason Works, Rochester, New York.
† Equation 3.37 is valid for the metric system by using millimeters instead of inches.

TABLE 3.3
Basic Tooth Proportions for Helical Gears

Tooth Form	Pitch of Teeth					Pressure Angle		Depth of Teeth		Helix Angle	Addendum	Tooth Thickness (Arc)	Edge Radius of Generating Rack
	Diametral		Circular										
	Normal	Transverse	Normal	Transverse	Axial	Normal	Transverse	Working	Whole				
No.	P_{nd}	P_d	p_n	p	p_x	ϕ_n	ϕ	h_k	h_t	ψ	a	t	r_T
1	1.03528	1	3.03454	3.14259	11.72456	19°22′12″	20°	2.0000	2.3500	15°	1.0000	1.5710	0.350
2	1.08360	1	2.89190	3.14259	7.40113	18°31′22″	20°	2.0000	2.3500	23°	1.0000	1.4460	0.350
3	1.08360	1	2.89185	3.14259	7.40113	18°31′22″	20°	1.8400	2.2000	23°	0.9200	1.4460	0.350
4	1.15470	1	2.72070	3.14259	5.44140	17°29′43″	20°	1.7400	2.0500	30°	0.8700	1.3604	0.300
5	1.22077	1	2.57340	3.14259	4.48666	16°36′06″	20°	1.6400	1.9500	35°	0.8200	1.2867	0.300
6	1.41421	1	2.22144	3.14259	3.14259	14°25′58″	20°	1.4200	1.7000	45°	0.7100	1.1107	0.250
7	1	–	–	–	–	23°14′07″	–	2.0000	2.2500	–	1.0000	–	0.240
8	1	–	–	–	–	23°14′07″	–	2.0000	2.3300	–	1.0000	–	0.270
9	1	–	–	–	–	21°58′50″	–	2.0000	2.2700	–	1.0000	–	0.310
10	1	–	–	–	–	21°58′50″	–	2.0000	2.3500	–	1.0000	–	0.300
11	1	–	–	–	–	21°58′50″	–	2.0000	2.4000	–	1.0000	–	0.290
12	1	–	–	–	–	21°58′50″	–	2.0000	2.4500	–	1.0000	–	0.275
13	1	–	–	–	–	18°31′35″	–	1.6300	2.0180	–	0.8150	–	0.204
14	1	–	–	–	–	24°39′57″	–	1.8280	2.1170	–	0.9140	–	0.158
15	1	–	–	–	–	24°51′32″	–	–	2.1570	–	0.9260	–	0.220
16	1	–	–	–	–	24°54′59″	–	–	2.1550	–	0.9230	–	0.264
17	1	–	–	–	–	15°00′00″	–	–	2.6150	–	1.1580	–	0.410
18	1	–	–	–	–	–	20°	2.0000	2.3500	–	1.0000	–	0.350
19	1	–	–	–	–	–	20°	2.0000	2.200 + $0.002P_d$	–	1.0000	–	–
20	1.15470	1	2.72070	3.14259	5.44140	22°30′00″	25°33′41″	1.7400	2.0500	30°	0.8700	–	0.250
21	1.10338	1	2.84725	3.14259	6.73717	25°00′00″	27°13′35″	1.5000	1.7500	25°	0.7500	–	0.200

If the tooth thickness (arc), at the pitch diameter (standard) is known, the following equation will give its thickness at the tip:

$$t_o = D_o\left(\frac{t}{D} + \operatorname{inv}\phi - \operatorname{inv}\phi_o\right), \tag{3.38}$$

$$\cos\phi_o = \frac{D\cos\phi}{D_o}, \tag{3.39}$$

where

t_o—tooth thickness at outside diameter D_o
D_o—outside diameter of gear, or diameter where tooth thickness is wanted
t—arc tooth thickness at D, or at known diameter
D—standard pitch diameter, or diameter where t is known
ϕ—standard pressure angel, or pressure angle where tooth thickness t is known
ϕ_o—pressure angle at outside diameter or at diameter where tooth thickness is wanted

3.1.4.11 Pointed Tooth Diameter

An independent method of checking the quality of long- and short-addendum gear designs is to calculate the diameter at which the teeth would come to a point. If this value is smaller than the value of the outside diameter chosen by other means, the design should be recalculated. The following equations each provide one method of calculating pointed tooth diameters.

$$\operatorname{inv}\phi_{oP} = \frac{t}{D} + \operatorname{inv}\phi, \tag{3.40}$$

$$D_{oP} = \frac{D\cos\phi}{\cos\phi_{oP}}, \tag{3.41}$$

where ϕ_{oP} is the pressure angle at pointed tooth diameter, and D_{oP} is the pointed tooth diameter. See Equation 3.39 for other symbols.

3.1.4.12 Purpose of Backlash

In general, backlash is the lost motion between mating gear teeth. It may be measured along the line of action or on the pitch cylinder of the gears (transverse backlash) and, in the case of helical gears, normal to the teeth.

In a set of meshing gears, the backlash that exists is the result of the actual center distance at which the gears operate, and the thickness of the teeth. Changes in temperature, which may cause differential expansion of the gears and mountings, can produce appreciable changes in backlash.

When establishing the backlash that a set of gears will require, the following should be considered:

- The minimum and maximum center distance values are the result of tolerance buildings in the distance between the bores supporting the bearings on which the gears are mounted as well as the basic or design values. Antifriction bearings, for example, have runout between the bore and the inner ball path. As the shaft and the inner race of the bearing rotate, and the outer race creeps in the housing, the center distance will vary by the amount of the eccentricities of these bearing elements.
- The thickness of the teeth, as measured at a fixed distance from the center on which the gear rotates, will vary because of gear runout. Also, the worker cutting the gear is given a tooth thickness tolerance to work with, which introduces tooth thickness variations from one gear to the next.

The minimum backlash will occur when all the tolerances react all at the same time to give the shortest center distance and the thickest teeth with the high points of gear runout. The maximum backlash will occur when all the tolerances move in the opposite directions.

Design backlash is incorporated into the mesh to ensure that contact will not occur on the nondriving sides of the gear teeth. Although backlash may be introduced by increasing the center distance, it is usually introduced by thinning the teeth. The minimum value should be at least sufficient to accommodate for a lubricating film on the teeth.

Sometimes a statistical approach is used, since there are a sufficient number of tolerances involved; thus, the introduced design backlash may not be numerically as large as the possible adverse buildup of tolerances. This approach is particularly handy in instrument gearing, since the allowable maximum backlash has to be usually held to a minimum.

By definition, backlash cannot exist in a single gear. Backlash is a function of the actual center distance on which the gears are operated, and the actual thicknesses of the teeth of each gear.

It is customary to use generally recognized values of center-distance tolerance and gear tooth tolerances for power gearing. If this is done, and the calculated tooth thicknesses are reduced by the amounts of design backlash as shown in Table 3.4, satisfactory gears should result. In the case of instrument gearing, either the values in Table 3.4 may be used or the tooth thickness actually required and the resulting maximum backlash may be calculated.

The backlash that is measured in gears under actual operation will in all probability be considerably larger than the values given in Table 3.4, since these values are for design and do not include a correction for normal machining tolerances.

3.1.4.13 Backlash: Recommended Values

The dimensions shown in Figure 3.2 are the distances from the axis of spur, helical, and internal gears at which the active profiles of the teeth begin. Form diameter is the lowest point (spur and helical gearings) at which the mating tooth can contact the active profile (see Figure 3.5).

TABLE 3.4
Recommended Backlash Allowance for Power Gearing

Normal Diametral Pitch, P_{nd}	Center Distance (in.) (3.93701×10^{-2} mm)						
	0–5	5–10	10–20	20–30	30–50	50–80	80–120
1/2	–	–	–	–	0.045	0.060	0.080
1	–	–	–	0.035	0.040	0.050	0.060
2	–	–	0.025	0.030	0.035	0.045	0.055
3	–	0.018	0.022	0.027	0.033	0.042	–
4	–	0.016	0.020	0.025	0.030	0.040	–
6	0.008	0.010	0.015	0.020	0.025	–	–
8	0.006	0.008	0.012	0.017	–	–	–
10	0.005	0.007	0.010	–	–	–	–
12	0.004	0.006	–	–	–	–	–
16	0.004	0.005	–	–	–	–	–
20	0.004	–	–	–	–	–	–
32	0.003	–	–	–	–	–	–
64	0.002	–	–	–	–	–	–

3.2 STANDARD SYSTEMS OF GEAR TOOTH PROPORTIONS

A standard or a system of gear tooth proportions provides a means of achieving engineering interchangeability for gears of all numbers of teeth of a given pitch and pressure angle. Because of the large variety of tooth proportions that are possible, it has been found desirable to standardize on a limited number of tooth systems. The systems specify the various relationships among tooth thicknesses, addendum, working depth, and pressure angle.

The AGMA and the American Standards Association (ASA) have provided standard systems for various types of gear teeth. When undertaking the design of new equipment, designers should be sure that they are working with the latest standards. The earlier systems are shown in the following sections to aid designers confronted with replacement gear problems. Designers should also be aware of the limitations of each of the systems. These are clearly stated in the discussion of each system. When a design falls outside these limitations, designers will have to employ tooth modi cations required by the design. Since these standards are not intended to be absolutely rigid, designers are at liberty to modify the tooth proportions within limits to achieve an optimum design. Previously, information was provided to indicate the more common tooth modi cations and the ends achieved by these modi cations. When departing from the proportions given in the standard, consideration should be given to the reasons giving rise to these departures and also the consequences. In some cases, departures will require special tooling, whereas others, if properly handled, will not. The following sections provide the basic data covered by most of the systems in present-day use. For additional information about these systems, reference should be made to the original standards listed. In each of the following systems, the tooth proportions are shown in terms of the basic rack of that system. In each case, all gears designed to the basic tooth proportions will have engineering interchangeability.

3.2.1 Standard Systems for Spur Gears

The following data are based on the information contained in AGMA standards for 20-Degree Involute Fine-Pitch System for Spur and Helical Gears, and for Tooth Proportions for Coarse-Pitch Involute Spur Gears.

3.2.1.1 Limitations in the Use of Standard Tables

Caution should be exercised in using the data contained in Table 3.5. The items shown apply only to gears that meet the following requirements:

- Standard-addendum gears must exceed the minimum numbers of teeth shown in Table 3.6.
- Long- and short-addendum designs, as derived from Tables 3.5, 3.7, and 3.8, are to be used for *speed-reducing drives only.*

The tooth proportions that result from the data shown in this section or from the application of the original standards will be suitable for most speed-reducing (torque-increasing) applications. All gears, including pinions with small numbers of teeth, designed in accordance with the procedure shown will be free from undercut. In order to avoid the problems of undercut in pinions having fewer than the minimum standard number of teeth, each system shows proportions for long- and short-addendum teeth. Gears designed with long- and short-addendum teeth cannot be interchangeably operated interchangeably on standard centers. In general, such gears should be designed to operate only as pairs. These gears can be cut with the same generating-type cutters and checked with the same equipment as standard-addendum gears. The proportions of long and short addenda shown in Table 3.1 are based on avoiding undercut.

TABLE 3.5
Basic Tooth Proportions of Spur Gears, AGMA and ASA Standard System

Symbol	Item	Coarse Pitch (Coarser than 20*P*), Full Depth		Fine Pitch (20*P* and Finer), Full Depth	Explanation No.
ϕ	Pressure angle	20°	25°	20°	1
a	Addendum (basic[a])	$\frac{1.000}{P}$	$\frac{1.000}{P}$	$\frac{1.000}{P}$	2
b	Dedendum (min.) (basic[b])	$\frac{1.250}{P}$	$\frac{1.250}{P}$	$\frac{1.200}{P}+0.002''$	3
h_k	Working depth	$\frac{2.000}{P}$	$\frac{2.000}{P}$	$\frac{2.000}{P}$	4
h_t	Whole depth (min.) (basic[b])	$\frac{2.250}{P}$	$\frac{2.250}{P}$	$\frac{2.200}{P}+0.002''$	5
t	Circular tooth thickness (basic[a])	$\frac{\pi}{2P}$	$\frac{\pi}{2P}$	$\frac{1.5708}{P}$	6
r_f	Fillet radius (in basic rack)	$\frac{0.300}{P}$	$\frac{0.300}{P}$	Not standardized	7
c	Clearance (min.) (basic[a])	$\frac{0.250}{P}$	$\frac{0.250}{P}$	$\frac{0.200}{P}+0.002''$	8
c	Clearance, shaved or ground teeth	$\frac{0.350}{P}$	$\frac{0.350}{P}$	$\frac{0.350}{P}+0.002''$	9
N'_P		**Min. Numbers of Teeth**			
	Pinion	18	12	18	10
	Pair	36	24	–	11
t_o	Min. width of top land	$\frac{0.250}{P}$	$\frac{0.250}{P}$	Not standardized	12
–		**Reference Standards**			
	AGMA	201.02	201.02	207.04	13
	ASA			B6.19	14

[a] These values are basic, for equal-addendum gearing. When the gearing is made long and short addendum, these values will be altered.
[b] These values are minimum. Shaved or ground teeth should be given proportions suitable to these processes.

TABLE 3.6
Minimum Number of Pinion Teeth versus Pressure Angle and Helix Angle Having No Undercut

	Minimum Number of Teeth to Avoid Undercut When Normal Pressure Angle ϕ_n (°)			
Helix Angle (°)	**14½**	**20**	**22½**	**25**
0 (spur gear)	32	17	14	12
5	32	17	14	12
10	31	17	14	12
15	29	16	13	11
20	27	15	12	10
23	25	14	11	10
25	24	13	11	9
30	21	12	10	8
35	18	10	8	7
40	15	8	7	6
45	12	7	5	5

Note: Addendum $1/P_d$; whole depth $2.25/P_d$.

Such teeth will not have the optimum balance of strength and wear. Slightly different proportions can be used to achieve equal sliding, balanced strength, or reduction in the tendency to score. Table 3.2 shows tooth proportions that have a good balance of strength and a minimum tendency to score.

3.2.1.2 Standard Tooth Forms That Have Become Obsolete

Because industry design standards are continuously reviewed by the sponsoring organizations to ensure that the standards embody the most modern technology, there now exists a group of obsolete tooth form standards. On occasion, a designer is confronted with a situation in which a replacement gear must be made to mesh in a gear train conforming to one of these earlier standards.

Table 3.9 shows basic data for some of the obsolete standards. The use of these data for new designs is not recommended.

3.2.1.3 Brown and Sharp System

The Brown and Sharp system (B and S system), see Table 3.9, was developed by the Brown and Sharp Company to replace

TABLE 3.7
General Recommendations on Numbers of Pinion Teeth for Spur and Helical Power Gearings

Range of No. of Pinion Teeth	Ratio, m_G	Diametral Pitch, P_d	Hardness
19–60	1–1.9	1–19.9	200–240 BHN[a]
19–50	2–3.9		
19–45	4–8		
19–45	1–1.9	1–19.9	Rockwell C 33–38
19–38	2–3.9		
19–35	4–8		
19–30	1–1.9	1–19.9	Rockwell C 38–63
17–26	2–3.9		
15–24	4–8		

Note: The maximum number of teeth in the range is based on providing a suitable balance between strength and wear capacity. The minimum number of teeth is intended to require either no enlargement to avoid undercutting or no more enlargement than necessary. Small numbers of teeth in the pinion require special tooth proportions to avoid undercut. Gears containing prime numbers of teeth of 101 and over may be dif cult to cut on the gear-manufacturing equipment available. In general, prime numbers over 101 and all numbers over 200 should be checked with the shop to be sure the teeth can be made.

[a] BHN: Brinell hardness number.

the cycloidal tooth system. It was therefore given similar tooth proportions. It was intended to be cut by form-milling cutters. The departure from the true involute curve in this system is made to avoid the problems of undercut in pinions having small numbers of teeth. Backlash is achieved by feeding the cutter deeper than the standard.

The AGMA adopted the principal features of this system in 1932 as the *composite system*. Present-day form-milling cutters are normally made to this system.

3.2.1.4 AGMA 14.5° Composite System

The AGMA 14.5° composite system, based on the B and S system, was standardized in 1932 as an AGMA standard. It is interchangeable with the B and S system. Replacement parts may be designed from the data in Table 3.9.

3.2.1.5 Fellows 20° Stub Tooth System

In order to achieve a stronger tooth form for special drives, the Fellows Gear Shaper Company developed a stub tooth system in 1898. This system avoided the problem of tooth interference by the combined means of higher pressure angle and smaller values of addendum and dedendum. The pitch was speci ed as a combination of two standard diametral pitches thus the 10/12 (read "ten-twelve"). The circular pitch, pitch diameter, and tooth thickness are based on the rst number, which is 10, and are the same as for a standard 10-diametral-pitch gear. The addendum, dedendum, and clearance, however, are based on the second number (12) and are the same as for a 12-diametral-pitch gear.

TABLE 3.8
Equations and References for Addendum Calculations, Spur Gears

Type of Tooth Design	Operating Condition	Pinion	Gear
Standard or equal addendum	Numbers of teeth in gear and pinion are greater than minimum numbers shown Tables 3.6 and 3.11.	(a) $\frac{1.00}{P_d}$; see Table 3.5 (b) Value from Table 3.13	(a) $\frac{1.00}{P_d}$; see Table 3.5 (b) Value from Table 3.13
Long and short addendum	Numbers of teeth in pinion are less than minimum numbers shown in Table 3.6 and numbers of teeth in gear are more than minimum numbers shown in Table 3.11.	(c) $\frac{\text{Table 3.1 value}}{P_d}$ or (d) $\frac{\text{Table 3.2 value}}{P_d}$ if not less than (c)	(c) $\frac{\text{Table 3.1 value}}{P_d}$ or (d) $\frac{\text{Table 3.2 value}}{P_d}$
	Numbers of teeth in pinion are less than minimum numbers shown in Table 3.11 and numbers of teeth in gear are less than minimum numbers of teeth shown in Table 3.11.	Suf ciently increase center distance to avoid undercut, in pinion and gear (e) $\frac{\text{Table 3.1 value}}{P_d}$	See Section 3.3
	Gearing designed for balanced strength	(f) $\frac{\text{Table 3.2 value}}{P_d}$ Note: If value is less than given by Table 3.1, check for undercut	$\frac{\text{Table 3.2 value}}{P_d}$ Note: If value is less than given by Table 3.1, check for undercut
	Designed for nonstandard center distance	(g) Calculate tooth thickness to meet required center distance from Equation 3.75.	

Note: The values in this table are for gears of 1 diametral pitch. For other sizes, divide by the required diametral pitch. The values of addendum shown are the minimum increase necessary to avoid undercut. Additional addendum can be provided for special applications to balance strength. See Table 3.2.

TABLE 3.9
Tooth Proportions of Spur Gears, Obsolete System

Symbol	Item	B and S System	AGMA 14.5° Composite System	Fellows 20° Stub System	AGMA Full-Depth Composite System
ϕ	Pressure angle (°)	14½	14½	20	14½
a	Addendum (basic[a])	$\frac{1.000}{P}$	$\frac{1.000}{P}$	$\frac{0.800}{P}$	$\frac{1.000}{P}$
b	Dedendum (basic[b])	$\frac{1.157}{P}$	$\frac{1.157}{P}$	$\frac{1.000}{P}$	$\frac{1.157}{P}$
h_k	Working depth	$\frac{2.000}{P}$	$\frac{2.000}{P}$	$\frac{1.600}{P}$	$\frac{2.000}{P}$
h_t	Whole depth (min.) (basic[b])	$\frac{2.157}{P}$	$\frac{2.157}{P}$	$\frac{1.800}{P}$	$\frac{2.157}{P}$
t	Circular tooth thickness (basic[a])	$\frac{\pi}{2P}$	$\frac{\pi}{2P}$	$\frac{\pi}{2P}$	$\frac{\pi}{2P}$
r_f	Fillet radius (in basic rack)	$\frac{0.157}{P}$	$\frac{0.157}{P}$	Not standardized	1.33 × clearance
c	Clearance (min.) (basic[a])	$\frac{0.157}{P}$	$\frac{0.157}{P}$	$\frac{0.200}{P}$	$\frac{0.157}{P}$
c	Clearance, shaved or ground teeth				
		Min. Numbers of Teeth[b]			
N_P'	Pinion	32	–	14	32
	Pair	64	–	–	64
	Reference standard AGMA	–	201.02A	–	201.02A

Note: Tooth proportions for fully interchangeable gears operate on standard center distances (see Table 3.5).
[a] These values are basic for equal-addendum gearing. When the gearing is made long- and short-addendum these values will be altered.
[b] These values are minimum. Shaved or ground teeth should be given proportions suitable to these processes

3.2.1.6 AGMA 14.5° Full-Depth System

In the AGMA 14.5° full-depth system, the tooth proportions are identical with those of the 14.5° composite system. The sides of the rack teeth are straight lines and therefore produce involute gear tooth pro les in the generating process. The minimum number of teeth to obtain full tooth action is 31 unless the teeth are modi ed.

3.2.1.7 Cycloidal Tooth Profiles

The cycloidal tooth pro le is no longer used for any types of gears except clockwork and certain types of timer gears. A combination of involute and cycloidal tooth pro les is found in the now-obsolete composite system. Clockwork gearing is based on the cycloid but has been greatly modi ed for practical reasons.

The cycloidal tooth is derived from the trace of a point on a circle (called the *describing circle*) rolling without slippage on the pitch circle of the proposed gear. The addendum portions of the tooth are the trace of a point on the describing circle rolling on the outside of the pitch circle. This trace is an epicycloid. The dedendum portion of the gear tooth is formed by a trace of a point on the describing circle rolling on the inside of the pitch circle. This trace is a hypocycloid.

Interchangeable systems of gears must have describing circles that are identical in diameter, and teeth that have the same circular pitch. The faces and the   anks of the teeth must be generated by describing circles of the same size. The tooth proportions for cycloidal teeth were made to the same proportions as were adopted by the B and S system for gears.

The cycloidal system does not have a standard pressure angle. The operating pressure angle varies from zero at the pitch line to a maximum at the tips of the teeth.

In order to achieve correct meshing, the gears must be operated on centers that will maintain the theoretical pitch circles in exact contact. This was one of the major disadvantages of the cycloidal tooth form.

If the diameter of the describing circles were made equal to the radius of the pitch circle of the smallest pinion to be used in the system, the   anks of the teeth would be radial. This member, in effect, establishes the describing circle diameters of each system. In general, the systems for industrial gears were based on pinions of 12 to 15 teeth.

3.2.1.8 Clockwork and Timer Tooth Profiles

The tooth form of most clockwork and timer gearings involving some type of escapement differs from other types of gearing because of the peculiar requirements imposed by the operating conditions. Two requirements of this type of gearing are outstanding:

- The gearing must be of the highest possible efficiency.
- The gearing is speed increasing, that is, larger gears driving smaller pinions, with high ratios (between 6:1 and 12:1) due to the need for minimizing the number of gear wheels and for economizing space.

As is discussed in Section 3.1.3.4, the most efficient mode of tooth engagement is found in the arc of recess. In the case of speed-increasing involute profile drives, this means short-addendum pinions. This requirement may be in direct conflict with the need to make the pinion long addendum to avoid undercut. Clock gearing also does not have the requirement that it transmits smooth angular motion. A typical watch train will come to a complete stop five times a second. Furthermore, the motion and force are in one direction, minimizing the need for accurate control of backlash.

The modified cycloidal tooth form is the most commonly used form for the going train of clocks, timers, and watches. Experience has shown that the various modifications to this basic tooth have little effect on the performance because of the scale effect of tolerances required.

There are no American standard tooth forms for clockwork gears.

3.2.1.9 Specific Spur Gear Calculation Procedure

The following directions give a step-by-step procedure for determining spur gear proportions:

1. The application requirements should give the requirements of ratio, input speed, and kind of duty to be performed. The duty may be one of power transmission or of motion transmission.
2. Based on application requirements, decide what pressure angle to use and plan to use a standard system (if possible) (see Table 3.5).
3. Pick the approximate number of pinion teeth.
   a. Consult Table 3.10 for general information.
   b. Check Tables 3.6 and 3.11 for undercut conditions.
   c. If power gearing, check Table 3.7 for data on balancing strength and wear capacity.
4. Having the approximate number of pinion teeth, determine the approximate center distance and the face width.

TABLE 3.10
Numbers of Pinion Teeth, Spur Gearing

No. of Teeth	Remarks
7–9	Smallest number of teeth recognized by AGMA
	(a) It requires long addendum to avoid undercut on all pressure angles.
	(b) If 20°, outside diameter should be reduced in proportion to tooth thickness to avoid pointed teeth.
	(c) It should be made with 25° pressure angle if feasible.
	(d) It may result in poor contact ratio in very fine diametral pitches because of accumulation of tolerances.
	(e) It may be subject to high specific sliding and usually have poor wear characteristics.
	(f) See Table 3.11 for minimum number of teeth in mating gear.
10	Smallest practical number with 20° pressure angle
	(a) It requires long addendum to avoid undercut if 20° pressure angle or less.
	(b) Contact ratio may be critical in very fine pitches.
	(c) See Table 3.11 for minimum number of teeth in mating gear.
12	Smallest practical number for power gearing of pitches coarser than 16 diametral pitch
	(a) It requires long addendum to avoid undercut if 20° pressure angle or less.
	(b) It is the smallest number of teeth that can be made standard if 25° pressure angle.
	(c) It is about the minimum number of teeth for any good fractional horsepower gear design where long life is important.
	(d) See Table 3.11 for minimum number of teeth in mating gear.
15	Used where strength is more important than wear
	(a) It requires long addendum to avoid undercut if 20° pressure angle or less.
	(b) See Table 3.11 for minimum number of teeth in mating gear.
19	Can be made standard addendum if 20° pressure angle or greater
25	(a) It allows good balance between strength and wear for hard steels.
	(b) Contact (contact diameter) is well away from critical base-circle region.
35	(a) If made of hard steels, strength may be more critical than wear.
	(b) If made of medium-hard (Rockwell C 30) steels, strength and wear are about equal.
50	(a) It has excellent wear resistance.
	(b) It is favored for high-speed gearing because of quietness.
	(c) It is critical on strength on all but low-hardness pinions.

TABLE 3.11
Numbers of Teeth in Pinion and Gear versus Pressure Angle and Center Distance

No. of Teeth in Pinion	No. of Teeth in Gear and When Pressure Angle Is Equal to: 14½ Coarse Pitch[a]	20 Coarse Pitch[b]	20 Fine Pitch[b]	25 Coarse Pitch[b]
7			42[c]	
8			39[c]	
9			36[c]	
10		25	33	15
11		24	30	14
12	52	23	27	12
13	51	22	25	
14	50	21	23	
15	49	20	21	
16	48	19	19	
17	47	18	18	
18	46			
19	45			
20	44			
21	43			
22	42			
23	41			
24	40			
25	39			
26	38			
27	37			
28	36			
29	35			
30	34			
31	33			

Note: Pinions having fewer than 10 teeth are not recommended in any AGMA standard. The ne-pitch AGMA standard gives data for pinions from 7 through 9 teeth but strongly discourages their use. Gears having fewer teeth than shown for any given pinion–gear combination will require an enlarged center distance for proper operation.

[a] Gears of this pressure angle are not recommended for new designs.

[b] Gears with these numbers of teeth can be made standard addendum and operated on standard center distances.

[c] Not recommended; but if essential, use these values.

5. Based on the number of pinion teeth and the center distance, determine the approximate pitch. Check Table 3.12 for standard pitches, and, if possible, use a standard pitch. Readjust pinion tooth numbers, center distance, and ratio to agree with the pitch chosen. See Chapter 2 for basic relations of pitch, center distance, and ratio. Also, use Equations 3.8 through 3.11 if special operating center distance is used.
6. Determine the whole depth of pinion and gear. Use Tables 3.5 and 3.13. Divide the tabular value for 1 diametral pitch by the actual diametral pitch to get the whole-depth design.

TABLE 3.12
Recommended Diametral Pitches

Coarse Pitch			Fine Pitch		
2	4	12	20	48	120
2.25	6	16	24	64	150
2.5	8		32	80	200
3	10		40	96	

Note: These diametral pitches are suggested as a means of reducing the great amount of gear-cutting tooling that would have to be inventoried if all possible diametral pitches were to be speci ed.

7. Determine the addenda of pinion and gear. Consult Tables 3.1, 3.2, 3.5, and 3.8. Follow these rules:
   a. Use enough long addendum to avoid undercut (Table 3.1).
   b. If a critical power job—speed decreasing—balance addendum for strength (Table 3.2).
   c. If no undercut or power problems, use standard addendum (Table 3.5).
8. Determine the operating circular pitch. If standard pitch is used, consult Table 3.14. If enlarged center distance is used, determine the operating circular pitch from the operating pitch diameters (Equations 3.8 through 3.11).
9. Determine the design tooth thickness. Decide first on how much to thin the teeth for backlash. Table 3.4 gives the recommended amounts for power gearing having in mind the normal accuracy of center distance and the normal operating temperature variations between gear wheels and casings. If special designs with essentially no backlash or unusual materials and accuracy are involved, consult Section 3.4 for special calculations. If the pinion and the gear have equal addendum, make the theoretical tooth thicknesses equal, and do this by dividing the circular pitch by two. If a long-addendum pinion is used with a short-addendum gear, adjust the tooth thicknesses by the following:

$$\Delta t = \Delta a \; 2 \tan \phi_n. \tag{3.42}$$

If only the pinion is enlarged and the center distance is enlarged to accommodate a standard gear, enlarge the pinion tooth thickness only (Table 3.15).

After the theoretical tooth thicknesses are obtained, subtract one-half the amount the teeth are to be thinned for backlash from the pinion and gear theoretical tooth thicknesses. This is the *maximum design* tooth thickness. Obtain the minimum design tooth thickness by subtracting a reasonable tolerance for machining from the maximum design tooth thickness.

TABLE 3.13
Equations and References for Whole-Depth Calculations

Pitch Range	Equation	Application	Remarks
Coarse pitch up to 19.99	$\frac{2.25}{P_d}$	General application	AGMA standard
	$\frac{2.35}{P_d}$	Gears to be given finishing operations such as shave or grind	AGMA standard
	$\frac{2.40}{P_d}$	Maximum strength for full-radius (tip) hobs	AGMA standard
Fine pitch[a] $20P_d$ and finer	$\frac{2.20}{P_d}+0.002$	Standard whole depth for general applications	AGMA standard
	$\frac{2.35}{P_d}+0.002$	Whole depth for preshaved cutters for shaving or grinding	AGMA standard

[a] Does not apply to pinions containing nine teeth or less.

TABLE 3.14
Values of Tooth Thicknesses for Pinions t_P and Gears t_G with Small Numbers of Teeth (Long and Short Addenda)

	Coarse-Pitch System				Fine-Pitch System	
	20° Pressure Angle		25° Pressure Angle		20° Pressure Angle	
No. of Teeth in Pinion	Pinion	Gear	Pinion	Gear	Pinion	Gear
7					2.0007	1.1409
8					1.9581	1.1835
9					1.9155	1.2261
10	1.9120	1.2300	1.7420	1.3990	1.8730	1.2686
11	1.8680	1.2730	1.6590	1.4820	1.8304	1.3112
12	1.8260	1.3150	1.5708	1.5708	1.7878	1.3538
13	1.7830	1.3580	1.5708	1.5708	1.7452	1.3964
14	1.7410	1.4000	1.5708	1.5708	1.7027	1.4389
15	1.6980	1.4430	1.5708	1.5708	1.6601	1.4815
16	1.6560	1.4860	1.5708	1.5708	1.6175	1.5241
17	1.6130	1.5290	1.5708	1.5708	1.5749	1.5667
18	1.5708	1.5708	1.5708	1.5708	1.5708	1.5708
19	1.5708	1.5708	1.5708	1.5708	1.5708	1.5708

Note: These tooth thicknesses go with Table 3.1 addenda. The above values are for 1 diametral pitch. These basic tooth thicknesses do not include an allowance for backlash.

10. Recheck the load capacity using design proportions just obtained. If it is not within allowable limits, change the design.
11. If the gear design is for critical power gears, additional items will need calculation:
    a. Root fillet radius (Equation 3.37)
    b. Form diameter
    c. Modification of profile (Equations 3.35 and 3.36)
    d. Diameter over pins (see Section 3.2.1.10)
12. Certain general dimensions must be calculated and toleranced:
    a. Outside diameter, $D + 2(a \pm \Delta a)$
    b. Root diameter, $D_o - 2h_t$
    c. Face width
    d. Chordal addendum and chordal thickness
13. It may be necessary to specify the following:
    a. Tip round (at outside diameter) (Table 3.15)
    b. Edge round (Table 3.15)
    c. Roll angle
    d. Base radius
14. In some unusual designs, it may be necessary to check the following to assure a sound design:
    a. Diameter at which teeth become pointed (Equation 3.41)

TABLE 3.15
Standard Tooth Parts

			Whole Depth[a]			Double Teeth	
Diametral Pitch, P_d (1/in.)	Circular Pitch, p (in.)	Module, $1/P_d$ Addendum, a (in.)	Shave or Grind, h_t (in.)	Cut Teeth, h_t (in.)	Tooth Thicknesses $(p/2)$, t (in.)	Cut Teeth $2h_t$ (in.)	Ground teeth $2h_t$ (in.)
1	3.1415927	1.0000000	2.35000	2.25000	1.5707963	4.5000	4.7000
1¼	2.5132741	0.8000000	1.88000	1.80000	1.2566371	3.6000	3.7600
1½	2.0943951	0.6666667	1.56667	1.50000	1.0471980	3.0000	3.1333
1¾	1.7951958	0.5714286	1.34286	1.28571	0.8975979	2.5714	2.6857
2	1.5707963	0.5000000	1.17500	1.12500	0.7853982	2.2500	2.3500
2½	1.2566371	0.4000000	0.94000	0.90000	0.6283185	1.8000	1.8800
3	1.0471976	0.3333333	0.78333	0.75000	0.5235988	1.5000	1.5667
3½	0.8975979	0.2857143	0.67143	0.64286	0.4487990	1.2857	1.3429
4	0.7853982	0.2500000	0.58750	0.56250	0.3926991	1.1250	1.1750
5	0.6283185	0.2000000	0.47000	0.45000	0.3141593	0.9000	0.9400
6	0.5235988	0.1666667	0.39167	0.37500	0.2617994	0.7500	0.7833
8	0.3926991	0.1250000	0.29375	0.28125	0.1963495	0.5625	0.5875
10	0.3141593	0.1000000	0.23500	0.22500	0.1570796	0.4500	0.4700
12	0.2617994	0.0833333	0.19583	–.18750	0.1308997	0.3750	0.3917
16	0.1963495	0.0625000	0.14688	0.14063	0.0981748	0.2813	0.2938
20	0.1570796	0.0500000	0.11950	0.11200	0.0785398	0.2240	0.2390
24	0.1308997	0.0416667	0.09992	0.09367	0.0654499	0.1873	0.1998
32	0.0981748	0.0312500	0.07544	0.07075	0.0490873	0.1415	0.1509
40	0.0785398	0.0250000	0.06075	0.05700	0.0392699	0.1140	0.1215
48	0.0654498	0.0208333	0.05096	0.04780	0.0327249	0.0956	0.1019
64	0.0490874	0.0156250	0.03872	0.03638	0.0245437	0.0728	0.0774
80	0.0392699	0.0125000	0.03138	0.02950	0.0196350	0.0590	0.0628
96	0.0327249	0.0104167	0.02648	0.02492	0.0163625	0.0498	0.0530
120	0.0261799	0.0083333	0.02158	0.02033	0.0130900	0.0407	0.0432
150	0.0209440	0.0066667	0.01767	0.01667	0.0104720	0.0333	0.0353

[a] h_t for ne pitch $(20P_d)$ is $2.2/P_d + 0.002''$.

b. Width of top land (Equation 3.38)
c. Effective contact ratio (Equations 3.3 and 3.4)
d. Undercut diameter

3.2.1.10 Explanation and Discussion of Items in Table 3.5

Table 3.5 shows the basic or the standard tooth proportions as given in the AGMA or the ASA standards listed at the end of the table.

1. *Pressure angle*: The pressure angle and the numbers of teeth in a given pair of gears are related in that the low pressure angles (14½) should be avoided for low numbers of teeth (see Tables 3.6 and 3.11).
2. *Addendum*: For general applications, use equal-addendum teeth when the numbers of teeth in the pinion and in the pair exceed the values shown in Explanations 10 and 11 of Table 3.5. Use long addenda for pinions having numbers of teeth, pressure angles, and diametral pitches as shown in Table 3.1. The amount of decrease of addendum in the mating gear is to be limited to values that will not produce undercut.

   The values of long addendum for pinions are limited to speed-decreasing drives. Data on speed-increasing drives are given in Section 3.1.3.2 but are beyond the scope of AGMA standards.
3. *Dedendum*: The values shown in the table are the minimum standard values. Shaved or ground teeth should be given a greater dedendum (whole depth and clearance values); see Explanation nos. 5 and 9. A constant value, 0.002 in., is added to the dedendum of ne-pitch gears, which allows space for the accumulation of foreign matter at the bottom of spaces. This provision is particularly important in the case of very ne diametral pitches.
4. *Working depth*: The working depth customarily determines the type of tooth; that is, a tooth with a $1.60/P$ working depth is called a *full-depth tooth*.
5. *Whole depth*: The value shown is the standard minimum whole depth. It will increase in proportion to the increase in backlash cut into the teeth (unless the outside diameter is also correspondingly adjusted). It

will also slightly increase if long- and short-addendum teeth are generated with pinion-shaped cutters. See also Explanations 3, 8, and 9 for increase due to manufacturing processing requirements.

6. *Circular tooth thickness, basic*: This is the basic circular tooth thickness on the standard pitch circle. These values will be slightly altered if backlash is introduced into the gears to allow them to mesh at a standard center distance. These values will be drastically altered in the case of long- and short-addendum designs. Table 3.14 gives values of tooth thickness corresponding to standard values of long and short addendum for small numbers of teeth.
7. *Fillet radius, basic rack*: The fillet radius shown is that in the basic rack. The tooth form standard directs that the edge radius on hobs and rack-type shaper cutters should be equal to the fillet radius in the basic rack. It also directs that pinion-shaped cutters should be designed using the basic rack as a guide so that the gear teeth generated by these cutters will have a fillet radius approximating those produced by hobs and rack-type shaper cutters. It can be approximately calculated by the data shown in Equation 3.42.

   In the case of 25° pressure angle teeth, the fillet radius shown must be reduced for teeth having a clearance of 0.250/*P*. This is discussed in Section 3.1.4.

   In the case of fine-pitch teeth, the fillet radius will usually be larger than the clearance customarily given as $c = 0.157/P$ because of edge breakdown in the cutter. The effects of tool wear on fine-pitch gears are proportionally larger than the effects produced by coarse-pitch tools.
8. *Clearance*: The value shown is the minimum standard. Greater clearance is usually required for teeth that are finished by grinding or shaving. In general, the value shown in Explanation no. 9 will be suitable for these processes. See also Explanations 3 and 5.
9. *Clearance for shaved or ground teeth*: This is the recommended clearance for teeth to be finished by shaving or grinding. In the case of 25° pressure angle teeth, the fillet radius shown must be reduced for teeth having a clearance of 0.250/*P*.
10. *Minimum number of teeth in pinion*: These are the lowest numbers of teeth that can be generated in pinions having standard addenda and tooth thicknesses that will not be undercut. Pinions having fewer teeth should be made long addendum in accordance with Table 3.1.
11. *Minimum numbers of teeth in pair*: This is the smallest number of teeth in a pinion and gear that can be meshed on a standard center distance without one member being undercut. For pairs with fewer teeth, the members will have to be meshed on a nonstandard (enlarged) center distance.
12. *Minimum width of top land*: This is the approximate minimum width of top land allowable in standard long-addendum pinions. Increases in addendum that cause the tops of the teeth to have less than this value should be generally avoided.
13. *Reference standard*: This is the source of the data in each column. AGMA standards are published by the AGMA.
14. *Reference standard*: This is the source of the data in each column. ASA standards are published by the American Society of Mechanical Engineers (ASME).

3.2.2 System for Helical Gears

The following data are based on the information contained in the AGMA standard for 20-Degree Involute Fine-Pitch System, for Spur and Helical Gears. In addition, tooth proportions are shown that are used by several of the larger gear manufacturers for the design of helical gears. These tooth proportions are shown since there are no AGMA standards for coarse-pitch helical gears.

In general, helical tooth proportions are based either on getting the most out of a helical gear design or on using existing tooling. The tooth proportions referred to earlier have been found to yield very good gears. In some cases, for less critical applications, tools on hand may be used. Very often hobs for spur gears are available, and gears for general service can be made with these tools. Since such hobs have no taper, they are not well suited to cut helix angles much over 30°.

In helical gear calculations, care should be taken to avoid confusion as to which plane the various tooth proportions are measured on. In some cases, the transverse plane is used. This plane is perpendicular to the axis of the helical gear blank. In some cases, it is desirable to work on the normal plane.

If a spur gear hob is used to cut helical teeth, the relationship between the transverse and normal pressure angles and the transverse and normal pitches as well as the base pitches should be established:

$$P_d = \frac{\pi \cos\psi}{p_n}, \qquad (3.43)$$

$$\tan\phi = \frac{\tan\phi_n}{\cos\psi}, \qquad (3.44)$$

$$p_N = p_n \cos\phi_n, \qquad (3.45)$$

where

P_d—diametral pitch in transverse plane
ψ—helix angle
p_n—circular pitch in normal plane
ϕ_n—pressure angle in normal plane
p_N—normal base pitch (normal to surface)

When gears mesh together with hobs, racks, shaper, or shaving cutters, their normal base pitches must be equal.

TABLE 3.16
Ranges of Gear Tooth Forms Applications

Type of Gear	Suggested Tooth Form
Single helix, low allowable thrust reaction	Type 1
Single helix, moderate allowable thrust reaction	Types 2 and 3
Double helix, general purpose	Types 4 and 5
Double helix, minimum noise	Type 6
Double helix, high load	Types 20 and 21

When a gear shaper is to be used to cut helical teeth, special guides are installed in the machine. These impart the twist into the cutter spindle, which produces the proper helix angle and lead. The cutter used has the same lead ground into the teeth as is produced by the guides. The cutter and the guides in turn produce a given lead angle to the gear being cut. Practical considerations limit the lead angle on the guides to roughly 35°. In order to control costs, helical gears that are to be shaped should be designed either to a standard series of values of leads or to values of lead available in existing guides.

The following equation shows the relationship of lead, number of teeth, diametral pitch, and helix angle in a given cutter, or helical gear:

$$\tan\psi = \frac{\pi N}{P_d L}, \tag{3.46}$$

where
ψ—helix angle
N—number of teeth, cutter, or gear
P_d—diametral pitch
L—lead of cutter

3.2.2.1 Selection of Tooth Form

The tooth forms shown in Table 3.3 are used for ranges of applications as follows (see Table 3.16). The foregoing types each require a separate set of tooling. They have the advantage of getting the most out of a good helical gear design.

When an existing spur gear hob is to be used to produce the gear, use types 18 and 19 tooth forms. The data in Table 3.17 will be helpful in making calculations for gears having types 18 and 19 tooth forms. Type 19 tooth form is similar to type 18 tooth form except that its proportions are based on the use of hobs designed to cut ne-pitch gears (20 diametral pitch or ner).

3.2.2.2 Selection of Helix Angle

Single-helical gears are usually given lower helix angles than double-helical gears in order to limit thrust loads. Typical helix angles are 15° and 23°. Low helix angles do not provide so many axial crossovers as can be achieved on high helix angle gears of a given face width. Double-helical gears have helix angles that typically range from 30° to 45°. Although higher helix angles provide smoother operation, the tooth strength is lower.

TABLE 3.17
Tooth Proportions for Helical Gears

Helix Angle (°)	Diametral Pitch, P_d	Circular Pitch, P_t	Axial Pitch, p_x	Pressure Angle[a]	Working Depth, h_k	Whole Depth, h_t[b]
0	1.000000	3.14159	∞	20°00′00″	2.000	2.250
5	0.996195	3.15359	36.04560	20°4′13.1″	2.000	2.250
8	0.990268	3.17247	22.57327	20°10′50.6″	2.000	2.250
10	0.984808	3.19006	18.09171	20°17′00.7″	2.000	2.250
12	0.978148	3.21178	15.11019	20°24′37.1″	2.000	2.250
15	0.965926	3.25242	12.13817	20°38′48.8″	2.000	2.250
18	0.951057	3.30326	10.16640	20°56′30.7″	2.000	2.250
20	0.939693	3.34321	9.18540	21°10′22.0″	2.000	2.250
21	0.933580	3.36510	8.76638	21°17′56.4″	2.000	2.250
22	0.927184	3.38832	8.38636	21°25′57.7″	2.000	2.250
23	0.920505	3.41290	8.04029	21°34′26.3″	2.000	2.250
24	0.913545	3.43890	7.72389	21°43′22.9″	2.000	2.250
25	0.906308	3.46636	7.43364	21°52′58.7″	2.000	2.250
26	0.898794	3.49534	7.16651	22°02′44.2″	2.000	2.250
27	0.891007	3.52589	6.91994	22°13′10.6″	2.000	2.250
28	0.882948	3.55807	6.69175	22°24′09.0″	2.000	2.250
29	0.874620	3.59195	6.48004	22°35′40.0″	2.000	2.250
30	0.866025	3.62760	6.28318	22°47′45.1″	2.000	2.250

[a] Pressure angle based on 20° normal pressure angle.
[b] The values shown for whole depth are for coarse-pitch gears. If the gears are to be shaved or ground, use $h_t = 2.35$. For ne-pitch gears, use $2.2/P_{nd} + 0.002$ for general-purpose gearing, and $2.35/P_{nd} + 0.002$ for gear to be shaved or ground.

In order to get the quietest gears and at the same time achieve good tooth strength, special cutting tools for each helix angle should be provided. Table 3.3 shows typical tooth proportions. Tooth types 1 through 5 require special cutters, and the helix angle shown should be used. These teeth are stubbed more and more as the helix angle is higher. Tooth types 6 through 12 can be made with any helix angle desired. Types 13 and 14 can also be made with almost any helix angle. These tooth proportions are based on the use of standard spur gear hobs. Since such hobs are not usually tapered, they do not do as god a job in cutting as do the specially designed helical hobs, and, as a result, they should be limited in use to helix angles less than 25°.

3.2.2.3 Face Width

In the design of helical gears, the face width is usually based on the need to achieve the required load-carrying capacity. In addition, the face width and the lead are interrelated in that it is necessary to obtain at least two axial pitches of face width ($F = 2p_x$) to get reasonable bene t from the helical action, and four or more if high speeds, noise, or critical designs are confronted.

Long- and short-addendum designs are not as common among helical gears as among spur gears. This is because a much lower number of teeth can be cut in helical gears without undercut. In low-hardness helical gearing, pitting, which is little affected by changes in addendum, is usually the limiting feature. In high-hardness designs, the same proportions as those used for spur gear addenda should be employed.

3.2.2.4 Specific Calculation Procedure for Helical Gears

The procedure for helical gears is very similar to that given in Section 3.2.1.9 for spur gears:

1. Determine the application requirements: ratio, power, speed, and so forth.
2. Decide on basic tooth proportions and helix angle (see Table 3.3).
3. Pick the appropriate number of pinion teeth (see Tables 3.6, 3.7, 3.10).
4. Determine the approximate center distance and face width.
5. Determine the pitch of teeth.
6. Determine the whole depth.
7. Determine the addenda of pinion and gear. Use equal addendum for pinion and gear except in special cases where the addendum must be increased to avoid undercut or special consideration must be given to increasing the pinion strength (see Table 3.18).
8. Determine the operating circular pitch, the operating helix angle, and the operating normal pitch if special center distance was used.
9. Determine the design tooth thickness. If long and short addenda, adjust theoretical tooth thicknesses accordingly. Thin teeth are for backlash. Consult Table 3.4 for power gearing backlash allowance.
10. Recheck the load capacity. Check the number of axial crossovers (Equation 3.5). Check the ability of

TABLE 3.18
Equations and References for Addendum Calculations, Helical Gears

Type of Tooth Design	Operating Conditions	Pinion	Gear
Standard or equal addendum	Number of teeth in gear and pinion are greater than minimum numbers shown in Figure 3.10.	a = Value from Table 3.3 $a = \dfrac{\text{types 1–6}}{P_d}$, $a = \dfrac{\text{types 7–19}}{P_{nd}}$, $a = \dfrac{\text{types 1–6}}{P_d}$ Value from special tools to be used	If standard center distance and values of a_P as shown at (a) left. If nonstandard center distance, see item below
Long and short addendum	Number of teeth in pinion are less than minimum numbers shown in Table 3.6, and numbers of teeth in gear are more than minimums shown in Figure 3.3.	Suf ciently increase addendum to avoid undercut. $a_P = a + K'_n/P_n$ See Figure 3.10 for K'_n.	Decrease addendum by amount pinion addendum is increased. If undercut, increase center distance, see item below
	Number of teeth in pinion are less than minimum numbers shown in Table 3.6, and numbers of teeth in gear are less than minimum numbers of teeth shown in Figure 3.3.	Suf ciently increase addendum to avoid undercut. $a_P = a + K'_n/P_n$	Suf ciently increase addendum to avoid undercut. $a_G = a + K'_n/P_n$
	Designed for nonstandard center distance.	Increase center distance by amount suf cient to accommodate increased addendum of both pinion and gear. Calculate tooth thickness to meet required center distance from Equation 3.75, then calculate required addendum from Equation 3.18.	

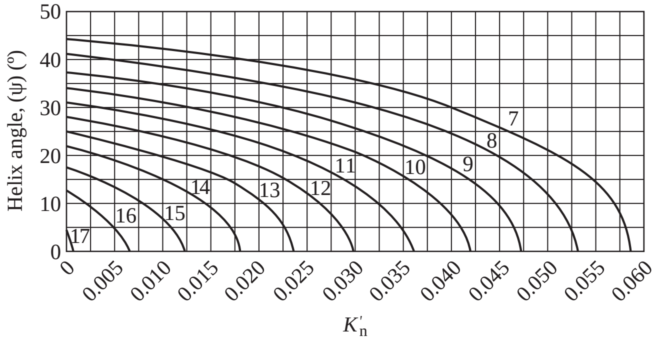


FIGURE 3.10 Addendum increment K'_n for 1 diametral pitch.

thrust bearings to handle thrust reactions (Chapter 11). If results of these checks are unsatisfactory, change the proportions.

11. If the gear design is for critical power gears, additional items may need calculation.
    a. Root fillet radius (Equation 3.37)
    b. Form diameter
    c. Modification of profile (Equations 3.35 and 3.36)
    d. Diameter over balls
12. General dimensions to be determined and toleranced.
    a. Outside diameter
    b. Root diameter
    c. Face width
    d. Chordal addendum and chordal tooth thickness
13. It may be necessary to specify.
    a. Tip round or chamber (Table 3.19)
    b. Edge round or end bevel (Table 3.19)
    c. Roll angle
    d. Base radius
14. Generally, helical gears are not designed for such critical applications as to be in danger of pointed teeth or undercut.

3.2.3 System for Internal Gears

In general, both spur internal meshes and helical internal meshes may be calculated by the same methods as external meshes. However, there are several problems unique to internal gear meshes.

The first of these is tip interference. In this type of interference, the pinion member cannot be radially assembled with the gear. Only axial assembly is possible, and it should be provided for in the design. If a shaper cutter having a number of teeth equal to or greater than the pinion is used to cut the internal gear, it will cut its way into the mesh but in so doing will remove some material from the flanks of a few of the teeth that should have been left in place in order to have good tooth operation. This cutting action is also known as trimming. Such teeth will have poor contact and will tend to be noisy. If the proper shaper cutter or a broach is used, this problem will not occur.

The second problem is sometimes known as *fouling*. In this case, the internal gear teeth interfere with the flanks of the external tooth pinion if there is too small a difference in numbers of teeth between the pinion and the gear members.

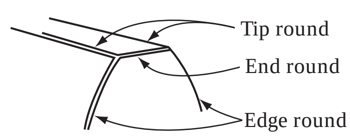
Both these problems can be avoided in most gear designs by reducing the addendum of the internal gear (increasing its inside diameter). Tables 3.20 and 3.21 show a group of tooth proportions that will avoid these problems. A rather complicated graphical layout or involved calculations may be required to determine the exact proportions to avoid these problems. Since such calculations are beyond the scope of this chapter, Tables 3.20 and 3.21 are offered as a guide.

3.2.3.1 Special Calculations

In general, the tip interference problem can be limited by providing more than a 17-tooth difference between gear and pinion.

The internal gear teeth have an addendum that extends toward the inside from the pitch circle. The critical dimension of addendum height is maintained by holding inside diameter.

TABLE 3.19
Values for Tip Round and Edge Round and End Round



	Edge Round			
Diametral Pitch	**General Applications**	**Medium Strength**	**High Strength**	**Tip Round and End Round**
20 and finer	Burr brush edges	0.001–0.005	0.005–0.010	0.001–0.005
16	Burr brush edges	0.003–0.015	0.010–0.025	0.003–0.010
12	Burr brush edges	0.005–0.020	0.012–0.030	0.005–0.015
10	Burr brush edges	0.010–0.025	0.015–0.035	0.005–0.015
8	Burr brush edges	0.010–0.025	0.020–0.045	0.010–0.030
5	Burr brush edges	0.010–0.025	0.025–0.060	0.010–0.030
3	Burr brush edges	0.015–0.035	0.040–0.090	0.010–0.050
2	Burr brush edges	0.015–0.035	0.060–0.125	0.010–0.050

TABLE 3.20
Addendum Proportions and Limiting Numbers of Teeth for Internal Spur Gears of 20° Pressure Angle

		Min. No. of Gear Teeth		Gear Addendum			
No. of Pinion Teeth	Pinion Addendum	Axial Assembly	Radial Assembly	Minimum Ratio[a]	Ratio of 2	Ratio of 4	Ratio of 8
12	1.350	19	26	0.472	0.510	0.582	0.616
	1.510	19	26	0.390	0.412	0.451	0.471
13	1.290	20	27	0.507	0.556	0.635	0.673
	1.470	20	27	0.419	0.445	0.488	0.509
14	1.230	21	28	0.543	0.601	0.688	0.729
	1.430	21	28	0.447	0.479	0.525	0.548
15	1.180	22	30	0.574	0.642	0.733	0.777
	1.400	22	30	0.470	0.506	0.554	0.577
16	1.120	23	32	0.608	0.688	0.786	0.834
	1.380	23	32	0.487	0.526	0.574	0.597
17	1.060	24	33	0.642	0.734	0.839	0.890
	1.360	24	33	0.505	0.546	0.594	0.617
18	1.000	25	34	0.676	0.779	0.892	0.947
	1.350	25	34	0.516	0.558	0.605	0.628
19	1.000	27	35	0.702	0.792	0.808	0.950
	1.330	27	35	0.539	0.578	0.625	0.648
20	1.000	28	36	0.713	0.802	0.903	0.952
	1.320	28	36	0.550	0.590	0.636	0.658
22	1.000	30	39	0.733	0.821	0.912	0.957
	1.290	30	39	0.577	0.621	0.666	0.688
24	1.000	32	41	0.750	0.836	0.920	0.960
	1.270	32	41	0.599	0.644	0.687	0.709
26	1.000	34	43	0.766	0.849	0.926	0.963
	1.250	34	43	0.620	0.666	0.709	0.729
30	1.000	38	47	0.792	0.870	0.936	0.968
	1.220	38	47	0.654	0.702	0.741	0.761
40	1.000	48	57	0.836	0.903	0.952	0.976
	1.170	48	57	0.718	0.764	0.797	0.814

[a] Minimum ratio of number of gear teeth divided by number of pinion teeth for axial assembly.

In general, the addenda of internal gears are made considerably shorter than those of equivalent external gears to avoid interference. The effect of the gear wrapping around the pinion tends to increase contact ratio, and also the chances for pinion- llet interference.

Internal gearsets may be designed so that the pinion can be introduced into the mesh at assembly by a radial movement. For any given number of teeth in the pinion, there must be a number of teeth in the gear that is somewhat greater than required if axial assembly were to be allowed.

If a minimum difference in numbers of pinion and gear teeth is desired, the designer must design the gear casings and bearings so that the pinion can be introduced into the mesh with the gear in an axial direction.

Tables 3.20 and 3.21 give the minimum numbers of teeth that a gear may have for any given pinion to achieve either radial or axial assembly.

The hand of helical internal gears is determined by the direction in which the teeth move, right of left, as the teeth recede from an observer looking along the gear axis. Whereas two external helical gears must be of opposite hand to mesh on parallel axes, an internal helical gear must be of the same hand as its mating pinion.

3.2.3.2 Specific Calculation Procedure for Internal Gears

Generally speaking, internal gears may be designed by the same procedure as outlined in Section 3.2.1.9 for spur gears or Section 3.2.2.4 for helical gears. However, there are some special considerations:

1. *Number of teeth*: The number of teeth in the pinion is based on the ratio required and also on tooth strength and wear considerations. In addition, there must be a suf ciently large difference between the numbers of teeth in the pinion and the gear to avoid problems of tip interference. Tables 3.20 and 3.21 give the minimum numbers of teeth in gear and pinion that can be speci ed and still be able to achieve either axial or radial assembly.
2. *Number of teeth*, *gear*: The minimum number of teeth in the gear member that can be used without

TABLE 3.21
Addendum Proportions and Limiting Numbers of Teeth for Internal Spur Gears of 25° Pressure Angle

		Min. No. of Gear Teeth		Gear Addendum			
No. of Pinion Teeth	Pinion Addendum	Axial Assembly	Radial Assembly	Minimum Ratio[a]	Ratio of 2	Ratio of 4	Ratio of 8
12	1.000	17	20	0.699	0.793	0.900	0.934
	1.220	17	21	0.601	0.656	0.720	0.740
13	1.000	18	22	0.718	0.810	0.908	0.940
	1.200	18	22	0.622	0.680	0.742	0.761
14	1.000	19	24	0.734	0.824	0.915	0.944
	1.180	19	24	0.644	0.703	0.763	0.782
15	1.000	20	25	0.748	0.837	0.921	0.948
	1.170	20	25	0.659	0.719	0.776	0.794
16	1.000	21	26	0.761	0.847	0.926	0.951
	1.150	21	26	0.679	0.741	0.797	0.815
17	1.000	22	27	0.773	0.857	0.930	0.954
	1.140	22	27	0.694	0.755	0.809	0.826
18	1.000	23	28	0.783	0.865	0.934	0.957
	1.130	23	28	0.708	0.769	0.820	0.837
19	1.000	24	29	0.793	0.873	0.938	0.959
	1.120	24	29	0.721	0.782	0.832	0.848
20	1.000	25	30	0.802	0.879	0.941	0.961
	1.110	26	29	0.741	0.795	0.843	0.859
22	1.000	28	32	0.824	0.891	0.947	0.965
	1.090	28	32	0.765	0.820	0.866	0.881
24	1.000	30	34	0.837	0.900	0.951	0.968
	1.080	30	34	0.782	0.836	0.879	0.893
26	1.000	32	37	0.847	0.908	0.955	0.970
	1.060	32	37	0.806	0.859	0.900	0.914
30	1.000	36	40	0.865	0.921	0.961	0.974
	1.050	36	40	0.829	0.879	0.915	0.927
40	1.000	46	51	0.896	0.941	0.971	0.981
	1.020	46	51	0.880	0.923	0.952	0.961

[a] Minimum ratio of number of gear teeth divided by number of pinion teeth for axial assembly.

getting into problems of assembly techniques or tip interference problems is shown in Tables 3.20 and 3.21. The manufacturing organization that will produce the gears should be checked to determine the availability of suitable equipment when the number of teeth in the gear exceeds 200 or when prime numbers over 100 are used. If the internal gear is to be broached, consideration should be given to minimizing the number of teeth in the gear so as to keep broach cost down.

3. *Helix angle*: Since shaper guides are usually required for the cutting of the internal gear, the rst choice in helix angles should be based on existing guides. Because of the kinematics of the gear shaper, helix angles above about 30° should be avoided. The helix angle of the pinion member is of the same hand as that of the gear.
4. *Diametral pitch*: Since special tools are usually required to produce the internal gear, thought should be given to standardizing on ranges of numbers of teeth and diametral pitches for internal gearing. Table 3.12 shows suggested diametral pitches. Internal gearing can be made long and short addenda so that the need for nonstandard pitches to meet special center distances is virtually nonexistent.
5. *Normal circular pitch*: In helical internal gears, the normal circular pitch may be speci ed on the internal gear cutter in the case of existing cutters.

3.2.4 Standard Systems for Bevel Gears

Over the years from 1950 to 1990 and later, Gleason Works, in Rochester, New York, United States, has published recommended standard systems for bevel gears. AGMA has also published standard data for bevel gears. At present, the older standards are being revised and updated.* The widespread

* It is instructive to note here that all standards on bevel and hypoid gears developed so far relate to approximate gears, and therefore, they are not applicable to precision gears and to gears with a few tooth count. This is because the base pitches of a gear and of a mating pinion, those designed in accordance with the acting standards, are not equal to the operating base pitch of the gear pair.

use of the computer has tended toward more complex calculation procedures.

The reader is advised to obtain the latest standards from both the AGMA and the Gleason Works. If a manufacturing equipment built in German, Switzerland, or Japanese companies is in use, appropriate standard data should be obtained from the manufacturer.

For the general guidance of the reader, the principal characteristics of systems now in use are discussed.

3.2.4.1 Discussion of 20° Straight Bevel Gear System

The tooth form of the gears in the straight bevel gear system is based on a symmetrical rack. In order to avoid undercut and to achieve approximately equal strength, a different value of addendum is employed for each ratio. If these gears are cut on modern bevel gear generators, they will have a localized tooth bearing. This resulting Coni ex tooth form is discussed in Section 3.4.1. The selection of addendum ratios and outside diameter is limited to a 1:1.5 ratio in width of top lands of pinion and gear. The face cone of the gear and pinion blanks is made parallel to the root cone elements to provide parallel clearance. This permits the use of larger edge radii on the generating tools with the attendant greater fatigue strength. Figure 3.11 shows data on the relation of dedendum angle to undercut for straight bevel gears.

3.2.4.2 Discussion of Spiral Bevel Gear System

Tooth thicknesses are proportioned so that the stresses in the gear and the pinion will be approximately equal with the LH pinion driving clockwise or a LH pinion driving counterclockwise. The values shown apply for gears operated below their endurance limits. For gears operated above their endurance limits, special proportions will be required. Special proportions will also be required for reversible drives on which optimum load capacity is desired. The method of establishing these balances is beyond the scope of this chapter but may be found in Gleason Works publication *Strength of Bevel and Hypoid Gears.*

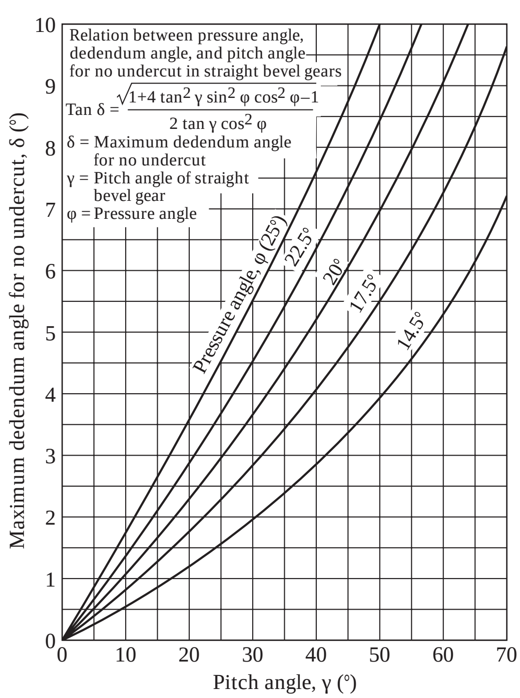


FIGURE 3.11 Relation between the dedendum angle and the pitch angle at which undercut begins to occur in generating straight bevel gears using sharp-cornered tools.

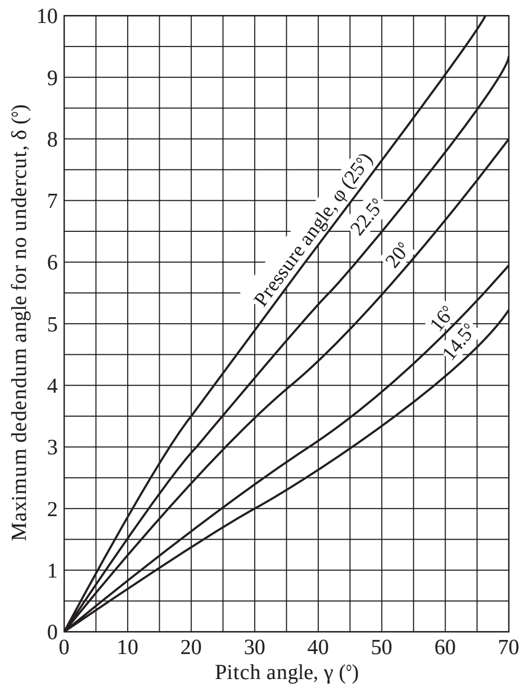


FIGURE 3.12 Relation between the dedendum angle and the pitch angle at which undercut begins to occur in generating spiral bevel gears at 35° spiral angle using sharp-cornered tools.

The tooth proportions are based on a 35° spiral angle. A smaller angle may result in an undercut and a reduction in contact ratio (Figure 3.12).

3.2.4.3 Discussion of Zerol Bevel Gear System

The considerations of tooth proportions to avoid undercut and loss of contact ratio as well as to achieve optimum balance of strength are similar in this system to those of the straight and spiral systems.

The Zerol bevel gear system is based on the duplex cutting method in which the root cone elements do not pass through the pitch cone apex. The face cone of the mating member is made parallel to the root cone to produce a uniform clearance.

3.2.4.4 Special Tooth Forms

The teeth of bevel gears can be given special form to reduce manufacturing costs or for engineering reasons.

Because of the exibility of the machining system used to cut most bevel gears, many forms are possible. In general, it is customary to produce a desired pro le on the teeth of the gear member and then to generate a conjugate pro le on the teeth of the pinion.

TABLE 3.22
Tooth Proportions of Standard Bevel Gears

	Type of Tooth		
Item	**Straight Teeth**	**Spiral Teeth**	**Zerol Teeth**
Pressure angle (°)	20° standard[a]	20° standard[d]	20° basic, 22½° or 25° where needed
Working depth	$2.000/P_d$	$1.700/P_d$	$2.000/P_d$
Clearance	$0.188/P_d + 0.002''$	$0.188/P_d$	$0.188/P_d + 0.002''$
Face width	$F \le A_o/3$ or $F \le 10/P_d$ does not exceed smaller value[b]	$F \le A_o/3$ or $F \le 10/P_d$ whichever is smaller[b]	$F \le 0.25\ A_o$ or $F \le 10/P_d$ whichever is smaller $F < 1$ on duplex Zerols[b,g]
Spiral angle	–	35°[e]	0°
Min. no. of teeth in system	13[c]; see Table 3.22	12	13
Whole depth	$\frac{2.18}{P_d} + 0.002''$	$\frac{2.18}{P_d}$[f]	$\frac{2.18}{P_d} + 0.002''$
Diametral range	–	12 and coarser	3 and finer
AGMA ref.	208.02	209.02	202.02

[a] Standard pressure angle for straight-tooth bevel gears is 20°. Table 3.23 shows ratios that may be cut with a 14½ degree pressure angle teeth.
[b] If the face width exceeds one-third the outer cone distance, the tooth is in danger of breakage in the event that tooth contact shifts to the small end of the tooth.
[c] This is the minimum number of teeth in the basic system (see Table 3.23 for equivalent minimum number of teeth in the gear member).
[d] Standard pressure angle for spiral bevel gears is 20°. Table 3.23 shows ratios that may be cut with a 16° degree pressure angle teeth.
[e] Standard pressure angle is 35°. If smaller spiral angles are used, undercut may occur and contact ratio may be less.
[f] For gears of 10 diametral pitch and coarser, the teeth are often rough cut 0.005 deeper to avoid having the finish blades cut on their ends.
[g] For duplex Zerol bevel gears, 1 in. is the maximum face width in all cases.

Although it is theoretically possible to use bevel teeth with an involute profile, this is rarely done. The most common curve for bevel gearing is the octoid, so called because of the shape of its line of action. This curve is much easier to generate than the spherical involute. It closely approximates the involute.

The Formate* tooth form is often used for gearing of high ratios because of its manufacturing economies. The teeth of the gear member are cut without generation, thus saving time, and the extra generation required to produce a conjugate pair is taken on the pinion. Since there are fewer pinion teeth, less time is spent in the generation than if both pinion and gear teeth were generated.

Another tooth form suitable for high-speed manufacture is the Revacile* straight bevel gear. This special tooth form is generated by a large disk cutter, one space being completed with each cutter revolution.

Both these forms are beyond the scope of this chapter.

3.2.4.5 Limitations in 20° Straight Bevel Gear System

The data contained in Table 3.22 apply only to gears that meet the following requirements:

- The standard pressure angle is 20°; in certain cases, depending on numbers of teeth, other pressure angles may be used (see Table 3.23).
- In all cases, full-depth teeth are used. Stub teeth are avoided because of the reduction in contact ratio, which may increase noise, and the reduction in wear resistance.
- Long- and short-addendum teeth are used throughout the system (except for 1:1 ratios) to avoid undercut and to increase the strength of the pinion.
- The face width is limited to between one-fourth and one-third cone distance. The use of greater face widths results in an excessively small tooth size at the inner ends of the teeth.

3.2.4.6 Limitations in Spiral Bevel Gear System

The spiral bevel gear system is more limited in its application than the straight tooth system. The data in this system do *not* apply to the following:

- Automotive rear-axle drives
- Formate pairs
- Gears and pinions of 12 diametral pitch and finer, which are usually cut with one of the duplex spread-blade methods
- Gear cut spread blade and pinion cut single side, with a spiral angle of less than 20°
- Ratios with fewer teeth than those listed in Table 3.23
- Larger spiral bevel gears cut on the planing-type generators where the spiral angles should not exceed 30°

* Registered trademark of Gleason Works, Rochester, New York.

TABLE 3.23
Pressure Angle and Ratio, Minimum Number of Teeth in Gear and Pinion That Can Be Used with Any Given Pressure Angle

	Type of Bevel Gear					
	Straight Tooth		Spiral Tooth		Zerol Tooth	
Pressure Angle (°)	Pinion	Gear	Pinion	Gear	Pinion	Gear
20 (standard)	16	16	17	17	17	17
	15	17	16	18	16	20
	14	20	15	19	15	25
	13	30	14	20	–	–
	–	–	13	22	–	–
	–	–	12	26	–	–
14½	29	29	28	28	Not used	
	28	29	27	29		
	27	31	26	30		
	26	35	25	32		
	25	40	24	33		
	24	57	23	36		
	–	–	22	40		
	–	–	21	42		
	–	–	20	50		
	–	–	19	70		
16	Not used		24	24	Not used	
			23	25		
			22	26		
			21	27		
			20	29		
			19	31		
			18	36		
			17	45		
			16	59		
22½	13	13	16	16	14	14
	–	–	16	17	13	15
	–	–	16	18	–	–
	–	–	16	19	–	–
	–	–	15	15	–	–
	–	–	15	16	–	–
	–	–	15	17	–	–
	–	–	15	18	–	–
	–	–	15	19	–	–
	–	–	15	20	–	–
	–	–	15	21	–	–
	–	–	15	22	–	–
	–	–	15	23	–	–
	–	–	15	24	–	–
	–	–	14	14	–	–
	–	–	13	15	–	–
25	12	12	13	13	13	13
	–	–	13	14	–	–
	–	–	12	12	–	–

Designs that fall in these restrictions should be referred to Gleason Works for assistance.

3.2.4.7 Limitations in Zerol Bevel Gear System

The data contained in Table 3.22 apply only to Zerol bevel gears that meet the following requirements:

- The (basic) standard pressure angle is 20°. Where needed to avoid undercut, 22½° and 25° pressure angles are standard (see Table 3.23).
- The face width is limited to 25% of the cone distance since the small-end tooth depth decreases even more rapidly as the face width increases because of the duplex taper. In duplex Zerol gears, 1 in. face is the maximum value in any case.

3.2.4.8 General Comments

Table 3.7 will be helpful in selecting the proper numbers of teeth for most power gearing applications. Table 3.23 gives the minimum numbers of teeth in the gear for each number of pinion teeth and pressure angle. In general, the more teeth that are in the pinion, the more quietly it will run and the greater will be its resistance to wear. The equipment used to produce bevel gearing imposes upper limits on numbers of teeth. In general, if the gear is to contain over 120 teeth if an even number or above 97 if a prime number, the manufacturing organization that is to produce the gears should be checked for capacity.

Table 3.23 gives the minimum numbers of teeth in the pinion for each number of teeth in the gear for each pressure angle. The minimum number of pinion teeth is based on the consideration of undercut.

Table 3.24 shows addendum values that have been used in the past. These values are at the large end of the tooth (outer cone distance). These values show what was usually used in older designs that are in production. New standards that are now being developed may not agree with these values. (If a new standard is used, all values should be taken from the new standard—including addendum and backlash.)

Table 3.25 shows typical backlash values for bevel gears. In the eld of bevel gears, mounting distance settings can change backlash and change the tooth contact pattern. Usually, the desired contact is achieved when a bevel gearset is mounted at the speci ed mounting distances. Figure 3.13 shows a typical mounting surface and the body dimensions for a bevel gear.

3.2.5 Standard Systems for Worm Gears

There have been AGMA standards for worm gear tooth proportions in the past. Like other types of gear teeth, the old standards are being modi ed. For general reference, Table 3.26 presents the tooth proportions for single-enveloping worm gears and for double-enveloping worm gears. These proportions are typical of past design practice.

TABLE 3.24
Bevel Gear Addendum (for 1 Diametral Pitch)

Ratio, m_G		Addendum		Ratio, m_G		Addendum	
From	To	Straight and Zerol	Spiral	From	To	Straight and Zerol	Spiral
1.00	1.00	1.000	0.850	1.52	1.56	0.730	0.620
1.01	1.02	0.990	0.840	1.56	1.57	0.720	0.620
1.02	1.03	0.980	0.830	1.57	1.60	0.720	0.610
1.03	1.04	0.970	0.820	1.60	1.63	0.710	0.610
1.04	1.05	0.960	0.820	1.63	1.65	0.710	0.600
1.05	1.06	0.950	0.810	1.65	1.68	0.700	0.600
1.06	1.08	0.940	0.800	1.68	1.70	0.700	0.590
1.08	1.09	0.930	0.790	1.70	1.75	0.690	0.590
1.09	1.11	0.920	0.780	1.75	1.76	0.690	0.580
1.11	1.12	0.910	0.770	1.76	1.82	0.680	0.580
1.12	1.13	0.900	0.770	1.82	1.89	0.670	0.570
1.13	1.14	0.900	0.760	1.89	1.90	0.660	0.570
1.14	1.15	0.890	0.760	1.90	1.97	0.660	0.560
1.15	1.17	0.880	0.750	1.97	1.99	0.650	0.560
1.17	1.19	0.870	0.740	1.99	2.06	0.650	0.550
1.19	1.21	0.860	0.730	2.06	2.10	0.640	0.550
1.21	1.23	0.850	0.720	2.10	2.16	0.640	0.540
1.23	1.25	0.840	0.710	2.16	2.23	0.630	0.540
1.25	1.26	0.830	0.710	2.23	2.27	0.630	0.530
1.26	1.27	0.830	0.700	2.27	2.38	0.620	0.530
1.27	1.28	0.820	0.700	2.38	2.41	0.620	0.520
1.28	1.29	0.820	0.690	2.41	2.58	0.610	0.520
1.29	1.31	0.810	0.690	2.58	2.78	0.600	0.510
1.31	1.33	0.800	0.680	2.78	2.82	0.590	0.510
1.33	1.34	0.790	0.680	2.82	3.05	0.590	0.500
1.34	1.36	0.790	0.670	3.05	3.17	0.580	0.500
1.36	1.37	0.780	0.670	3.17	3.41	0.580	0.490
1.37	1.39	0.780	0.660	3.41	3.67	0.570	0.490
1.39	1.41	0.770	0.66	3.67	3.94	0.570	0.480
1.41	1.42	0.770	0.650	3.94	4.56	0.560	0.480
1.42	1.44	0.760	0.650	4.56	4.82	0.560	0.470
1.44	1.45	0.760	0.640	4.82	6.81	0.550	0.470
1.45	1.48	0.750	0.640	6.81	7.00	0.540	0.470
1.48	1.52	0.740	0.630	7.00	8.00	0.540	0.460

Note: In the earlier AGMA standards and Gleason handbooks, the equation for addendum for all types of bevel gears was a_{oG} = values in this table/P. The values used in this table should be used only when checking calculations based on earlier standards.

3.2.5.1 General Practice

The following rules apply to conventional single-enveloping worm gears:

- The worm axis is at right angle to the worm gear axis (90°).
- The worm gear is hobbed. Except for a small amount of oversize, the worm gear hob has the same number of threads, the same tooth profile, and the same lead as that of the mating worm. (A slight change in lead may be made to compensate for oversize effects.)

TABLE 3.25
Design Backlash for Bevel Gearing (in.)

Diametral Pitch	Range of Design Backlash	Diametral Pitch	Range of Design Backlash
1.00–1.25	0.020–0.030	3.50–4.00	0.007–0.009
1.25–1.50	0.018–0.026	4.00–5.00	0.006–0.008
1.50–1.75	0.016–0.022	5.00–6.00	0.005–0.007
1.75–2.00	0.014–0.018	6.00–8.00	0.004–0.006
2.00–2.50	0.012–0.016	8.00–10.00	0.002–0.004
2.50–3.00	0.010–0.013	10.00–12.00	0.001–0.003
3.00–3.50	0.008–0.011	12 and finer	

Note: In the earlier AGMA standards and Gleason handbooks, the equation for addendum for all types of bevel gears was a_{oG} = Table 3.24 value/P. The values used in this table should be used only when checking calculations based on earlier standards.

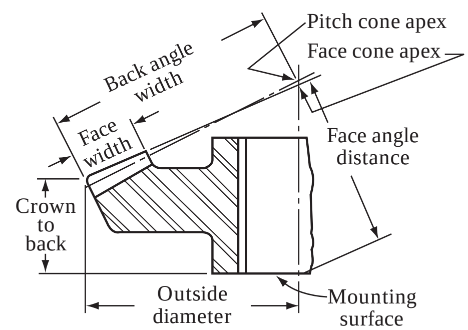


FIGURE 3.13 Bevel gear dimensions.

- Fine-pitch worms are usually milled or ground with a double-conical cutter or grinding wheel with an inclined angle of 40° (tool pressure angle is 20°).
- Coarser-pitch worms are often made with a straight-sided milling or grinding tool—in American practice. Some European countries favor making the worm member in involute helicoid. Functionally, either type will work well in practice providing that the accuracy and the fit of mating parts are of equal quality.

For double-enveloping worm gears, the defined profile is that of the worm gear. The worm is made with a special tool that has a shape similar to that of the worm gear. For the double-enveloping worm gear, the usual shape is straight-sided in the axial section and tangent to the defined basic circle.

3.2.5.2 Basic Tooth Forms for Worm Gearing

The tooth forms of worm gearing have not been standardized to the same degree as the tooth forms for spur gearing, for example. Since a special hob has to be made to cut the gears that will mesh with each design of worm and since the elements that constitute good worm gearing design are less well understood, there has been little incentive to standardize.

The most general practice in worm gear design and manufacture is to establish the shape of the worm thread and then to design a hob that will generate teeth on the gear that are conjugate

TABLE 3.26
Typical Tooth Proportions for Worms and Worm Gears

Application	No. of Worm Threads	Cutter Pressure Angle	Addendum	Working Depth	Whole Depth
Tooth Proportions for Single-Enveloping Worms and Worm Gears					
Index or holding mechanism	1 or 2	14.5°	1.000	2.000	2.250
Power gearing	1 or 2	20°	1.000	2.000	2.250
	3 or more	25°	0.900	1.800	2.050
Fine pitch (instrument)	1–10	20°	1.000	2.000	2.200 + 0.002 in. (English)
	1–10	20°	1.000	2.000	2.200 + 0.002 mm (Metric)
Tooth Proportions for Double-Enveloping Worms and Worm Gears					
Item	**No. of Worm Threads**	**Pressure Angle**	**Addendum**	**Working Depth**	**Whole Depth**
Power gearing	1–10	20°	0.700	1.400	1.600

Note: The addendum, working-depth, and whole-depth values are for 1 normal diametral pitch. (The normal diametral pitch of the worm is equal to 3.141593 divided by the axial pitch of the worm and then the result divided by cosine of the worm lead angle.) In the English system, for normal diametral pitches other than 1, divide by the normal diametral pitch. In the metric system, the values are for 1 normal module. For modules other than 1, multiply by the normal module.

to those on the worm. This practice is not followed in the case of double-enveloping gearing since special tooling based on the shape of the gear member is required to generate the worm.

The shape of the teeth of the worm member is dependent on the size of the tool and the method used to cut the threads. These threads may be cut with a straight-sided V-shaped tool in a lathe, milled with double conical milling cutters of up to 6 in. in diameter in a thread mill, hobbed with a special hob, or ground in a thread-grinding machine with a grinding wheel having a diameter up to 20 in. In each case, the worm thread pro le will be noticeably different, especially in the case of the higher lead angles. Worms can also be rolled, in which case, the thread pro le will still be different. It is essential, therefore, for the designer of worm gearing either to specify the manufacturing method to be used to make the worm or to specify the coordinates of the worm pro le.

3.2.5.3 Specific Calculations for Worm Gears

In designing worm gear units and making calculations, several considerations should be kept in mind.

- The ratio is the number of worm gear teeth divided by the number of worm threads. (The pitch diameter of the gear divided by the pitch diameter of the worm is almost never equal to the ratio!)
- Due to the tendency of the worm gear to wear in to best t the worm, common factors between the number of worm threads and the number of worm gear teeth should be avoided.
- The normal circular pitches of the worm and the worm gear must be the same. Likewise, the normal pressure angle of worm and the normal pressure angle of the worm gear must be the same.
- The axial pitch of the worm and the transverse circular pitch of the worm gear must be the same. In a like manner, the axial pressure angle of the worm must be the same as the transverse pressure angle of the worm gear.
- The lead of the worm is equal to the worm axial pitch multiplied by the number of threads. The lead can be thought of as the axial advance of a worm thread in one turn (360°) of the worm.
- It has been customary to thin the worm threads to provide backlash but not thin the worm gear teeth.
- There is a small difference between the pressure angle of a straight-sided tool used to mill or grind worm threads and the normal pressure angle of the worm thread.

Table 3.26 shows typical tooth proportions for single-enveloping worm gearsets and for double-enveloping worm gearsets. This table shows that the addendum is one-half the working depth. In some cases, it may be described to make the worm addendum larger than the addendum of the worm gear.

Table 3.27 shows recommended minimum numbers of teeth for single-enveloping worm gears. Since the pressure angle changes going across the face width of the worm gear, larger numbers of teeth are needed to avoid undercut problems in worm gears than in spur gears.

Table 3.28 shows that lead angle needs to increase with the numbers of threads. If, for instance, a lead angle of 20° is

TABLE 3.27
Minimum Number of Worm Gear Teeth for Standard Addendum

Pressure Angle (°)	Min. No. of Teeth
14½	40
20	25
25	20
30	15

TABLE 3.28
Suggested Limits on Lead Angle

No. Worm Threads	Lead Angle (°)
1	Less than 6
2	3 to 12
3	6 to 18
4	12 to 24
5	15 to 30
6	18 to 36
7 and higher	(not over 6° per thread)

desired to get good ef ciency, the designer should plan to use four or ve threads on the worm.

Worm gears are normally sized by picking the number of threads and teeth and then choosing a normal circular pitch that will give enough center distance to have the necessary load-carrying ability. Table 3.29 shows suggested normal circular pitches.

Table 3.30 shows nominal backlash values for worm gearsets. These values represent thinning of worm threads and tolerances on worm thread thickness and gear tooth thickness.

Table 3.31 shows recommended cutter or grinding wheel outside diameters for the making of single-enveloping worms.

TABLE 3.29
Some Recommended Normal Circular Pitches

Fine Pitch		Coarse Pitch	
English (in.)	Metric (mm)	English (in.)	Metric (mm)
0.030	1.000	0.200	5.000
0.040	1.250	0.250	6.500
0.050	1.500	0.300	8.000
0.065	2.000	0.400	10.000
0.080	2.500	0.500	12.000
0.100	3.000	0.625	15.000
0.130	3.500	0.750	20.000
0.160	4.000	1.000	25.000
		1.250	30.000
		1.500	40.000

Note: The normal circular pitch of the worm gear is equal to the axial pitch of the worm multiplied by the cosine of the lead angle.

TABLE 3.30
Recommended Values of Backlash for Single-Enveloping Worm Gearing and Double-Enveloping Worm Gearing

Center Distance	Backlash (Amount by Which Worm Should Be Reduced)	
2	0.003	0.008
6	0.006	0.012
12	0.012	0.020
24	0.018	0.030

TABLE 3.31
Recommended Values of Cutter Diameter (Single-Enveloping Gearsets)

Type of Worm	Suggested Cutter Diameter, D_c (in.)	Process
Low-speed power	4	Milled
High-speed power	20	Ground
	Fine Pitch	
Commercial quality	3	Milled
Precision quality	20	Ground

Further details in worm gear design and rating are given in consequent chapters of the book.

3.2.6 Standard System for Face Gears

Fine-pitch (20 diametral pitch and ner) face gears have been standardized by the AGMA. Coarse-pitch face gears have not been standardized. The following data are arranged to provide a logical method of calculating the tooth proportions of face gears. Such gears will be suitable for most applications. For more complete treatment of the subject, the reader is referred to the current AGMA standards. Table 3.32 shows basic tooth proportions for face gearsets.

TABLE 3.32
Tooth Proportions for Pinions Meshing with Face Gears

Item	Coarse Pitch, $P_d \le 20$		Fine Pitch, $P_d > 20$	
No. of Teeth in Pinion, N_P	$N_P < 18$	$N_P \ge 18$	$N_P < 18$	$N_P \ge 18$
Pressure angle, ϕ (°)	20	20	20	20
Addendum, a	See Table 3.34	$\frac{1}{P_d}$	See Table 3.34	$\frac{1}{P_d}$
Standard pitch diameter, D_p	$\frac{N}{P_d}$	$\frac{N}{P_d}$	$\frac{N}{P_d}$	$\frac{N}{P_d}$
Working depth, h_k	–	$\frac{2.0}{P_d}$	See Table 3.35	$\frac{2.0}{P_d}$
Whole depth, h_t	–	$\frac{2.25}{2P_d}$	See Table 3.35	$\frac{2.20}{2P_d}+0.002$
Circular tooth thickness[a], t	–	$\frac{\pi}{2P_d}$	See Table 3.35	$\frac{\pi}{2P_d}$
Clearance, c	–	$\frac{0.25}{P_d}$	$\frac{0.2}{P_d}+0.002$	$\frac{0.2}{P_d}+0.002$
Reference standard	None		AGMA 203.01	

[a] Thin teeth for backlash. See Section 3.2.1 for general information on spur gear backlash.

Caution should be exercised in using Table 3.32. The items shown apply only to gears that meet the following requirements:

- The axes of the gear and the pinion must intersect at an angle of 90°.
- The gear must be generated by means of a reciprocating pinion-shaped cutter having the same diametral pitch and pressure angle as the mating pinion and must be substantially the same size.
- The pinion should be sized to meet requirements on load-carrying capacity.
- The minimum number of teeth in the pinion in this system is 12, and the minimum cutter pitch diameter is 0.250 in.
- The minimum gear ratio is 1.5:1 and the maximum ratio is 12.5:1.
- The long- and short-addendum designs for the pinion with less than 18 teeth should not generally be used on speed-increasing drives.

3.2.6.1 Pinion Design

The pinion member in case of numbers of teeth less than 18 has an enlarged tooth thickness to avoid undercut when cut with a standard cutter (Tables 3.33 and 3.34). Compared with the pinions designed in accordance with data in Section 3.2.1, the outside diameter is somewhat less. This is to avoid the necessity of cutting the gear with shaper cutters that have excessively pointed teeth. The limit set is top land with ≥$0.40/P_d$. Table 3.35 gives the tooth proportions for face gear pinions.

TABLE 3.33
Minimum Numbers of Teeth in Pinion and Face Gear

	Min. Numbers of Teeth[a]	
Diametral Pitch Range	**Pinion**	**Gear**
20–48	12	18
49–52	13	20
53–56	14	21
57–60	15	23
61–64	16	24
65–68	17	26
69–72	18	27
73–76	19	29
77–80	20	30
81–84	21	32
85–88	22	33
89–92	23	35
93–96	24	36
97–100	25	38

[a] The minimum numbers of teeth in the pinion are limited by the design requirements of the gear cutter. These requirements, in addition to the minimum gear ratio limitation, limit the numbers of teeth in the gear.

TABLE 3.34
Addendum of Face Gears and Pinions

	Coarse Pitch		
No. of Teeth in Pinion	**Pinion Addendum**	**Gear Addendum**	**Fine Pitch Pinion Addendum**
12	1.120	0.700	$\frac{1.1215}{P_d} - 0.002$
13	1.100	0.760	$\frac{1.1050}{P_d} - 0.002$
14	1.080	0.820	$\frac{1.0865}{P_d} - 0.002$
15	1.060	0.880	$\frac{1.0650}{P_d} - 0.002$
16	1.040	0.940	$\frac{1.0420}{P_d} - 0.002$
17	1.020	0.980	$\frac{1.0175}{P_d} - 0.002$
18 and 19	1.000	1.000	$\frac{1.0000}{P_d} - 0.002$
	1.250	0.750	
20 and 21	1.000	1.000	
	1.250	0.750	
22 through 29	1.000	1.000	
	1.200	0.800	
30 through 40	1.000	1.000	
	1.150	0.850	
41 and higher	1.000	1.000	
	1.100	0.900	

TABLE 3.35
Tooth Proportions for Fine-Pitch Face Pinions

No. of Teeth	Tooth Thickness at Standard Pitch Diameter	Whole Depth	Working Depth[a]
12	1.7878	2.0234	1.8234
13	1.7452	2.0654	1.8604
14	1.7027	2.1054	1.9054
15	1.6601	2.1424	1.9424
16	1.6175	2.1778	1.9778
17	1.5749	2.2118	2.0118
18	1.5708	2.2000	2.0000
19 and up	1.5708	2.2000	2.0000

[a] Divide by diametral pitch and subtract 0.002″.

3.2.6.2 Face Gear Design

The face gear member is generated by a cutter having proportions based on the pinion with which the face gear will operate. The most important specification for the shape of the teeth of a face gear is a complete specification of the cutter to be used to cut it or, next best, a detailed specification of the mating pinion.

TABLE 3.36
Tooth Proportions and Diameter Constants for 1 Diametral Pitch Face Gears, 20° Pressure Angle

	Gear Diameter Constraints							
	$m_g = 1.5$		$m_g = 2$		$m_g = 4$		$m_g = 8$	
No. of Pinion Teeth, N_P	m_o	m_i	m_o	m_i	m_o	m_i	m_o	m_i
12	–	–	1.221	1.020	1.221	0.960	1.221	0.945
13	1.202	1.064	1.202	1.015	1.202	0.959	1.202	0.945
14	1.187	1.062	1.187	1.011	1.187	0.958	1.187	0.944
15	1.174	1.052	1.174	1.007	1.174	0.957	1.174	0.944
16	1.161	1.051	1.161	1.004	1.161	0.956	1.161	0.944
17	1.156	1.041	1.156	1.000	1.156	0.955	1.156	0.944
18	1.150	1.039	1.150	0.997	1.150	0.954	1.150	0.943
	1.176	1.042	1.176	0.999	1.176	0.955	1.176	0.943
20	1.144	1.030	1.144	0.991	1.144	0.953	1.144	0.943
	1.166	1.032	1.166	0.993	1.166	0.953	1.166	0.943
22	1.140	1.022	1.140	0.987	1.140	0.952	1.140	0.943
	1.156	1.024	1.156	0.988	1.156	0.952	1.156	0.943
24	1.133	1.015	1.133	0.983	1.133	0.951	1.133	0.943
	1.150	1.017	1.150	0.984	1.150	0.951	1.150	0.943
30	1.121	1.001	1.121	0.975	1.121	0.949	1.121	0.942
	1.131	1.001	1.131	0.975	1.131	0.949	1.131	0.942
40	1.109	0.986	1.109	0.966	1.109	0.946	1.109	0.941
	1.113	0.986	1.113	0.966	1.113	0.946	1.113	0.941

The face gear has two dimensions unique to face gears which control the face width of the teeth: the outer and inner diameters of the face gear.

The maximum usable face width may be estimated from Table 3.36. The outside diameter of the gear should not exceed m_oD. The inside diameter should not be less than m_iD. Thus,

$$\text{Outer diameter} = m_oD = D_o, \tag{3.47}$$

$$\text{Inner diameter} = m_iD = D_i, \tag{3.48}$$

$$\text{Pitch diameter} = D, \tag{3.49}$$

$$\text{Face width} = \frac{D_o - D_i}{2}. \tag{3.50}$$

3.2.7 System for Spiroid and Helicon Gears

At present, there are no AGMA standards covering Spiroid, Helicon, or Planoid* gears. The following material is based on information covered in Spiroid Gearing, Paper 57-A-162 of the ASME, and on additional information supplied by its author W. D. Nelson.

Although specialized designs can be developed that will best suit a given application, the standardized procedure presented here will yield designs that will meet the needs of most Spiroid and Helicon gear applications.

Planoid gears are often associated with Spiroid and Helicon gearings but are an entirely different form of gearing, used for ratios generally under 10:1 where maximum strength and ef ciency are required. Their design is beyond the scope of this section.

The basic approach to the establishment of the Spiroid or the Helicon tooth form is to establish the pinion tooth form and then develop a gear tooth form that is conjugate to it.

The tooth form of the Spiroid gear is compromised of teeth having different pressure angles on each side of the teeth: a low-pressure-angle side and a high-pressure-angle side. The choice of pressure angle will control the extent of the elds of the conjugate action of the teeth. The values of pinion taper angle have been standardized by Illinois Tool Works in order to achieve the best general-purpose gearing. These standard values are shown in Table 3.37.

The design of Helicon gearing is closely related to Spiroid gearing. Table 3.38 shows the general equations used in the design of both kinds of gearing. The general Spiroid formulas become Helicon formulas when the pinion taper angle τ is set to be equal to zero. Figure 3.14 shows a Spiroid gearset. A Helicon pinion and a Helicon gear are similar except that the pinion taper angle τ and the gear face angle γ are both *zero* in Helicon gearing.

3.2.7.1 Spiroid Gearing

Spiroid gearing is suitable for gear ratios of 10:1 or higher. The numbers of teeth in the gear can range from 30 to 300.

The design procedure for Spiroid gearing somewhat differs from other types of gearing discussed in this chapter. The following are the basic steps:

- Center distance is the basic starting point for the calculation of Spiroid tooth proportions.

* Spiroid, Helicon, and Planoid are trademarks registered by Illinois Tool Works, Chicago, Illinois.

TABLE 3.37
Standard Tooth and Gear Blank Relationships, Spiroid and Helicon Gearings

Spiroid Gearing

1. Sigma angle $\sigma_P = 40°$ (standard)
2. Pinion taper angle τ versus gear face angle γ

Pinion Taper Angle, τ (°)	Gear Face Angle[a], γ (°)
5	8 preferred
7	11
10	14

3. Gear ratio m_G = 10:1 higher
4. Number of teeth in gear:
   Varies with center distance (see Table 3.17).
   Hunting ratios are desirable with all multiple-thread pinions.
5. Tentative pressure angle selection

	Pressure Angle Selection	
Ratio	**Low Side**	**High Side**
$m_G \leq 16{:}1$	15°	35°
$m_G > 16{:}1$	10°	30°

[a] Approximate values, for $\sigma_P = 40°$.

- Lead and pinion zero plane radius are then calculated.
- Tooth proportions and blank proportions are based on the pinion zero plane radius and are then calculated.

From this procedure, it can be seen that the pitch of the teeth is an end result of the calculating procedure for Spiroid gearing, rather than the beginning of other types of gearing. Center distance governs the horsepower capacity of the gearset.

Table 3.37 shows the basic tooth proportions for Spiroid gearing. Table 3.38 gives the general equations used to determine Spiroid tooth proportions.

If optimum ef ciency is desired, the size of the pinion relative to the gear should be kept small by using values of R_G/R_P which are larger than those shown in Figure 3.15. This has the effect of producing an increased lead angle for a given center distance and gear ratio. Figures 3.16 and 3.17 show recommended data for Helicon and Spiroid calculations.

If control gear applications are being considered, in which runout is critical, the driving side should be the low-pressure-angle side of the teeth. This minimizes the effects of runout in both members. A 10° taper angle τ may be used, since it requires only one-half the axial movement of the pinion to produce a given change in backlash.

For gearsets of the highest strength, the number of teeth in the gear should be kept low (30 to 40 for lower ratios) and a stub tooth form employed. Such designs are beyond the scope of this section.

Straddle-mounted pinions may require a shaft of larger size (greater stiffness) than would result from the data shown in Figure 3.14. In such cases, the values of R_G and R_P are

TABLE 3.38
General Equations for Spiroid and Helicon Gearings

Spiroid Gearing

$$\sin\phi_P = \frac{\tan\tau}{\tan\sigma_P} \tag{3.51}$$

$$R_P = \frac{C}{\sin\phi_P + \left(\dfrac{R_G}{R_P}\right)\cos\sigma_P} \tag{3.52}$$

$$R_G = \frac{R_G}{R_P} R_P \tag{3.53}$$

$$L = \frac{2\pi R_G \cos\sigma_P}{m_G - \left(\dfrac{R_G}{R_P}\right)\sin\sigma_P \cos\phi_P} \tag{3.54}$$

$$x_P = R_G \sin\sigma_P \tag{3.55}$$

$$r_o = R_P - x_P \tan\tau \tag{3.56}$$

$$D_W = \frac{0.6\left(L/N_P\right)\sec\tau}{\left[\sin\psi_1/\cos(\psi_1+\tau)\right]+\left[\sin\psi_2/\cos(\psi_2+\tau)\right]} \tag{3.57}$$

$$D_N = D_W \cos\tau \tag{3.58}$$

$$CLR = 0.07\frac{L}{N_P} + 0.002 \tag{3.59}$$

$$\psi_{2L} = \tan^{-1}\left[\frac{RP}{x_P} + \frac{Cz_P}{R_P(k_{yP} - x_P)}\right] \tag{3.60}$$

$$\tan\lambda_m = \frac{L\sec\tau}{2\pi r_m} \tag{3.61}$$

$$\psi_2 = \psi_{2L} + 5° \tag{3.62}$$

Helicon Gearing

$$R_P = \frac{C}{\left(\dfrac{R_G}{R_P}\right)\cos\sigma_P} \tag{3.63}$$

$$L = \frac{2\pi R_G \cos\sigma_P}{m_G - \left(\dfrac{R_G}{R_P}\right)\sin\sigma_P} \tag{3.64}$$

$$D_W = \frac{0.6\dfrac{L}{n}}{\tan\psi_1 + \tan\psi_2} \tag{3.65}$$

$$\psi_{2L} = \tan^{-1}\frac{R_P}{x_P} \tag{3.66}$$

$$\psi_{2L} = \tan^{-1}\frac{1}{\left(\dfrac{R_G}{R_P}\right)\sin\sigma_P} \tag{3.67}$$

$$\psi_2 = \psi_{2L} + 7° \tag{3.68}$$

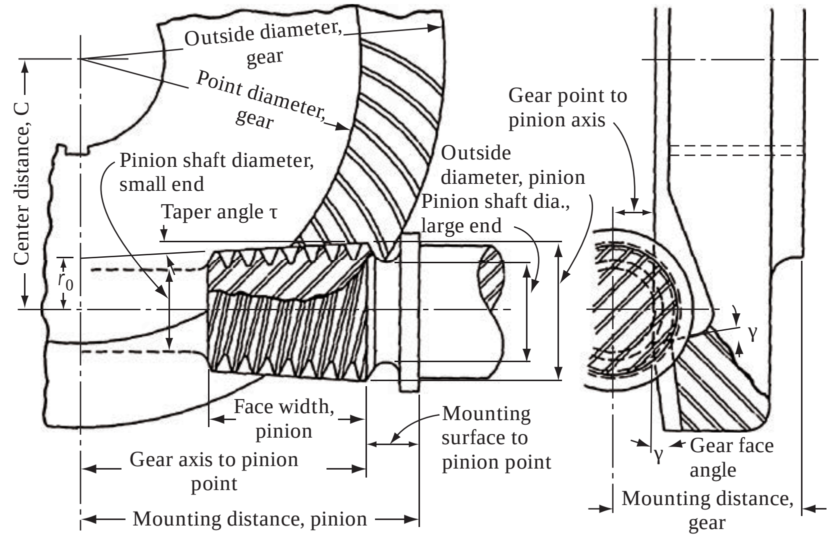


FIGURE 3.14 Spiroid gearset—mounting and gear blank nomenclature.

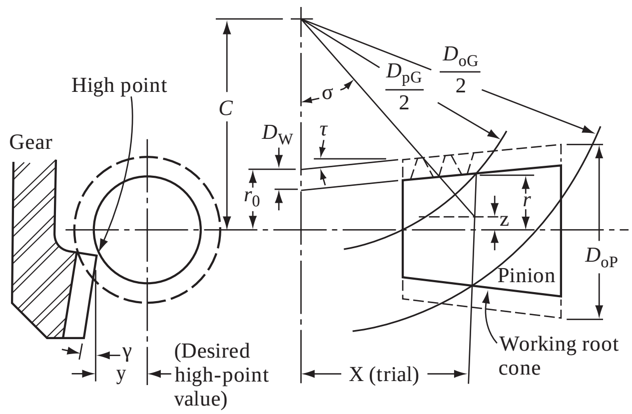


FIGURE 3.15 Gear tooth high point—Spiroid gearing.

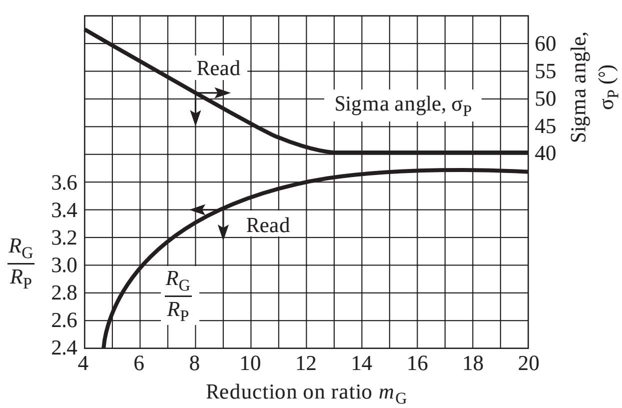


FIGURE 3.16 Sigma angle for different ratios—Helicon gearing.

decreased until a shaft of the desired size can be achieved. There is a de nite limit to the maximum size of pinion for a given pressure angle. The size of the pinion may be increased until a limiting pressure condition is reached. Limiting pressure angle can be calculated by means of Equation 3.62.

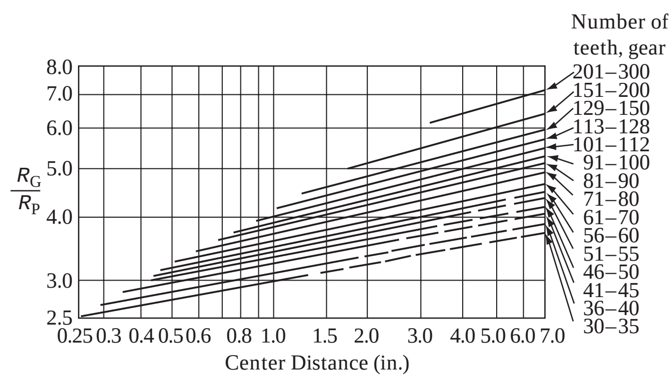


FIGURE 3.17 R_G/R_P versus center distance and number of teeth—Spiroid gears.

3.2.7.2 Helicon Gearing

Helicon gearing is suitable for gear ratios from 4:1 to 400:1 in light- to medium-loaded applications.

The design procedure for Helicon gears is somewhat similar to that for Spiroid gears but starts with the gear outside diameter as the basis of load capacity instead of the center distance as is used in Spiroid gearing.

The following are the basic steps:

- The gear ratio m_G, pinion r/min, and horsepower are used to establish the necessary gear outside diameter.
- Pinion pitch radius, center distance, lead, and working depth are next established.
- From these values, the gear and pinion blank dimensions are established.

3.2.7.3 Detailed Calculations of Spiroid and Helicon Tooth Data

In the earlier edition of this book (1962), Tables 5.41 and 5.42 were presented to show detail calculations of 59 items

of specific tooth data relating to the specific geometry of the teeth. Several pages of instructions were also given on how to use or interpret the various items.

Computer programs for gear tooth data were not in use in the 1960s. Now, computer programs are in general use. The software of these programs is established and kept up to date by the patent holders and developers of special gear types.

A lengthy detail calculation procedure is not presented in the book. For Spiroid and Helicon gears, the reader is advised to obtain the latest technical publications of the Spiroid Division of the Illinois Tool Works. When a gear application gets down to final design calculations, arrangements can be made to have a computer run made with the latest and best computer software.

3.3 GENERAL EQUATIONS RELATING TO CENTER DISTANCE

This section of Chapter 3 deals with the distance between the shafts of meshing gears. This distance is called *center distance* in the case of gearing operating on nonintersecting shafts. Spur and helical gearings, both external and internal, worm gearing, hypoid gearing,* spiral gearing, Spiroid[†] gearing, and Helicon[‡] gearing must all operate at specific center distances. Certain types of gears which operate on intersecting shafts, such as bevel gearing, do not have a center-distance dimension. However, their pitch surfaces must be maintained in the correct relationship; hence, the axial position of these gears along their shafts is critical. Such gears, therefore, have a mounting distance that defines the axial position of the gears.

Certain types of gears, such as hypoid gears, Spiroid gears, throated worms and worm gears, face gears, Helicon gears, and Planoid[§] gears, have both a center distance and a mounting distance.

When establishing the center distance for a set of gears, it is first customary to determine the theoretical center distance for the gears. The actual operating center distance is next determined. This center distance includes the considerations of tolerance and the interchangeability of the various parts that may be included in the final assembly such as bearing, mounting brackets, and housing.

In spur and helical gears, it may be necessary or desirable to operate the gears on an appreciably larger center distance than theoretical to get improved load-carrying capacity. For instance, gears can be designed so that 20° hobs cut the tooth, but the spread center is calculated so as to make the gears operate at a 25° pressure angle. See Sections 3.3.4 through 3.3.6.

Section 3.3.1 covers the calculation of the theoretical center distance at which various types of gearing will operate. The most generally used equations are summarized in Table 3.39. In the sections that follow, the special consideration on which the equations are based are covered in detail.

* In the case of hypoid gearing, *offset* is the correct term for the distance between gear and pinion shafts.
† Spiroid is a trademark of Illinois Tool Works, Chicago, Illinois.
‡ Helicon is a trademark of Illinois Tool Works, Chicago, Illinois.
§ Planoid is a trademark of Illinois Tool Works, Chicago, Illinois.

TABLE 3.39
Center-Distance Equations

Standard Center Distance, *C*		
Spur, helical, and worm gears	$C = \frac{d+D}{2}$	(3.69)
	$C = \frac{n+N}{2P}$[a]	(3.70)
Internal gears	$C = \frac{D-d}{2}$	(3.71)
	$C = \frac{N-n}{2P}$	(3.72)
Operating Center Distance, *C′*		
Spur, helical, and worm gears	$C' = \frac{C\cos\phi}{\cos\phi'}$	(3.73)
Spur helical	$C' = \frac{d'+D'}{2}$	(3.74)
Internal	$C' = \frac{D'-d'}{2}$	(3.75)
General Equations Relating Tooth Thickness and Center Distance for Parallel Axis Gearing		
External spur and helical gears	$\text{inv}\,\phi' = \frac{n(t_P + t_G + B) - \pi d}{(n+N)\,d} + \text{inv}\,\phi$	(3.76)
Internal gears	$\text{inv}\,\phi' = \frac{\pi d - n(t_P + t_G + B)}{(N-n)d} + \text{inv}\,\phi$	(3.77)
General Equations Relating Tooth Thickness and Center Distance for Nonparallel Nonintersecting Axis Gearing		
	$K_a = \frac{n}{N} = \frac{1}{m_G}$	(3.78)
	$K_b = \text{inv}\,\phi_G + K_a \text{inv}\,\phi_P - \frac{\pi(p_n - t_{nP} - t_{nG})}{Np_n}$	(3.79)
	$K_d = \frac{\text{inv}\,\phi_G}{\text{inv}\,\phi_P}$	(3.80)
	$K_c = K_a + K_d$	(3.81)
	$\text{inv}\,\phi_P = \frac{K_b}{K_c}$	(3.82)
	$\text{inv}\ \phi_G = K_b - K_a\ \text{inv}\ \phi_P$	(3.83)
Check Equations: To Check Equations 3.78 through 3.83		
	$K'_d = \frac{\text{inv}\,\phi_G}{\text{inv}\,\phi_P}$	(3.84)
	$K'_d = K_d$ (check equation)	(3.85)
	$D'_P = \frac{D_{bP}}{\cos\phi_P}$	(3.86)
	$D'_G = \frac{D_{bG}}{\cos\phi_G}$	(3.87)

(*Continued*)

TABLE 3.39 (CONTINUED)
Center-Distance Equations

Use Values from Equations 3.84 and 3.85 in Equation 3.74, where

$$K_c = \frac{\sin\phi_G}{\sin\phi_P} = \frac{\sin\Phi_G}{\sin\Phi_P} \tag{3.88}$$

P—transverse diametral pitch
C—standard center distance (de ned by Equations 3.69 and 3.70) (see also Section 3.3.2)
C′—operating center distance
d—standard (n/P) pitch diameter of pinion (see also Section 3.3.3)
d′—operating pitch diameter of pinion (see also Section 3.3.4)
D—standard (N/P) pitch diameter of gear (see also Section 3.3.3)
D′—operating pitch diameter of gear (see also Section 3.3.4)
n—number of teeth in pinion
N—number of teeth in gear
ϕ—cutting pressure angle
B—design backlash, the total amount the teeth in both members are thinned for backlash considerations
ϕ'—operating pressure angle (see also Section 3.3.5)
t_P—circular (transverse) thickness of pinion tooth
t_G—circular (transverse) thickness of gear tooth
t_{nP}—circular (normal) thickness of pinion tooth
t_{nG}—circular (normal) thickness of gear tooth
ψ_P—helix angle of pinion
ψ_G—helix angle of gear
Φ_P—rolling pressure angle of pinion
Φ_G—rolling pressure angle of gear

Note: To obtain C', use ϕ' from Equation 3.76 or 3.77 in Equation 3.73.
[a] Does not apply to worm gearing.

3.3.1 Center-Distance Equations

Table 3.39 is a summary of equations convenient to use to obtain the values of center distance for the various types of gearing shown in Figure 3.18.

Center distance and tooth thickness are inseparable. In most case, however, standard tooth proportions are used, and the simpli ed equations of center distance (Equations 3.69 through 3.72) are adequate. Standard tooth proportions are de ned in Section 3.3.2. When the sum of the tooth thickness of pinion and gear is not equal to the circular pitch, nonstandard center distance, as obtained from equations such as Equations 3.73 and 3.76, will be required. Most problems involving center distance and tooth thickness fall into one of the following categories:

- The thickness of the teeth of the gear and of the pinion is xed. The center distance at which the gears will properly mesh is to be established.
- The sum of the tooth thicknesses of both members plus the design backlash is equal to the circular pitch. In this case, standard center distance is correct. Use Equations 3.69 and 3.72.
- The sum of the tooth thickness of both members plus the design backlash is *not* equal to the circular pitch. In this case, a nonstandard center distance is required. Use Equations 3.73 and 3.76. This problem is covered in greater detail in Section 3.3.9.
- The center distance is xed. The tooth thicknesses for gears that will operate on the given center distance are to be established. If the center distance for the given diametral pitch and number of teeth is different from that obtained from Equation 3.70, then Equation 3.73 must be solved and the operating pressure angle thus found used in Equation 3.76), which is then solved for the term ($t_P + t_G + B$). In Section 3.1.4.2, the ways in which this total tooth thickness can be divided between gear and pinion are discussed.
- Neither the tooth thickness nor the center distance is xed; the best values for both are to be established. In this happy case, the tooth thickness is usually established rst on the basis of strength or some other appropriate consideration and the center distance required is then based on Equations 3.76 and 3.73.
- Both the center distance and the tooth thickness are xed. The amount of backlash or the degree of tooth interference is to be established. This is a frequent check problem of an existing gear design. Equations 3.73 and 3.76 are used and solved for *B*. Minus values of *B* indicate tooth interference.

3.3.2 Standard Center Distance

Most gear designs are based on standard tooth proportions. Such gears are intended to mesh on standard center distances. The equations that establish standard center distances are based on the following assumptions:

1. The sum of the circular tooth thicknesses (effective) equals the circular pitch minus the backlash:

$$t_P + t_G = p_t - B. \tag{3.89}$$

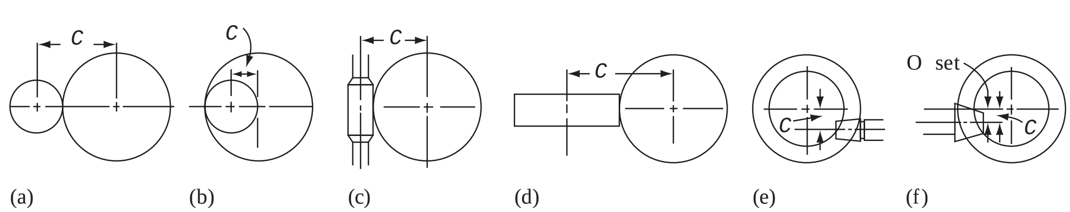


FIGURE 3.18 Center distance, shown for various kinds of gears, is the distance between the axes of the shafts measured on a common normal. (a) Spur and helical gearings—external; (b) spur and helical gearings—internal; (c) worm gearing—single- and double-enveloping; (d) spiral gearing; (e) Spiroid, Planoid, and Helicon gearings; and (f) hypoid gearing.

This rule covers the case of long and short addenda in which the gear teeth are corrected for such things as undercut or balanced strength. In such cases, the tooth thicknesses of both members are suf ciently altered to permit to mesh on standard center distances.

2. The tooth thickness of an individual gear is one-half the circular pitch (transverse) minus one-half the backlash:

$$t = \frac{p - B}{2}. \tag{3.90}$$

This covers the more common case. It represents the approach used by most makers of catalog-type gears. Such gears are all expected to operate on standard center distances regardless of the number of teeth in the pinion. The center distances on which such gears will operate may be calculated by means of Equations 3.69 and 3.70.

The tolerance on standard center distance can be bilateral (thus, 5.000 ± 0.002 in.) if the value of *B* was chosen suf - ciently large. This is the most convenient way from the standpoint of the manufacturer. If *B* is not large enough, a unilateral tolerance (thus, 5.000 in. + 0.004 in. − 000 in.) may be used.

3.3.3 Standard Pitch Diameters

The standard pitch diameter of a gear is a dimension of a theoretical circle. It is given for each type of gear by the following relations:

Spur gearing:

$$\frac{N}{P_d}, \tag{3.91}$$

where P_d is the diametral pitch.

Helical gearing:

$$\frac{N}{P_t}, \tag{3.92}$$

where P_t is the transverse diametral pitch.

Bevel gearing:

$$\frac{N}{P_d} \tag{3.93}$$

Worm gearing:

$$\frac{N_G p}{\pi}, \tag{3.94}$$

where N_G equals the number of worm gear teeth, and *p* equals the circular pitch of the worm gear.

Since the diametral pitch of a gear is xed by the tool that is used to cut it (hob, shaper cutter, shaving cutter, and so forth), and since the number of teeth in the gear is a whole number, the standard pitch diameter is an imaginary circle. It can have no tolerance, regardless of the variations in the tooth thickness or in the center distance on which it is to operate.

3.3.4 Operating Pitch Diameters

As shown elsewhere, it is entirely practical to operate involute gears of speci c diametral pitch and numbers of teeth at various center distances. It is convenient in such cases to calculate operating pitch diameters for such gears. See Equations 3.95 through 3.98.

External spur and helical pinions:

$$d' = \frac{2C'}{m_G + 1} \tag{3.95}$$

Internal spur and helical pinions:

$$d' = \frac{2C'}{m_G - 1} \tag{3.96}$$

External spur and helical gears

$$D' = \frac{2C'}{m_G + 1} \tag{3.97}$$

Internal spur and helical gears:

$$D' = \frac{2C'}{m_G - 1} \tag{3.98}$$

where

C'—operating center distance
m_G—gear ratio (N/n)
d'—operating pitch diameter of pinion
D'—operating pitch diameter of gear

These equations de ne the operating pitch diameters as being proportional to the transmitted ratio and the instantaneous center distance.

Since gears with thicker-than-standard teeth must operate on enlarged center distances, they will run on operating pitch diameters that are larger than their standard pitch diameters. The operating pitch diameter should not be speci ed as a drawing dimension on the detail drawing of the gear, since it will vary for every different gear and center distance. The best place to show the operating center distance is on an assembly drawing that shows both the pinion and the gear that mesh together.

3.3.5 Operating Pressure Angle

The pressure angle of an individual gear is based on the diameter of the base circle of the gear and on the identi cation of

the speci c radius at which the pressure angle is to be considered. In standard gears, this radius is customarily the standard pitch radius. It is convenient to consider the pressure angle of gears operating on nonstandard center distances at the point of intersection of the line of action and the line of centers. This is the de nition of the operating pressure angle which may be calculated by means of Equation 3.99:

$$\cos\phi' = \frac{C}{C'}\cos\phi, \tag{3.99}$$

where
- ϕ'—operating pressure angle
- C—standard center distance (see Equation 3.70)
- C'—operating center distance
- ϕ—standard pressure angle

It can be seen from the foregoing that a gear can be cut with a cutter of one pressure angle and operate at a different pressure angle. This exibility causes much of the confusion in gear design. It is necessary to accurately de ne each of the standard elements of a nonstandard gear—pitch and pressure angle. A speci cation of base-circle diameter is highly desirable.

3.3.6 Operating Center Distance

The actual center distance at which a gear will operate will have a large in uence on the way in which the gear will perform in service. The actual operating center distance is made up of the combined effects of manufacturing tolerances, basic center distance, differential expansions between the gears and their mountings, and de ections in the mountings due to service loads.

The items that should be considered when determining the minimum and maximum operating center distances for any given gear design are discussed in detail in Sections 3.3.9 and 3.4.1.

In any critical evaluation of a gear design, particularly in the eld of control gearing, the minimum and the maximum operating center distance should be used in equations covering backlash, contact ratio, tooth tip clearance, and so forth.

In this chapter, the concept *operating center distance* is used in two ways. In one case, it is considered to be the center distance that results from the buildup of all tolerances that in uence center distance in any one case. This concept is illustrated in Case 2 in Section 3.4.1. Thus, it is the largest or the smallest actual center distance that could be encountered in a given design. In the other case, it is the one covered by Equations 3.73 and 3.76. For example, the operating center distance is the center distance at which gears of a speci ed tooth thickness and backlash will operate. Depending on the problem encountered, the correct concept to use will have to be selected by the designer.

3.3.7 Center Distance for Gears Operating on Nonparallel Nonintersecting Shafts

The most frequently encountered example of this type of gear operation is in the shaving or the lapping of gears. Here, a cutter, usually a helical-toothed member, is run in tight mesh with a spur or a helical gear. The operating center distance is dependent on the tooth thickness of each member. Equations 3.78 through 3.87 should be solved in sequence and the result is used in Equation 3.73 to obtain the operating center distance. Equation 3.88 gives the ratio constant for the transverse pressure angles for crossed axis gearing.

3.3.8 Center Distance for Worm Gearing

The center distance for worm gearing is based on the sum of the standard pitch diameters of the worm and the worm gear (Equation 3.100). The standard pitch diameter of the worm is obtained from Equation 3.101. This circle has no kinematic signi cance but is the basis for worm tooth proportions:

$$C = \frac{d_w + D}{2}, \tag{3.100}$$

$$d_w = \frac{L}{\pi \tan\lambda}, \tag{3.101}$$

where
- C—center distance
- d_w—pitch diameter of worm
- D—pitch diameter of gear
- L—lead
- λ—lead angle

3.3.9 Reasons for Nonstandard Center Distances

On some occasions, a set of gears must operate on a center distance that is not one-half the sum of the standard pitch diameter of the meshing gears. The designer is confronted with nonstandard center distance in several situations, the more important of which are the following:

- Gear trains in which the teeth are made to standard tooth thickness and backlash is introduced by slightly increasing the standard center distance.
- Gear trains in which the number of teeth and pressure angle relationship requires a long addendum (enlarged tooth thickness design) to avoid undercut, and yet the number of teeth in the gear is so small that making the gear suf ciently short addendum to compensate for the pinion enlargement would cause undercut. In such cases, an enlarged center distance is usually indicated.

- Gear trains in which the sum of the tooth thicknesses of pinion and gear is not equal to the circular pitch for reasons of tooth strength, wear, or scoring.
- Gear trains in which a minor change in ratio (total number of teeth in mesh) has been made without a change in center distance.

In each of these cases, the calculation of operating center distance is performed using Equations 3.73 and 3.76, or 3.77.

3.3.10 Nonstandard Center Distances

By proper adjustments of the thickness of the teeth on each gear of a meshing pair, it is possible to achieve gear designs that will meet most nonstandard center distances. In Section 3.1.3, the limitations governing tooth thickness are outlined. In cases where maximum strength is not critical, it will be found that gears not exceeding these limitations will satisfy most nonstandard center distance problems encountered. This is more fully discussed in method 3.

Sometimes, designers who do not realize the possibilities of gear design will select such things as fractional diametral pitches (10.9652, etc.) in order to make gears fit a nonstandard center distance. This is poor practice in that it necessitates special tooling, which may be costly. It is better to attempt to utilize standard pitch tooling and, by proper adjustments of tooth thickness, to establish gear designs that will meet the nonstandard distances.

In cases where in the center distance is nonstandard, the designer has four possible methods of designing gears to meet the given center distance:

1. A number of teeth in gear and pinion different from the numbers originally selected may be found which will more nearly meet the given center distance. A Brocot table is very useful in finding the numbers of teeth that will give different gear ratios ($1/m_G$). In this case, it is assumed that any of several ratios may be good enough. If not, the following alternatives, which all assume that a specific ratio must be obtained, may be considered.
2. Equation 3.69, 3.70, 3.71, or 3.72 may be rearranged to solve for P_d. Thus, a diametral pitch that will meet the gear ratio and the center distance is found. This method will yield nonstandard diametral pitches for spur gears, which will almost always lead to large manufacturing costs because of the need for nonstandard tooling, cutters, and master gears. This method should be used only if methods 2, 3, and 4 do not meet the needs of application.
3. The gear and the pinion can be made with teeth that are thicker or thinner than the standard. The easiest approach when using this method is to follow method 2, and then use the results as the basis for a selection of the standard diametral pitch nearest to the one found. Equation 3.73 is then used, which will yield the operating pressure angle for the gears based on the new selection of diametral pitch and the desired center distance C'. The value of C in this equation is determined from Equation 3.69 or 3.70. Equation 3.76 is then used and solved for the term ($t_P + t_G$). The selection of values for each term, t_P and t_G, should be somewhat based on the ratios of the number of teeth. In general, pinions cannot be changed from standard as much as gears with large numbers of teeth. The selected value of tooth thickness should be checked for undercut in the event that t_P is less than $p/2$. If the thickness is appreciably greater than $p/2$, the pinion should be checked for pointed teeth. The same should be done for the gear, especially if the changed tooth thickness was not based on the gear ratio. Once the values of C', P, n, N, t_P, and t_G have been established, the remainder of the tooth proportions can be calculated from these values. If the addendum and the whole depth are properly adjusted from the tooth thickness, standard tools can be used to cut and to inspect these gears.
4. If helical gears can be used, the helix angle can be adjusted to obtain the required tooth thickness–center distance relationship. In effect, the designer is following method 3 but is adjusting the transverse tooth thickness by the selection of the required helix angle. The procedure in this case is to use Equation 3.70 or 3.72 to establish the diametral pitch required in the transverse plane P to suit given center distance. The nearest finer (smaller) diametral pitch for which tools are available is selected (P_n), and these values are used in the following equation to establish the required helix angle:

$$\cos\psi = \frac{P}{P_n}. \tag{3.102}$$

Although not an essential part of center-distance problem, somewhat smoother-running gears will result if the designer can manage to pick diametral pitches that will result in a helical overlap of a least two. After the helix angle has been found, the designer can determine the remaining tooth proportions from data found in this chapter. This method can be economically used if the hobbing process of cutting gears is to be used. Special guides and shaper cutters are required for every different lead of helical gear for gears cut by the shaping process. This then fixes the limits on the values of helix angle that any given shop could cut.

3.4 ELEMENTS OF CENTER DISTANCE

Up to this point, this chapter has considered only the means of determining the theoretical center distance that is required

by a set of gears. The theoretical center distance is either the basic center distance, as established by Equation 3.69, 3.70, 3.71, or 3.72, or the theoretical tight-mesh center distance, as established by Equations 3.73 through 3.88. In an actual gearset, however, the gears will be forced to operate on a real center distance that may be larger or smaller than the theoretical center distance by amounts that are based on the way that the part tolerances (bearings, casing, bores, and so forth) will add up.

3.4.1 Effects of Tolerances on Center Distance

In all but the very simplest gear casing designs, there are many tolerances that will govern the operating position of the gear and the pinion shafts. Usually power gearing is not designed to small values of backlash. It is customary, therefore, to apply a generous amount of backlash to the gear teeth, a value that has been found by experience suf cient to prevent the binding of the teeth. In control gearing, however, particularly in those applications that are operated in both directions and are intended to have a minimum of lost motion a critical study of the effects of part tolerances is usually required. The tooth thickness can then be made as large as possible, ensuring a minimum of backlash with little risk of binding. The following two examples indicate a very simple case and a more complex case of center-distance calculation.

Case 1

Figure 3.19 shows a spur gear pair mounted on a cast-iron gear case. The loads and speeds are such that the bearings are simply smooth bores in the cast iron with shafts running in them. Figure 3.20 is a vector diagram showing the forces produced on the bearings as a result of the gear tooth loads. Figure 3.21 shows the displacements of the shafts in the bearing resulting from these loads.

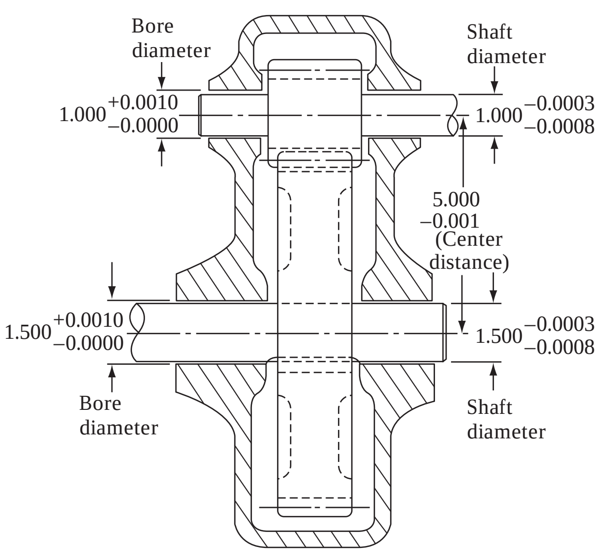


FIGURE 3.19 Spur or helical gears operated in bearings bored in the gear casing exhibit the simplest case of tolerance buildup on center distance.

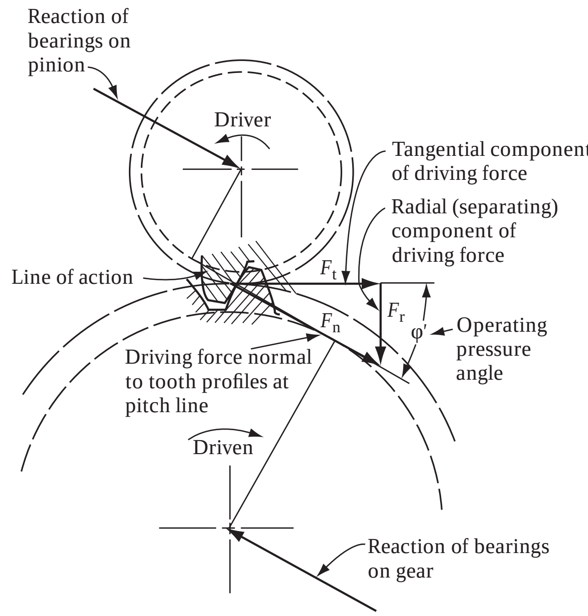


FIGURE 3.20 The reactions forces developed by the bearings to overcome the gear teeth driving forces are parallel to the line of action.

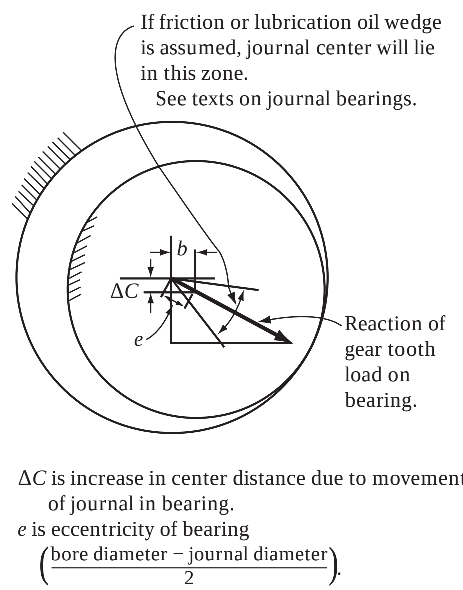


FIGURE 3.21 The position occupied by the journal in a bearing is a function of the gear tooth loads along the shaft and the effects of the friction or of the oil wedge in the bearing.

In this example, it is desired to establish the largest and the smallest center distances that the gears will ever experience in order to determine the range of backlash existing in the gearset. Table 3.40 outlines the calculations made to establish the various center distances at which the gears may be expected to operate in service. Part A of Table 3.40 shows the calculation of the maximum distance that the axis of either shaft can move away from the axis of its bore. This is given as the eccentricity of the shaft within the bearing. It is assumed that the bearing is dry. For a very rough approximation of

TABLE 3.40
Summary of Calculations for Case 1

A. Calculation of Bearing Clearance and Shaft Eccentricity (See Figure 3.19 for Drawing Dimensions)

Bore for pinion shaft	Max. 1.0010	Min. 1.0000
Pinion shaft diameter	Min. 0.9992	Max. 0.9997
Clearance in bearing (c)	Max. 0.0018	Min. 0.0003
Eccentricity of shaft position in bearing (c/2)	Max. 0.0009	Min. 0.00015
Bore for gear shaft	Max. 1.5010	Min. 1.5000
Gear shaft diameter	Min. 1.4992	Max. 1.4997
Clearance in bearing (c)	Max. 0.0018	Min. 0.0003
Eccentricity of shaft position in bearing (c/2)	Max. 0.0009	Min. 0.00015

B. Calculation of Theoretical Maximum and Minimum Center Distances (Assuming Reaction Forces Acting along Line of Centers; Values from Part A)

Center distance for bores (machining tolerance)	Max. 5.0010	Min. 5.0000
Eccentricity of pinion bearing	0.0009	−0.0009
Eccentricity of gear-bearing center distance	0.0009	−0.0009
	Max. 5.0028	Min. 4.9982

C. Calculation of Theoretical Maximum and Minimum Center Distance (Assuming Reaction Forces Acting along Line of Action; Values from Part B)

Center distance of bores (machining tolerance)	Max. 5.0010	Min. 5.0000
Eccentricity, radial component of pinion bearing		
(0.0009) sin 20°	0.0003	–
(0.00015) sin 20°	–	0.0005
Eccentricity, radial component of gear bearing		
(0.0009) sin 20°	0.0003	–
(0.00015) sin 20°	–	0.0005
	Max. 5.0016	Min. 5.0001

maximum and minimum possible center distances, these values of eccentricity can be added to and subtracted from the boring center distance plus its tolerance. This is shown in part B of Table 3.40. This assumes that the forces acting on the shafts are tooth-load reactions that act along the line of centers. In the case of journal bearings that are properly lubricated, the calculated value of eccentricity of operation can be used. This calculation is discussed in texts on journal bearing design. In most cases, if the designer can select the tolerances for the gear tooth thickness, and also the tolerances on the various parts of the casing and the bearings that control the center distance, such that the backlash is not excessive when the calculations shown thus far are carried out, no further calculations need be made. However, when the design is to achieve an absolute minimum of lost motion, the more accurate evaluation of center distance should be carried out.

Part C shows a calculation assuming that the gear tooth reactions will control the position of the journal in the bearing. The minimum center distance will occur when the minimum boring center distance is achieved and the minimum bearing clearance occurs. The tooth loads tend to increase the center distance in this example so that less than the minimum boring center distance cannot occur.

In actual service, the maximum and minimum operating center distances will rarely reach these limits. In the case of maximum center distance, a properly lubricated shaft will develop an oil wedge that keeps the journal away from the walls of the bore. Second, if the only separating loads on the shafts are those produced by gear tooth reactions, the radial movement of the shaft along the line of centers will be equal to $e(\sin \Phi)$. The proper evaluation of operating center distance will depend on whether the application is intermittent in rotation and on the direction of rotation.

Case 2

A more complex case occurs when the pinion and the gear shafts are mounted in separate units. The following example illustrates a case in which a motor having a pinion cut integral with the shaft is mounted by means of a rabbet in a gear case. This type of construction is often found in power hand tools and in aircraft actuators. This example also illustrates a critical evaluation of antifriction bearings. It is desired to determine the largest and the smallest center distances that can occur in order to establish the maximum and minimum tooth thicknesses and the amounts of backlash that result (see Figure 3.22).

Table 3.41, which summarizes the calculations, shows a total of 16 different elements that combine to control the maximum and minimum actual center distances. Since there are so many tolerances, the use of the root mean square of these values is suggested as a reasonable approximation to operating conditions, and it is used here to establish the maximum and minimum probable center distances. This method gives over 90% assurance that the values shown will not be exceeded.

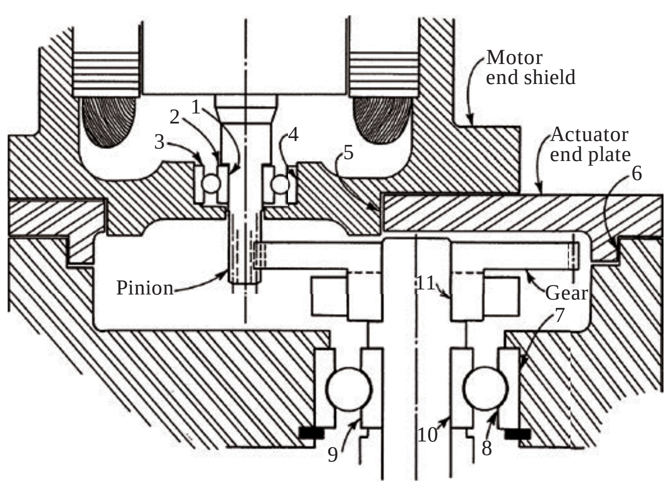


FIGURE 3.22 Cross section of the rst stage of gearing in an aircraft-type actuator. The members refer to the surfaces discussed in Table 3.41.

TABLE 3.41
Summary of Calculations for Case 2

No.	Item Discussed in Footnotes	Surface Designation (see Figure 3.22) (1)	Tolerance (Clearance) (in.) (2)	Equivalent Change in Center Distance (3)	Min. Center Distance (Col. (3)^c × 10^{-6})		Max. Center Distance (Col. (3)^c × 10^{-6})	
					(4a)	(4b)	(5a)	(5b)
1	Fit of motor shaft in inner race of ball bearing[a]	(1)	0.0000	0.0000	−0.00	0.00	+0.00	+0.00
2	Eccentricity of inner ring bearing[b]	(1–2)	0.0002	0.0001	−0.01	–0.01	+0.01	+0.01
3	Radial clearance in bearing[c]	(2–3)	0.0004–0.0000	0.0002	−0.04	+0.04	−0.04	+0.04
4	Eccentricity of outer ring of bearing[d]	(3–4)	0.0004	0.0002	−0.04	−0.04	+0.04	+0.04
5	Fit of outer race of bearing into motor end shield[e]	(4)	0.0008–0.0000	0.0004–0.0000	−0.16	−0.16	+0.16	+0.16
6	Concentricity of axis of bore with axis of rabbet on end shield[f]	(4–5)	0.0020	0.0010	−1.00	−1.00	+1.00	+1.00
7	Clearance between rabbet of motor end shield and bore of actuator end plate[g]	(5)	0.0025	0.00125	−1.56	−1.56	+1.56	+1.56
8	Concentricity of motor bore in actuator end plate and rabbet[h]	(5–6)	0.0010	0.0005	−0.25	−0.25	+0.25	+0.25
9	Distance between axis of gear shaft bore and rabbet in actuator housing[i]	(6–7)	0.0020	0.0010	−1.00	−1.00	+1.00	+1.00
10	Clearance between bore for bearing and bearing outer race[e]	(7)	0.0008	0.0004	−0.16	−0.16	+0.16	+0.16
11	Eccentricity, outer race of bearing[d]	(7–8)	0.0004	0.0002	−0.04	−0.04	+0.04	+0.04
12	Radial clearance in bearing[c]	(8–9)	0.0004–0.0000	0.0002–0.0000	−0.04	−0.04	+0.04	+0.04
13	Eccentricity, inner race of bearing[b]	(9–10)	0.0002	0.0001	−0.01	−0.01	+0.01	+0.01
14	Fit of gear shaft in inner race of bearing[a]	(10)	0.0000	0.0000	−0.00	−0.00	+0.00	+0.00

Note: The maximum values of center distance (columns 4b and 5b) are based on the assumption that the separating forces between pinion and gear act to hold the shafts at their maximum separation within the limits of bearing clearance. The minimum values of center distance (columns 4a and 5a) are based on the assumption that the external forces on gear and pinion act to hold the shafts at their minimum separation within the limits of bearing clearance. The minimum center distance (column 4) is based on the assumption that all parts, made to their maximum clearances, are assembled to achieve the minimum possible center distance. The maximum center distance (column 5) is based on the assumption that all the parts, made to their maximum clearances, are assembled to achieve the maximum possible center distance. Column 1 shows the surface designated on Figure 3.22 by a number in a circle; column 2 shows the tolerance (usually on a diameter); column 3 shows the amount this tolerance can contribute to center distance (±); column 4 shows the square of the tolerance contributing to the minimum center distance; and column 5 shows the square of the tolerance contributing to the maximum center distance. The values in columns 4 and 5 are algebraically added and their square roots are shown at the foot of the column.

[a] Most bearing catalogs indicate bearing and shaft tolerances that will produce an interference t. This t cannot, therefore, contribute to the maximum center distance at which the gears will operate.

[b] Most bearing catalogs indicate a tolerance on runout between the bore and the raceway of the inner ring. This eccentricity contributes to the instantaneous center distance when the inner face is the moving portion of the bearing.

[c] Most bearing catalogs include a certain radial clearance between the inner and the outer race of a bearing. This clearance usually acts to allow an increase in center distance due to the reaction between the two meshing gears.

[d] Most bearing catalogs indicate a tolerance on runout between the raceway and the outer diameter of the outer ring. This eccentricity contributes to the center distance at which the gears will operate. This value will change during running since the outer race is often permitted to creep.

[e] Most bearing catalogs indicate a bore tolerance on the hole that the bearing is tted in. Usually, the tolerance will provide clearance between the bearing and the housing. When clearance exists, the gear reaction acts to increase the center distance to the limit allowed by the clearance.

[f] Many manufacturers of pilot-mounted motors specify a value of runout between the motor shaft and the pilot surface. It is customary to measure this by holding the motor shaft xed (as between centers) and by rotating the motor about its shaft. A dial indicator running on this surface will show runout (twice eccentricity).

[g] A clearance t is usually speci ed on the hole into which the pilot of the motor is to t. This clearance allows the motor to be somewhat shifted and, as a result, has a direct effect on center distance. The value shown assumes a tolerance of $^{+0.0005\text{ in.}}_{-0.0005\text{ in.}}$ on the diameter of the pilot on the motor.

[h] In an assembly, such as shown in this example, a tolerance must be placed on the location of the bore to receive the motor relative to the rabbet on the end plate.

[i] This, in effect, is the same type of tolerance as discussed under g above.

3.4.2 Machine Elements That Require Consideration in Critical Center-Distance Applications

As illustrated in Cases 1 and 2, the operating center distance for a pair of gears is made up of several elements, each of which contributes to the overall center distance. The accumulation of tolerances on each of these elements must be considered in application in which a minimum of backlash is to be established. The following is a consideration of the elements that have the largest contribution to variations in center distance:

- *Rolling-element bearings*: Ball-and-roller bearings consist of three major elements that contribute to backlash and to changes in center distance. Some of these are illustrated in Case 2. The outer race has machining tolerances that cause eccentricity of the axes of the inner raceway and the outer bore. Depending on how this member is installed, the center distance established by the bores in which this element is fitted will be increased or decreased. It is customary to let this element creep; thus, the eccentricity will go through all positions. There is also a tolerance on the outer diameter of this member. This tolerance, plus the tolerance on the bore in which this element is fitted, can cause varying degrees of looseness and therefore changes in operating center distance.

  The inner race has machining tolerances that cause eccentricity in the same manner as in the outer race member. In cases where the shaft rotates, it is customary to use a light interference fit between the inner race and the shaft. In such cases, the center of the gear will move in a path about the center of the ball or a roller path of the bearing. The eccentricity of the gear will be that of the inner race of the bearing. This will cause a once-per-revolution change in center distance.

  The clearance in the bearing is controlled by the selection of the diameter of the balls. This clearance allows greater or lesser changes in the center distance of the gears supported by the bearings.
- *Journal bearings*: Journal bearings can change the center distance at which the gear was intended to operate by an amount that is a function of the clearance designed into the bearing, its direction or rotation, the speed of rotation, and the lubricant used. The change from the distance actually established by the distance between the centers of the bores is due to the oil wedge developed. Handbooks give convenient methods of evaluating the eccentricity that can be expected of a given bearing design. When necessary, these refinements can be introduced into the calculations illustrated by Cases 1 and 2.
- *Sleeve bearings*: The terms *sleeve bearings* is used here to identify the journal-type bearing in which a sleeve of some bearing material is pushed into the gear casing or frames. The tolerance of the eccentricity of the outside diameter and the inside bearing surface is the element to consider in this type of bearing. In some cases, the clearance between the sleeve and the shaft is affected because of the interference fit between the sleeve and the bore in the casing.
- *Casing bores*: The distance between bearing bores in a gear casing can be made up of several elements. In the simplest case, a single frame or casing has two holes drilled at a specified center distance. The tolerance is selected to give the machine operator the necessary working allowance (see Figure 3.19). This tolerance has a direct effect on the operating center distance of the gears.

In some designs, the casing is made up of several parts bolted together. One part may carry the bore for the gear shaft, and another may carry the bore for the pinion shaft. The center distance is then made up of a series of parts each having dimensions and tolerances that can add up in various ways to give maximum and minimum center distances. Case 2 illustrates this type of gearing.

3.4.3 Control of Backlash

In cases where it is necessary to control the amount of backlash introduced by the mountings, two courses of action may be employed. In the first, the center distance may be made adjustable. That is, provisions may be made so that, at assembly, the centers may be moved until the desired mesh is obtained. This entails a method of moving the parts through very small distances, and then being able to securely fix them when the desired distance has been reached. Provision must also be made for assemblers to see what they are doing when adjusting the mesh. In this approach, the difference between the smallest and the largest backlash in the adjusted mesh is only the effect of total composite error (runout) in both meshing parts. Size tolerance on the teeth can be generous, since this is one of the tolerances adjusted out of the mesh by an assembler.

After the center-distance adjustment is completed, the parts may be drilled and doweled, although this causes difficulties if replacement gears are ever to be used on these centers.

In the second approach, very close tolerances are held both in the mountings (boring center distance, bearings, etc.) and on the tooth thickness of the gear. This approach is used whenever interchangeability is specified and in mass production in which assembly costs is to be minimized.

3.4.4 Effects of Temperature on Center Distance

Many gear designs consist of gears made of one material operating in mountings made of another material having a markedly different coefficient of expansion. Since it is customary to manufacture and assemble gears at room temperature of about 68°F, an analytical check of the effective center distance present in the gears at their extremes of operating temperature is desirable.

Power gears may heat up because of the frictional losses in the mesh and may achieve an operating temperature above that

of their casing. If made of the same materials as their mountings, such gears would have a larger apparent pitch diameter* relative to the center distance and would therefore have a smaller effective center distance at their running temperature. Some gears, particularly those in military applications, may be subjected to extreme cold for some time. Often, these are steel gears operating in aluminum or magnesium casings. In this case, the center distance will shrink to a greater degree than the apparent pitch diameter of the gears, and the resulting backlash will be less at room temperature.

If evaluating the effective center distance of a mesh at an operating condition other than room temperature, the temperature of the gear blanks as well as that of the mountings, which may be considerably different, must be considered, as well as the coef cient of expansion of the gear and the mounting materials. There are six operating conditions in which temperature can have an important effect on gear performance:

1. Gears operating at temperatures *higher* than their mountings:
   a. Operating temperatures lower than assembly room temperature
   b. Operating temperatures higher than assembly room temperature
2. Gears operating at temperatures *lower* than their mountings:
   a. Operating temperatures lower than assembly room temperature
   b. Operating temperatures higher than assembly room temperature
3. Gears operating at temperatures essentially the *same* as their mountings:
   a. Operating temperatures lower than assembly room temperature
   b. Operating temperatures higher than assembly room temperature

Equations 3.103 and 3.104 evaluate each of these possibilities and establish the most critical extremes at which minimum or maximum effective center distance will occur.

The signi cance of these possibilities can be appreciated by considering the following service conditions. The rst case is typical of power gearing under steady-state conditions. Condition 1a is a possibility at start-up under a cold operating environment. The case is usually a transitory condition where the gears may have to perform adjacent to an external source of heat. The third condition is most typical of control gears that do not transmit enough energy to be at a temperature greatly different from their mountings.

The procedure recommended here is to calculate a minimum and a maximum effective center distance. These are based on the extremes of operating temperatures. From these values, the designer can calculate either a value of tooth thickness that will give the necessary operating backlash under the tightest mesh conditions, or the amounts of backlash that will be found in a given gearset.

The following equations give minimum and maximum effective center distances:

$$C_e \text{ min} = C_d + C'_t = \Delta C_{T_T}, \tag{3.103}$$

$$C_e \text{ max} = C_d + C'_{t'} = \Delta C_{T_L}, \tag{3.104}$$

where

C_e min—minimum effective center distance that occurs under extreme temperature
C_d—basic or nominal center distance
C'_t—minimum tolerance on center distance
ΔC_{T_T}—change in center distance (see note following Equations 3.105 and 3.106)
C_e max—maximum effective center distance that occurs under extreme temperature
$C'_{t'}$—maximum tolerance on center distance
ΔC_{T_L}—change in center distance (see note following Equations 3.105 and 3.106)

The following equations indicate the amount that the center distance (effective) will change because the gears and their mountings shrink or expand at different rates:

$$\Delta C'_T = \frac{D}{2}\left(\Delta T'_M K_M - \Delta T'_G K_G\right) + \frac{d}{2}\left(\Delta T'_M K_M - \Delta T'_G K_P\right), \tag{3.105}$$

$$\Delta C''_T = \frac{D}{2}\left(\Delta T''_M K_M - \Delta T''_G K_G\right) + \frac{d}{2}\left(\Delta T''_M K_M - \Delta T''_G K_P\right), \tag{3.106}$$

Note: Compare $\Delta C'_T$ and $\Delta C''_T$. Assign the smallest positive number or the largest negative number to ΔC_{T_T}. Assign the largest positive number or the smallest negative number to ΔC_{T_L}. If signs are opposite, use this negative value for ΔC_{T_T}, and use the positive value for ΔC_{T_L}. These values are used in Equations 3.103 and 3.104 to obtain effective center distance:

$$\Delta T'_M = T_M - T_R, \tag{3.107}$$

$$\Delta T''_M = T'_M - T_R, \tag{3.108}$$

$$\Delta T'_G = T_G - T_R, \tag{3.109}$$

$$\Delta T''_G = T'_G - T_R, \tag{3.110}$$

where

K_P—coef cient of expansion of pinion material (in./in./°F)
K_G—coef cient of expansion of gear material
K_M—coef cient of expansion of mounting material
T_M—minimum mounting temperature (operating)

* Apparent pitch diameter is the diameter at which a given value of tooth thickness is found.

T_R—assembly room temperature (68°F)
T'_M—maximum mounting temperature (operating)
T_G—minimum gearing temperature (operating)
T'_G—maximum gearing temperature (operating)

3.4.5 Mounting Distance*

Bevel gearing, worm gearing, face gearing, and Spiroid gearing must be given close control on axial positioning (mounting distance) if good performance is to be achieved (see Figure 3.23). Most of the elements that must be given control to achieve proper center distance must also be given close control to achieve proper mounting distance. If any of the mentioned types of gearing can move along the axis of its shaft relative to the mating gear, the teeth will either bind or in the opposite case, have excessive backlash.

Straight-toothed bevel gears are critical in respect to axial position, since backlash and tooth bearing pattern are affected by changes in axial position of the pinion and the gear. Spiral bevel gears, hypoid gears, and Spiroid gears are also critical in that they tend to screw into the mesh in one direction of rotation, which can cause binding, and will unscrew in the other direction of rotation, causing backlash, unless the mounting system is stiff enough to prevent axial shift. All properly designed mountings for gears of this type have bearings capable of withstanding all axial thrust loads imposed both by the gears and by possible external loads. Occasionally, a design is attempted in which the designer relies on the separating forces produced in the mesh to keep the gears away from a binding condition. Only a few of such designs are truly successful.

Worm gearing must also be accurately positioned if of the single- or the double-enveloping design. If a throated member is permitted to move axially, the frictional force developed in the mesh will tend to move the member, and the geometry of the throated design will cause a reduction in backlash, sometimes to the point of binding.

Mounting distance is usually specified for each gear member as the distance from a specific mounting surface on the gear blank, a face of a hub, for example, to the axis of the mating gear. Bevel gears are usually stamped with the correct mounting distance for the pair. This distance was established in a bevel gear test fixture at the time of manufacture.

The design of mountings for gearing requiring a mounting distance should include provisions for shimming or otherwise adjusting the position of the members at assembly. The calculation of mounting distance is accomplished in two steps.

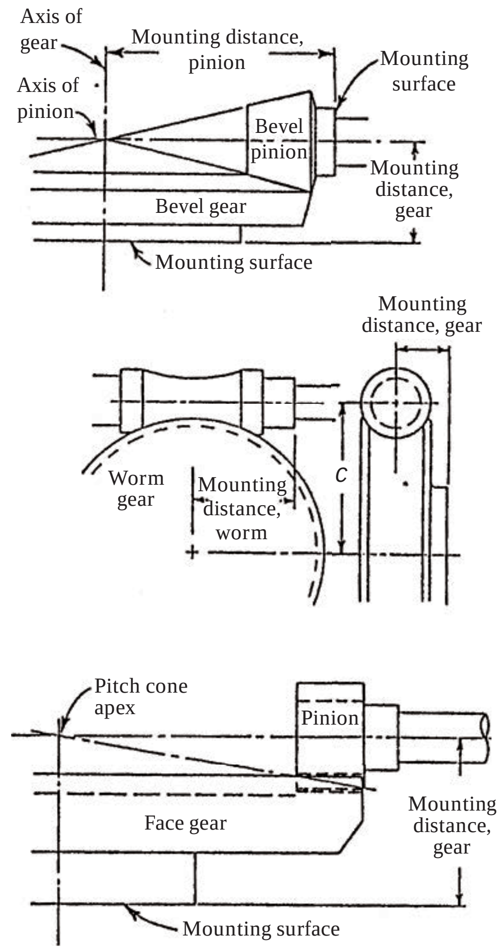


FIGURE 3.23 Mounting distance, shown for various kinds of gears, is the distance from the axis of a gear to the mounting surface of its mating gear. This distance is measured parallel to the shaft of the subject gear.

The design of the gear teeth (earlier in this chapter) includes a distance from the axes or the pitch cone apex to a given surface of the gear. To this distance is added the distance to the closest bearing face that is used to provide the axial location of the gear and its shaft. Provision should be made in the design to secure the bearing seats that provide axial location at the specified mounting distances.

In Chapter 2, radial clearance in rolling-element bearing was considered. In gear designs requiring a mounting distance, the axial clearance in one bearing on each shaft must be controlled. Reference to a bearing catalog or a handbook is recommended.

* See Chapter 14 for general information on gear mounting tolerances and practices.

4 Preliminary Design Considerations

In Chapter 1, we reviewed the design trends in different fields of gear work. General information was given that would help the designer pick the kind of gear that might be most suitable for a particular application.

In this chapter we shall begin to get into the detail work of designing a gear. Usually, the gear designer roughs out a preliminary design before doing all the work involved in a final design. The preliminary design stage involves consideration of the kinds of stress in the gears, an estimate of the approximate size, and consideration of the kind of data that will be required on the gear drawings. Each of these preliminary design considerations will be discussed in this chapter. Tables 4.1 and 4.2 are included for easy reference to the important gear nomenclature and symbols in this chapter.

4.1 STRESS FORMULAS

The gear designer's first problem is to find a design which will be able to carry the power required. The gears must be big enough, hard enough, and accurate enough to do the job required.

There are several kinds of stresses present in loaded and rotating gear teeth. The designer must consider all the possibilities so that the gears are proportioned to keep all the stresses within design limits.

4.1.1 Calculated Stresses

The stresses calculated in gear-design formulas are not necessary *true* stresses. For instance, the tensile stress at the root of a gear tooth might be calculated at 275 N/mm^2 (40,000 lb/in.^{2} [psi]) using the formula for a cantilever beam. If the tooth was very hard and there were a large number of cycles, there might be an effective stress-concentration factor of about 2 to 1. This would tend to raise the stress to an effective value of perhaps 550 N/mm^2 (80,000 psi) in this example. If the part was case-hardened, there might be a residual compressive stress in the outer fiber of the root fillet of as much as 140 N/mm^2 (20,000 psi). If the root fillet was large and well polished and the number of cycles was low, the effective stress concentration might be as low as 1.0. In this case, a calculated stress of 275 N/mm^2 (40,000 psi) might be effectively reduced to as little as 140 N/mm^2 (20,000 psi)!

It can be seen from the example just considered that things such as stress concentration and residual stress can make it difficult to get correct answers about gear-tooth stresses.

Other things also make it hard to get correct calculated stresses. The load that the gear teeth are transmitting may be known. However, whether this load is uniformly distributed (see Figure 4.1) across the face width and whether this load is properly shared by the two or more pairs of teeth that are in mesh at the same time may not be known. Errors in tooth spacing not only disrupt the sharing of tooth loads but may cause accelerations and decelerations which will cause a dynamic overload. The masses of the rotating gears and connected apparatus resist velocity changes. Tachometer gears made of hardened steel have been known to pit in high-speed aircraft applications even when the transmitted load was negligible.

Several assumptions have to be made to permit the calculation of stresses in gear teeth. It can be seen from the previous discussion that it is difficult to make assumptions that will properly allow for such things as stress concentration, residual stress, misalignment, and tooth errors. This means that the stress calculated is probably not true stress.

Once a "stress" is calculated, there is usually no sure way of knowing how this stress is related to the physical properties of material. Ordinarily, the only properties of gear material known with some certainty are the ultimate strength and the yield point. Endurance-limit values may be available, but these will usually be taken from reverse-bending tests of small bars. The gear tooth is essentially a cantilever beam which is bent in *one direction only*. This is not as hard on the tooth as fully reversed bending would be.

Laboratory fatigue tests are made on small test specimens with a uniform and smoothly polished test section. In contrast, the actual gear tooth is usually a part of a large piece of metal of more or less nonuniform structure. The critical section of the gear tooth may have tear marks, rough finish, and possibly corrosion on the surface. Its failure is usually a *fatigue* failure. All these things make it hard for the gear designer to take ordinary handbook data on the strength of materials and design gears. The best way to find out how much load gears will carry is to build and test gears. Then the designer can work backward and calculate what stress was present when the gear worked properly.

4.1.2 Gear-Design Limits

Despite the difficulty of calculating of real stresses, gear-stress formulas are a valuable and necessary design tool. When (a) materials, (b) quality of manufacture, and (c) kind of design stay quite constant, then formulas can be used quite successfully to determine the proper size of new designs. Essentially the formula is used as a yardstick to make a new design a scale model of an old design which was known to be able to carry a certain load successfully.

TABLE 4.1
Glossary of Gear Nomenclature, Chapter 4

Term	Definition
Backlash	The amount by which the sum of the circular tooth thicknesses of two gears in mesh is less than the circular pitch. Normally, backlash is thought of as the freedom of one gear to move while the mating gear is held stationary.
Bottom land	The surface at the bottom of the space between adjacent teeth.
Crown	A modification that results in the flank of each gear tooth having a slight outward bulge in its center area. A crowned tooth becomes gradually thinner toward each end. A *fully crowned* tooth has a little extra material removed at the tip and root areas also. See Figure 10.50 for the shape of a crowned helix. The purpose of crowning is to ensure that the center of the flank carries its full share of the load even if the gears are slightly misaligned or distorted.
Face width	The length of the gear teeth as measured along a line parallel to the gear axis. (A gap between the helices of double-helical gears is excluded unless "total face width" is specified.)
Fillet	The rounded portion at the base of the gear tooth between the tooth flank and the bottom of the land.
Flank	The working, or contacting, side of the gear tooth. The flank of a spur gear usually has an involute profile in a transverse section.
Form diameter	The diameter set by the gear designer as the limit for contact with a mating part. Gears are manufactured so that the tooth profile is suitable to transmit load between the top of the tooth and the form diameter.
Involute curve	A mathematical curve which is commonly specified for gear-tooth profiles. See Section B.1 for details.
Journal surfaces	The finished surfaces of the part of a gear shaft which has been prepared to fit inside a sleeve bearing or ball bearing.
K factor	An index of the intensity of tooth load from the standpoint of surface durability.
Overhung	A gear with two bearings (for support) at one end of the face width and no bearing at the other end.
Pi	A dimensionless constant which is the ratio of the circumference of a circle to its diameter. Pi is denoted by the Greek latter π and is approximately equal to 3.14159265.
Pitch-line velocity	The linear speed of a point on the pitch circle of a gear as it rotates. Pitch-line velocity is equal to rotational speed multiplied by the pitch-circle circumference.
Q factor	A quantity factor from the weight of a gearset, defined by Equation 4.47.
Sliding velocity	The linear velocity of the sliding component of the interaction between two gear teeth in mesh. The rate of sliding changes constantly; it is zero at the pitch line, and it increases as the contact point travels away from the pitch line in either direction.
Straddle mount	A method of gear mounting in which the gear has a supporting bearing at each end of the face width.
Throated	A gear is throated when the gear blank has a smaller diameter in the center than at the ends of the "cylinder." This concave shape causes the gear to partially envelope its mate and thus increases the area of contact between them. This design is often used to increase the load-carrying capacity of worm gearsets.
To land	The top surface of a gear tooth. See Figure 1.21.
Undercut	When part of the involute profile of a gear tooth is cut away near its base, the tooth is said to be *undercut*. See Figure 5.4. Undercutting becomes a problem when the numbers of teeth are small. See Section 5.5 for more details. Also, preshave or pregrind may be designed to produce undercut, even when no undercut would be present because of the small number of teeth.

It is particularly important to base new designs on old designs which have been successful. If the designer has an application which is not similar to anything built in the past, it is important to realize that a *new* design cannot be rated with certainty. Formulas may be used to estimate how much the new gear will do. However, it should be kept in mind that one can *only estimate* the performance of a new gear design. Field experience will be required to give the final answer about what the gears will do. In many cases, it has been possible to raise the power rating of gearsets after a sufficient amount of field experience was obtained. Several aircraft engines today have gears in them which are running at 25% to 50% more power than the gears were rated at when they were first designed.

The design engineer is obligated to design a gear unit so that all criteria are met in a reasonable fashion. Besides stress limits, there are temperature limits and oil-film-thickness limits. In addition to these first-order requirements, there are secondary considerations such as vibration, noise, and environment.

The general procedure in design is to first make the gears large enough to keep the tooth-bending stresses and surface compressive stresses within allowable limits. Further calculations are made to check the risks of scoring and overtemperature. Frequently, refinements in details of tooth design, kind of lubricant, temperature of lubricant, and accuracy of the gear tooth are required to meet all the design limits.

TABLE 4.2
Gear Terms, Symbols, and Units, Chapter 4

Term	Metric Symbol	Metric Unit	English Symbol	English Unit	Reference or Comment
Module, transverse	m_t	mm	–	–	$m_t = 25.4/P_t$
Module, normal	m_n	mm	–	–	
Diametral pitch, transverse	–	–	P_t of P_d	in.	$P_t = 25.4/m_t$
Diametral pitch, normal	–	–	P_{nd}	in.	
Circular pitch	p	mm	p	in.	
Pitch diameter, pinion	d_{p1}	mm	d	in.	
Pitch diameter, gear	d_{p2}	mm	D	in.	
Pitch radius, pinion	r_{p1}	mm	r	in.	Figure 4.14
Pitch radius, gear	r_{p2}	mm	R	in.	Figure 4.14
Outside radius, pinion	r_{a1}	mm	r_o	in.	Figure 4.14
Outside radius, gear	r_{a2}	mm	R_o	in.	Figure 4.14
Face width	b	mm	F	in.	
Tooth ratio (gear ratio)	u	–	m_G	–	No. of gear teeth/No. of pinion teeth
Center distance	a	mm	C	in.	Figure 4.14
Pressure angle	α	deg	ϕ	deg	Figure 4.10
Helix angle (or spiral)	β	deg	ψ	deg	
Pitch angle, pinion	δ'	deg	γ	deg	Figure 4.21
Outer cone distance	R_a	mm	A_o	in.	Figure 4.21
Bending stress	s_t	N/mm²	s_t	psi	Section 4.3
Contact stress (Hertz)	s_c	N/mm²	s_c	psi	Compressive stress also; Section 4.4
Modulus of elasticity	x_E	N/mm²	E	psi	
Poisson’s ratio	ν	–	ν	–	
Load	W	N	W	lb	Section 4.3
Tangential load (force)	W_t	N	W_t	lb	Figure 4.4
Power	P	kW	P	hp	
Torque	T	N m	T	in. lb	Equations 4.59, 4.60
Radius of curvature	ρ	mm	ρ	in.	Equations 4.14, 4.36
Radius of curvature, root llet	ρ_f	mm	ρ_f	in.	Figure 4.6
Roll angle (involute)	θ_r	deg	ε_r	deg	Figure 4.10
Zone of action	g_a	mm	Z	in.	Figure 4.14, Equation 4.38
Rotational speed, pinion	n_1	rpm	n_P	rpm	
Rotational speed, gear	n_2	rpm	n_G	rpm	
Pitch-line velocity	v_t	m/s	v_t	fpm	Rotational speed × pitch circumference
Sliding velocity	v_s	m/s	v_s	fpm	Section 4.5
Number of cycles	n_c	–	n_c	–	Section 4.4
Size factor for gearbox	Q	–	Q	–	Section 4.8
Loading index for strength	U_l	N/mm²	U_l	psi	Section 4.9
Loading index for surface durability	K	N/mm²	K	psi	Section 4.9
Scoring-criterion number	Z_c	°C	Z_c	°F	Equation 4.48
Contact ratio (pro le or transverse)	ε_a	–	m_p	–	Equation 4.43
Band of contact width (between two cylinders)	–	–	B	in.	Figure 4.11, Equation 4.9
Speci c lm thickness	–	–	Λ	–	Equation 4.35
Oil- lm thickness (EHD)	h_{min}	μm	h_{min}	μin.	Equation 4.35, Figure 4.11
Surface roughness	S	μm	S	μin.	Equation 4.35

(*Continued*)

TABLE 4.2 (CONTINUED)
Gear Terms, Symbols, and Units, Chapter 4

Note: Abbreviations for units are as follows:

Metric		English	
mm	millimeters	in.	inches
deg	degrees	deg	degrees
N	newtons	lb	pounds
N m	newton-meters	in. lb	inch-pounds
N/mm^2	newtons per square millimeter	psi	pounds per square inch
kW	kilowatts	hp	horsepower
rpm	revolutions per minute	rpm	revolutions per minute
m/s	meters per second	fps	feet per second
°C	degrees Celsius	°F	degrees Fahrenheit
μm	micrometers (10^{-6} m)	μin.	microinches (10^{-6} in.)

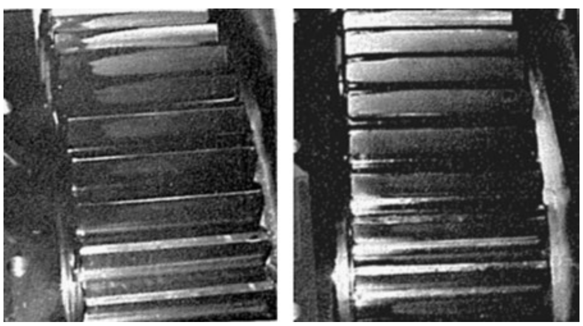


FIGURE 4.1 Contact checks at full torque for two sets of gears. *Left:* misalignment resulting in overload at end of tooth more than twice what it should be; *right:* essentially uniform load distribution across the face width.

Table 4.3 outlines the design limits generally used. Sections 4.1.3 through 4.1.6 explain these limits and present background information about how the limits have developed.

4.1.3 Gear-Strength Calculations

A gear tooth is essentially a stubby cantilever beam. At the base of the beam, there is tensile stress on the loaded side and compressive stress on the opposite side. When gear teeth break, they usually fail by a crack at the base of the tooth on the tensile-stress side. The ability of gear teeth to resist tooth breakage is usually referred to as their *beam strength* or their *exural strength*.

TABLE 4.3
Gear-Design Limits

Item	Symbol: Metric	Symbol: English	Prime Variables	Cross Reference in This Book
Tooth bending stress	s_t	s_t	Size of teeth, metal hardness, tooth design, tooth accuracy	Section 5.2.2
Surface contact stress	s_c	s_c	Pinion diameter, ratio of pinion to gear, metal hardness, tooth design, tooth accuracy	Section 5.2.2
Oil- lm thickness	h_{min}	h_{min}	Pitch-line speed, oil viscosity, pinion diameter, ratio, load intensity	Section 5.2.8
Speci c lm thickness	–	Λ	Surface nish, oil- lm thickness	Section 5.2.8
Blank temperature	T_b	T_b	Oil inlet temperature, temperature rise of part	Section 5.2.8
Flash temperature	T_F	T_F	Blank temperature, tooth loading, pinion rotational speed, tooth-design details, tooth size	Section 5.2.8

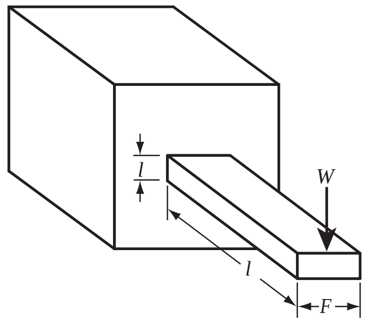


FIGURE 4.2 A loaded cantilever beam.

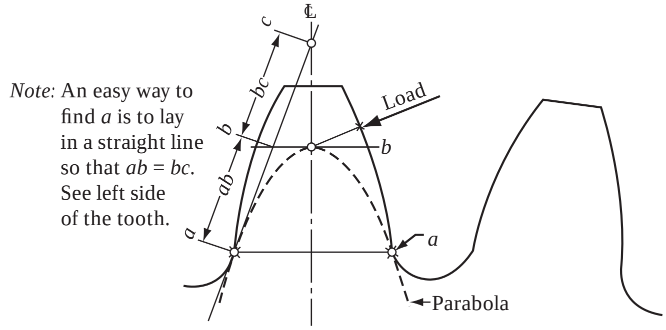


FIGURE 4.3 When gear tooth is loaded at point b, point a is the most critically stressed point. Inscribed parabola is tangent to root fillet at point a and has its origin where load vector cuts the centerline.

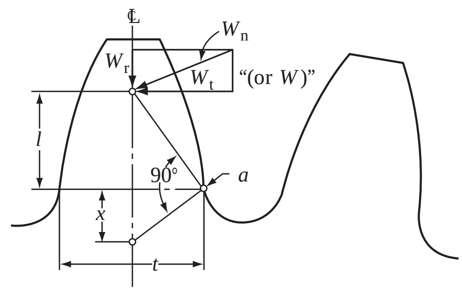


FIGURE 4.4 Determination of x dimension from a gear-tooth layout.

The flexural strength of gear teeth was first calculated to a close degree of accuracy by Wilfred Lewis* in 1893. He conceived the idea of inscribing a parabola of uniform strength inside a gear tooth. It happens that when a parabola is made into a cantilever beam, the stress is *constant* along the surface of the parabola. By inscribing the largest parabola that will fit into a gear-tooth shape, one immediately locates the most critically stressed position on the gear tooth. This position is at the point at which the parabola of uniform strength becomes *tangent* to the surface of the gear tooth.

The Lewis formula can be derived quite simply from the usual textbook formula for the stress at the root of a cantilever beam. Figure 4.2 shows a rectangular cantilever beam. The tensile stress at the root of this beam is

$$s_t = \frac{6Wl}{Ft^2}. \tag{4.1}$$

If we substitute a gear tooth for the rectangular beam, we can find the critical point in the root fillet of the gear by inscribing a parabola. This is point a in Figure 4.3.

The next step is to draw some construction lines to get a dimension x. By similar triangles in Figure 4.4, it is apparent that

$$x = \frac{t^2}{4l}. \tag{4.2}$$

By substituting x into Equation 4.1, we get

$$s_t = \frac{Wl}{F(2x/3)}. \tag{4.3}$$

The circular pitch p may be entered into both the numerator and the denominator without changing the value of the equation. This gives

$$s_t = \frac{Wlp}{F(2x/3)p}. \tag{4.4}$$

The term $2x/3p$ was called y by Lewis. This was a factor that could be determined by a layout of the gear tooth. Since the factor was *dimensionless*, it could be tabulated and used for any pitch.

By using y, the Lewis formula may be written as

$$s_t = \frac{W}{Fpy}, \tag{4.5a}$$

or

$$W = s_t pFy. \tag{4.5b}$$

In the present-day work, most engineers prefer to use module or diametral pitch instead of circular pitch in making stress calculations. This can be done by substituting Y for y ($Y = \pi y$) and P_d for p ($P_d = \pi/p$). The Lewis formula becomes

$$s_t = \frac{WP_d}{FY} \text{ (psi)}. \tag{4.6}$$

The original Lewis formula was worked out for the transverse component of the applied load. In Figure 4.4 the load normal to the tooth surface has a tangential and a radial component. The radial component produces a small compressive stress across the root of the gear tooth. When this component is considered, the tensile stress is reduced by a small amount, and the compressive stress on the opposite side of the tooth is increased by a slight amount. Seemingly, this would indicate that the tooth would be most critically stressed on

* The Lewis paper and other references that apply to this chapter are given in the Literature section, at the end of the book.

the compression side. This is not the case, however. In most materials, a tensile stress is more damaging than a somewhat higher compressive stress.

Lewis took the application of load to be at the tip of the tooth. In his day, even the best gears were not very accurate. This meant that the load was usually carried by a single tooth instead of being shared between two teeth on a gear. If a single tooth carried full load, it is obvious that the greatest stress would occur when the tooth had rolled to a point at which the tip was carrying the load.

4.1.3.1 Worst Load

When gears are made accurate enough for the teeth to share load, the tip-load condition is not the most critical. In nearly all gear designs, the contact ratio is high enough to put a second pair of teeth in contact when one pair has reached the tip-load condition on one member. The *worst-load* condition occurs when a single pair of teeth carry full load and the contact has rolled to a point at which a second pair of teeth is just ready to come into contact. Figure 4.5 shows how to locate the worst-load condition on a spur pinion. Note that the contact point has advanced to where it is just one base pitch away from the rst point of contact.

In spur gears, the worst-load condition can be used in the Lewis formula by simply making the tooth layout with the load applied at the worst point instead of at the tip. Table 4.4 shows how this affects the *Y* factor for a few 20° spur pinions made to a standard depth of 2.250 in. for 1 pitch. The addendum is 1.000 in. for 1 pitch.

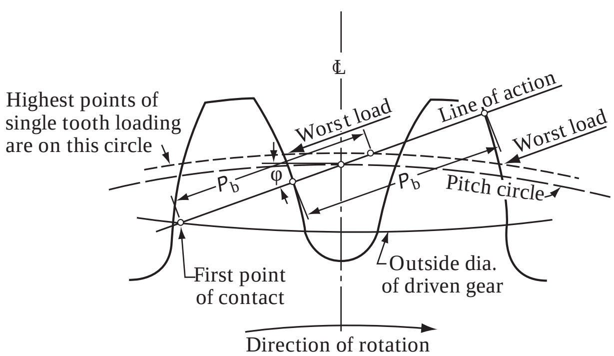


FIGURE 4.5 Worst application of beam loading on precision spur gear is one base pitch above the rst point of contact.

TABLE 4.4
***Y* Factors for Standard 20° Spur Pinions**

		Y Factor	
No. Pinion Teeth	**No. Gear Teeth**	**Tip Load**	**Worst Load**
20	20	0.287	0.527
20	60	0.287	0.527
20	120	0.287	0.600
25	25	0.310	0.583
25	60	0.310	0.657
25	120	0.310	0.693
30	30	0.332	0.640
30	60	0.332	0.673
30	120	0.332	0.740

In helical and spiral bevel gears, the teeth are usually accurate enough to share load. The geometry of the teeth, however, is such as to make any position of load which can accurately be labeled the "worst application" of load impossible to nd. In all positions of contact, the design usually makes it possible for two or more pairs of teeth to share the load. This situation is handled by calculating a *geometry factor* for strength. See Section 4.2.3 for further details.

4.1.3.2 Stress Concentration

In Lewis's time, stress-concentration factors were not used in engineering calculations. In present gear work, almost all designers realize the necessity of using a stress-concentration factor. Dolan and Broghamer (1942) made an extensive study of photoelastic stress-concentration factors in plastic models of gear teeth. Their work has been widely used in recent years.

Figure 4.6 shows the dimensions used to calculate stress concentration. The equations established from the experimental work of Dolan and Broghamer are

$$K_r = 0.18 + \left(\frac{t}{\rho_f}\right)^{0.15} \left(\frac{t}{h}\right)^{0.45} \tag{4.7}$$

for 20° involute spur teeth and

$$K_r = 0.22 + \left(\frac{t}{\rho_f}\right)^{0.20} \left(\frac{t}{h}\right)^{0.40} \tag{4.8}$$

for 14.5° involute spur teeth.

In the above equations, K_r is the stress-concentration factor. The radius ρ_f is the radius of curvature of the root llet at the point at which the llet joins the root diameter.

Tests of actual metal gear teeth show that the real stress-concentration factor is not the same as the photoelastic value determined on models made of plastic material.* If the root llet has deep scratches and tool marks, it may be higher. Some materials are more brittle than others. In general, high-hardness steels show more stress-concentration effects than low-hardness steels. Some of the case-hardened and induction-hardened steels are an exception of this, however. When a high residual compressive stress is obtained in the surface layer of the material, stress-concentration effects are reduced.

Stress concentration is also in uenced by the number of cycles. Some of the low-hardness steels show little or no effect of stress concentration when loaded to destruction statically. Yet when fatigued for a million or more cycles, they show a de nite reduction in strength due to stress concentration.

* The Drago paper P229.24 (1982) gives some very interesting research test data on stress in gear-tooth samples.

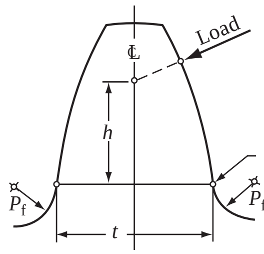


FIGURE 4.6 Dimensions used to calculate stress concentration.

4.1.3.3 Load Distribution

The face width used by Lewis was the *full* face width. Actual gears are seldom loaded uniformly across the face width. Errors in shaft alignment and helix angles tend to increase the load on one end of the tooth or the other. The present practice is to use a *load-distribution factor* to account for the extra load imposed somewhere in the face width as a result of nonuniform load distribution.

4.1.3.4 Dynamic Load

The load applied to the gear tooth is greater than the transmitted load based on horsepower. In general, the faster the gears are running, the more shock due to tooth errors and the more dynamic effects due to imbalance and torque variations in the driving and driven apparatus. Lewis intended to allow for this situation by reducing the safe working stress as pitch-line velocity was increased.

A considerable amount of research has been carried out to determine the amount of *dynamic* gear-tooth loads. A research committee of American Society of Mechanical Engineers—headed by Earle Buckingham—published the first authoritative work on dynamic loads in 1931. This work gave what was believed to be an accurate method for calculating dynamic load. A simplified formula that makes possible a quick but approximate calculation of dynamic load was developed from this work. This method has been widely published and used.

In his later work, Buckingham gave equations that permit dynamic load to be calculated using actual shaft stiffnesses and inertias. This work covers dynamic-load calculations on several kinds of gear teeth.

The problem of gear-tooth dynamic load received much attention during the 1950s and 1960s. Many papers were published showing test data. The latest and probably the most technically accurate data is from the work done by Dr. Aizoh Kubo* and others (Kubo et al., 1980).

In design work, the overload on gear teeth is often handled by a *dynamic factor*. This factor is used as a multiplier of the transmitted load.

Tucker and many others (including Buckingham) point out that the dynamic overload is really an *incremental* amount of load that is *added* to the transmitted load rather than being a multiplier of the transmitted load. For instance, low-hardness gears idling at little or no transmitted load may be inaccurate enough to develop such serious dynamic loads that they fail prematurely!

Using the dynamic overload as an adder is somewhat unhandy in rating equation. The usual rating formula is worked out for full rated power, and the dynamic factor—used as a multiplier—adds on what is considered to be the extra amount for dynamic load. Such rating formulas, of course, give the wrong answer at light loads, but they can be quite accurate at full load.

4.1.3.5 Finite Element

The German practice in calculating tooth strength has developed into taking the tangency point of a 30° angle as the critical stress point on the root fillet. Stress-concentration effects are determined by the *finite-element* method of stress analysis instead of by photoelastic results.

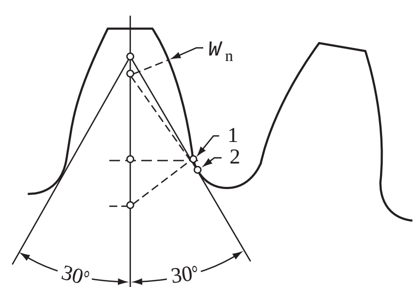
The method of gear rating for tooth strength proposed by the International Standards Organization (ISO) uses the 30° angle and equations for stress concentration based on finite-element studies of gear teeth. The standards of AGMA use strength formulas derived from the work of Lewis and Dolan and Broghamer.

German gear people feel that the 30° angle method is simpler than the inscribed parabola method and that the results are close enough to be suitable for any normal design work. The finite-element method of getting stress-concentration values, though, gives results that are often considerably different from those found using the Dolan–Broghamer method. Figure 4.7 shows a comparison of the 30° angle method and the inscribed parabola method.

In 1980 Dr. Giovanni Castellani and V. Parenti Castelli made a considerable study of the proposed ISO strength rating method compared with established standard methods. They presented comparative geometry factors for strength based on several spur- and helical-gear designs. The differences in results were so great that it is now felt that more test work must be done.

4.1.4 Gear Surface-Durability Calculations

Gears fail by pitting and wear as well as by tooth breakage. Frequently gears will wear to the point where they begin to run roughly. Then the increased dynamic load and the stress-concentration effects of the worn tooth surface both cause



1: Most critically stressed point by inscribed parabola method
2: Most critically stressed point by 30° angle method

FIGURE 4.7 Comparison of 30° method of finding most critically stressed point with inscribing parabola method.

* See Appendix B for a review of Kubo's dynamic-load work.

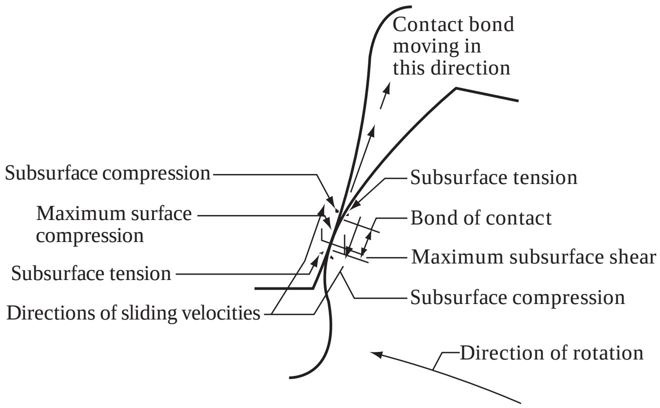


FIGURE 4.8 Stress in region of tooth contact.

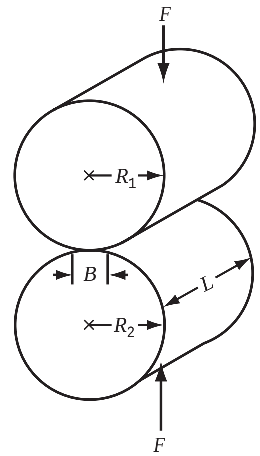


FIGURE 4.9 Parallel cylinders in contact and heavily loaded.

the teeth ultimately to fail by breakage. Figure 4.8 shows the kinds of stress that are present in the region of the contact band. In the center of the band, there is a point of maximum compressive stress. Directly underneath this point, there is a maximum subsurface shear stress. The depth to the point of maximum shear stress is a little more than one-third the width of the band of contact.

The gear-tooth surfaces move across each other with a combination of rolling and sliding motion. The sliding motion plus the coef cient of friction tends to cause additional surface stresses. Just behind the band of contact, there is a narrow region of tensile stress.

A bit of metal on the surface of a gear tooth goes through a cycle of compression and tension each time a mating gear tooth passes over it. If the tooth is loaded heavily enough, there will usually be evidence of both surface cracks and plastic ow on the contacting surface. There may also be a rupturing of the metal as a result of subsurface shear stresses.

4.1.4.1 Hertz Derivations

The stress on the surface of gear teeth are usually determined by formulas derived from the work of H. Hertz (of Germany) (1881a,b). Frequently, these stresses are called Hertz stresses.

Hertz determined the width of the contact band and the stress pattern when various geometric shapes were loaded against each other. Of particular interest to gear designers is the case of two cylinders with parallel axes loaded against each other.

Figure 4.9 shows the case of two cylinders with parallel axes. The applied force is F pounds and the length of the cylinders is L inches. The width of the band of contact is B inches.

The Hertz formula for the width of the band of contact is

$$B = \sqrt{\frac{16F(K_1 + K_2)R_1R_2}{L(R_1 + R_2)}}, \qquad (4.9)$$

where

$$K_1 = \frac{1 - \nu_1^2}{\pi E_1},$$

$$K_2 = \frac{1 - \nu_2^2}{\pi E_2}.$$

In the above equations, ν is *Poisson's* ratio and E is the modulus of elasticity.

The maximum compressive stress is

$$s_c = \frac{4F}{L\pi B}. \qquad (4.10)$$

The maximum shear stress is

$$s_s = 0.295 s_c. \qquad (4.11)$$

The depth to the point of maximum shear is

$$Z = 0.393B. \qquad (4.12)$$

Equations 4.9 and 4.10 may be combined and simpli ed to give the following when Poisson's ratio is taken as 0.3:

$$s_c = \sqrt{0.35\frac{F(1/R_1 + 1/R_2)}{L(1/E_1 + 1/E_2)}}. \qquad (4.13)$$

The Hertz formulas can be applied to spur gears quite easily by considering the contact conditions fog eras to be equivalent to those of cylinders that have the same radius of curvature at the point of contact as the gears have. This is an approximation because the radius of curvature of an involute tooth will change while going across the width of the band of contact. The change is not too great when contact is in the region of the pitch line. However, when contact is near the base circle, the change is rapid, and contact stresses calculated by the Hertz method for cylinders are not very accurate.

Figure 4.10 shows how the radius of curvature may be determined at any point of an involute curve. At the pitch line, the radius of curvature is

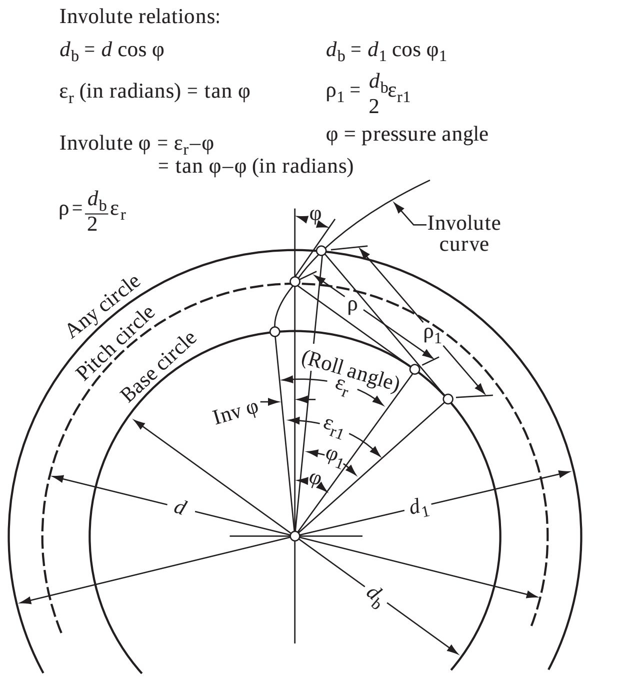


FIGURE 4.10 Radius of curvature of an involute and other basic relations of the involute curve.

$$\rho = \frac{d \sin \phi}{2}. \tag{4.14}$$

At any other diameter, such as d_1, the radius of curvature is

$$\rho_1 = \frac{d_1 \sin \phi_1}{2}. \tag{4.15}$$

The angle ϕ_1 can be found out by the relation

$$\phi_1 = \cos^{-1}\left(\frac{d \cos \phi}{d_1}\right). \tag{4.16}$$

The compressive stress at the pitch line of a pair of spur gears can be obtained by substituting the following into Equation 4.13:

$$F = \frac{W_t}{\cos \phi}, \tag{4.17}$$

$$L = F, \tag{4.18}$$

$$R_1 = \frac{d \sin \phi}{2}, \tag{4.19}$$

$$R_2 = m_G R_1. \tag{4.20}$$

Note: At this point, we are switching from the symbols of conventional mechanics to the symbols used by gear engineers. This can be confusing, unfortunately, because F is *applied force* in Equations 4.9 through 4.17, but F is *face width* in Equation 4.18 and following equations.

In Equation 4.17, W_t is the tangential driving pressure in pounds. It can be obtained by dividing the pinion torque by the pitch radius of the pinion. In Equation 4.19, F is the face width in inches and d is the pitch diameter of the pinion. The ratio of gear teeth to pinion teeth is m_G.

Combining Equations 4.13, 4.17, and 4.19 gives

$$s_c = \sqrt{\frac{0.70}{(1/E_1 + 1/E_2)\sin\phi\cos\phi}} \times \sqrt{\frac{W_t}{Fd}\left(\frac{m_G+1}{m_G}\right)}. \tag{4.21}$$

4.1.4.2 *K* Factor Derivations

For a steel spur pinion meshing with a steel gear, we may substitute the nominal value of 30,000,000 for E. Then

$$s_c = 5715\sqrt{\frac{W_t}{Fd}\left(\frac{m_G+1}{m_G}\right)} \tag{4.22}$$

when the teeth have a 20° pressure angle.

Many gear designers have found it convenient to call the term under the square-root sign in Equation 4.22 the K factor. This makes it possible to reduce Equation 4.22 to

$$s_c = 5715\sqrt{K}, \tag{4.23}$$

where

$$K = \frac{W_t}{Fd}\left(\frac{m_G+1}{m_G}\right). \tag{4.24}$$

The compressive stress on helical-gear teeth can be obtained by nding the radius of curvature of the teeth in a section *normal* to the pitch helix. This section has a *pitch ellipse* instead of pitch circle. Using an equation for an ellipse, we can bet values to use for the R values in Equation 4.13. These are

$$R_1 = \frac{d \sin \phi_n}{2\cos^2 \psi}, \tag{4.25}$$

$$R_2 = m_G\left(\frac{d \sin \phi_n}{2\cos^2 \psi}\right). \tag{4.26}$$

In accurate helical gears, the load is shared by several pair of teeth. The average length of tooth working is equal to the face width multiplied by the contact ratio and divided by the cosine of the helix angle. The *normal* load (load in normal section) applied is equal to the tangential load in the transverse plane divided by the cosine of the helix and divided by the cosine of the normal pressure angle. Substituting these

values into Equation 4.21 gives a basic equation for the compressive stress in helical-gear teeth:

$$s_c = \sqrt{\frac{0.70\cos^2\psi}{(1/E_1 + 1/E_2)m_p \sin\phi_n \cos\phi_n}} \times \sqrt{\frac{W_t}{Fd}\left(\frac{m_G + 1}{m_G}\right)}, \qquad (4.27)$$

where m_p is pro le contact ratio (pro le contact ratio in a transverse section).

By introducing the K factor and a constant C_k, most of the terms in the equation can be eliminated. This gives us a simpli ed equation:

$$s_c = C_k\sqrt{\frac{K}{m_p}} \text{ (psi)}. \qquad (4.28)$$

Table 4.5 gives some values of C_k and m_p for typical gears made to full-depth proportions in the normal section. The pro le-contact-ratio values were calculated for a 25-tooth pinion meshing with a 100-tooth gear. For other numbers of teeth, the contact ratio will change by a small amount; however, it makes very little difference in Equation 4.28, since the contact-ratio term is *under* the square-root sign.

Bevel and worm gears can be handled in a somewhat similar manner.

4.1.4.3 Worst-Load Position

In most cases, the compressive stress on spur-gear teeth is calculated at the lowest point on a pinion tooth at which full load is carried by a single pair of teeth. This situation cannot occur in wide-face helicals, but it can occur in narrow helicals, bevels, and spur gears. Theoretically, if one pair of teeth carries full load and the position of loading is at the lowest possible position on the pinion, there is a worst-load condition for Hertz stress corresponding to the worst-load position for strength described in the preceding section.

Different formulas for contact stress may include factors to account for the increase in load due to velocity and tooth accuracy. The same questions of dynamic load and misalignment across the face width are present in calculating contact stress as in calculating the root stress. Service factors allow for torque pulsations and the length of service required from the gears.

4.1.4.4 Endurance Limit

The tendency of gear teeth to pit has traditionally been thought of as a surface fatigue problem in which the prime variables were the compressive stress at the surface, the number of repetitions of the load, and the endurance strength of the gear material. In steel gears, the surface endurance strength is quite closely related to hardness, and so stress, cycles, and hardness then become the key item.

It was also believed that there was an *endurance limit* for surface durability at about 10 million cycles (10^7 cycles). For case-carburized steel gears, fully hardened, typical design values are listed in Table 4.6.

Gear work in the 1970s led to two very important conclusions (these conclusions had been suspected in the 1960s):

1. Pitting is very much affected by the lubrication conditions.
2. There is no endurance limit against pitting.

Work on the theory of elastohydrodynamic (*EHD*) lubrication showed that gears and rolling-element bearings often developed a very thin oil lm that tended to separate the two contacting surfaces so that there was little or no metallic

TABLE 4.6
Typical Design Values for Case-Carburized Steel Gears

	Metric		English	
Item	**Symbol**	**Value**	**Symbol**	**Value**
Maximum allowable stress	s_c	1724 N/mm²	s_c	250,000 psi
Number of cycles	n_c	10^7	n_c	10^7
Surface hardness	HV	700 minimum	HRC	60 minimum

Note: In this chapter, most of the data are given in English units to agree with previously published derivations. From Chapter 5 onward, the data will be primarily given in the metric system. For the convenience, equivalent English values will often be given as a second set of values.

TABLE 4.5
Values of C_k and m_p for Helical Gears

Normal Pressure	Spur		15° Helix		30° Helix		45° Helix	
Angle, ϕ_n	C_k	m_p	C_k	m_p	C_k	m_p	C_k	m_p
14.5°	6581	2.10	6357	2.01	5699	1.71	4653	1.26
17.5°	6050	1.88	5844	1.79	5240	1.53	4278	1.13
20°	5715	1.73	5520	1.65	4949	1.41	4041	1.05
25°	5235	1.52	5057	1.45	4534	1.25	3702	0.949

Note: The above values are illustrative of the development of gear-rating equations, but should not be used for design purposes. Chapter 5 gives different values that are appropriate for actual design work.

contact. When this favorable situation was obtained, the gear or the bearing could either carry *more load* without pitting or run for a *longer time* without pitting at a given load.

The idea that there was an endurance-limit pitting grew from test-stand data, where tests were generally discontinued after 10^7 or 2×10^7 cycles. (If a test is being run at 1750 cycles per minute, it takes 190 hours to reach 2×10^7 cycles.)

Real gears in service frequently run for several thousand hours before pitting starts—or becomes serious. A gear can often run for up to a billion (10^9) cycles with little or no pitting, but after two or three billion (2 or 3×10^9) cycles, pitting—and the wear resulting from pitting—makes the gears un t for further service.

4.1.4.5 Regimes of Lubrication

To handle the problem of EHD lubrication effects, the designer needs to think of three *regimes* of lubrication:

- Regime I: No appreciable EHD oil lm (boundary)
- Regime II: Partial EHD oil lm (mixed)
- Regime III: Full EHD oil lm (full lm)

In regime I, the gears may be thought of as running wet with oil, but the thickness of the EHD oil lm developed is quite compared with the surface roughness. Essentially full metal-to-metal contact is obtained in the hertzian contact band area. Regime I is typical of slow-speed, high-load gears running with a rough surface nish. Hand-operated gears in winches, food presses, and jacking devices are typical of slow-speed regime I gears.

Regime II is characterized by partial metal-to-metal contact. The asperities of the tooth surfaces hit each other, but substantial areas are separated by a thin lm. Regime II is typical of medium-speed gears, highly loaded, running with a relatively thick oil and fairly good surface nish. Most vehicle gears are in regime II. (Tractors, trucks, automobiles, and off-road equipment are vehicle gear applications.)

In regime III the EHD oil lm is thick enough to essentially avoid metal-to-metal contact. Even the asperities miss each other. The well-designed and well-built high-speed gear is generally in regime III. Turbine-gear applications in ship drives, electric generators, and compressors are good examples of high-speed gears. In the aerospace gearing eld, turboprop drives are high-speed and in regime III. Helicopter main rotor gears are in the high-speed gear region, except for some nal-stage gears that may be slow enough to be out of the high-speed domain and into medium speed with regime II conditions. (Some of the best pioneering work in de ning regimes of lubrication was done by the late Charles Bowen (1977), working on helicopter-gearing development.)

Figure 4.11 shows a schematic representation of the three regimes of lubrication. Note the details of the hertzian contact band for each regime. Figure 4.12 shows the "nominal" zones for the three regimes.

The quality of the surface nish of the new gear, the degree of nish improvement achieved in breaking in new gears, the thickness and kind of lubricant, the operating temperature,

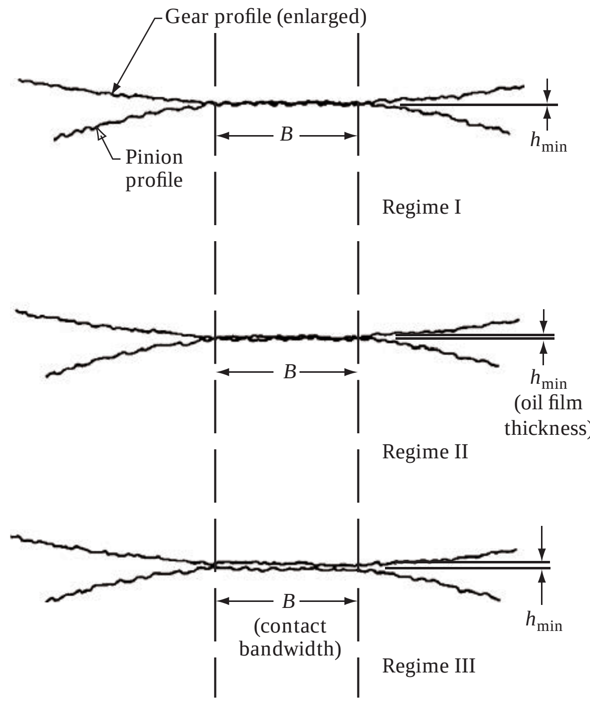


FIGURE 4.11 Schematic representation of the three regimes of gear-tooth lubrication.

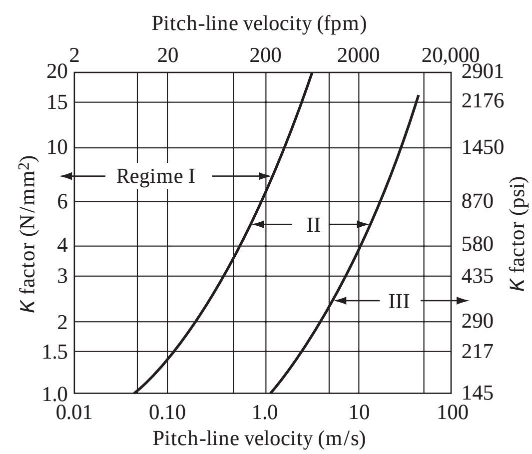


FIGURE 4.12 Location of regimes of lubrication for average gears.

the pitch-line velocity, and the load intensity all enter into the determination of which regime the gear pair operates in. Means of calculating EHD oil- lm thickness and design for different regimes of lubrication are given in Chapter 5. Figure 4.12 should be used only as a rough guide.

The surface durability in the different regimes of lubrication varies considerably. Figure 4.13 shows the general trend* of surface contact stress capacity at different numbers of cycles for each of the three regimes of lubrication. The three curves have these equations at 10^7 cycles:

* Figure 4.13 is a nominal curve intended to show average conditions for reasonably good steel and nominal quality. These curves should not be used for nal design.

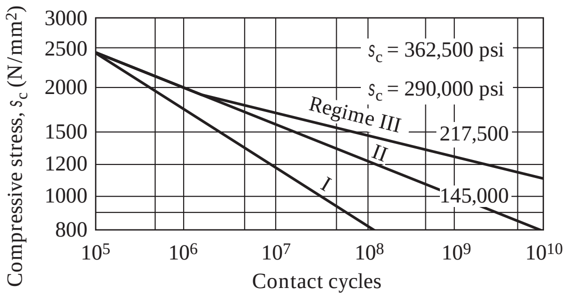


FIGURE 4.13 Trend of contact stress for regimes I, II, and III. Carburized gears.

$$\frac{L_b}{L_a} = \left(\frac{K_a}{K_b}\right)^{3.2} \quad \text{for regime I,} \tag{4.29}$$

$$\frac{L_b}{L_a} = \left(\frac{K_a}{K_b}\right)^{5.3} \quad \text{for regime II,} \tag{4.30}$$

$$\frac{L_b}{L_a} = \left(\frac{K_a}{K_b}\right)^{8.4} \quad \text{for regime III,} \tag{4.31}$$

where

L_b—cycles (any number from 10^6 to 10^8)
L_a—10^7 cycles
K_a—allowable load intensity at 10^7 cycles
K_b—allowable load intensity at L_b cycles

The contact stress s_c is proportional to the square root of the load intensity. For simplified gear rating,

$$s_c = C_k\sqrt{KC_d}\,, \tag{4.32}$$

where C_k is tooth geometry constant for the particular design.

K factor can be calculated from the equations

$$K = \frac{W_t}{Fd}\left(\frac{m_G + 1}{m_G}\right) \quad \text{for external teeth,} \tag{4.33}$$

$$K = \frac{W_t}{Fd}\left(\frac{m_G - 1}{m_G}\right) \quad \text{for internal teeth.} \tag{4.34}$$

The tangential driving force W_t is equal to W_t = torque × $2/d$, and C_d is overall derating factor for gear and gearbox design imperfections.

The data shown in Figure 4.13 can be interpreted to show the results of Table 4.7.

Table 4.7 shows the substantial difference in load-carrying capacity of the different regimes. Regime I loses over 50% of its capacity every time there is a 10-fold increase in life (number of cycles). Regime II loses about 35% of its load capacity for a 10-fold increase. In comparison, Regime III loses only about 24% of its capacity with a 10-fold increase.

TABLE 4.7
Nominal Comparison of Load-Carrying Capacity of the Regimes of Lubrication

	Load Capacity for Different Regimes		
No. of Cycles	**I**	**II**	**III**
10^5	100%	100%	100%
10^6	48.7%	64.8%	76.0%
10^7	23.7%	41.9%	57.8%
10^8	–	27.2%	44.9%
10^9	–	–	33.4%
10^{10}	–	–	25.4%

The damaging situation in regime I can be considerably offset by substituting a *chemical additive* film on the tooth surface for an EHD oil film. Special extreme pressure (*EP*) oils develop chemical compounds on the contacting gear-tooth surfaces. When the lubricant additive is working, it is possible for the gear-tooth operation to shift from regime I to regime II with load-carrying capacity improving to regime II. Likewise, appropriate additives can help gear teeth in regime II move over into regime III. This subject will be treated in depth in Sections 5.2.8, 9.8, and 9.15.

4.1.5 Gear Scoring

When excessive compressive stresses are carried on a gear toot for a long period, pits will develop. If the gear is kept in operation after pitting starts, the whole surface of the tooth will eventually be worn away. Severe pitting leads to rough running and "hammering" of the gear teeth. This in turn leads to other types of surface wear, such as scoring, swaging, and abrasion. *Scoring* is characterized by radial scratch lines, *swaging* is an upsetting process similar to cold rolling, while *abrasion* is the tearing away of small particles when rough surfaces are rubbed across each other.

Gear teeth may score when no pitting has taken place. This may occur when the gears are first put into operation. Sometimes the cause of scoring—when gears have not pitted first—is simply that the accuracy is not sufficient. The teeth do not have a good enough surface finish or good enough spacing and profile accuracy.

Aircraft gears and other types of heavily loaded high-speed gears tend to fail by scoring even when accuracy is good. Apparently, the combination of high surface pressure and high sliding velocity can be severe enough to vaporize the oil film and cause instantaneous welding of the surfaces. The continued rotation of the gear teeth causes a radial tearing action, which makes the characteristic score marks.

4.1.5.1 Hot and Cold Scoring

The scoring problem is not limited to high-speed gears. Slow-speed gears may also score. The score marks on slow-speed gears look somewhat different. The failure mechanism seems to be more one of filing or abrading away the metal rather

than welding and tearing. European gear people—for some reason—call the slow-speed scoring *cold scuffing** and the high-speed scoring *hot scuffing.*

Cold scoring is basically a problem of gears that are running in regime I or regime II conditions with oil that does not have enough chemical additives to protect the surface. The key variable is *specific film thickness* Λ, which is defined as

$$\Lambda = \frac{h_{\min}}{S'}, \tag{4.35}$$

where

$h_{\min}$—EHD oil-film thickness
S'—composite surface roughness of the gear pair, $\left(S' = \sqrt{S_1^2 + S_2^2}\right)$
S_1—surface roughness of pinion
S_2—surface roughness of gear

The surface roughness values used for S_1 and S_2 are normally the arithmetic-average (*AA*) values. Formerly, root-mean-square (rms) values were used for surface finish. The rms value for finish is normally about 1.11 times the AA value.

The surface roughness that is used may be different from the value measured on new gears just finished in the gear shop. Most gears will wear in and improve their finish in the first 100 or so hours of operation. This process is helped by not loading the gears too heavily or running them at maximum temperature until they are well broken in. A special lubricant with extra additives or more viscosity may also be used to help lessen the danger of scoring when new gears are first put into service.

Assuming that a favorable break-in will be achieved (in Equation 4.35), the designer can use the surface roughness values after break-in rather than those from the gear shop. For instance, a precision-ground gear with a 20 AA surface roughness may wear into a 15 AA finish.

It is necessary to calculate a *scoring factor* that will evaluate the combined effects of surface pressure, sliding velocity, coefficient of friction, kind of metals, and kind of oil from the standpoint of scoring.

The gear trade has used several methods of calculating scoring risk. No method has yet proved to be entirely reliable. The principal methods that have been used are shown in Table 4.8. The remainder of this section will cover the basics of *PVT,* flash temperature, and scoring index. Specific design recommendations will be given in Section 5.2.8.

TABLE 4.8
Scoring Calculation Methods

Method	When Developed	Kind of Scoring	Where Covered in This Book
PVT	1940s	Hot scoring	Chapter 2
Flash temperature	1940s, 1950s	Hot scoring	Chapters 2, 5
Scoring criterion	1960s	Hot scoring	Chapters 2, 5
Specific film thickness	1960s, 1970s	Cold scoring	Chapters 2, 5
Integral temperature	1970s	Hot scoring	Chapters 2, 5

4.1.5.2 PVT Formula

A PVT formula was used with considerable success in designing small aircraft gears to be built by automotive-gear manufacturers. During World War II, a number of automotive-gear plants made large numbers of aircraft gears. The PVT formula developed as a result of this wartime experience.

The factors in the PVT formula are as follows[†]:

- P—Hertz contact pressure. This is usually figured for the tip of the pinion and figured again for the root of the pinion (tip of the gear). The applied load is divided by the profile contact ratio to approximate the way load is shared by two pairs of teeth.
- V—sliding velocity in feet per second at the point at which *P* is figured.
- T—distance along the line of action from the pitch point to the point at which *P* is calculated.

The quantity PVT is always zero at the pitch point on spur or helical gears. It increases steadily as contact moves away from the pitch point, reaching a maximum at the point at which contact is at the tip of the tooth. For this reason, PVT is ordinarily calculated only for the tips of the teeth.

The equations and nomenclature used in the following equations follow the procedure customarily used by automotive-gear designers. The reader will recognize that general equations such as Equations 4.13 and 4.15 could have been used as well.

The calculation of PVT involves solving a series of equations. First, the radius of curvature at the pinion tip is calculated (see Figure 4.14):

$$\rho_P = \sqrt{r_0^2 - (r\cos\phi_t)^2}\,. \tag{4.36}$$

Similarly, the radius of curvature at the gear tip is

$$\rho_G = \sqrt{R_0^2 - (R\cos\phi_t)^2}\,. \tag{4.37}$$

The length of the line of action is

$$Z = \rho_P + \rho_G - C\sin\phi_t. \tag{4.38}$$

* The terms *scuffing* and *scoring* are used somewhat interchangeably. Common practice recommends the word *scoring.* Other engineering groups in the United States and abroad often prefer *scuffing* over *scoring.*

† PVT nomenclature was established in the early 1940s and does not agree with current nomenclature. See Table 4.2 for symbols in use now.

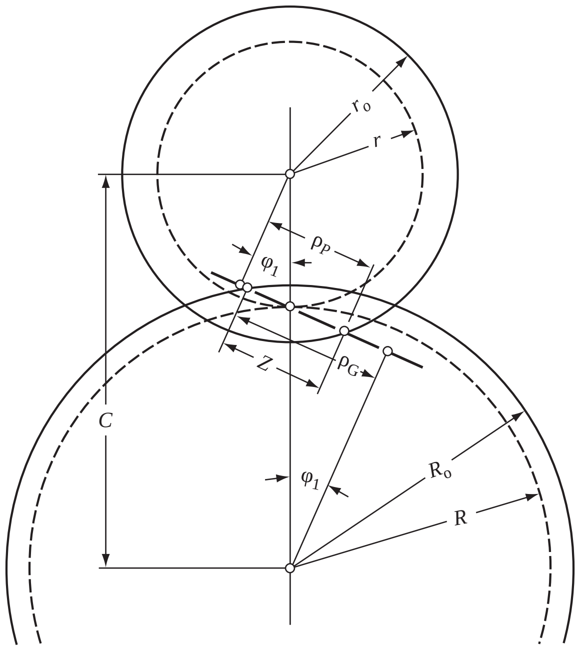


FIGURE 4.14 Dimensions used in scoring-factor calculation.

The Hertz compressive stress for the tip of the pinion is

$$P_P = 5740\sqrt{\frac{T_P}{FZN_P} \times \frac{C\sin\phi_n}{\rho_P(C\sin\phi_t - \rho_P)}}\,, \tag{4.39}$$

where
T_P—pinion torque (in. lb)
F—face width (in.)
N_P—number of pinion teeth
C—center distance

Similarly, the stress at the gear tip is

$$P_G = 5740\sqrt{\frac{T_G}{FZN_G} \times \frac{C\sin\phi_n}{\rho_G(C\sin\phi_t - \rho_G)}}\,. \tag{4.40}$$

The scoring factor for the pinion tip is

$$PVT_P = \frac{\pi n_P}{360}\left(1 + \frac{N_P}{N_G}\right)(\rho_P - r\sin\phi_t)^2 P_P\,, \tag{4.41}$$

where n_P is the pinion speed (rpm).

At the gear tip, the scoring factor is

$$PVT_G = \frac{\pi n_P}{360}\left(1 + \frac{N_P}{N_G}\right)(\rho_G - R\sin\phi_t)^2 P_G\,. \tag{4.42}$$

The equations will work for either spur or helical gears. In the case of spur gears, there is only one pressure angle. This makes $\phi_n = \phi_t$.

The pro le contact ratio can be obtained easily from the Z value in Equation 4.38. Since this value is needed frequently, we shall give an equation for it here:

$$m_p = \frac{ZN_P}{2\pi r\cos\phi_t}\,. \tag{4.43}$$

Equation 4.43 holds for either spur or helical gears. Literally, the contact ratio represents the length* of the line of action divided by the base pitch. It is the average number of teeth that are in contact in the transverse plane. When the contact ratio comes out to some number such as 1.70, it does not actually mean that 1.70 teeth are working. If the ratio is between 1 and 2, there are alternately one pair and two pair of teeth working. Form a *time* standpoint, though, it would work out that on the *average* there were 1.70 pair of teeth working.

A design limit of 1,500,000 has been used frequently for a safe limit on PVT. This value works reasonably well with case-hardened gears that are of good accuracy and are lubricated with medium-weight petroleum oil. Pitch-line velocity should be above 2000 fpm.

4.1.5.3 Flash Temperature

The concept of ash temperature was rst presented by Professor Herman Blok (1937). The basics of this formula are

$$T_f = T_b + \frac{c_f\, f\, W_t(v_1 - v_2)}{F_e(\sqrt{v_1} + \sqrt{v_2})\sqrt{B/2}\cos\phi_t}\,, \tag{4.44}$$

where
T_f— ash temperature (°F)
T_b—temperature of blank surface in contact zone (often taken as inlet oil temperature) (°F)
c_f—material constant for conductivity, density, and speci c heat
f—coef cient of friction
W_t—tangential driving load (lb)
v_1—rolling velocity of pinion at point of contact (fps)
v_2—rolling velocity of gear at point of contact (fps)
$v_s = (v_1 - v_2)$—sliding velocity (fps)
F_e—face width in contact (in.)
B—width of band of contact (in.)
ϕ_t—transverse pressure angle (°)

The rolling velocity may be obtained from the rpm of the pinion or gear by

$$v_1 = \frac{n_P \pi \rho_1}{360}\,, \tag{4.45}$$

$$v_2 = \frac{n_P \pi \rho_2}{360}\,. \tag{4.46}$$

Originally, ash temperatures were calculated for the pinion tip and the gear tip. In this case,

* Length between the rst point of contact and the last point of contact is the *length* used here.

Pinion tip:

$$\rho_1 = \rho_P \text{ (see Equation 4.36)}, \ \rho_2 = \rho_G - Z \text{ (see Figure 4.14)}, \quad (4.47)$$

Gear tip:

$$\rho_1 = \rho_G \text{ (see Equation 4.37)}, \ \rho_2 = \rho_P - Z \text{ (see Figure 4.14)}, \quad (4.48)$$

Later development of the ash-temperature formula led to scoring being considered most apt to occur at the *lowest point of single* tooth contact on the pinion or the *highest point of single* tooth contact on the pinion. This situation is most apt to occur when very accurate aircraft spur gears have a small amount of pro le modi cation at the tip to relieve tip loading.

Figure 4.15 shows the location of the highest and lowest points of single tooth contact for the pinion. Normally, scoring calculations are made for the *pinion only*. The gear is handled indirectly, since the pinion can score only when in contact with the gear. (If the pinion is OK, the gear should be OK.)

Table 4.9 shows how to calculate the radii of curvature of the highest and lowest points of single tooth contact.

The material constant c_f was taken as 0.0528 for straight petroleum oils. The coef cient of friction was taken as 0.06.

The width of the band of contact for steel gears was obtained from

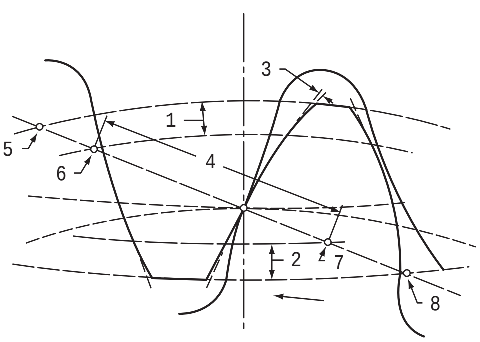


FIGURE 4.15 Layout of teeth to locate critical points for scoring calculations.

TABLE 4.9
Radii of Curvature of Involute Spur or Helical Teeth

Contact Position	Pinion Curvature, ρ1	Gear Curvature, ρ2
Lowest single tooth contact	$\sqrt{r_o^2 - (r\cos\phi_t)^2} - p\cos\phi_t$	$C \sin \phi_t - \rho_1$
Highest single tooth contact	$C \sin \phi_t - \rho_2$	$\sqrt{R_o^2 - (R\cos\phi_t)^2} - p\cos\phi_t$

Note: The outside radius of the pinion is r_o and the pitch radius is r. The radii of the gear are capitalized. The circular pitch is p.

TABLE 4.10
Maximum Design Limit of Flash Temperatures to Prevent Scoring of Spur Gears

Kind of Oil	Specification	T_f (°F)
Petroleum	SAE 10	250
	SAE 30	375
	SAE 60	500
	SAE 90 (gear lubricant)	600
Diester, compounded	75 SUS at 100°F	330
Petroleum	SAE 30 plus mild EP	425

Note: SUS: Saybolt universal second.

$$B = 0.00054\sqrt{\frac{W_t \rho_1 \rho_2}{F_e(\rho_1 + \rho_2)\cos\phi_t}}. \quad (4.49)$$

Table 4.10 shows some typical design values for maximum ash temperatures used in the 1950s. These values are of some historical importance, but they are not recommended for current designs. More complex procedures, involving surface nish, overload of teeth due to errors, special lubricant effects, and other such factors are needed. (See Chapter 5, Section 5.2.8.)

4.1.5.4 Scoring Criterion

Some gears have a high risk of hot scoring, while others have little or no risk of hot scoring. To judge the risk of scoring, the *scoring criterion* was developed by Dudley.

The formula for this index number is

$$\text{Scoring-criterion number} = \left(\frac{W_t}{F_e}\right)^{3/4} \sqrt[4]{\frac{(n_P)^2}{P_d}}. \quad (4.50)$$

Table 4.11 shows scoring-criterion numbers published in 1964. If the calculated scoring value exceeds those in Table 4.11, there is a risk of scoring, and more complex calculations should be made. If the value is below those in the table, the risk of scoring is low, but it may still exist as a result of such things as poor nish, appreciable overload due to inaccuracy, and inadequate lubricant proportions.

The scoring-criterion number is a useful guide for spur, helical, and spiral bevel gears.

The scoring criterion was derived from the ash-temperature equation, Equation 4.44. By mathematical manipulation it is possible to write the ash-temperature equation in the form

$$T_f = T_b + Z_t \left(\frac{W_t}{F_e}\right)^{3/4} \sqrt[4]{\frac{(n_P)^2}{P_d}}, \quad (4.51)$$

where

$$Z_t = 0.0175 \frac{\left(\sqrt{\rho_1} - \sqrt{\rho_2/m_G}\right)\sqrt[4]{P_d}}{[\rho_1\rho_2/(\rho_1+\rho_2)]^{1/4}(\cos\phi_t)^{3/4}}. \quad (4.52)$$

TABLE 4.11
Critical Scoring-Criterion Numbers

	Critical Scoring-Index Numbers at Blank Temperatures				
Kind of Oil	**100°F**	**150°F**	**200°F**	**250°F**	**300°F**
AGMA 1	9000	6000	3000	–	–
AGMA 3	11,000	8000	5000	2000	–
AGMA 5	13,000	10,000	7000	4000	–
AGMA 7	15,000	12,000	9000	6000	–
AGMA 8A	17,000	14,000	11,000	8000	–
Grade 1065, Mil-O-6082B	15,000	12,000	9000	6000	–
Grade 1010, Mil-O-6082B	12,000	9000	6000	2000	–
Synthetic (Turbo 35)	17,000	14,000	11,000	8000	5000
Synthetic Mil-L-7808D	15,000	12,000	9000	6000	3000

Note: See Section 5.2.8 for more data on rating gears for scoring and the use of scoring-criterion number. See Section 12.3.3 for general data on gear lubricants and the hazard of lubrication failures.

The constant Z_t is dimensionless when used with the scoring-criterion number. This makes it possible to tabulate Z_t values for different proportions and styles of tooth design. The term Z_t can be thought of as a *tooth geometry* factor* for scoring.

4.1.6 Thermal Limits

The design of gear drives involves more than just making the gear teeth able to carry bending stresses and contact stresses and to resist scoring. Along with such things as bearing capacity, shaft design, and spline capacity, the designer must consider the thermal limits.

4.1.6.1 Thermal Limits at Regular Speed

Small gear drives (particularly those under 200 hp) are often splash lubricated by a quantity of liquid oil in the gearbox. Without a pumped oil system and oil coolers, the gearbox is cooled by the surrounding air. Such a gear drive will have a *thermal rating* as well as a mechanical rating. The application of the gear unit to a job should be such that neither the thermal rating nor the mechanical rating is exceeded.

The customary procedure in calculating thermal ratings is based on nding the maximum horsepower that a unit can carry for 3 hours without the sump temperature exceeding 93°C (200°F) when the ambient air temperature is not over 38°C (100°F).

Long trade experience has led to the development of a great deal of expertise in the thermal capacity of helical-gear units, spiral-bevel-gear units, and worm-gear units. Trade practices have developed to set approximate methods of calculation for a standard thermal rating. No doubt, further experience will develop ways of handling thermal ratings that are more complex than those now used. (For instance, special steels and oils can be used at quite high temperatures, and so trade practices may change to recognize special designs that can run hotter than is common in industrial gearing today.)

Equations and calculation procedures for setting thermal ratings will not be given in this book. Instead, Table 4.12 is a rough guide to what is involved in thermal rating.

The prime variables in determining thermal capacity are the gearbox size, the input pinion speed, and the ratio. Table 4.12 shows that if a gear unit is built twice as large, then the thermal rating increases about 3 to 1. The mechanical rating increases almost 8 to 1 when the size doubles, so this predicts that large gear units will be short of thermal capacity.

Table 4.12 shows that for the same size unit, the thermal rating drops quite rapidly as the speed is increased. In contrast, the mechanical rating increases somewhat in proportion to an increase in speed.

There is an in uence from the ratio, but it is quite mild. From Table 4.12, at 400 rpm it might be possible to design medium-hard gears up to 600 mm (24 in.) center distance and still have enough thermal rating to match the mechanical rating. At 1200 rpm, though, it would probably not be possible to exceed 200 mm (8 in.) center distance and still have enough thermal capacity to match the mechanical capacity.

TABLE 4.12
Some Approximate Values of Thermal Rating in Horsepower

	Center Distance (in.)			
Pinion Speed (rpm)	**4**	**8**	**16**	**24**
	Ratio, 2 to 1			
400	80	240	720	1200
1200	40	110	315	500
2400	12	30	80	55
	Ratio, 4 to 1			
400	80	240	720	1200
1200	42	115	325	525
2400	15	40	110	85

* See Appendix B for general calculation method for Z_t which can be used to obtain values at any point on the tooth pro le.

When the normal thermal capacity is exceeded, the thermal capacity may be improved by using one or more fans mounted on input shafts. A favorable fan arrangement can as much as double the normal thermal capacity of a gear unit.

For the large and more powerful gear units, the thermal capacity is completely inadequate. Pumped-oil lubrication systems with oil coolers must be used.

With a pumped-oil system, calculations must be made to assure that the bearings and gear teeth are fed enough oil to adequately cool and lubricate all parts. The design of gear-lubrication systems will not be covered here.

4.1.6.2 Thermal Limits at High Speed

When spur-gear teeth run faster than 10 m/s (2000 fpm) or helical teeth run faster than 100 m/s (20,000 fpm), there may be problems with the trapping of air and oil in the gear mesh. Special design features have allowed small spur teeth in aerospace applications to run up to around 100 m/s reasonably successfully. Likewise, special designs of helical gears have permitted successful operation up to 200 m/s.

Some further data on problems with fast-running gears are given in Section 12.3.4 (Chapter 12).

4.2 STRESS FORMULAS

Section 4.1 has shown the general nature of the various kinds of stress formulas that may be used in designing a gearset. This material will probably help the reader to understand the problem of designing a gearset, but it does not give much help in knowing what to do *first* in designing a set after choosing the kind of gear.

There are many ways to start the design of a set. Perhaps the easiest way is to skip all the detail design work and immediately estimate how big the gears must be. If a size that is close to being right can be chosen in the beginning, the designer can work through the dimensional calculations with the prospect of only minor adjustments after the design is checked with appropriate rating formulas.

In this part of the chapter we shall take up the problem of estimating gear size.

4.2.1 Gear Specifications

Gears are used to transmit power from one shaft to another and to change rotational speed. The designer needs to have specifications for the following:

- The amount of power to be transmitted
- The pinion speed (or gear speed)
- The required ratio of input and output speeds
- The length of time the gears must operate

Frequently, it is hard to find out how much power a gearset must carry. Take the example of a gear driven by a 10 hp motor. In some applications, the motor might be expected to run every day of the year at 10 hp. In other cases, the motor might run only intermittently at powers well below 10 hp. In still others the motor might be started every day and have to pull up to 20 hp for a short period while the driven machinery was warming up.

The example just cited shows that the gear designer's first problem may be to establish a proper power specification. In doing this, it is necessary to check both the driving and the driven apparatus. It may be that the driven apparatus tends to stall or suffer severe shock at infrequent intervals. This might give rise to peak torques as much as 5 or 10 times the full-load torque capacity of the driving motor. Although the gearset does not have to be the strongest of the three connected pieces of apparatus, neither should it be the weakest—unless there is good reason.

Where power and speeds may be quite variable, the designer should strive to reduce the specifications to simple conditions. First, the maximum *continuous* load that the gear might be expected to handle should be established. Next, the maximum *torque* should be determined. This load will probably last for only a short period. In many designs, it is necessary to make stress calculations for only these two conditions. In some cases, though, there may be a high *intermediate* load which is less than the maximum but does not last so long as the continuous load. Here it is necessary to calculate stresses for the intermediate load also.

In complex design situations, it is necessary to construct a gear *histogram*. The best procedure is to calculate the pinion torque for each design condition. The results are plotted on log–log graph paper, with the highest loads being put at the lowest number of cycles. Figure 4.16 shows two examples of load histograms. For the vehicle gear shown, it turns out that the critical design condition is low gear and maximum torque. In comparison, the turbine application has its most critical situation at the maximum continuous torque, not the highest torque seen under starting conditions.

4.2.2 Size of Spur and Helical Gears by *Q* Factor Method

It is fairly easy to estimate the sizes of spur and helical gears. After the proper specifications have been established, a tentative gear size can be obtained by the *Q factor method*. This method was originally developed to estimate gear weights. It is very handy, though, in estimating center distance and face width.

The *Q* factor method of estimating gear size is simply a method whereby the power, speed, and ratio of a gearset are all reduced to a single number. This number, called *Q* for quantity, is a measure of the size of the job the gear has to do. The value of the *Q* factor as an index can be demonstrated quite readily from the weight curves shown in Figures 4.17 and 4.18. These show how the average weight of complete single-reduction gearsets compares with calculated *Q* factors:

$$Q = \frac{\text{kilowatt power}}{\text{pinion rpm}} \times \frac{(u+1)^3}{u} \quad \text{(metric)}, \tag{4.53}$$

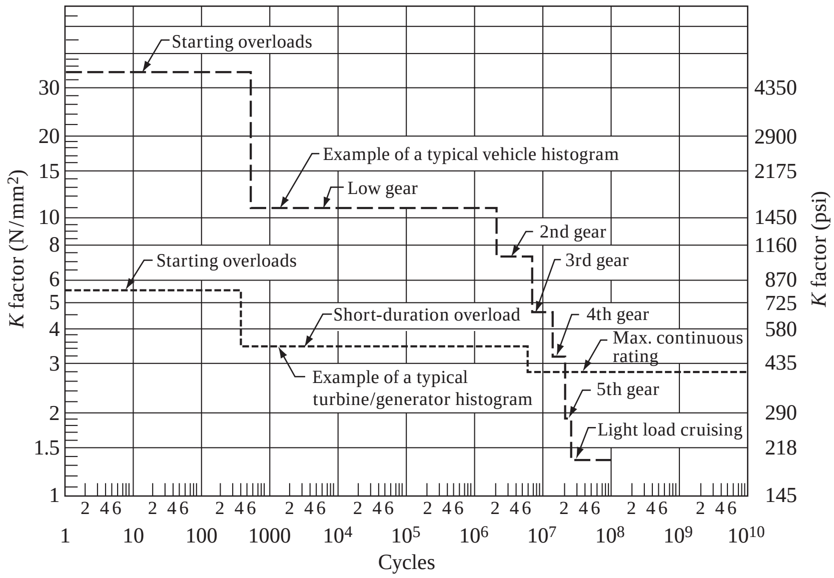


FIGURE 4.16 Histogram of load intensity (*K* factor) plotted against contact cycles for a vehicle gear mesh and a turbine gear mesh. Note that the vehicle is critical for the low gear rating and the turbine is critical for its maximum continuous rating.

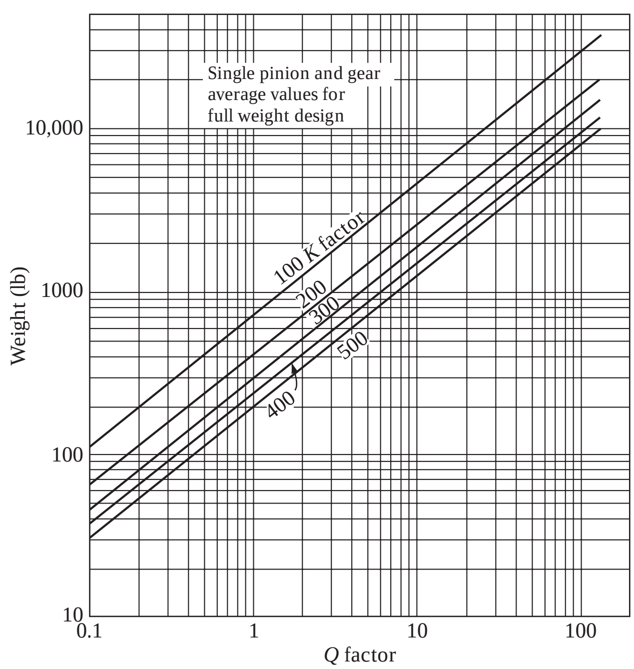


FIGURE 4.17 Gear weights plotted against *Q* factor for different intensities of tooth loading (English units).

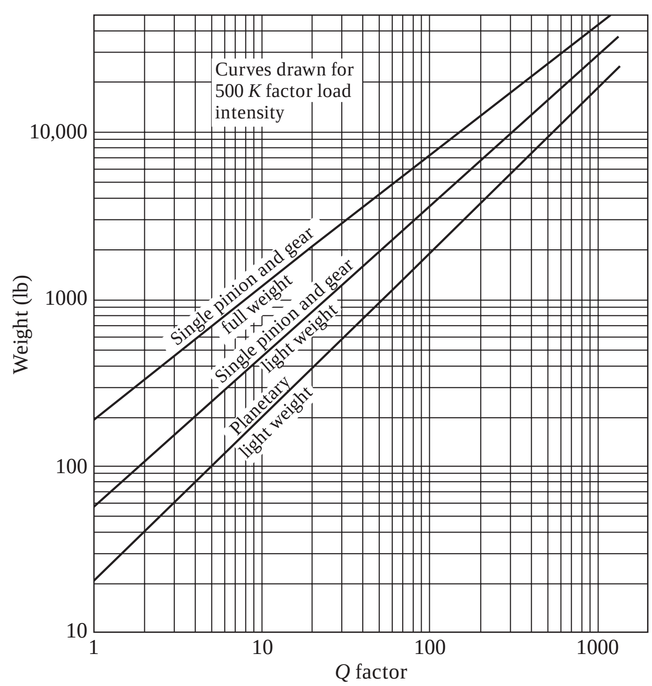


FIGURE 4.18 Comparison of gear weights for different designs.

$$Q = \frac{\text{horse power}}{\text{pinion rpm}} \times \frac{(m_G + 1)^3}{m_G} \text{ (English).} \qquad (4.54)$$

The ratio factor in Equations 4.53 and 4.54 has been tabulated in Table 4.13 for ratios from 1 to 10.

The required center distance and face width can be obtained from the *Q* factor as soon as the designer decides how heavily it is safe to load the gears. For spur and helical gears, the *K* factor is a convenient index for measuring the intensity of tooth loads. Equations 4.67 through 4.70 de ne* the *K* factor, and Table 4.14 gives some values of *K* factor that can be used for preliminary estimates of gearset sizes. (See Section 4.2.4.)

* The derivation of *K* factor is given in Section 4.1.4. For estimating gear size, disregard the equation in Section 4.4 and work with the material in this section. Do the nal design using the methods in Chapter 5.

TABLE 4.13
Ratio Factors for Single-Reduction Gears

Speed Ratio, *u*	Ratio Factor, $(u + 1)^3/u$	Speed Ratio, *u*	Ratio Factor, $(u + 1)^3/u$
1.00	8.000	3.00	21.333
1.20	8.873	3.50	26.036
1.40	9.874	4.00	31.250
1.60	10.985	4.50	36.972
1.80	12.195	5.00	43.200
2.00	13.500	6.00	57.167
2.20	14.895	7.00	73.143
2.40	16.377	8.00	91.125
2.60	17.945	9.00	111.11
2.80	19.597	10.00	133.10

As soon as the amount of *K* factor has been decided on, the following equation can be solved:

$$a^2 b = \frac{4,774,650Q}{K} \text{ (metric)}, \tag{4.55}$$

$$C^2 F = \frac{31,500Q}{K} \text{ (English)}. \tag{4.56}$$

4.2.2.1 Face-Width Considerations

In spur or single-helical gears, using a face width which exceeds the pinion pitch diameter is often not advisable. If the face width is wider, torsional twist concentrates the load quite heavily on one end. In many applications, it is not even possible to effectively use a face width as wide as the pinion diameter. Errors in toot alignment and shaft alignment may make it impossible to get tooth contact across this much face width.

When a face width equal to the pinion diameter is intended, Equations 4.55 and 4.56 may be modi ed to the following:

$$a^3 = \frac{2,387,325Q(u+1)}{K} \text{ (metric)}, \tag{4.57}$$

$$C^3 = 15\left[\frac{31,750Q(m_G+1)}{K}\right] \text{ (English)}, \tag{4.58}$$

TABLE 4.14
Indices of Tooth Loading for Preliminary Design Calculations

	Minimum Hardness of Steel Gears				*K* Factor		Unit Load	
Application	Pinion	Gear	No. of Pinion Cycles	Accuracy	N/mm²	psi	N/mm²	psi
Turbine driving a generator	225 HB	210 HB	10^{10}	High precision	0.69	100	45	6500
	335 HB	300 HB	10^{10}	High precision	1.04	150	59	8500
	59 HRC	58 HRC	10^{10}	High precision	2.76	400	83	12,000
Internal combustion engine driving a compressor	225 HB	210 HB	10^9	High precision	0.48	70	31	4500
	335 HB	300 HB	10^9	High precision	0.76	110	38	5500
	58 HRC	58 HRC	10^9	High precision	2.07	300	55	8000
General-purpose industrial drives, helical (relatively uniform torque for both driving and driven units)	225 HB	210 HB	10^8	Medium high precision	1.38	200	38	5500
	335 HB	300 HB	10^8	Medium high precision	2.07	300	48	7000
	58 HRC	58 HRC	10^8	Medium high precision	5.52	800	69	10,000
Large industrial drives, spur—hoists, kilns, mills (moderate shock in driven units)	225 HB	210 HB	10^8	Medium precision	0.83	120	24	3500
	335 HB	300 HB	10^8	Medium precision	1.24	180	31	4500
	58 HRC	58 HRC	10^8	Medium precision	3.45	500	41	6000
Aerospace, helical (single pair)	60 HRC	60 HRC	10^9	High precision	5.86	850	117	17,000
Aerospace, spur (epicyclic)	60 HRC	60 HRC	10^9	High precision	4.14	600	76	11,000
Vehicle transmission, helical	59 HRC	59 HRC	4×10^7	Medium high precision	6.20	900	124	18,000
Vehicle transmission, spur	59 HRC	59 HRC	4×10^6	Medium high precision	8.96	1300	124	18,000
Small commercial (pitch-line speed less than 5 m/s)	335 HB	Phenolic/laminate		Medium precision	0.34	50	–	–
	335 HB	Nylon		Medium precision	0.24	35	–	–
Small gadget (pitch-line speed less than 2.5 m/s)	200 HB	Zinc alloy	10^6	Medium precision	0.10	15	–	–
	200 HB	Brass or aluminum	10^6	Medium precision	0.10	15	–	–

Note: The above indices of tooth loading assume average conditions. With a special design and favorable application, it may be possible to go higher. With an unfavorable application and/or a design that is not close to optimum, the indices of tooth loading shown will be too high for good practice. The table assumes that the controlling load must be carried for the pinion cycles shown.

TABLE 4.15
Guide for Choice of Face Width in Spur or Helical Gears

		Numerical Constant for Center Distance, Equations 4.57 and 4.58	
Aspect Ratio[a], m_a	Situation	Metric	English
0.4	Lower-accuracy gears, appreciable error in mounting dimensions; may need crowning, but helix modi cation not needed	59,683,125	39,375
1.0	Medium accuracy, good mounting accuracy; helix modi cation generally not needed; may need crowning	2,387,325	15,750
1.5	High-accuracy parts and mounting; helix modi cation generally not needed	1,591,550	10,500
1.75	Very high accuracy; probably needs helix modi cation for single-helical gears; may not need for double helix	1,364,186	9000
2.00	Very high accuracy; will need helix modi cation in single-helix designs, and will probably need helix modi cation in double-helix designs	1,193,662	7875
2.25	Very high accuracy; may not be practical to use, due to criticalness of helix modi cation required	1,061,003	7000

[a] b/d_{p1} (metric); F/d (English).

In double-helical gears, the face width may be as wide as 1.75 times the pinion pitch diameter before the problems of torsional twist and beam bending get too serious. This results from the fact that double-helical pinions shift axially under load to equalize the loading on each helix. This shifting compensates for most of the torsional twist.

With both single- and double-helix gears, the face width can be made relatively wide, provided proper helix modi cation is made to compensate for the de ections involved.

Table 4.15 shows guideline information on how to make an initial choice of face width. A constant is also given to permit the center distance to be immediately obtained from the Q factor for the chosen avspect ratio. Use this constant in Equations 4.55 and 4.56 in place of the numerical constant shown.

After the center distance is obtained, the pitch diameters are obtained by Equations 1.5 and 1.6.

4.2.2.2 Weight from Volume

When other than simple gear pairs or simple planetary units are involved, the Q factor method of weight estimating rather becomes impractical. A simple alternative based on a summation of the face widths multiplied by the pitch diameters squared works quite well. In 1963, R. J. Willis published a paper showing how to pick gear ratios and gear arrangements for the lightest weight. His work was based on this concept. The basic weight equation is

$$\sum\left(\frac{bd_{\mathrm{P}}^2 \times \text{weight constant}}{36{,}050}\right) = \text{weight} \quad \text{(kg; metric)}, \tag{4.59}$$

$$\sum(Fd^2 \times \text{weight constant}) = \text{weight} \quad \text{(kg; English)}. \tag{4.60}$$

The weight constant to be used is the same for metric or English units and is given in Table 4.16.

Figure 4.19 shows in schematic form a few of the many possible gear arrangements. To get a rough approximation of the weight, pitch diameters and face widths are obtained for all gear parts. Then, Equations 4.55 and 4.56 are used to get the summation of the face widths times the pitch diameters squared.

Table 4.16 shows some typical values of the weight constant needed for Equations 4.55 and 4.56. The data in this table are based on general experience in gear design. Special situations, of course, may make the real weight of a well-designed unit quite different from the rst estimate. For instance, a gearbox may support the motor or engine and need extra weight for the frame structure. Extra weight may be needed for oil-pump equipment or other accessory parts added to the preliminary gear unit.

TABLE 4.16
Weight Constants for Use in Preliminary Estimates

Application	Factor	Typical Conditions
Aircraft	0.25–0.30	Magnesium or aluminum casings Limited-life design High-stress levels Rigid weight control
Hydrofoil	0.30–0.35	Lightweight steel casings Relatively high stress levels Limited-life design Rigidity desired
Commercial	0.600–0.625	Cast or fabricated steel casings Relatively low stress levels Unlimited-life design Solid rotors and shafts

Note: Use these factors in Equations 4.59 and 4.60.

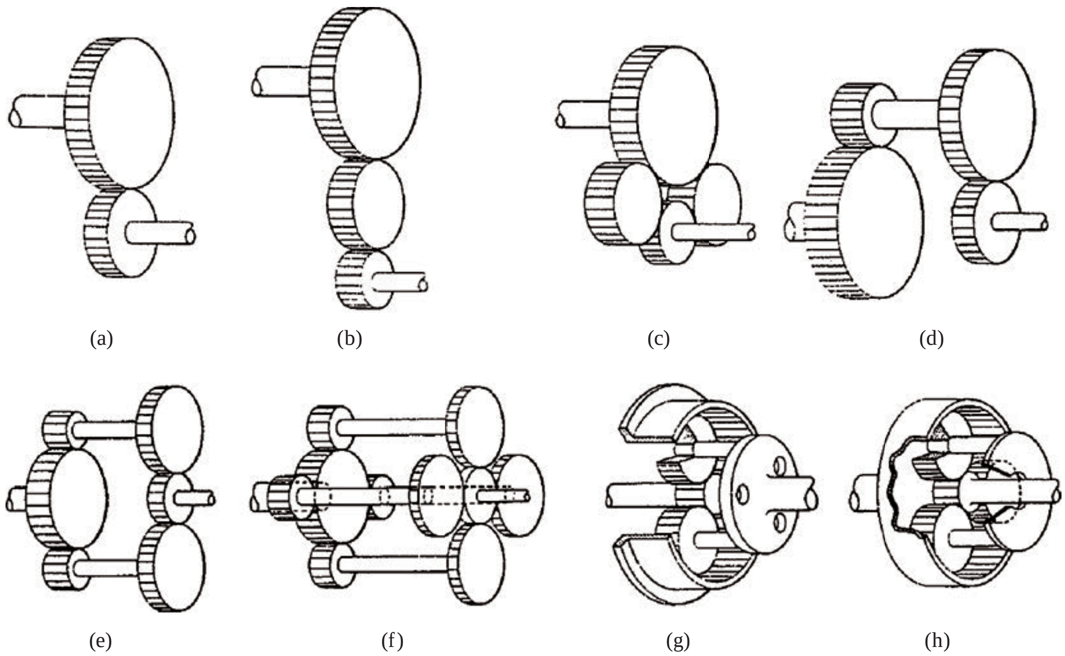


FIGURE 4.19 Eight kinds of gear arrangements for spur or helical gears: (a) offset; (b) offset with idler; (c) offset with two idlers; (d) double-reduction; (e) double-reduction double-branch; (f) double-reduction four-branch; (g) planetary; and (h) star. (Reprinted from *Product Engineering*, January 21, 1963.)

When weight is critical, special design efforts that use lightweight bearings, hollow shafts, thin casing walls, and gears with all excess material removed from the gear bodies may achieve surprisingly low weights.

4.2.3 Indices of Tooth Loading

There are two indices of tooth loading that are very important in gear design. *Unit load* is an index of tooth loading from the standpoint of tooth strength. The *K* factor is an index of tooth loading from the standpoint of tooth surface durability. To say it another way, the higher the unit load, the more risk of tooth breakage, and the higher the *K* factor, the more risk of tooth pitting.

Both of these index numbers are calculated from the transmitted power. The normal calculation method for spur or helical gears is

$$\text{Torque } T_\text{P} = \frac{P \times 9549.3}{n_1} \quad \text{(N m; metric),} \tag{4.61}$$

where P is power in kilowatts,

$$\text{Torque } T_\text{P} = \frac{P \times 63{,}025}{n_\text{P}} \quad \text{(in. lb; English),} \tag{4.62}$$

where P is power in horsepower.

The tangential driving load is

$$W_\text{t} = T_\text{P} \times \frac{2000}{d_\text{p1}} \quad \text{(N; metric)} \tag{4.63}$$

$$W_\text{t} = T_\text{P} \times \frac{2}{d} \quad \text{(lb; English),} \tag{4.64}$$

where d_{p1} is pinion pitch diameter (mm), and d is pinion pitch diameter (in.).

The unit load index is derived from the Lewis formula for tooth strength. (See Equation 4.6.) It is

$$U_\text{l} = \frac{W_\text{t}}{b} \times m_\text{n} \quad \text{(N/mm}^2\text{; metric)} \tag{4.65}$$

$$U_\text{l} = \frac{W_\text{t}}{F} \times P_\text{nd} \quad \text{(psi; English).} \tag{4.66}$$

The *K* factor is based on the Hertz stress formula. It is rst shown in Equation 4.24. It will now be given in both metric and English forms and for both external and internal gears:

External:

$$K = \frac{W_\text{t}}{d_\text{p1} b}\left(\frac{u+1}{u}\right) \quad \text{(N/mm}^2\text{; metric)} \tag{4.67}$$

$$K = \frac{W_\text{t}}{dF}\left(\frac{m_\text{G}+1}{m_\text{G}}\right) \quad \text{(psi; English)} \tag{4.68}$$

Internal:

$$K = \frac{W_t}{d_{p1}b}\left(\frac{u-1}{u}\right) \text{ (N/mm}^2\text{; metric)} \tag{4.69}$$

$$K = \frac{W_t}{dF}\left(\frac{m_G-1}{m_G}\right) \text{ (psi; English)} \tag{4.70}$$

The load indices show the *intensity* of loading that the teeth are trying to carry. They are based on real quantities, and they measure what the user of gearing is getting out of the mesh. For instance, a *K* factor of 5 means that a gear mesh is carrying 5 N of tooth load for each millimeter of pinion pitch diameter and each millimeter of face width in contact, with an appropriate adjustment for the relative size of the gear that is in mesh with the pinion.

The gear user can understand unit load and *K* factor as measures of how much load is being carried per unit of size in the gear mesh, from bending strength and surface loading viewpoints. In comparison, the stress formulas have factors in them that are related to the quality and geometry of the application. The calculated stress number is, of course, very useful, but it does not directly tell the user the relative intensity of loading. For instance, a high stress may occur in a situation in which moderate load intensity is coupled with a poor geometric design and low quality. Obviously, less gear-unit size is needed if acceptable stress levels can be achieved when the design has a relatively high intensity of loading and good enough quality and geometric design of teeth to keep these factors favorable.

The general procedure in preliminary design is to size the gears based on the *K* factor. Then the tooth size in module (or pitch) is determined by guring the unit load and making the teeth large enough to get an acceptable unit-load value. For instance, a spur pinion with 36 teeth might be OK on *K* factor but too high on unit load. If 18 teeth are put on the same pitch diameter (tooth size twice as great), the unit load is reduced from 2 to 1 (50% as much unit load).

4.2.4 Estimating Spur- and Helical-Gear Sizes by *K* Factor

Since the *K* factor is so important in determining gear size, it is necessary to know how much *K* factor can be carried in different applications.

The teeth of low- and medium-hard-steel gears usually have more strength than they have capacity to resist pitting. Hence, the index of surface durability becomes the limiting factor in determining the load-carrying capacity of the gears. If very thin oil is used or there is inadequate provision to cool the gearset, scoring might be a limiting condition. Generally, though, the designer will use heavy enough oil and provide enough cooling to get all the capacity out of the gears that is in the metal. Oil and oil-cooling systems are usually cheaper than larger-sized gears.

In fully hardened gearing, the strength of the teeth may become as important as, or even more important than, surface durability. Even in this case, the gear designer can often so proportion the design that there will be enough strength available to match the capacity of the teeth to withstand pitting. If strength is more limiting than wear, a proper reduction in *K* factor can still aid in obtaining teeth of adequate strength. With this in mind, it is possible to use approximate *K* factors as a measure of the amount of load that can be carried on many types of spur and helical gears.

Table 4.14 shows a study of *K* factor and unit-load values that are typical of nominal design practice for a variety of gear applications. These values are intended for use in the preliminary sizing of a gearset. Figure 4.20 shows in pictorial

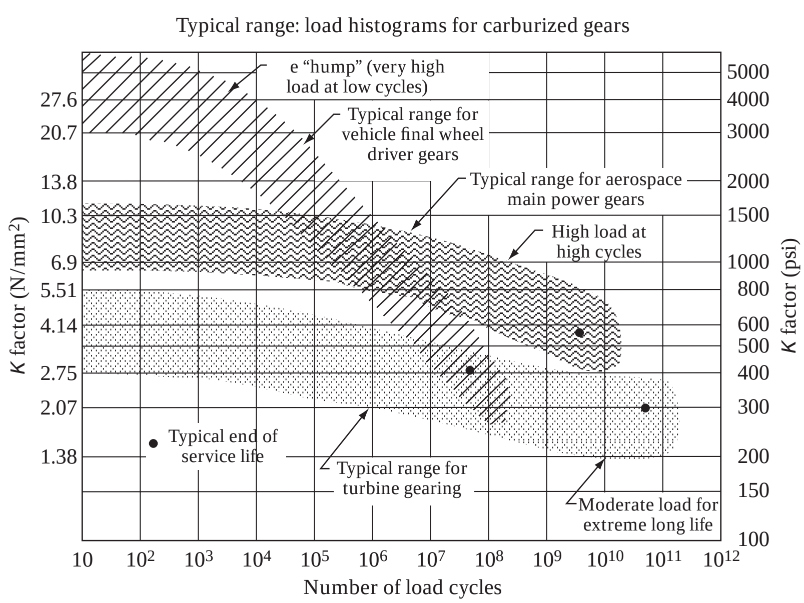


FIGURE 4.20 Rough comparison of design *K* factors for carburized gears used in vehicle, aerospace, and turbine applications. The bandwidths result from quality of gearing, tooth design, application, level of risk, etc.

fashion the nominal design range for vehicle, aerospace, and turbine gears.

After the preliminary sizing of gears, detail design work must be done to establish the pressure angle, helix angle, tooth addenda, tooth whole depth, and tooth thickness. Then, when complete tooth geometry has been picked, the designer should calculate tooth stresses and compare them with allowable values for the material being used and the degree of reliability required. Chapter 5 covers the determination of gear-tooth geometry and the calculation of load-carrying capacity to meet approximate gear-rating criteria. In this chapter, the data on how to use index values to size gears are intended only as a means of establishing the rst approximation of an appropriate gear size.

4.2.5 Estimating Bevel-Gear Size

It is possible to estimate bevel-gear sizes by the *Q*-factor method. So far this method has had limited usage. It appears that the method will work fairly well for bevel gears but will not give quite as good results as for spur and helical gears.

The geometry of bevel gears is more complicated than that of spur or helical gears. Under load, bevel gears tend to shift position more than gears on parallel shafts. It is frequently necessary to design a “mismatch” into the teeth. The mismatch concentrates the tooth load in the center of the face width and allows some shifting of shaft alignment to occur before the load is concentrated too heavily at one end of the tooth. These things make it hard for a simple formula to estimate capacity correctly.

Some changes are required before the formulas in Section 4.2.2 can be applied to bevel gears. Bevel gears do not have any *center-distance* dimensions. This means that center distance must be removed from Equations 4.55 and 4.56. This can be done with the help of Equation 1.5. The result is

$$d_{\mathrm{p1}}^2 b = \frac{1.91\times 10^7 Q}{(u+1)^2 K} \quad \text{(metric)} \tag{4.71}$$

$$d^2 F = \frac{126{,}000Q}{(m_{\mathrm{G}}+1)^2 K} \quad \text{(English).} \tag{4.72}$$

Equations 4.71 and 4.72 can be used for both bevel and spur gears. It is handy to use when the design requirements have been reduced to a *Q* factor, but the designer has not yet decided whether spur, helical, or bevel gears are to be used. These equations can help the designer estimate the sizes of all three kinds.

The calculation of unit load and *K* factor for bevel gears is somewhat different for spur and helical gears. Figure 4.21 shows how these are calculated for the bevel gearset.

The *K* factor in Equations 4.71 and 4.72 is the same value that was used previously. However, it is sometimes necessary to use lower values than those shown in Table 4.14 to compensate for some of the special problems in bevel gears. For instance, if mountings do not maintain good tooth contact at full load, the *K* factor should be reduced.

Bevel-gear designers have tended to reduce capacity for both size of pinion and increase in pitch-line velocity.

Table 4.17 shows a study of bevel-gear *K* factors based on load-rating formulas in current use. These values represent averages of different rating designs for a *uniform* power source and a *mild shock* power-absorbing device (such as an electric motor driving a well-designed pump).

If the designer has already decided to use a bevel gearset, it is not necessary to go to the trouble of calculating a *Q* factor. The equation can be simpli ed to

$$d_{\mathrm{p1}}^2 b = \frac{1.910}{K}\times\frac{10^7 P}{n_1}\times\frac{(u+1)}{u} \quad \text{(metric),} \tag{4.73}$$

$$d^2 F = \frac{126{,}000}{K}\times\frac{P}{n_{\mathrm{P}}}\times\frac{(m_{\mathrm{G}}+1)}{m_{\mathrm{G}}} \quad \text{(English).} \tag{4.74}$$

Both Equations 4.73 and 4.74 determine the quantity pitch diameter squared times face width. To get a complete solution, it is necessary to know the relation of pitch diameter to face width. Gleason Works recommends that the face width of straight and spiral bevel gears not exceed 0.3 times the cone distance or 10 in. divided by the diametral pitch. For Zerol gears, the only difference in the limits is that the cone-distance constant is 0.25 instead of 0.30.

Table 4.18 shows how these limits work out in terms of pitch diameter or the pinion for different ratios. A shaft angle of 90° is assumed.

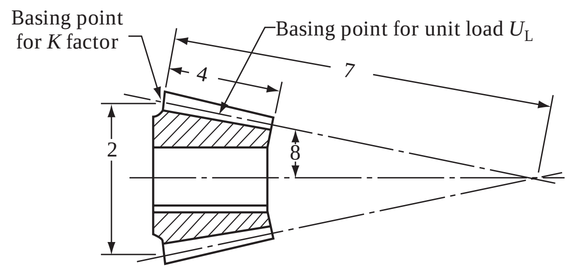
Table 4.18 shows that the face width for a low ratio such as 1 to 1 will probably be limited by the cone distance. For a high ratio such as 5 to 1, the pitch will probably limit the face width. An appropriate value of face width can be picked from Table 4.18. Then this value can be used in Equations 4.73 and 4.74.

Hypoid gears are hard to estimate. As a general rule, a hypoid pinion will carry about the same power as a bevel pinion. Since the hypoid pinion is bigger—for the same ratio—than a bevel pinion, the hypoid set will carry more power as a set.

Face gears may be handled somewhat similarly as done with straight bevel gearsets. Generally, it will be necessary to use les face width for the face gear than would be allowed as a maximum for the same ratio of bevel gears (see Section 5.1.19).

4.2.6 Estimating Worm-Gear Size

It is much harder to estimate the capacity of worm gears. In spur, helical, and bevel gears, the magnitude of the surface compressive stress is the major thing that determines capacity. The limits of strength and scoring seldom determine the gear size—unless the set is poorly proportioned. In worm gearing, the tendency to score is often as important as the tendency to pit in determining capacity. Since scoring depends on both



Item	Metric symbol	English symbol
1. Number of teeth	z_1	n or N_P
2. Pitch diameter	d_{p1} (mm)	d (in.)
3. Spiral angle	β (°)	ψ (°)
4. Face width	b (mm)	F (in.)
5. Number of teeth in mate	z_2	N or N_G
6. Pinion torque	T_1 (N m)	T_P (in. lb)
7. Outer cone distance	R_a (mm)	A_o (in.)
8. Pitch angle	δ_1 (°)	γ (°)
9. Tooth ratio	u	m_G

Equations

Tooth ratio = no. gear teeth ÷ no. pinion teeth (1)

Pitch angle = arctan 1.0 ÷ tooth ratio (2)

$$\text{Outer cone distance} = \frac{\text{pinion pitch dia.}}{2 \sin \text{pitch angle}} \quad (3)$$

$$\text{Tan. driving load, } W_t = \frac{2.0 \text{ pinion torque}}{\text{pinion pitch dia.}} \quad (4)$$

$$K \text{ factor} = \begin{cases} \dfrac{W_t}{d_{p1} b}\left(\dfrac{u+1}{u}\right) & \text{Metric (5a)} \\[2ex] \dfrac{W_t}{dF}\left(\dfrac{m_G+1}{m_G}\right) & \text{English (5b)} \end{cases}$$

$$\text{Unit load} = \begin{cases} \dfrac{W_t}{b \cos \beta} \times \dfrac{z_1}{d_{p1}} \times \left(\dfrac{R_a}{R_a - 0.5b}\right)^2 & \text{Metric (6a)} \\[2ex] \dfrac{W_t}{F \cos \psi} \times \dfrac{n}{d} \times \left(\dfrac{A_o}{A - 0.5F}\right)^2 & \text{English (6b)} \end{cases}$$

FIGURE 4.21 The method for calculating indices of tooth loading for straight or spiral bevel gears at a 90° shaft angle.

compressive stress and rubbing (or sliding) velocity, a rating formula has to be based on more than a K factor.

Worm-gear sizes can be estimated reasonably well from a table of power versus center distance for a series of worm speeds. Table 4.19 shows nominal capacity of a range of sizes of single-enveloping worm gears. It is assumed that the worm is case-hardened and ground to good accuracy and nish. To meet the table speci cations, the worm gear should be made of a good grade of chill-cast phosphor bronze and cut to give a good bearing with the worm. The ratings in the table should be regarded as *nominal*. Several manufacturers have been able to carry up to 100% more load than those shown in the table by the use of special materials and by obtaining a very high degree of precision in the worm and gear. Conversely, when accuracy has not been the best and when operating conditions have not been good (when such conditions as shock loads, vibration, or overheating have been present), it has been necessary to rate worm gearsets substantially lower than the values shown in Table 4.19.

The nominal capacity of double-enveloping worm gears of the Cone Drive design is shown in Table 4.20.

Tables 4.19 and 4.20 are intended for worm gears subject to shock-free loading and in service for not more than 10 hours per day. If service is 24 hours per day with some shock loading, the table ratings should be reduced to about 75% of the values shown.

TABLE 4.17
Approximate *K* Factor Values for Bevel Gears

Pinion Pitch Diameter mm	in.	Gear Ratio, z_2/z_1 (N_G/N_P)	*K* Factor Values for Pitch-Line Speed: 5 m/s (N/mm²)	1000 fpm (psi)	20 m/s (N/mm²)	4000 fpm (psi)
Case 1: Industrial Spiral Bevel Gears at HRC 58						
50	2.0	1	3.93	570	3.45	500
		3	3.38	490	2.96	430
100	5.0	1	3.27	475	2.90	420
		3	2.79	405	2.48	360
200	10.0	1	2.72	395	2.41	350
		3	2.38	345	2.10	305
Case 2: Industrial Spiral Bevel Gears, Pinion BHN 245, Gear BHN 210						
50	2.0	1	1.83	265	1.62	235
		3	1.65	239	1.46	212
100	5.0	1	1.41	205	1.24	180
		3	1.23	179	1.08	157
200	10.0	1	1.12	163	0.99	143
		3	0.94	137	0.83	120
Case 3: Industrial Straight Bevel Gears, Pinion BHN 245, Gear BHN 210						
50	2.0	1	0.86	125	–	(See Note 1)
		3	0.77	112	–	(See Note 1)
100	5.0	1	0.83	120	–	–
		3	0.76	110	–	–
200	10.0	1	0.79	115	–	–
		3	0.72	105	–	–

Note: 1. Straight bevel gears are usually not used above 10 m/s (2000 fpm). At 20 m/s (4000 fpm), spiral bevel gears are the normal choice; 2. The above *K* factor values assume average conditions. With a special design and a favorable application and/or design that is not close to being optimum, the *K* factor values shown will be too high for good practice. These values are also based on 10^8 cycles and an application factor K_a of 1.5. This is equivalent to an electric motor driving a piece of equipment that has mild shock.

TABLE 4.18
Ratios of Maximum Face Width to Bevel-Pinion Pitch

Ratio	Face Width Based on 0.3 Cone Distance	Face Width Based on 10 in. per Diametral Pitch: 15 Teeth	20 Teeth	25 Teeth
1	0.212*d*	0.667*d*	0.500*d*	0.400*d*
1.5	0.270*d*	0.667*d*	0.500*d*	0.400*d*
2	0.335*d*	0.667*d*	0.500*d*	0.400*d*
3	0.474*d*	0.667*d*	0.500*d*	0.400*d*
4	0.618*d*	0.667*d*	0.500*d*	0.400*d*
5	0.765*d*	0.667*d*	0.500*d*	0.400*d*
6	0.912*d*	0.667*d*	0.500*d*	0.400*d*
7	1.061*d*	0.667*d*	0.500*d*	0.400*d*

Note: *d* is the pinion diameter.

The horsepower ratings shown in Tables 4.19 and 4.20 are *mechanical* ratings. The mechanical rating is the amount of power that the set is expected to carry without excessive wear or tooth breakage when the set is kept reasonably cool. In many cases, worm gearsets will overheat because there is not enough cooling of the gear case or the oil supply to remove the heat generated by the set. This makes it necessary to calculate a *thermal* rating. The thermal rating is the maximum amount of power that the set can carry before a dangerous operating temperature is reached. Quite obviously, the thermal rating depends as much on casing design and lubrication system as it does on the size of the gears themselves. In many cases, a worm-gear design will not carry so much thermal rating as mechanical rating. However, if adequate oil pumps, heat exchangers, and oil jets are used, it should be possible to operate any worm gearset up to its full mechanical rating. Standard thermal ratings for cylindrical worm gearsets are shown in AGMA standards.

TABLE 4.19
Nominal Capacities of Cylindrical Worm Gearing

Ratio, m_G	Center Distance, C (in.)	Worm Pitch Diameter, d (in.)	Effective Face Width, F_e (in.)	Lead Angle, λ (°)	Output (hp) at Different Worm Speeds (rpm)				
					100	720	1750	3600	10,000
5	2	0.825	0.46875	37.6	0.19	1.20	2.10	2.90	4.40
8	2		0.50000	25.7	0.15	0.90	1.70	2.30	3.60
15	2		0.46875	14.4	0.08	0.51	0.95	1.30	2.10
25	2		0.46875	8.75	0.05	0.31	0.59	0.83	1.30
50	2		0.46875	4.40	0.02	0.15	0.28	0.39	0.62
5	4	1.525	0.87500	40.3	1.60	8.00	12.00	17.00	22.00
8	4		0.93750	28.0	1.20	6.40	9.90	14.00	19.00
15	4		0.93750	15.8	0.72	3.90	6.10	8.70	12.00
25	4		0.93750	9.64	0.44	2.40	3.80	5.40	7.40
50	4		0.93750	4.85	0.21	1.20	1.80	2.60	3.60
5	8	2.800	1.68750	43.3	11.00	39.00	60.00	75.00	–
8	8		1.81250	30.5	8.10	33.00	50.00	64.00	–
15	8		1.81250	17.4	4.80	21.00	31.00	41.00	–
25	8		1.81250	10.7	3.00	13.00	19.00	26.00	–
50	8		1.81250	5.39	1.40	6.20	9.30	12.00	–
5	16	5.100	3.12500	46.5	63.00	186.00	254.00	–	–
8	16		3.37500	33.4	50.00	156.00	221.00	263.00	–
15	16		3.37500	19.4	30.00	98.00	143.00	169.00	–
25	16		3.37500	11.9	19.00	62.00	90.00	107.00	–
50	16		3.37500	6.02	8.90	30.00	43.00	52.00	–

Note: Service factor K_s has a value of 1.0 for this table (see service factors in Table 5.47). The sliding velocity is not over 6000 fpm.

TABLE 4.20
Nominal Capacities of Double-Enveloping Worm Gearing

Ratio, m_G	Center Distance, C (in.)	Worm Pitch Diameter, d (in.)	Output (hp) at Different rpm's of Worm Speeds (rpm)				
			100	720	1750	3600	10,000
5	2	0.830	0.40	2.19	3.82	5.53	7.93
15	2	0.830	0.17	0.99	1.79	2.61	3.83
50	2	0.850	0.05	0.31	0.56	0.82	1.22
5	4	1.730	3.65	16.90	26.60	34.60	–
15	4	1.550	1.65	8.15	13.10	17.30	22.90
50	4	1.660	0.52	2.56	4.11	5.47	7.28
5	8	3.450	25.90	95.20	135.00	163.00	–
15	8	2.940	11.80	48.10	70.10	90.90	–
50	8	2.900	3.67	15.20	22.50	28.90	–
5	16	5.143	180.00	513.00	678.00	–	–
15	16	5.143	83.20	273.00	369.00	439.00	–
50	16	5.143	26.10	87.00	119.00	140.00	–
5	24	7.333	473.00	1194.00	1488.00	–	–
15	24	7.333	227.00	636.00	829.00	–	–
50	24	7.333	71.70	204.00	270.00	–	–

Note: Service factor K_s is 1.0 for this table (see service factors in Table 5.52). The sliding velocity is not over 6000 fpm.

In each of the designs shown in the tables, an arbitrary size of worm was used. The size chosen represents good design practice. In many instances, though, it will be necessary to use differently sized worms. Often a worm is made of a "shell" design to slip over a large shaft. Large turbine shafts may have large worms mounted on them to drive small worm gears attached to oil pumps or governors. Good designs of this type can be made, but they are not as ef cient as worm gearsets in which the worm and gear sizes can be more properly proportioned.

TABLE 4.21
Nominal Capacities of Spiroid Gears (Pinion and Gear Case-Hardened, 60 Rockwell C Minimum)

Ratio, m_G	Center Distance, C	Pinion OD, d_o	Gear OD, D_o	hp at Different Pinion Speeds (rpm) 100	720	1750	3600	10,000
10.250	0.500	0.437	1.500	0.0142	0.0697	0.129	0.1948	0.3354
14.667		0.421		0.0120	0.0589	0.109	0.1645	0.2834
25.500		0.423		0.0088	0.0432	0.080	0.1208	0.2080
47.000		0.427		0.0064	0.0313	0.058	0.0876	0.1508
10.250	1.000	0.853	3.000	0.0937	0.4601	0.852	1.2870	–
14.667		0.821		0.0759	0.3726	0.690	1.0420	
25.500		0.827		0.0550	0.2700	0.500	0.7550	
47.000		0.837		0.0377	0.1852	0.343	0.5179	
71.000		0.761		0.0299	0.1469	0.272	0.4107	
10.250	1.875	1.507	5.625	0.4961	2.4350	4.510	6.8100	–
14.667		1.463		0.4125	2.0250	3.750	5.6630	
25.500		1.435		0.2915	1.4310	2.650	4.0020	
47.000		1.448		0.1804	0.8856	1.640	2.4760	
71.000		1.308		0.1441	0.7074	1.310	1.9780	
106.000		1.215		0.1188	0.5832	1.080	1.6310	
10.250	3.250	2.395	9.750	2.1010	10.3100	19.000	–	–
14.667		2.465		1.760	8.6400	16.000		
25.500		2.319		1.2100	5.9400	11.000		
47.000		2.344		0.7205	3.5370	6.550		
71.000		2.090		0.5544	2.7220	5.040		
106.000		1.933		0.4455	1.1870	4.050		
10.250	5.125	3.342	15.375	7.0070	34.4000	63.700	–	–
14.667		3.342		5.9180	29.0500	53.800		
25.500		3.092		3.9490	19.3900	35.900		
47.000		2.910		2.2550	11.0700	20.500		
71.000		3.097		1.7270	8.4780	15.700		
106.000		2.841		1.3860	6.8040	12.600		

Note: Class 1 AGMA service. Pitch-line speed not over 1700 fpm. No allowance for shock loads, $K_s = 1.0$. Based on tooth proportions recommended by Spiroid Division of Illinois Tool Works, Chicago, Illinois, United States.

4.2.7 Estimating Spiroid Gear Size

Spiroid gears have less sliding than worm gears but much more sliding than spur or helical gears. It is not practical to estimate their size by a Q factor method.

In general, both members of the Spiroid set are carburized. Final nish is done by grinding or lapping. Normally, the lubrication is handled by EP lubricants.

Table 4.21 shows the nominal capacity of a range of Spiroid gearset sizes. This table can be used to make a preliminary estimate of the size needed for Spiroid gearset.

4.3 DATA NEEDED FOR GEAR DRAWINGS

After the size of a gear train has been determined, the designer must work out all the dimensional speci cations and tolerances that are necessary to de ne exactly the gears required. Data on the gear material required and the heat treatment must also be given.

Many years ago it was customary to specify only a few major dimensions, such as pitch, number of teeth, pressure angle, and face width. It was assumed that the skilled mechanic in the shop would know what tooth thickness, whole depth, addendum, and degree of accuracy were required. Little or no consideration was given to such things as root llet radius, surface nish, and pro le modi cation.

Today the designer of gears for a highly developed piece of machinery such as an airplane engine, an ocean-going ship, or an automobile nds it necessary to specify dimensions and tolerances covering all features of the gear in close detail. There is often the risk that a buyer of gears will purchase thousands of dollars' worth of gears and nd that their quality is unsuitable for the application.

4.3.1 Gear Dimensional Data

The dimensional data which may be required to make a gear drawing can be broken down into blank dimensions and tooth data. The blank dimensions are usually shown in cross-sectional views. The tooth data are either tabulated or shown directly on an enlarged view of one or more teeth. Some of the common blank dimensions are the following:

- Outside diameter
- Face width
- Outside cone angle (bevel gears)
- Back cone angle (bevel gears)
- Throat diameter (worm gears)
- Throat radius (worm gears)
- Root diameter
- Bore diameter (internal gears)
- Mounting distance (bevel gears)
- Inside rim diameter
- Web thickness
- Journal diameter

The tabulated gear-tooth data will cover items such as the following:

- Number of teeth
- Module (or diametral pitch)
- Pitch diameter
- Circular pitch
- Linear pitch (worm gears)
- Pressure angle
- Normal pressure angle (helical gears)
- Normal circular pitch (helical gears)
- Normal module or pitch (helical gears)
- Addendum
- Whole depth
- Helix angle
- Hand of helix
- Lead angle (worm gears)
- Pitch cone angle (bevel gears)
- Root cone angle (bevel gears)
- Tooth thickness
- Lead (worm gears)

Special views of gear teeth may be used to show things such as minimum root fillet radius, form diameter, tip radius, end radius, and surface finish. Notes may be added to the drawing to specify the heat treatment and to define the accuracy limits for checking the gear teeth. Further notes often define the reference axis used in checking and refer to drawings of cutting tools or processing procedures needed to make the gear come out right.

The description just given of what may go on a gear drawing may leave the impression that gear drawings have to be very complicated. This is not necessary the case. The designer should consider the gear quality required to meet design requirements, together with the responsibility that the shop making the gear will assume for making gears that will work satisfactorily. If the gear shop will assume responsibility for making a gear that will operate satisfactorily, and they understand the design requirements, a very simple drawing may be sufficient. Many fine gears are made from drawings that are very simple. The problem of the gear designer is to determine just how detailed the drawing must be *to give the gear maker the responsibility of producing a gear that will do the job.*

Figure 4.22 shows how a complex drawing is made for a pinion used with a high-speed turbine. These aspects of the drawing shown in Figure 4.22 are worth noting:

- Basic tooth data for gear teeth and spline teeth are shown in tabular form.
- A cross-sectional view (and an end view if needed) shows the body dimension of the gear.
- Enlarged tooth views may be needed to define finish, critical radii, and instructions on where grinding is permitted.
- Special rounding or breaking of sharp end corners is covered in an enlarged tooth view.
- A series of notes covers requirements of accuracy, metallurgy, and (perhaps) special processing procedures.

For several years, a major effort has been made to establish drafting standards for gears. This work has led to gear and spline standards issued as American National Standards Institute (*ANSI*).

Figure 4.23 shows examples of a spur gear and helical gear. Note that formats A, B, C, and D are recognized, and that there are quite a few special instructions for those who do the design and drafting work.

Examples of straight bevel gear and a spiral bevel gear are shown in Figures 4.24 and 4.25. Note the long list of tabulated data items. Also note the special data shown in the cross-sectional view to define *crossing point*, *face apex*, and other terms peculiar to bevel gears.

Standard worm and worm-gear drawing examples are shown in Figures 4.26 and 4.27. Note the difference in practice between single-enveloping gearing and double-enveloping worm gearing.

Spiroid and Helicon gears also need special drafting treatment. Figure 4.28 shows an examples of a Spiroid pinion and a Spiroid gear.

Those involved in the details of gear and spline* drafting should study carefully all the special data and instructions given in ANSI standards. After years of work, a general agreement has been reached on an international practice on defining and depicting the whole family of gears and splines.

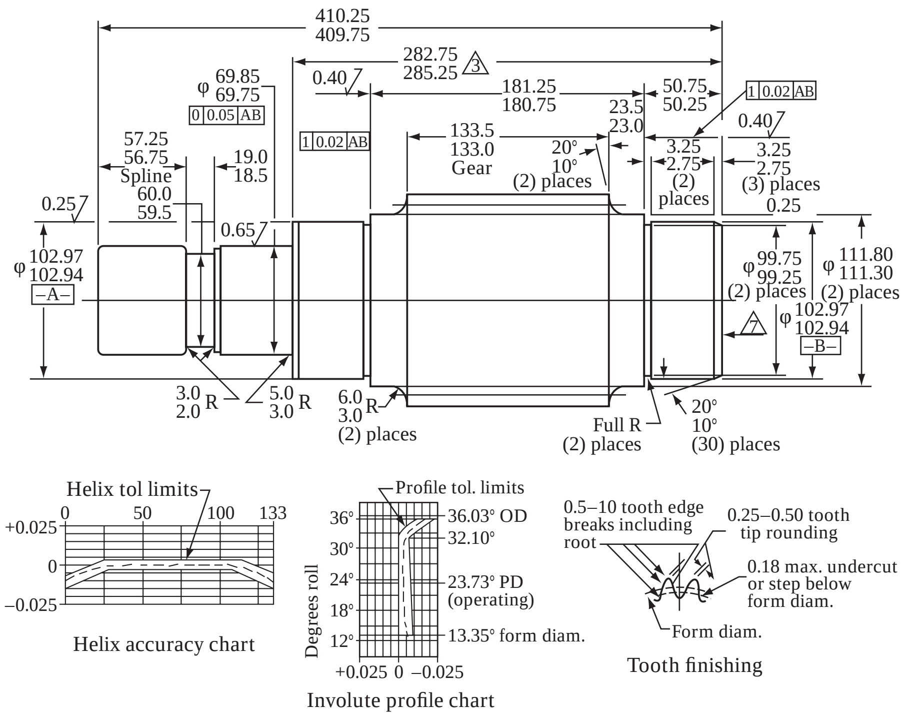
4.3.2 Gear-Tooth Tolerances

Gear tooth tolerances are a difficult and controversial subject. Much has been written about gear tolerances, and yet there are no clear-cut answers. Space will not permit detailed study of tolerances in this book.

From a general standpoint, tolerances on a gear must meet two kinds of requirements:

1. The tolerances must be broad enough to be met by the method of manufacture and the artisanship of the plant undertaking the manufacture.
2. Tolerances must be close enough so that the gears will carry the required loads for a sufficient length of

* A spline with gear teeth is closely related to gearing in its design and manufacture. Functionally, though, a spline is a joint connection rather than a gear mesh.



External helical gear manufacturing data

Module, normal	5.05
Module, transverse	5.1764
Pressure angle, normal	20°
Pressure angle, transverse	20.647°
Helix angle and hand	15°, right
Number of teeth	24
Pitch diameter	124.233
Base circle diameter	116.254
Base helix angle	14.076°
Helix lead	1456.58
Major diameter	137.33·137.07
Root diameter	112.60·116.68
Form diameter, max.	119.37
Roll angle at form	13.356°
Roll angle at OD	36.033°
Root fillet radius, min.	1.75
Arc tooth thickness, norm.	8.928·8.801
Pin diameter	8.64001
Dimension over pins	138.67·138.40

Operating conditions (ref.)

No. teeth in mating gear	80
Center distance	272.637
Pitch diameter	125.832
Module, transverse	5.2430
Pressure angle, transverse	22.500°
Addendum	5.7500
Backlash	0.265.0.530

Gear quality

Material grade	1
Quality level, tooth geometry	A
Tooth to tooth spacing limits	0.0061
Cumulative tooth to tooth	0.0127
Total composite variation	0.0256
Profile tolerance	see chart
Helix accuracy	see chart
Profile surface finish, microns	0.5
Root fillet surface finish, microns	2.0

External involute spline data flat root side fit

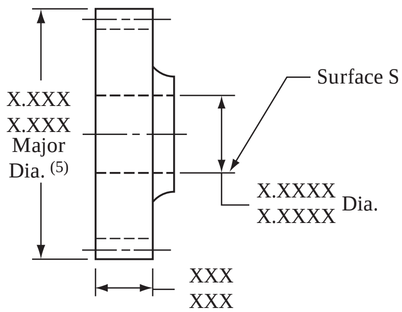
Number of teeth	26
Spline module	2.54/1.27
Pressure angle	30°
Pitch diameter	66.040
Base diameter	63.924
Major diameter	68.58·67.82
Form diameter, max.	63.348
Minor diameter, min.	62.230
Circular tooth thickness	
Max. e ective	3.990
Min. actual	3.909
Max. actual	3.945
Pin diameter	4.8768
Min. measurement over pins	73.482

Notes:
1. Diameters –A– and –B– form axis of gear.
2. Material: AISI 4320, make from forging no. 1002.2.
3. Heat treatment:
   - Carburize and HDN, noted area
   - Case hardness HRC 59 minimum
   - E ective case depth after grinding 1.0–1.5
   - Core hardness HRC 35–42
4. Grinding prohibited in roots of teeth
5. Magnetic particle inspect. Indications prohibited in gear or spline teeth
6. Nital etch inspect, grind burns or retempered areas prohibited
7. Brand part no. and serial no.
8. (1) required per assembly

FIGURE 4.22 Sample gear drawing for high-precision turbine gear work.

time and without objectionable noise or vibration. In timing and control applications, suitable accuracy of motion and freedom from "lost motion" on reversal must be obtained.

The designer has several sources of information on tolerances. If the product is in production, the accuracy already achieved and the performance in the eld can be studied. This should give an answer to the questions of what can be done and what is needed. On entirely new products, studies of dynamic loads, effects of misalignment, and effects of surface nish may be required. Sections 5.2.4 and 5.2.5 in Chapter 5 and Section 10.4.1 in Chapter 10 show some of the considerations that will help a designer to estimate accuracy requirements. Trade standards are very helpful in showing trade practices in regard to tolerances and inspection of different kinds of gears.



Involute spur gear data (1)

— 12' — root
Number of teeth XX
Diametral pitch (1)XX
Pressure angle XX XX°
Pitch dia. X XXXXXXXX ref
Base dia. X XXXXXXXX ref
Form dia. (4) X.XXX
Minor dia. (5) X.XXX–X.XXX
Actl circ. tooth thk. at pitch dia. XXXX–.XXXX
Meas. over two .XXXXX dia. pins (6)
X.XXXX–X.XXXX ref
Max. runout XXXX fir (7)
All tooth element specifications are from datum
established by the axis of surface S

(a) Format A

Show data as in Format A, and add:
Pitch tol. .XXXX

(b) Format B

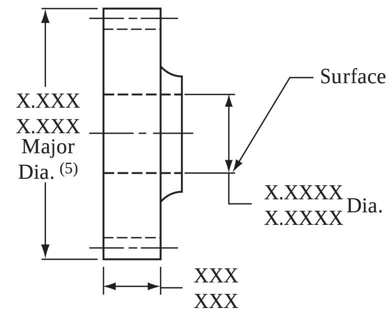
Show data as in Format A, and add:
Pitch tol. .XXXX
Profile tol. .XXXX

(c) Format C

For Format D, see paragraph 7-5.4

(d) Format D

1 When view does not clearly delineate the gear, chart title shall be supplemented by the word "external."
2 Show here, either "full fillet", "optional" or "flat."



Involute helical gear data (1)

— 2' — root
Number of teeth XX
Diametral pitch-norm. (3)XX
Pressure angle-norm. XX XX°
Helix angle X.XXXXXXXX° ref
Hand of helix X
Lead XX.XXX (8)
Pitch dia. X.XXXXXXXX ref
Base dia. X.XXXXXXXX ref
Form dia. (4) X.XXX
Minor dia. (5) X.XXX–X.XXX
Actl circ. tooth thk.-norm. at pitch dia.
.XXXX–.XXXX
Meas. over two .XXXXX dia. balls (6)
X.XXXX–X.XXXX ref
Max. runout .XXXX fir (7)
All tooth element specifications are from datum
established by the axis of surface S

(a) Format A

Show data as in Format A, and add:
Pitch tol. .XXXX

(b) Format B

Show data as in Format A, and add:
Pitch tol. .XXXX
Profile tol. .XXXX

(c) Format C

For Format D, see paragraph 7-5.4 of American National Standard Drafting Practice, Gear Drawing Standards

(d) Format D

1 When view does not clearly delineate the gear, chart title shall be supplemented by the word "external."
2 Show here, either "full fillet," "optional" or "flat."

FIGURE 4.23 Standard drawing formats for spur and helical gears.

Machine-tool builders can usually give good data as to the accuracy that their products can produce.

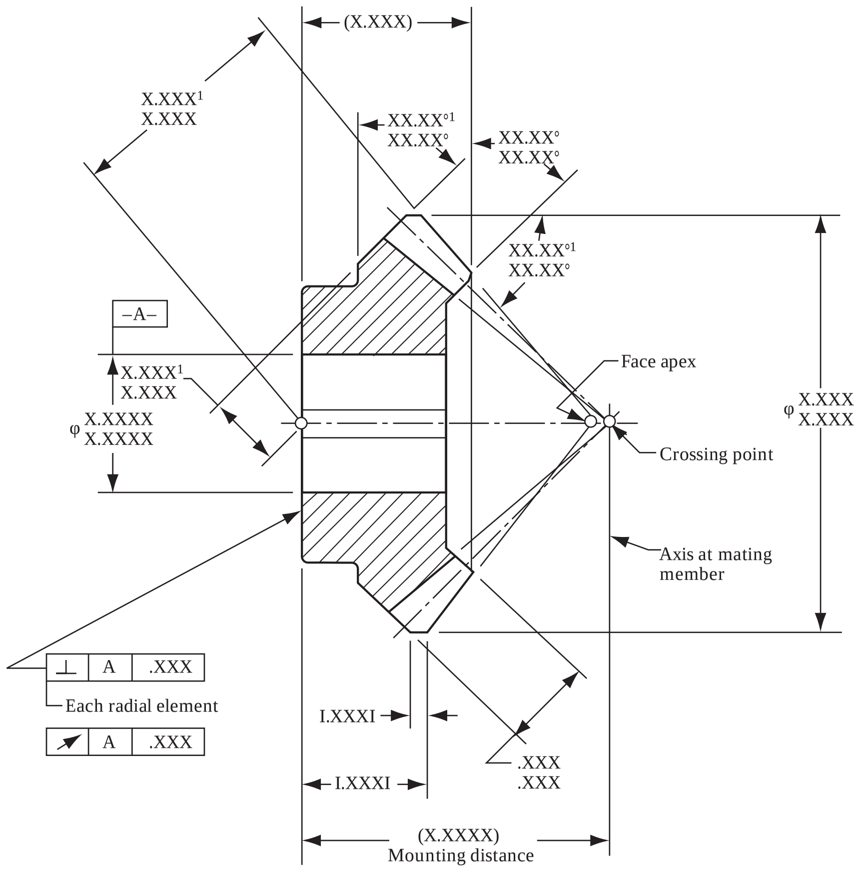
The quality of gear teeth cannot be completely controlled until appropriate tolerances are speci ed on the following items:

- Tooth spacing
- Tooth pro le
- Concentricity of teeth with axis
- Tooth alignment (or lead, or helix)
- Tooth thickness (or backlash)
- Surface nish of ank and llet

The best tooth-to-tooth spacing accuracy obtainable is about 2.5 μm (0.0001 in.). Very careful grinding or very good cutting and shaving is required to get this extreme degree of accuracy. Only a few of the best-designed machine tools are capable of this kind of work. This kind of accuracy is needed in a few very high-speed gears for marine and industrial uses. Tooth-to-tooth accuracy of most *precision* gear applications is on the order of 5 μm (0.0002 in.) or 8 μm (0.0003 in.). Good commercial gears range from 12 μm (0.0005 in.) to 40 μm (0.0016 in.) in tooth-to-tooth accuracy.

In few cases, the involute pro le is held to 5 μm (0.0002 in.) variation. Most precision gears have their involutes true within 12 μm (0.0005 in.). Good general-purpose gears range from 25 μm (0.001 in.) to 12 μm (0.0005 in.) on involute.

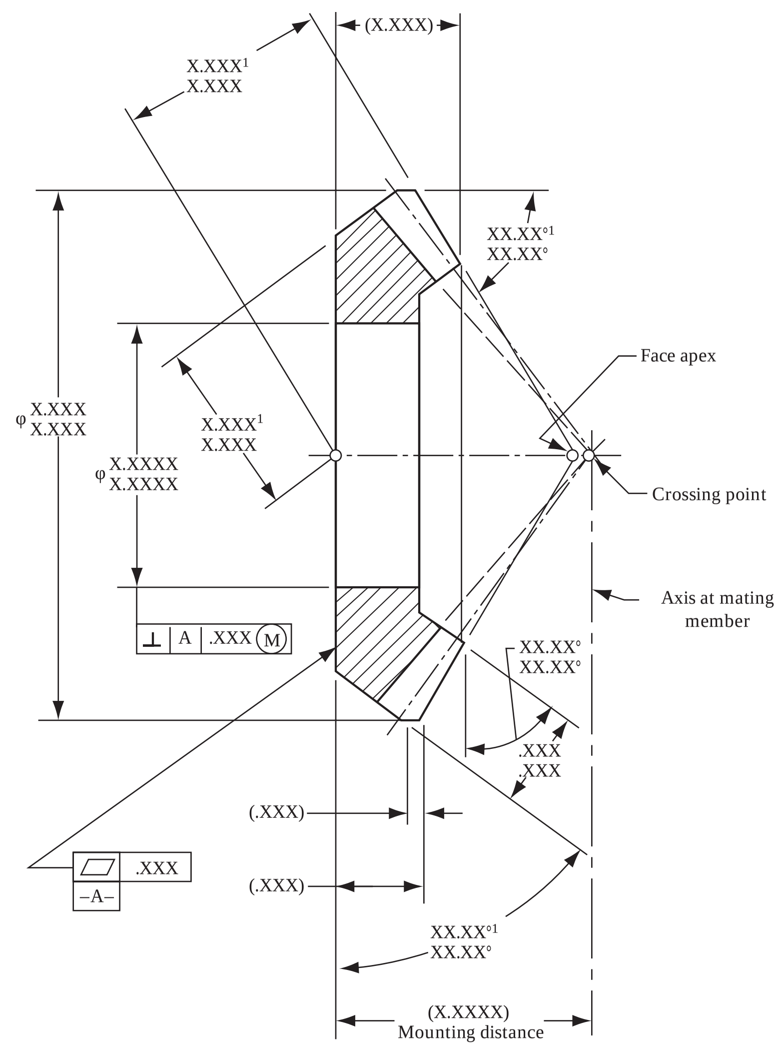
The concentricity of small control gears is held to 12 μm for some applications. Most precision gears are in the range of 25 to 50 μm. Good commercial gears range up to about



Number of teeth	XX
Diametral pitch[1]	(XX.XXX)
Pressure angle	(XX.XX°)
Pitch diameter	(X.XXXX)
Addendum	(.XXX)[2]
Working depth	(.XXX)[2]
Whole depth	.XXX–.XXX[1]
eoretical outside diameter	(X.XXX)[2]
eoretical crown to back	(X.XXX)[2]
Pitch angle	(XX.XX°)[2]
Root angle	(XX.XX°)
Circular thickness	(.XXXX)[3]
Mean measuring addendum	XXX[2]
Mean measuring thickness	.XXX–.XXX[2]
Normal, backlash with mate	.XXX–.XXX[2]
Backlash variation tolerance[1]	.XXXX
Shaft angle	(XX.XX°)
Fillet radius	.XXX–.XXX[1]
Runout tolerance[3]	.XXXX
Pitch tolerance[3]	.XXXX
Index tolerance[1]	.XXXX
Tooth surface texture	XX AA or R,
AGMA quality class	XX
Tooth form	Coniflex© or Revacycle©
Driving member	Pinion or gear
Direction of rotation	CW and/or CCW
MFG summary number	XXXXXX
Part number of mate	XXXXXX
Number of teeth in mate	XX

[1] For metric drawings, specify module in place of diametral pitch.
[2] For metric drawings, the number of decimal places to the right of the decimal point should be reduced by one.
[3] Sections 10.3.3 and 10.4.1 concerning when to specify these values on the gear drawing.

FIGURE 4.24 Standard drawing format for straight bevel gears. MFG, manufacture.



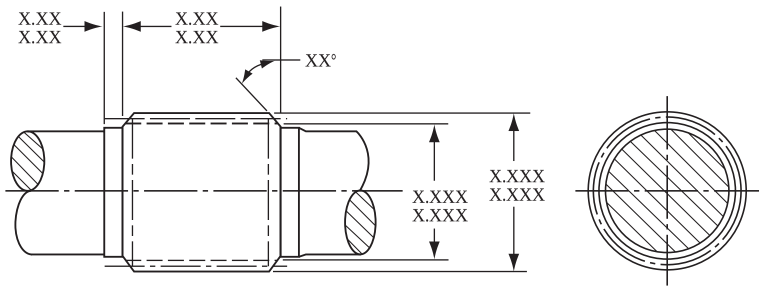
Number of teeth	XX
Diametral pitch[1]	(XX.XXX)
Normal pressure angle	(XX.XX°)
Mean spiral angle	(XX.XX°)
Hand of spiral	Left or Right
Pitch diameter	(X.XXXX)
Addendum	(.XXX)[2]
Working depth	(.XXX)[2]
Whole depth	(.XXX)[2]
eoretical outside diameter	(X.XXX)[1]
eoretical crown to back	(X.XXX)[2]
Pitch angle	(XX.XX°)
Root angle	(X.XXX)[2]
Circular thickness	.XXX[2]
Mean measuring addendum	.XXX–.XXX[2]
Mean measuring thickness	.XXX–.XXX[2]
Normal backlash with mate	.XXX–.XXX[2]
Backlash variation tolerance[2]	X.XXX[2]
Shaft angle	(XX.XX°)
Fillet radius	.XXX–.XXX[2]
Runout tolerance[3]	.XXXX
Pitch tolerance[3]	.XXXX
Index tolerance[1]	.XXXX
Tooth surface texture	XX AA or R
AGMA quality class	XX
Tooth form	Generated or Formate© or HELIXFORM©
Driving member	Pinion or gear
Direction of rotation	CW and/or CCW
MFG summary number	XXXXXX
Part number of mate	XXXXXX
Number of teeth in mate	XX

V and H check in thousandths of an inch (or hundredths of a millimeter) for finished gears[4]

Gear convex	Toe	Heel	Total		Gear concave	Toe	Heel	Total
V	XX	XX	XX		V	XX	XX	XX
H	XX	XX	XX		H	XX	XX	XX

[1] For metric drawings, specify module in place of diametral pitch.
[2] For metric drawings, the number of decimal places to the right of the decimal point should be reduced by one.
[3] See Sections 10.3.3 and 10.4.1 concerning when to specify these values on the gear drawing.
[4] May be optional for matched sets Specify on matched set drawing.

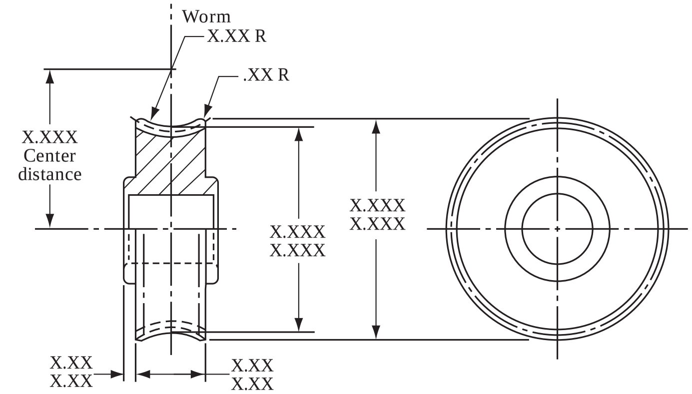
FIGURE 4.25 Standard drawing format for spiral bevel gears.



Worm data

Number of threads (starts)	XX
Pitch diameter	X.XXX
Axial pitch	X.XXXX
Lead angle	XX.XXXXXXX°
Hand of lead	Right or left
Axial pressure angle	XX.XX°
Normal pressure angle	XX.XX°
Addendum	X.XXX
Whole depth (minimum)	X.XXX
Axial tooth thickness at PD	X.XXX/X.XXX
Tooth profile form	Involute or other
Mating part number	XXXXX

For metric drawings, specify module in place of pitch diameter, and the number of decimal places to the right of the decimal point should be reduced by one.



Worm gear data

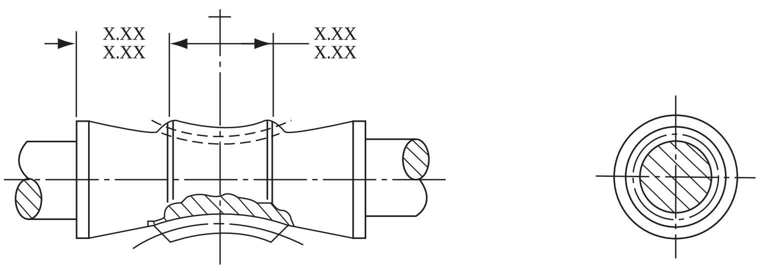
Number of teeth	XX
Pitch diameter	X.XXX
Addendum	X.XXX
Whole depth (minimum)	X.XXX°
Backlash in assembly	X.XXX/X.XXX
Mating part number	XXXXX

Mating worm data (reference)

Number of threads (starts)	XX
Pitch diameter	X.XXX
Axial pitch	X.XXXX
Lead angle	XX.XXXXXXX°
Hand of lead	Right or left
Axial pressure angle	XX.XX°
Normal pressure angle	XX.XX°
Addendum	X.XXX
Axial tooth thickness at PD	X.XXX/X.XXX

For metric drawings, specify module in place of pitch diameter, and the number of decimal places to the right of the decimal point should be reduced by one.

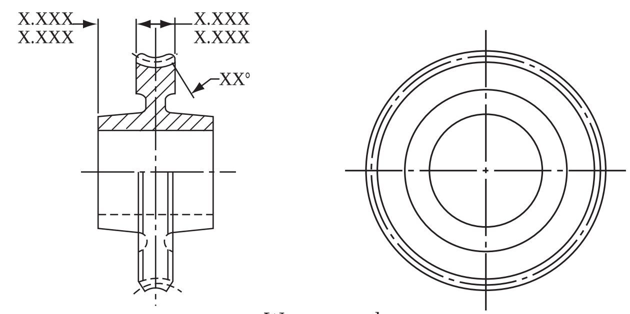
FIGURE 4.26 Standard drawing format for single-enveloping worm gears.



Worm data

Pitch diameter	X.XXX
Circular pitch	X.XXX
Normal pressure angle	XX.XX°
Number of threads (starts)	XX
Lead angle average	XX.XX°
Hand of lead	Right or left
Backlash with mate	X.XXX/X.XXX
Center distance	X.XXX
Ratio	XX to 1
Outside diameter	X.XXX/X.XXX
roat diameter	X.XXX/X.XXX
Working depth	X.XXX
Whole depth (minimum)	X.XXX
Tooth thickness X.XXX normal at X.XXX depth when throat diameter is	X.XXX
Base circle diameter	X.XXX
Mating gear number	XXXXX

For metric drawings, specify module in place of pitch diameter, and the number of places to the right of the decimal point should be reduced by one.



Worm gear data

Pitch diameter	X.XXX
Circular pitch	X.XXX
Normal pressure angle	XX.XX°
Number of teeth	XX
Lead angle average	XX.XX°
Hand of lead	Right or left
Backlash with mate	X.XXX/X.XXX
Center distance	X.XXX
Ratio	XX to 1
Outside diameter	X.XXX/X.XXX
roat diameter	X.XXX/X.XXX
Working depth	X.XXX
Whole depth (minimum)	X.XXX
Tooth thickness X.XXX normal at X.XXX depth when throat diameter is	X.XXX
Base circle diameter	X.XXX
Mating worm number	XXXXX

For metric drawings, specify module in place of pitch diameter, and the number of places to the right of the decimal point should be reduced by one.

FIGURE 4.27 Standard drawing format for double-enveloping worm gears.

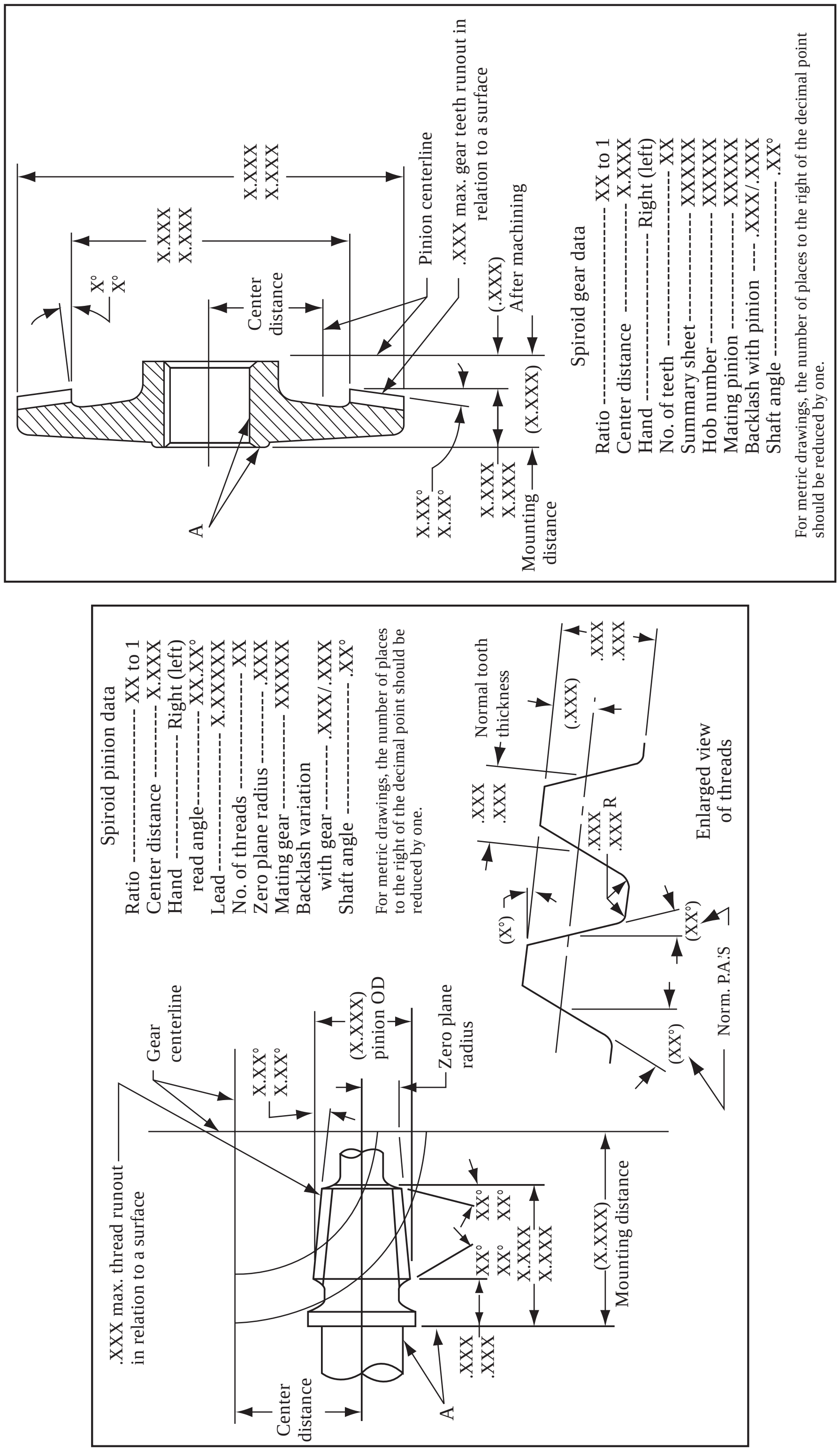


FIGURE 4.28 Standard drawing format for Spiroid gears.

120 μm. Concentricity is ordinarily measured as *twice* the eccentricity (full indicator reading used).

Tooth alignment in very critical applications is held to as little as 15 and 10 μm over 250 to 500 mm. Extraordinary equipment and effort are required to hold a tight control over wide-face-width gears such as the 500 mm–per–helix gears used in ship propulsion. Many commercial gears are held to limits such as 12 to 20 μm for 25 mm face width.

Power gears allow from 50 to 120 μm (0.0020 to 0.0050 in.) tooth-thickness variation. This variation, of course, is *not within one* gear but from one gear to another. All teeth of one gear will be very close to the same size if tooth spacing is close. Some control gears are ground or shave to as little as 5 μm tooth-thickness tolerance. This is very hard to do.

Tooth-surface finish can be held within 0.4 μm (16 μin.) AA by very skilled shaving or grinding. Many precision gears are held to about 0.8 μm AA. Commercial gears are apt to be around 1.2 to 2 μm or more.

There is a general trend to use limits-of-accuracy sheets to define tolerances. This saves writing out all the tolerances on each drawing. All that has to be put on the drawing is the limits column that the gear is to be checked to.

When a company issues its own engineering limits for gear accuracy, several things can be covered:

- Several classes or grades of accuracy can be set. High-accuracy grades can be set for long-life, high-speed gears. Lower-accuracy grades will cover medium- or low-speed gears.
- Things not covered in trade standard on accuracy can be covered. These may be things such as root fillet radius tolerances, allowable mismatch between side of tooth profile and root fillet, or allowable waviness or irregularity in profile and helix.
- An engineering limits document can define checking machine procedures to get tooth-to-tooth spacing, accumulated spacing, and runout readings. Procedures can be defined to handle allowed error in master gears used to check production gears. Stylus size, stylus pressure, and other mechanics of checking can be defined to fit the checking machinery intended to be used in manufacture.

Many major companies that build a goodly volume of important geared machinery have found a well-developed engineering document on gear checking essential.

Trade standard are valuable guidelines for setting company engineering standards for gears. And, of course, it is quite appropriate to work directly to trade standards on gearing without special extra data that may be peculiar to an individual company.

Table 4.22 shows typical gear tolerances for quality numbers 9 to 13. In comparison, Table 4.23 shows Deutsches Institut für Normung (DIN) tolerances from 10 to 3. All AGMA standards and DIN standards show a considerable amount of additional gear tolerance and inspection data. Also, these documents are revised every few years. Those designing and building gears are strongly advised to get the latest issues of these standards and keep abreast of the best current practices in gear inspection.

In many cases, gear quality can be measured by simple functional tests. In some applications, the ability to run quietly in a noise-testing machine is a good check. Some power gearing is checked by running at full load and full speed for some time. If the gear runs smoothly and the surface polishes up without wear, it can be assumed that the gear will do its job in service. In control gearing, the measurement of backlash in different positions may serve to control quality. Since it costs a lot to give gears detailed checks, the designer should choose the least expensive checking system which will ensure proper quality. Then the gear drawing should be toleranced in keeping with the inspection that is planned.

4.3.3 Gear Material and Heat-Treatment Data

Historically, the geometric quality of gears (tooth tolerances) received much attention between 1940 and 1980. As Section 4.3.2 has just described, there are trade standards for gear accuracy and a well-developed gear industry practice. The *metallurgical* quality of gears has not (so far) received equal attention.

The best gear people around the world are now coming to realize that metallurgical quality is just as important as geometric quality. A gear of good accuracy made from material that has a poor hardness pattern or substantial metallurgical flaws will not last as long as it should or will be unable to carry full load without serious distress.

Take the case-hardened gear, for example. If the case is too thin, the tooth strength and wear resistance will be unsatisfactory. If the case is too deep, the tooth is apt to be too brittle and subject to high internal stresses (the case will be tend to break away from the core material). If the carburizing gas is too rich, the outer case will contain too much carbon and is apt to spall under heavy load. If the gas is too lean, the tooth surface will not develop full hardness and wear resistance.

If the case-carburized gear is unprotected just prior to quench, it may "out of gas" carbon and have deficient hardness right at the surface. If the quench is too slow for the alloy content and the size of the part, the core may lack strength and hardness and be unable to support the case under severe load conditions.

The raw steel used to make forgings for carburized gears may be dirty or nonhomogeneous. The best carburizing possible will not make a good gear if the machined forging has serious internal flaws before the final heat-treating operation.

Those making drawings for carburized gears need to cover items such as these:

- Steel composition (maximum and minimum limits for all elements and impurities)
- Cleanness of steel (vacuum-arc-remelt steel is often used where even the best air-melt steel is not clean enough)
- Case depth (control is needed at tip, flank, and root)

TABLE 4.22
Typical Standard Gear Tolerances for Quality Numbers 9 to 13

AGMA Quality Number	Normal Diametral Pitch	Runout Tolerance for Pitch Diameter (in.)										Pitch Tolerance for Pitch Diameter (in.)				
		¾	1½	3	6	12	25	50	100	200	400	¾	1½	3	6	12
9	½					104.7	124.7	147.0	173.4	204.5	241.2					13.4
	1				63.5	74.8	89.1	105.1	124.0	146.2	172.4				10.2	11.5
	2			38.5	45.4	53.5	63.7	75.2	88.6	104.5	123.3			7.7	8.7	9.8
	4		23.3	27.5	32.4	38.3	45.6	53.7	63.4	74.7	88.1		5.8	6.6	7.4	8.4
	8	14.1	16.7	19.7	23.2	27.4	32.6	38.4	45.3	53.4	63.0	4.4	5.0	5.6	6.4	7.2
	12	11.6	13.7	16.2	19.1	22.5	26.8	31.6	37.2	43.9	51.8	4.0	4.6	5.1	5.8	6.6
	20	9.1	10.7	12.6	14.9	17.6	20.9	24.7	29.1	34.3	40.4	3.6	4.1	4.6	5.2	5.9
10	½					74.8	89.0	105.0	123.8	146.1	172.3					9.4
	1				45.3	53.5	63.7	75.1	88.5	104.4	123.2				7.2	8.1
	2			27.5	32.4	38.2	45.5	53.7	63.3	74.7	88.1			5.4	6.1	6.9
	4		16.7	19.6	23.2	27.3	32.5	38.4	45.3	53.4	63.0		4.1	4.6	5.2	5.9
	8	10.1	11.9	14.0	16.6	19.5	23.3	27.4	32.4	38.2	45.0	3.1	3.5	4.0	4.5	5.1
	12	8.3	9.8	11.5	13.6	16.1	19.1	22.6	26.6	31.4	37.0	2.8	3.2	3.6	4.1	4.6
	20	6.5	7.6	9.0	10.6	12.5	14.9	17.6	20.8	24.5	28.9	2.5	2.9	3.2	3.7	4.1
11	½					53.4	63.6	75.0	88.5	104.3	123.0					6.6
	1				32.4	38.2	45.5	53.6	63.2	74.6	88.0				5.0	5.7
	2			19.6	23.1	27.3	32.5	38.3	45.2	53.3	62.9			3.8	4.3	4.9
	4		11.9	14.0	16.6	19.5	23.2	27.4	32.3	38.1	45.0		2.9	3.3	3.7	4.2
	8	7.2	8.5	10.0	11.8	14.0	16.6	19.6	23.1	27.3	32.2	2.2	2.5	2.8	3.2	3.6
	12	5.9	7.0	8.2	9.7	11.5	13.7	16.1	19.0	22.4	26.4	2.0	2.3	2.6	2.9	3.3
	20	4.6	5.5	6.4	7.6	9.0	10.7	12.6	14.8	17.5	20.6	1.8	2.0	2.3	2.6	2.9
12	½					38.1	45.4	53.6	63.2	74.5	87.9					4.7
	1				23.1	27.3	32.5	38.3	45.2	53.3	62.8				3.5	4.0
	2			14.0	16.5	19.5	23.2	27.4	32.3	38.1	44.9			2.7	3.0	3.4
	4		8.5	10.0	11.8	13.9	16.6	19.6	23.1	27.2	32.1		2.0	2.3	2.6	2.9
	8	5.2	6.1	7.2	8.5	10.0	11.9	14.0	16.5	19.5	23.0	1.5	1.7	2.0	2.2	2.5
	12	4.2	5.0	5.9	6.9	8.2	9.8	11.5	13.6	16.0	18.9	1.4	1.6	1.8	2.0	2.3
	20	3.3	3.9	4.6	5.4	6.4	7.6	9.0	10.6	12.5	14.7	1.3	1.4	1.6	1.8	2.0

(*Continued*)

TABLE 4.22 (CONTINUED)
Typical Standard Gear Tolerances for Quality Numbers 9 to 13

AGMA Quality Number	Normal Diametral Pitch	Runout Tolerance for Pitch Diameter (in.)										Pitch Tolerance for Pitch Diameter (in.)				
		¾	1½	3	6	12	25	50	100	200	400	¾	1½	3	6	12
13	½					27.2	32.4	38.3	45.1	53.2	62.8					3.3
	1				16.5	19.5	23.2	27.4	32.3	38.1	44.9				2.5	2.8
	2			10.0	11.8	13.9	16.6	19.6	23.1	27.2	32.1			1.9	2.1	2.4
	4		6.1	7.2	8.4	10.0	11.9	14.0	16.5	19.5	22.9		1.4	1.6	1.8	2.1
	8	3.7	4.3	5.1	6.0	7.1	8.5	10.0	11.8	13.9	16.4	1.1	1.3	1.4	1.6	1.8
	12	3.0	3.6	4.2	5.0	5.9	7.0	8.2	9.7	11.4	13.5	1.0	1.1	1.3	1.4	1.6
	20	2.4	2.8	3.3	3.9	4.6	5.4	6.4	7.6	8.9	10.5	0.9	1.0	1.1	1.3	1.4

Note: Tolerance values are in ten-thousandths of an inch.

					Profile Tolerance for Pitch Diameter (in.)											Lead Tolerance for Face Width (in.)				
25	50	100	200	400	¾	1½	3	6	12	25	50	100	200	400	≤1	2	3	4	5
15.3	17.3	19.5	22.1	24.9					30.4	34.1	37.9	42.2	46.9	52.2	4	7	9	11	13
13.1	14.8	16.7	18.9	21.4				20.2	22.5	25.2	28.1	31.2	34.7	38.6					
11.2	12.7	14.3	16.2	18.3			13.5	15.0	16.7	18.6	20.7	23.1	25.7	28.6					
9.6	10.8	12.2	13.8	15.7		8.9	10.0	11.1	12.3	13.8	15.3	17.1	19.0	21.1					
8.2	9.3	10.5	11.9	13.4	5.9	6.6	7.4	8.2	9.1	10.2	11.4	12.6	14.1	15.6					
7.5	8.5	9.6	10.8	12.2	5.0	5.5	6.2	6.9	7.6	8.6	9.5	10.6	11.8	13.1					
6.7	7.6	8.5	9.7	10.9	4.0	4.4	4.9	5.5	6.1	6.8	7.6	8.5	9.4	10.5					
10.8	12.2	13.7	15.5	17.6					21.7	24.3	27.1	30.1	33.5	37.3	3	5	7	9	10
9.2	10.4	11.8	13.3	15.0				14.5	16.1	18.0	20.0	22.3	24.8	27.6					
7.9	8.9	10.1	11.4	12.9			9.6	10.7	11.9	13.3	14.8	16.5	18.3	20.4					
6.7	7.6	8.6	9.8	11.0		6.4	7.1	7.9	8.8	9.9	11.0	12.2	13.6	15.1					
5.8	6.5	7.4	8.3	9.4	4.2	4.7	5.3	5.9	6.5	7.3	8.1	9.0	10.0	11.2					
5.3	6.0	6.7	7.6	8.6	3.6	4.0	4.4	4.9	5.5	6.1	6.8	7.6	8.4	9.4					
4.7	5.3	6.0	6.8	7.7	2.9	3.2	3.5	3.9	4.4	4.9	5.4	6.1	6.7	7.5					

(*Continued*)

TABLE 4.22 (CONTINUED)
Typical Standard Gear Tolerances for Quality Numbers 9 to 13

					Profile Tolerance for Pitch Diameter (in.)										Lead Tolerance for Face Width (in.)				
25	50	100	200	400	¾	1½	3	6	12	25	50	100	200	400	≤1	2	3	4	5
7.6	8.6	9.7	10.9	12.4					15.5	17.4	19.3	21.5	24.0	26.7	3	4	6	7	8
6.5	7.3	8.3	9.4	10.6				10.3	11.5	12.9	14.3	15.9	17.7	19.7					
5.6	6.3	7.1	8.0	9.1			6.9	7.6	8.5	9.5	10.6	11.8	13.1	14.6					
4.8	5.4	6.1	6.9	7.8		4.6	5.1	5.6	6.3	7.0	7.8	8.7	9.7	10.8					
4.1	4.6	5.2	5.9	6.6	3.0	3.4	3.8	4.2	4.6	5.2	5.8	6.4	7.2	8.0					
3.7	4.2	4.7	5.4	6.1	2.5	2.8	3.1	3.5	3.9	4.4	4.9	5.4	6.0	6.7					
3.3	3.7	4.2	4.8	5.4	2.0	2.3	2.5	2.8	3.1	3.5	3.9	4.3	4.8	5.4					
5.3	6.0	6.8	7.7	8.7					11.1	12.4	13.8	15.4	17.1	19.0	2	3	5	6	7
4.6	5.2	5.8	6.6	7.5				7.4	8.2	9.2	10.2	11.4	12.7	14.1					
3.9	4.4	5.0	5.6	6.4			4.9	5.5	6.1	6.8	7.6	8.4	9.4	10.4					
3.3	3.8	4.3	4.8	5.5		3.3	3.6	4.0	4.5	5.0	5.6	6.2	6.9	7.7					
2.9	3.2	3.7	4.1	4.7	2.2	2.4	2.7	3.0	3.3	3.7	4.1	4.6	5.1	5.7					
2.6	3.0	3.3	3.8	4.3	1.8	2.0	2.2	2.5	2.8	3.1	3.5	3.9	4.3	4.8					
2.3	2.6	3.0	3.4	3.8	1.5	1.6	1.8	2.0	2.2	2.5	2.8	3.1	3.4	3.8					
3.8	4.2	4.8	5.4	6.1					7.9	8.9	9.9	11.0	12.2	13.6	2	3	4	4	5
3.2	3.6	4.1	4.6	5.3				5.3	5.9	6.6	7.3	8.1	9.0	10.1					
2.8	3.1	3.5	4.0	4.5			3.5	3.9	4.3	4.9	5.4	6.0	6.7	7.4					
2.4	2.7	3.0	3.4	3.8		2.3	2.6	2.9	3.2	3.6	4.0	4.4	4.9	5.5					
2.0	2.3	2.6	2.9	3.3	1.5	1.7	1.9	2.1	2.4	2.7	3.0	3.3	3.7	4.1					
1.8	2.1	2.4	2.7	3.0	1.3	1.4	1.6	1.8	2.0	2.2	2.5	2.8	3.1	3.4					
1.6	1.9	2.1	2.4	2.7	1.0	1.2	1.3	1.4	1.6	1.8	2.0	2.2	2.5	2.7					

TABLE 4.23
Typical DIN Gear Tolerances for Grades 10 to 3

			Pitch Diameter																	
			Over 50–125 (Over 2–4.9)						Over 125–280 (Over 4.9–11)						Over 280–560 (Over 11–22)					
					Spacing						Spacing						Spacing			
DIN	Normal Tooth Size		Runout		t-to-t		Cum.		Runout		t-to-t		Cum.		Runout		t-to-t		Cum.	
Grade No.	Module	Diametral Pitch	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.
10	10–16	1.6–2.5	110	43	63	25	140	55	125	49	71	28	180	71	140	55	71	28	200	79
	6–10	2.6–4.4	100	39	56	22	140	55	110	43	56	22	160	63	125	49	63	25	180	71
	3.55–6	4.2–7.15	90	35	50	20	125	49	100	39	50	20	140	55	110	43	56	22	180	71
	2–3.55	7.15–12.7	80	31	40	16	125	49	90	35	45	18	140	55	100	39	45	18	160	63
9	10–16	1.6–2.5	80	31	40	16	90	35	90	35	45	18	110	43	100	39	45	18	125	49
	6–10	2.6–4.4	71	28	36	14	90	35	80	31	36	14	100	39	90	35	40	16	110	43
	3.55–6	4.2–7.15	63	25	32	13	80	31	71	28	32	13	90	35	80	31	36	14	110	43
	2–3.55	7.15–12.7	56	22	25	10	71	28	63	25	28	11	90	35	71	28	28	11	100	39
8	10– 16	1.6–2.5	56	22	32	13	63	25	63	25	32	13	80	31	71	28	36	14	90	35
	6–10	2.6–4.4	50	20	25	10	63	25	56	22	25	10	71	28	63	25	28	11	80	31
	3.55–6	4.2–7.15	45	18	20	8	56	22	50	20	22	9	71	28	56	22	25	10	80	31
	2–3.55	7.15–12.7	40	16	18	7	50	20	45	18	20	8	63	25	50	20	20	8	71	28
7	10–16	1.6–2.5	40	16	22	9	45	18	45	18	22	9	56	22	50	20	25	10	63	25
	6–10	2.6–4.4	36	14	18	7	45	18	40	16	18	7	56	22	45	18	20	8	63	25
	3.55–6	4.2–7.15	32	13	16	6	40	16	36	14	16	6	45	18	40	16	18	7	56	22
	2–3.55	7.15–12.7	28	11	12	5	36	14	32	13	14	5.5	45	18	36	14	16	6	50	20
6	10–16	1.6–2.5	28	11	16	6	32	13	32	13	16	6	40	16	36	14	18	7	45	18
	6–10	2.6–4.4	25	10	12	5	32	13	28	11	14	5.5	36	14	32	13	14	5.5	40	16
	3.55–6	4.2–7.15	22	9	11	4	28	11	25	10	11	4	36	14	28	11	12	5	40	16
	2–3.55	7.15–12.7	20	8	9	3.5	28	11	22	9	10	4	32	13	25	10	10	4	36	14

(Continued)

TABLE 4.23 (CONTINUED)
Typical DIN Gear Tolerances for Grades 10 to 3

			Pitch Diameter											
			Over 50–125 (Over 2–4.9)						Over 125–280 (Over 4.9–11)						Over 280–560 (Over 11–22)					
					Spacing						Spacing						Spacing			
DIN	Normal Tooth Size		Runout		t-to-t		Cum.		Runout		t-to-t		Cum.		Runout		t-to-t		Cum.	
Grade No.	Module	Diametral Pitch	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.	μm	10^{-4} in.
5	10–16	1.6–2.5	20	8	11	4	25	10	22	9	11	4	28	11	25	10	12	5	32	13
	6–10	2.6–4.4	18	7	9	3.5	22	9	20	8	10	4	25	10	22	9	10	4	28	11
	3.55–6	4.2–7.15	16	6	8	3	20	8	18	7	9	3.5	25	10	20	8	9	3.5	28	11
	2–3.55	7.15–12.7	14	5.5	6	2	20	8	16	6	8	3	22	9	18	7	8	3	25	10
4	10–16	1.6–2.5	14	5.5	8	3	18	7	16	6	8	3	20	8	18	7	9	3.5	22	9
	6–10	2.6–4.4	12	5	6	2	16	6	14	5.5	7	3	20	8	16	6	8	3	22	9
	3.55–6	4.2–7.15	11	4	5	2	16	6	12	5	5.5	2	18	7	14	5.5	6	2	20	8
	2–3.55	7.15–12.7	10	4	4.5	2	14	5.5	11	4	5	2	16	6	12	5	5	2	18	7
3	10–16	1.6–2.5	10	4	5.5	2	12	5	11	4	5.5	2	14	5.5	12	5	6	2	16	6
	6–10	2.6–4.4	9	3.5	4.5	2	11	4	10	4	5	2	14	5.5	11	4	5	2	16	6
	3.55–6	4.2–7.15	8	3	4	1.5	10	4	9	3.5	4	1.5	12	5	10	4	4.5	2	14	5.5
	2–3.55	7.15–12.7	7	3	3	1	10	4	8	3	3.5	1	12	5	9	3.5	3.5	1	14	5.5

Source: DIN Standard 3962, Beuth Verlag GmbH, Berlin 30, Germany.

Note: Tolerance values are given in both microns (μm) and ten-thousandths of an inch (10^{-4} in.). For de nition of tolerances, see Figure 10.50; t-to-t stands for tooth-to-tooth spacing tolerance. Cum. stands for cumulative spacing tolerance. Runout values are about 10% lower than concentricity values determined with a master gear.

(*Continued*)

TABLE 4.23 (CONTINUED)
Typical DIN Gear Tolerances for Grades 10 to 3

Pitch Diameter																					
Over 560–1000 (Over 22–39)						Over 1000–1600 (Over 39–63)						Profile Slope (Total)		Helix Slope for Face Width							
Runout		Spacing				Runout		Spacing						20–40 mm (0.8–1.6 in.)		40–100 mm (1.6–3.9 in.)		100–160 mm (3.9–6.3 in.)		160+ mm (6.3+ in.)	
		t-to-t		Cum.				t-to-t		Cum.											
µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{-4} in.	µm	10^{4} in.	µm	10^{-4} in.
160	63	80	31	220	87	180	71	80	31	250	98	90	35	50	20	63	25	80	31	80	31
140	55	71	28	200	79	160	63	71	28	220	87	71	28								
125	49	56	22	200	79	140	55	63	25	220	87	56	22								
110	43	50	20	180	71	125	49	56	22	200	79	45	18								
110	43	50	20	140	55	125	49	50	20	160	63	56	22	32	13	40	16	50	20	50	20
100	39	40	16	125	49	110	43	45	18	140	55	45	18								
90	35	36	14	125	49	100	39	40	16	140	55	36	14								
80	31	32	13	110	43	90	35	36	14	125	49	28	11								
80	31	36	14	100	39	90	35	36	14	110	43	40	16	20	8	25	10	32	13	32	13
71	28	28	11	90	35	80	31	32	13	100	39	32	13								
63	25	25	10	90	35	71	28	28	11	100	39	25	10								
56	22	22	9	80	31	63	25	25	10	90	35	20	8								
56	22	25	10	71	28	63	25	28	11	80	31	28	11	15	6	18	7	22	9	22	9
50	20	20	8	71	28	56	22	22	9	71	28	22	9								
45	18	20	8	63	25	50	20	20	8	71	28	18	7								
40	16	16	6	56	22	45	18	18	7	63	25	14	5.5								
40	16	18	7	50	20	45	18	20	8	56	22	22	9	10	4	12	5	16	6	16	6
36	14	14	5.5	45	18	40	16	16	6	50	20	16	6								
32	13	14	5.5	45	18	36	14	16	6	50	20	12	5								
28	11	11	4	40	16	32	13	12	5	45	18	10	4								
28	11	12	5	36	14	32	13	14	5.5	40	16	16	6	8	3	10	4	12	5	12	5
25	10	11	4	32	13	28	11	11	4	36	14	12	5								
22	9	10	4	32	13	25	10	10	4	36	14	9	3.5								
20	8	8	3	28	11	22	9	9	3.5	32	13	7	3								
20	8	9	3.5	25	10	22	9	10	4	28	11	11	4	6	2	8	3	10	4	10	4
18	7	8	3	25	10	20	8	9	3.5	28	11	8	3								
16	6	7	3	22	9	18	7	8	3	25	10	7	3								
14	5.5	5.5	2	20	8	16	6	7	3	22	9	5	2								
14	5.5	6	2	18	7	16	6	7	3	20	8	8	3	5	2	6	2	8	3	8	3
12	5	5.5	2	18	7	14	5.5	6	2	18	7	6	2								
11	4	5	2	16	6	12	5	5	2	18	7	5	2								
10	4	4.5	2	16	6	11	4	5	2	18	7	4	1.5								

- Case and core hardness
- Test for grinding burns
- Metallurgical structure (proper martensitic quality is needed in both case and core)

Through-hardened gears have their own metallurgical problems. If the quench is not fast enough for the size and alloy content of the gear part, there is apt to be a lack of hardness in the tooth root area. The structure may also be improper.

General information on material and heat treatments for gears will be given in Chapter 6. See in particular Section 6.2.6 for detail information on gear quality.

4.3.4 Enclosed Gear Unit Requirements

The gear designer has much more to do than just design gear teeth and gear parts. The gear unit will have gear casings, shafts, bearings, seals, and a lubrication system. All these things are part of the gear product and therefore become the responsibility of the gear designer.

Those in gear-design work soon find that they have to become highly skilled in bearing selection, bearing design, and the fit of bearings on shafts and into casings.

The load on the bearings from a gear mesh is not a steady load like a weight load, but a somewhat pulsating load because of tooth-error effects and tooth-stiffness effects as pairs of teeth roll through the meshing zone. Wear particles from contacting gear teeth tend to get into the lubricant and go through the bearings. Temperature changes resulting from changing gear loads or changing ambient conditions cause somewhat nonuniform expansions and contractions that disturb the rather critical gear-bearing fits. All these things make the design of gear bearings more critical than the design of bearings for a machinery application usually is.

The lubrication system for a gear unit needs to keep all the tooth surfaces wet with lubricant and reasonably cool. In high-horsepower, high-speed gear units, oil-jet design, oil cooling by heat exchangers, oil-pump design, and oil-filter provision to remove dirt and wear particles all become very critical. Even in slow-speed designs with heavy oil or grease lubrication, where a pumped system is not needed, keeping the teeth wet and providing a lubricant that will work over a wide temperature range (from start-up on a cold day to maximum continuous load on a hot day) may be critical.

It is beyond the scope of this book to go into gear lubrication in detail. Section 12.3.3 of this book covers some of the lubrication problems that lead to gear failures.

The gear casing must support the gears and provide accurate alignment. For face widths up to about 10 mm, it is usually possible to make the gears and casing accurate enough to achieve satisfactory alignment with parts made to normal gear trade tolerances. Above 100 mm this becomes difficult, and at 250 mm face width it is usually impossible to be assured of a satisfactory tooth fit just by making all gears, bearings, and casings to close tolerances.

The solution to this problem is to assemble a gear unit and check contact across the face width. In a critical application, contact checks may be needed at no load, ¼ load, ½ load, ¾ load, and full load. (When allowance for deflections under load is made by changing the helix angle, full contact under light load is not wanted, since the parts are meant to deflect and shift contact to achieve the desired load distribution as torque is increased.)

When contact checks show an unsatisfactory condition under torque, the pinion or gear may be recut to better fit its mate. Sometimes casing bores are scrapped or remachined to correct small errors in the original casing boring.

In some situations, there may be several gear casings and several sets of gears available on the assembly floor. If the first gearset in a casing does not fit quite right, a second or third set may be tried. With selective assembly, it is often possible to fit up most of the units without doing any corrective work on gear parts or casings.

In addition to fitting all right under torque loads, a critical gear unit may need to be tested at full speed to make sure that it runs without undue vibration and that all bearing temperatures and gear-part temperatures are OK. In addition, the run test will show whether or not the gear teeth are tending to score or have some serious local misfit condition. (Generally, after full-speed power testing, the unit is disassembled and all gear parts and bearings are inspected carefully by someone experienced with gears to see if there is any evidence of serious distress—that requires correction—in either the gear teeth or the bearings.)

Those designing gear units have the responsibility of determining what testing may be needed to get satisfactorily under power. (Of course, if the gear unit is small and does not run too fast, the risk of an assembled unit not being OK when all parts are appropriately checked to part drawings may be relatively negligible.)

To summarize this section, the gear unit designer has these major tasks:

- Design of gear parts
- Design of casing structures
- Design of bearings
- Design of lubrication system
- Design of seals, bolts, dowel pins, etc.
- Specification of assembly procedures
- Specification of gear running test procedure
- Definition of acceptable or unacceptable results from run tests and inspection or parts after testing

5 Design Formulas

The material presented so far has shown how to choose a kind of gear and how to make a preliminary estimate of the gear size required. The general nature of the detailed information required on gear drawings has also been discussed.

In this chapter, we shall take up the calculation of the detail dimensions and the checking of the design against gear rating formulas. The designer following this procedure should get almost the right size design on the rst try. If the load rating calculations show that the design is not quite right, it will usually be possible to make only one or two changes, such as a change in pitch or a change in face width, to adjust the capacity within the proper limits. It should not be necessary for the designer to scrap the whole design and start over from scratch after checking the load rating.

5.1 CALCULATION OF GEAR TOOTH DATA

In this part of the chapter, we shall go over all the numbers and the dimensions that will be needed on the drawings of the various kinds of gears. Designers who are not familiar with the limitations on gear size and shape that different materials or different manufacturing equipment may impose should consider the design worked out in this chapter as *tentative* until they have considered the things that will be brought up in the later chapters of the book.

Table 5.1 is a glossary of gear terms used in Chapter 5. Table 5.2 shows the metric and English symbols for the principal terms used in calculating gear data. (Many secondary terms are de ned in gures and tables used for speci c calculations.)

5.1.1 Number of Pinion Teeth

In general, the more teeth a pinion has, the more quietly it will run and the better its resistance to wear will be. On the other hand, a smaller number of pinion teeth will give increased tooth strength, lower cutting costs, and larger tooth dimensions. In spur work, for instance, a seven-tooth pinion of 64 diametral pitch has been found quite useful in some fractional horsepower applications.

Spur pinions used on railroad traction motors usually have 12 to 20 teeth. High-load, high-speed aircraft pinions run from about 18 to 30 teeth. High-speed helical pinions for ship propulsion frequently have 35 to 60 teeth.

In general, low-ratio sets can stand more teeth than high-ratio sets. A 1-to-1 ratio set with 35 teeth will have about the same tooth strength as a 5-to-1 ratio set with 24 pinion teeth.

Table 5.3 may be used as a general guide for numbers of teeth for spur and helical gears.

Table 5.4 is worked out to give an approximate balance between gear surface durability capacity and gear tooth strength capacity. To understand this table, let us consider a high-speed gear drive at 3-to-1 ratio with case-hardened and ground teeth at a surface hardness of 60 HRC (equivalent to about 600 HB or about 725 HV). The table says that a 25/75 tooth ratio ought to be OK. This means that when a large enough center distance and face width are picked based on surface durability calculations, further calculations on tooth strength ought to be OK if the module (or pitch) is chosen to get 25 pinion teeth. Gear people would say that it was a "balanced design between durability and strength."

For our 3-to-1 example with fully hard gears, the balance would be reasonably good in short-life vehicle gears or somewhat in longer-life aircraft gears. For turbine power gears that would run for many more hours and would need a capability of around 10^{10} or more pinion cycles, the durability calculations would considerably reduce the intensity of tooth loading. A balanced design might then be achieved at higher numbers of teeth, like 30 to 38 pinion teeth.

In high-speed turbine gearing, scoring hazards and noise and vibration considerations make it desirable to use as small a tooth size as possible. The designer might get a good durability/strength balance at something like 35 teeth for the 3-to-1 example, but decide to go up to 40 pinion teeth and put a little more face width and/or center distance in to keep the rating within acceptable limits. This design would not have the lightest weight possible, but would compromise enough to avoid scoring and noise hazards reasonably well.

In contrast to the example of high-speed turbine gear just cited, a vehicle designer with hard gears at a 3-to-1 ratio might go to as low as 20/60 teeth rather than 25/75. The much shorter-life, slow-speed vehicle gears can stand quite a little surface damage, but a broken tooth immediately puts the gear drive out of action. The designer of vehicle gears needs extra strength and can get it by using fewer and larger teeth. There is often a scoring hazard in vehicle gears, but this is handled more by using special extreme pressure (EP) lubricants than by using smaller teeth.

To sum it up, Table 5.4 is a general guide to where to start on tooth numbers, but a complete design study may show that it is desirable to use pinion tooth numbers that differ from the table by a modest amount.

5.1.2 Hunting Teeth

The numbers of pinion and gear teeth must be whole numbers. It is generally desirable—particularly with low-hardness parts—to obtain *hunting ratio* between gear and pinion teeth. With a hunting ratio, any tooth on one member will—in time—contact all the teeth on the mating part. This tends to equalize wear and improve spacing accuracy.

TABLE 5.1
Glossary of Gear Nomenclature, Chapter 5

Term	Definition
Approach action	Involute action before the point of contact between meshing gears has reached the pitch point. (A driving pinion has approach action on its dedendum.)
Breakage	A gear tooth or a portion of a tooth breaking off. Usually, the failure is from fatigue; pitting, and scoring, and wear may weaken a tooth so that it breaks even though the stresses on the tooth were low enough to present no danger of tooth breakage when it was new. Sometimes, a relatively new tooth will break as a result of a severe overload or a serious defect in the tooth structure.
Derating	Reducing the power rating of a gearset to compensate for tooth errors and for irregularities in the power transmission into and out of the gear drive. (When derated 2 to 1, a gear unit is rated to transmit only one-half the power that it might have transmitted under perfect conditions.)
Edge radius	A radius of curvature at the end corner or top corner of a gear tooth.
End easement	A tapering relief made at each end of a gear tooth while the middle portion of the tooth length is made to the true helix angle. The relief stops at some specified distance from the tooth end, such as one-eighth to one-sixth of the face width. End easement protects the teeth from misalignment and from stress concentrations peculiar to tooth ends. End easement is used on wide-face-width gears, while crowning may be used on narrow gears. (See Figure 10.50 for comparison.)
Flash temperature	The temperature at which a gear tooth surface is calculated to be hot enough to destroy the oil film and allow instantaneous welding at the contact point.
Full-depth teeth	Gear teeth with a working depth of 2.0 times the normal module (or 2.0 divided by the normal pitch).
Helix correction or helix modification	When pinions are wide in face width for their diameter, there is appreciable bending and twisting of the pinion in mesh. A small change in helix angle (called *helix correction* or *helix modification*) may be used to compensate for the bending and twisting. This correction tends to give better load distribution across the face width.
Hunting ratio	A ratio of numbers of gear and pinion teeth, which ensures that each tooth in the pinion will contact *every* tooth in the gear before it contacts any gear tooth second time. (An example of a hunting ratio is 13 to 48; 12 to 48 is not a hunting ratio.)
Lead	The axial advance of a thread or a helical spiral in 360° (one turn about the shaft axis).
Lead angle	The inclination of a thread at the pitch line from a line at 90° to the shaft axis.
Limit diameter	The diameter at which the outside diameter of the mating gear crosses the line of action. The limit diameter can be thought of as a *theoretical* form diameter. See Figure 5.11.
Pitch point	The point on a gear tooth profile which lies on the pitch circle of that gear. At the moment that the pitch point of a gear contacts its mating gear, the contact occurs at the pitch point of the mating gear, and this common pitch point lies on a line connecting the two gear centers.
Pitting	A fatigue failure of a contacting tooth surface which is characterized by little bits of metal breaking out of the surface.
Profile modification	Changing a part of the involute profile to reduce the load in that area. Appropriate profile modifications help gears to run more quietly and better resisting scoring, pitting, and tooth breaking.
Recess action	Involute action after the point of contact between meshing gears has passed the pitch point. (A driving pinion has recess action on its addendum.)
Runout	A measure of eccentricity relative to the axis of rotation. Runout is measured in a radial direction, and the amount is the difference between the highest and the lowest reading in 360° (one turn). For gear teeth, runout is usually checked by either putting pins between the teeth or using a master gear. Cylindrical surfaces are checked for runout by a measuring probe that reads in a radial direction as the part turned on its specified axis.
Scoring	A failure of a tooth surface in which the asperities tend to weld together and than tear, leaving radial scratch lines. Scoring failures quickly come and are thought of as lubrication failures rather than metal fatigue failures.
Zone of action	The distance, on the line of action between two gears, from the start of contact to the end of contact for each tooth.

To illustrate this point, let us consider a tooth ratio of 21 to 76. The factors of 21 are 3 and 7. The factors of 76 are 2, 2, and 19. This ratio will hunt because *the parts have no common factor*. The gear should not be cut with a double-thread hob. A shaving cutter with 57 teeth would be a poor choice for either part. The cutter has factors of both 3 and 19.

As a general rule, tooth numbers should be selected so that there is no common factor between the number of teeth of a pinion and a gear that mesh together and there should be no common factor between the numbers of the teeth of a gear and of a cutting tool that has a gear-like meshing action with the part being cut.

Technical studies by Ishibashi and Hoyashita (1980) and by Ichimaru et al. (1980) show that, in certain cases, improved gear load-carrying capacity can be obtained with a nonhunting ratio. Tests were made of spur gears having a hunting ratio of 25/27 and an integer ratio of 26/26. Pitch-line speed was about 7 m/s (1400 fpm).

At a low hardness of 185 HV (about 185 HB), Ishibashi and Hoyashita found that the 26/26 ratio would carry a little over 1000 N/mm^2 surface-compressive stress before pitting, while the 25/27 ratio would only carry about 800 N/mm^2 before pitting. The pitting limit was defined as the highest loading the gear pair would carry for 10^7 cycles without pitting. The

TABLE 5.2
Gear Terms, Symbols, and Units, Chapter 5

	Metric		English		
Term	**Symbol**	**Units**	**Symbol**	**Units**	**Reference or Formula**
Number of teeth, pinion	z_1	–	N_P or n	–	Section 5.1
Number of teeth, gear	z_2	–	N_G or N	–	Sections 5.1 and 5.2
Number of threads, worm	z_1	–	N_W	–	Table 5.30
Number of crow teeth	z	–	N_c	–	Equation 5.46 and following
Tooth ratio	u	–	m_G	–	z_2/z_1 (or N_G/N_P)
Addendum, pinion	h_{a1}	mm	a_P	in.	Sections 5.3 and 5.5
Addendum, gear	h_{a2}	mm	a_G	in.	Sections 5.3 and 5.5
Addendum, chordal	$\bar{h}_a$	mm	a_c	in.	Figure 5.7, Equation 5.4
Rise of arc	–	mm	–	in.	Figure 5.7
Dedendum	h_f	mm	b	in.	Equation 5.1
Working depth	h'	mm	h_k	in.	2.0 × module (for full-depth teeth)
Whole depth	h	mm	h_t	in.	Section 5.3, Equation 5.33
Clearance	c	mm	c	in.	Equation 5.33, Section 5.32
Tooth thickness	s	mm	t	in.	Section 5.6
Arc tooth thickness, pinion	s_1	mm	t_P	in.	Sections 5.6 and 5.11
Arc tooth thickness, gear	s_2	mm	t_G	in.	Sections 5.6 and 5.14, Equation 5.23
Tooth thickness, chordal	$\bar{s}$	mm	t_c	in.	Figure 5.7, Equation 5.5
					Section 5.15, Equation 5.24
Backlash, transverse	j	mm	B	in.	Section 5.6, Table 5.22
Backlash, normal	j_n	mm	B_n	in.	Section 5.6
Pitch diameter, pinion	d_{p1}	mm	d	in.	Table 5.12
Pitch diameter, gear	d_{p2}	mm	D	in.	Table 5.12
Pitch diameter, cutter	d_{p0}	mm	d_c	in.	Equation 5.36
Base diameter, pinion	d_{b1}	mm	d_b	in.	Table 5.12
Base diameter, gear	d_{b2}	mm	D_b	in.	Table 5.12
Outside diameter, pinion	d_{a1}	mm	d_o	in.	Pitch diameter + (2 × addendum)
Outside diameter, gear	d_{a2}	mm	D_o	in.	Pitch diameter + (2 × addendum)
Inside diameter, face gear	d_{i2}	mm	D_i	in.	Equation 5.31
Root diameter, pinion or worm	d_{f1}	mm	d_R	in.	Equation 5.39
Root diameter, gear	d_{f2}	mm	D_R	in.	Equation 5.39
Form diameter	d_f'	mm	d_f	in.	Sections 5.9 and 5.11
Limit diameter	d_l	mm	d_l	in.	Figure 5.11, Section 5.8
Excess involute allowance	Δd_l	mm	Δd_l	in.	Allows for runout, Δ center distance, Sections 5.9 and 5.11
Ratio of diameters	ε	–	m	–	Ratio, any diameter to pitch diameter
Center distance	a	mm	C	in.	Sections 4.7 through 4.13
Face width	b	mm	F	in.	Section 4.8
Net face width	b'	mm	F_e	in.	Meshing face width
Module, transverse	m or m_t	mm	–	–	mm of pitch diameter per tooth
Module, normal	m_n	mm	–	–	Module in normal section
Diametral pitch, transverse	–	–	P_d or p_t	in.$^{-1}$	Teeth per in. of pitch diameter
Diametral pitch, normal	–	–	p_n	in.$^{-1}$	Diametral pitch in normal section
Circular pitch	p	mm	p	in.	Pitch-circle arc length per tooth
Circular pitch, transverse	p_t	mm	p_t	in.	
Circular pitch, normal	p_n	mm	p_n	in.	
Base pitch	p_b	mm	p_b	in.	Equation 5.17
Axial pitch	p_x	mm	p_x	in.	p_n/sin of helix angle
Lead (length)	p_z	mm	L	in.	p_x × no. of threads
Pressure angle	α or α_t	deg	ϕ or ϕ_t	deg	Section 5.3
Pressure angle, normal	α_n	deg	ϕ_n	deg	Section 5.12
Pressure angle, axial	α_x	deg	ϕ_x	deg	Equation 5.40
Pressure angle of cutter	α_0	deg	ϕ_c	deg	Section 5.22
Helix angle	β	deg	ψ	deg	Section 5.12

(*Continued*)

TABLE 5.2 (CONTINUED)
Gear Terms, Symbols, and Units, Chapter 5

	Metric		English		
Term	**Symbol**	**Units**	**Symbol**	**Units**	**Reference or Formula**
Lead angle	γ	deg	λ	deg	Complement of helix angle
Shaft angle	Σ	deg	Σ	deg	Angle between gear shaft and pinion shaft
Roll angle	θ_r	deg	ε_r	deg	Involute roll angle
Pitch angle, pinion	δ_1'	deg	γ	deg	Equation 5.27
Pitch angle, gear	δ_2'	deg	Γ	deg	Equation 5.26
Pi	π	–	π	–	Constant, about 3.14159265…
Contact ratio	ε_α	–	m_p	–	Section 5.9
Zone of action	g_α	mm	Z	in.	Figure 5.12
Edge radius, tool	r_{a0}	mm	r_T	in.	Generating tool, Equations 5.1 and 5.22
Radius of curvature, root fillet	ρ_f	mm	ρ_f	in.	Generated root radius, Equations 5.1 and 5.22
Circular thickness factor	k	–	k	–	(Bevel gears), Section 5.14
Cone distance	R	mm	A	in.	(Bevel gears), Table 5.24
Outer cone distance	R_a	mm	A_o	in.	Table 5.24
Mean cone distance	R_m	mm	A_m	in.	$\frac{R_a + R_f}{2}\left(\text{or } \frac{A_o + A_i}{2}\right)$
Inner cone distance	R_f	mm	A_i	in.	$R_a - b$ (or $A_o - F$)

Note: See Table 5.7 for terms, symbols, and units in load rating of gears.

numbers just quoted are based on a part being considered pitted when 1% of the contact area is pitted.

To say it another way, the tests showed that the integer ratio could carry about 25% more hertz stress. Since hertz stress is proportional to the square root of K factor, the integer ratio carried about 60% more K factor. At a higher hardness of 300 HB for one part and over 600 HB for the other part, the hunting ratio carried about 6% more stress or about 12% more K factor.

The Ichimaru et al. work showed somewhat similar results. Of particular interest in this study were the data on how the surface finish wore in and how an elastohydrodynamic (EHD)

TABLE 5.3
General Guide to Selection of Number of Pinion Teeth

No. of Pinion Teeth	Design Considerations
7	Requires at least 25° pressure angle and special design to avoid undercutting. Poor contact ratio. Use only in fine pitches.
10	Smallest practical number with 20° teeth. Takes about 145% long addendum to avoid undercut. Poor wear characteristics.
15	Use where strength is more important than wear. Requires long addendum.
19	No undercutting with 20° standard-addendum design.
25	Good balance between strength and wear for hard steels. Contact kept away from critical base-circle region.
35	Strength may be more critical than wear on hard steels—about even on medium-hard steels.
50	Probably critical on strength on all but low-hardness pinions. Excellent wear resistance. Favored in high-speed work for quietness.

Note: The data given in this table are rather general in nature. They are intended to give the reader a general view of the considerations involved in picking numbers of teeth. For somewhat more specific guidance in designing spur or helical gears, Table 5.4 shows how the choice tends to shift as hardness and ratio are varied.

TABLE 5.4
Guide to Selecting Number of Pinion Teeth z_1 (or N_P) for Good Durability and Adequate Strength

	Long-Life, High-Speed Gears Having Brinell Hardness				Vehicle Gears, Short Life at Maximum Torque Having Brinell Hardness			
Ratio, u (m_G)	**200**	**300**	**400**	**600**	**200**	**300**	**400**	**600**
1	80	50	39	35	50	37	29	26
1.5	67	45	32	30	45	30	24	22
2	60	42	28	27	42	27	21	20
3	53	37	25	25	37	24	18	18
4	49	34	24	24	34	23	17	17
5	47	32	23	23	32	22	17	17
7	45	32	22	22	31	21	16	16
10	43	30	21	21	30	20	16	16

Note: Typical high-speed applications that fit this table are turbine-driven helicopter gears. For turbine-driven industrial gears, more pinion teeth can be used because of lower allowable surface loading. (About 25% more is typical.) Typical vehicle gears are spur and helical gears used in final wheel drives. (Considerably fewer pinion teeth are used in hypoid and spiral bevel final drives.)

TABLE 5.5
Hunting Tooth Considerations

Approximate Hardness (HB)		Lubrication Regime		
Pinion	**Gear**	**I**	**II**	**III**
200	200	1	2	3
300	300	4	5	6
600	300	7	8	9
600	600	10	11	12

Situation

1, 2, 4, 5	Substantial gain in load capacity with integer ratio. After serious pitting, failure may be hastened as a result of growth of spacing errors and rough running of integer ratio.
3, 6	Hunting ratio probably best if parts kept in service after some teeth pitted more than 1%. No data available to prove gain in load carrying for integer ratio parts to be taken out of service when a worst tooth pits more than 5%.
7, 8	Substantial gain in load capacity with integer ratio. After serious pitting, failure may be quick as a result of spacing error effects.
9	Hunting ratio probably best. Worst error spots on lower-hardness gear will have fewer cycles and more chance to be worked into a t by the hard pinion teeth.
10, 11	Possibly a small gain in load capacity with integer ratio. Hinting ratio probably best. Wear-in effects will be quite small.
12	Worst tooth pair (with highest stresses resulting) will contact much less frequently.

oil lm was quickly formed with the integer ratio but slowly with the hunting ratio.

The explanation seems to be that a single pair of teeth meshing with each other wears down asperities (or coin a t between the surfaces) quite quickly. In the hunting ratio, a tooth on one part has to get worn and wear *all* the teeth on the other part into a t with itself. Thus, a full t cannot occur until all pinion teeth are worn alike, all gear teeth are worn alike, and the pinion-worn pro le is a very close surface t to the gear-worn pro le.

The decision on whether or not to use hunting ratios becomes much more complex. Several pros and cons need to be considered. Table 5.5 shows the range of considerations.

5.1.3 Spur Gear Tooth Proportions

The proportions of addendum equal to $1.000m_n$ and whole depth equal to $2.250m_n$ have been used for many years. This design allows only a very small root llet radius of curvature, and it is a hard design to work with when designing shaper-cutters, preshaved hobs, or shaving cutters.

Table 5.6 shows the more popular spur gear tooth proportions. Note that teeth nished by shaving or grinding require more whole depth than gears nished by cutting only (hobbing, shaping, or milling).

When maximum load capacity is desired, the teeth need as large a root llet radius as possible. This in turn leads to a need for more whole depth and a relatively large corner radius on the hob, the grinding wheel, or other tool used to make the root area of the tooth.

The most common pressure angle in use now for spur gears is 20°. Case-hardened aircraft or vehicle gears very often use a 25° pressure angle. The 25° form makes the teeth thicker at the base, and this improves the bending strength. In addition, the 25° teeth have larger radii of curvature at the pitch line, and this enables more load to be carried before the contact stress exceeds allowable limits.

The 25° teeth generally run with more noise, as a result of the lower contact ratio. The tips of the teeth are thinner, and this may lead to fracturing of the tip when the case-hardened teeth are cased too deep or the metallurgical structure is faulty.

When made just right, and when the application can stand somewhat rougher running, the 25° tooth form will carry about 20% more torque (or power in kilowatts) than a 20° tooth form.

TABLE 5.6
Spur Gear Proportions

Use	Pressure Angle, α (ϕ)	Working Depth, h' (h_k)	Whole Depth, h (h_t)	Edge Radius of Generating Rack, r_{a0} (r_T)
General purpose	20	2.000	2.250	0.300
Extra depth for shaving	20	2.000	2.350	0.350
Aircraft, full llet, high fatigue strength	20	2.000	2.400	0.380
Alternative high-strength design	25	2.000	2.250	0.300
Fine-pitch gears	20	2.000	2.200 + (constant)[a]	–

Note: For metric design, the depth values and the tool radius are in millimeters and for 1 module. (For other modules, multiply by module.) For English design, the depth values and the tool radius are in inches and for 1 diametral pitch. (For other diametral pitches, divide by pitch.)

[a] The extra amount of whole depth is 0.05 mm or 0.002 in. This constant is added to the calculated whole depth for small teeth 1.27 module or smaller. (In the English system, ne pitch is 20 pitch and larger pitch numbers.)

A good compromise design that is easier to successfully make than 25° and has more capacity than 20° is a 22.5° tooth form. This design has about 11% more capacity than a 20° design.

Figure 5.1 shows comparisons of tooth forms when they are cut with a rack (tooth number = ∞).

In the gear trade, there was considerable use of stub teeth in early days. A typical stub tooth would have a working depth of around 1.60 (1.60 × m). When teeth are quite inaccurate, load sharing between two pairs of teeth in the meshing zone may not exist. This means that one pair has to carry full load right out to the very tip of the pinion or the gear. In such a situation, it is obvious that a shorter and stubbier tooth can take more load without breakage than taller, full-depth tooth.

The use of stub teeth has declined to where they are seldom used. In general, gear teeth are accurately designed and built enough to share load when they should. Test data and field experience show that full-depth teeth, made accurately, will carry more load and/or last longer than stub teeth.

This leads to the consideration of *extra-depth teeth*. (If accurate, full-depth teeth are better than stub teeth; why not go on to even deeper teeth than full-depth teeth?)

Some gears are in production and doing very well with a whole depth as great as 2.30. At this much depth, a 25° pressure angle is impossible and 22.5° is difficult. Pressure angles in the 17.5° to 20° range are reasonably practical with 2.30 working depth.

The extra-depth teeth have problems with thin tips and smaller root fillet radii. Also, the effect on radii of curvature of the lower pressure angle somewhat offsets the gain from the higher contact ratio of the extra depth. In addition, very high–speed extra-depth teeth are more sensitive to scoring troubles.

In some situations, a less than 20° pressure angle may be used to get smoother and quieter running. If the pinion has more than 30 teeth, a fairly good load capacity can be obtained with a pressure angle as low as 14.5°. Also, in instrument and control gears, center-distance changes have less effect on backlash at 14.5° than at 20°.

Stub teeth, extra-depth teeth, and low-pressure-angle teeth all have possible advantages for limited applications. For general use, though, the full-depth 20° design presents the best compromise.

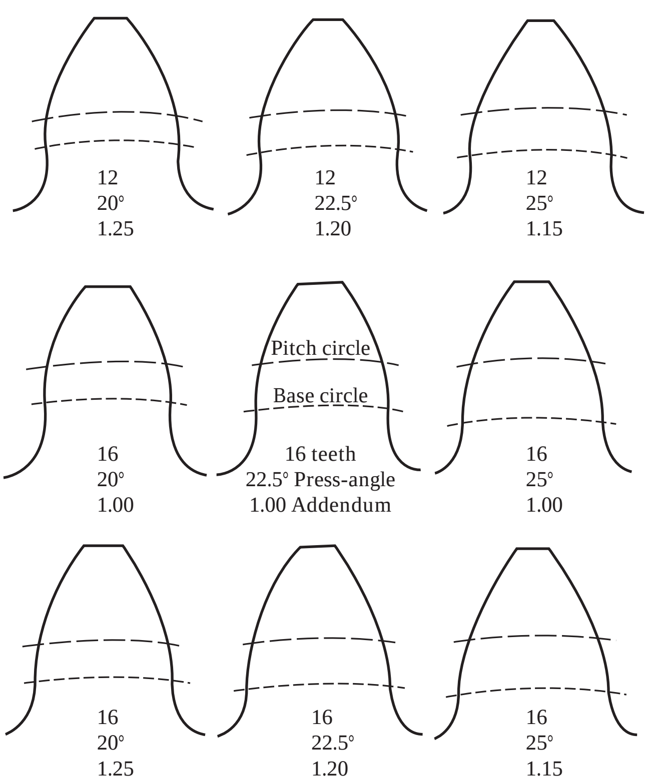


FIGURE 5.1 Comparison of tooth forms.

5.1.4 Root Fillet Radii of Curvature

To get adequate tooth strength, it is desirable to cut or grind gear teeth to get either a full-radius root fillet or one that is almost full radius.

Figure 5.2 shows an example of a 25-tooth pinion with a 22.5° pressure angle, as it would be cut with a hob having a 0.36 edge radius (0.36 × module). The pinion has a whole depth of 2.35 and a long addendum of 1.2. Note the shape of the trochoidal root fillet and the path of the hot tip. Note also that the generated fillet of this type has its smallest radius of curvature near the root diameter.

For design purposes, a 22.5° spur tooth would not be made exactly as shown in Figure 5.2. A slightly smaller whole depth of 2.3 and a preshaved or preground edge radius of 0.35 would be used. These changes facilitate the overall tool design and make it possible to use normal tolerances. The fillet obtained is still very good and has a relatively low stress concentration factor.

The formula for calculating the minimum radius of curvature produced by hobbing or generating grinding is as follows:

$$\rho_f = r_{a0} + \frac{(h_f - r_{a0})^2}{0.5\,d_{p1} + (h_f - r_{a0})} \quad \text{(metric)} \tag{5.1}$$

$$\rho_f = r_T + \frac{(b - r_T)^2}{0.5\,d + (b - r_T)} \quad \text{(English).} \tag{5.2}$$

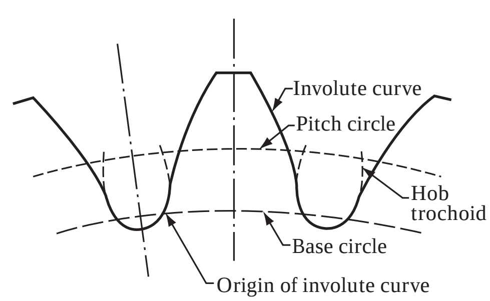


FIGURE 5.2 25-tooth pinion with 22.5° pressure angle.

TABLE 5.7
Amount of Long Addendum to Avoid Undercut with Hobbed Teeth

Pressure Angle	At 10 Teeth	At 12 Teeth	At 15 Teeth
20°	1.50	1.40	1.27
22.5°	1.37	1.22	1.00 OK
25°	1.20	1.03	1.00 OK

Note: For metric design the addendum of the pinion is the long addendum given earlier multiplied by the module. The gear addendum = 2.00m − pinion addendum. For English design, divide the number given by the diametral pitch for the pinion addendum. For gear addendum, divide 2.00 by diametral pitch and subtract pinion addendum.

The term ρ_f is the minimum radius of curvature in the generated trochoidal fillet. The edge radius of the hob or the grinding tool is r_{a0} (r_T). The dedendum of the part is h_f (b). For a list of gear symbols and units used in this chapter, see Tables 5.2 and 5.7.

For general-purpose work, the value of the minimum root fillet radius on the drawing should not be more than about 70% of the calculated minimum. This allows the toolmaker a reasonable margin of error in not obtaining the design tool radius.

When gears are shaped, form ground, or milled, it is possible to get about the same minimum radius as would be obtained in hobbing. The shape of the fillet will be slightly different, though. From a practical standpoint, the designer can design to the method shown earlier and be assured that the gears can be produced by any of the cutting methods if the tools are properly designed.

Figure 5.3 shows curve sheet based on Equations 5.1 and 5.2 for teeth with 20° and 25° pressure angles. Values for 22.5° are about midway between the values for 20° and 25°.

Figure 5.3 shows an example of special curve sheet paper used to plot gear data. The abscissa distance is proportional to the *reciprocal* of the tooth number. Note that the distance from 12 to ∞ is just *twice* the distance the distance from 24 to ∞.

The effect* of tooth numbers on many variables having to do with involute curvature or trochoidal curvature is somewhat proportional to the reciprocal of the tooth number. Hence, this graph paper results in relatively straight-line curves for many tooth variables.

5.1.5 Long-Addendum Pinions

A *long-addendum pinion* is one that has an addendum which is longer than that of the mating gear. Since the working depth is 2.000m for full-depth teeth, long-addendum pinions—in a full-depth design—have an addendum *greater* than 1.000m.

Long-addendum pinions mate with short-addendum gears. The standard design practice is to make the gear addendum short by the same amount that the pinion addendum is made long.

The most compelling reason to use long-addendum pinions is to avoid undercut.[†] An undercut design is bad for several reasons. From a load-carrying standpoint, the undercut pinion is low in strength and easily wears at the point at which the undercut ends. In addition, there is the danger of *interference*. If the cutting tool does not make a big enough undercut, the mating gear may try to enlarge the undercut. Since the mating gear is not a cutting tool, it does a poor job of removing metal. It tends to bind in the undercut. This creates an interference condition which is very detrimental.

The more teeth on the cutting tool, the greater the undercut in the gear. Since a hob corresponds to a rack (a *rack* is a section of a gear with an infinite number of teeth), the hob produces the most undercut. As a general rule, it can be stated that *hobbed* teeth will not have interference due to lack of undercut.

Besides avoiding undercut with low numbers of teeth, the long-addendum design, together with a proportional increase in tooth thickness at the pitch line (see Equations 5.5 and 5.6), makes the pinion tooth stronger and the gear tooth weaker. Thus, a long addendum can be used to balance strength. Figure 5.4 shows the remarkable difference that a 35% long addendum makes on a 12-tooth pinion.

The highest sliding velocity and the greatest compressive stress occur at the bottom of the pinion tooth. This condition can be helped by using a long-addendum design to get the start of the active involute farther away from the pinion base circle.

Figure 5.5 shows a curve sheet giving amounts of addendum for 20° standard teeth. The dashed curve on the left-hand side shows the bare minimum which is necessary to avoid undercut. Neither gear nor pinion should have lesser addendum than that given by this curve.

The solid lines in Figure 5.5 show the addendum for both pinion and gear. Suppose a 20-tooth pinion was meshing with a 50-tooth gear. The curve would be read for 20 meshing with 50, and then again for 50 meshing with 20. The answers would be 1.16m for the pinion and 0.84m for the gear.

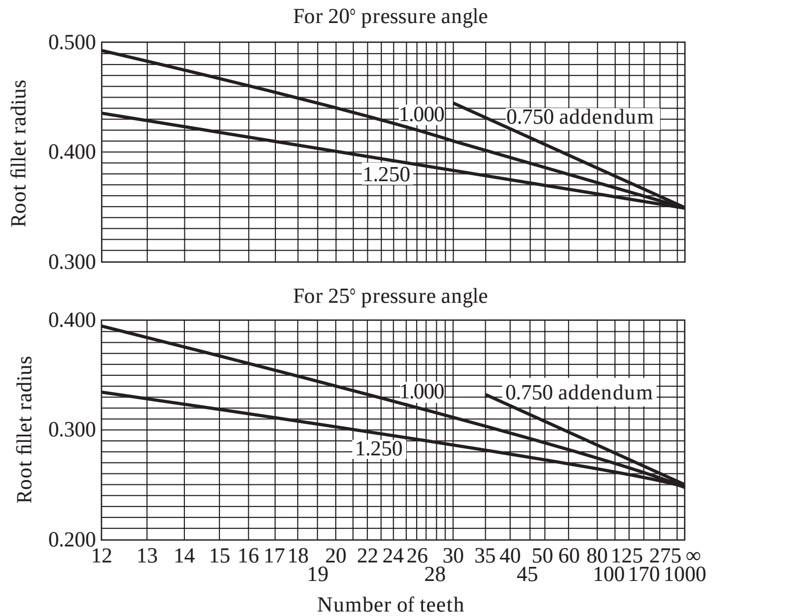
If the dashed curve is read for pinion addendum, there is no place to read the gear addendum. In this case, the designer should *subtract from the gear addendum the same amount that is added to the pinion addendum*. The sum of the pinion addendum and the gear addendum must always be equal to the working depth.

The curve sheet of Figure 5.5 was drawn to give an approximate balance between the strength of the pinion and the strength of the gear. It also took care of the problem of pinion undercut.

The problem of undercut is not as critical with 22.5° teeth, and it almost ceases to exist with 25° teeth. The general

* The reader should note from this graph paper that a change of one tooth at the 12-tooth point means a change of more than 20 teeth at the 80-tooth point!

† See Figure 13.1 for an example of a 16-tooth standard-addendum pinion that is undercut.



Note: e above curves are for 1 module if metric or for 1 diametral pitch if English.
At 20° the tip radius of the generating rack is 0.350 mm (metric) or 0.350 in. (English).
At 25° the tip radius is 0.250.
Read values in millimeters or in inches.

FIGURE 5.3 Minimum root llet radius graphs.

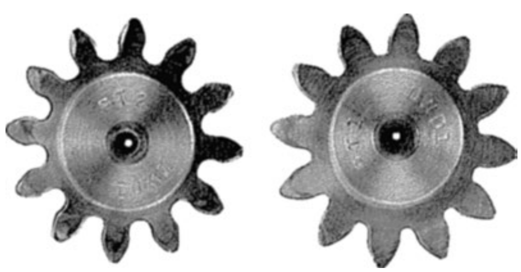


FIGURE 5.4 How long addendum eliminates undercut. Pinion on the left, with an addendum of 0.100 in., is badly undercut, but pinion on the right, with 0.135 in. addendum, has *no* undercut. Both of these 10-pitch, 12-tooth pinions were cut with the same hob and shaving cutter to the same whole depth.

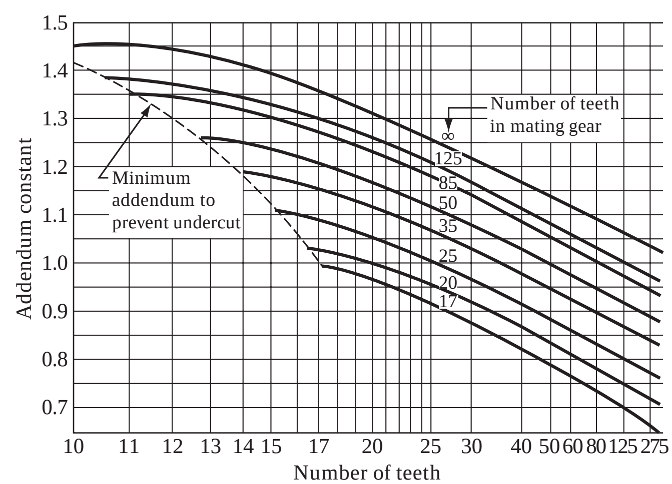


FIGURE 5.5 Recommended addendum constant for pinions and gears of 20° pressure angle. The addendum constant is for 1 module or for 1 diametral pitch.

formula to nd the minimum number of teeth needed to avoid undercut in spur gears is as follows:

$$z = \frac{2h_{fx}}{m \sin^2 \alpha} \quad \text{(metric)}, \tag{5.3}$$

where $h_{fx} = h_f - r_{a0}(1 - \sin \alpha)$,

$$N = \frac{2XP_t}{\sin^2 \phi} \quad \text{(English)}, \tag{5.4}$$

where $X = b - r_T(1 - \sin \phi)$.

When the pinion has a standard addendum of 1.000*m* and the spur teeth are designed to the proportions shown in Table 5.6, the minimum number of teeth that can be used without undercut is as follows:

20° pressure angle	19 teeth
22.5° pressure angle	15 teeth
25° pressure angle	13 teeth

TABLE 5.8
Long and Short Addendum for Speed-Reducing Spur Gears

	α = 20° (ϕ = 20°)		α = 25° (ϕ = 25°)	
Tooth ratio, z_1/z_2 (N_P/N_G)	Pinion Addendum, h_{a1} (a_P)	Gear Addendum, h_{a2} (a_G)	Pinion Addendum, h_{a1} (a_P)	Gear Addendum, h_{a2} (a_G)
12/35	1.24	0.76	1.16	0.84
12/50	1.32	0.68	1.22	0.78
12/75	1.38	0.62	1.25[a]	0.75
12/125	1.44	0.56	1.25[a]	0.75
12/∞	1.48	0.52	1.25[a]	0.75
16/35	1.115	0.85	1.10	0.90
16/50	1.22	0.78	1.15	0.85
16/75	1.27	0.73	1.18	0.82
16/125	1.32	0.68	1.21	0.79
16/∞	1.36	0.64	1.24	0.76
24/35	1.06	0.94	1.04	0.96
24/50	1.11	0.89	1.08	0.92
24/75	1.15	0.85	1.10	0.90
24/125	1.20	0.80	1.13	0.87
24/∞	1.24	0.76	1.16	0.84

Note: Data in millimeters for 1 module teeth. (For 1 pitch teeth, read addendum data in inches.)

[a] It is not practical to use more addenda here because the tip of the tooth is as thin as good practice will allow.

When using a long addendum to avoid undercut, the amounts of long addendum shown in Table 5.8 are needed.

When the pinion has enough teeth to avoid undercut, the question of how much long addendum to use becomes more complex. A modest amount of long addendum will tend to balance strength between the pinion and the gear. Some bene t also results from the standpoint of surface durability and scoring.

If a large amount of long addendum is used, the pinion is apt to be substantially stronger than the gear, and the tendency to score at the pinion tip becomes much greater.

Some gear designers have used and advocated a 200% pinion addendum and a 0% gear addendum. This kind of design has all recess action* when the pinion drives and no approach action. If gears are somewhat accurate, they will run more smoothly and quietly as the arc of recess is increased and the arc of approach decreased.

There are special cases in which the design of a gearset justi es a very long pinion addendum and a very short gear addendum. For most designs of power gears, only a modest amount of long addendum is recommended. This somewhat balances pinion and gear strengths and gives favorable results from the standpoint of surface durability and scoring. In service, the dedendum of the pinion and the gear may extensively pit (see Chapter 12). If the gear dedendum is not unduly large, the gear can survive heavy pitting fairly well.

Table 5.9 shows some guideline values for amounts of long addendum. These relatively low amounts of long addendum will give good results in rating calculations.

TABLE 5.9
Suggested Backlash When Assembled

Metric		English	
Module	Backlash (mm)	Diametral Pitch	Backlash (in.)
25	0.63–1.02	1	0.025–0.040
18	0.46–0.69	1½	0.018–0.027
12	0.35–0.51	2	0.014–0.020
10	0.28–0.41	2½	0.011–0.016
8	0.23–0.36	3	0.009–0.014
6	0.18–0.28	4	0.007–0.011
5	0.15–0.23	5	0.006–0.009
4	0.13–0.20	6	0.005–0.008
3	0.10–0.15	8 and 9	0.004–0.006
2	0.08–0.13	10–13	0.003–0.005
1	0.05–0.10	14–32	0.002–0.004

From a practical standpoint, these gears will run backward (gear driving rather than pinion) relatively well. Most gear drives have a power reversal under *coast* conditions, so the ability to run in reverse is usually a design requirement. In addition, the gear dedendum is not large enough to be a major problem when the gear member happens to have premature pitting.

In speed-increasing drives, the gear is the driver. A long pinion addendum (in this case) makes the drive run more roughly and noisily. As a general rule, only enough long addendum is used to avoid undercut. Generally, it is possible—and also advisable—to design the speed-increasing drive with enough pinion teeth to be out of the undercut problem area. This

* See Figure B.15 for de nition of arc of approach and arc of recess.

means that speed-increasing gear drives will usually have the same addendum for the pinion and the gear.

5.1.6 Tooth Thickness

The tooth thickness of standard-addendum pinions and gears can be obtained by subtracting the minimum backlash from the circular pitch and dividing by two. When long- and short-addendum proportions are used, it is necessary to adjust the tooth thicknesses. Usually, this adjustment is made with a formula that will permit a shaper-cutter to cut long and short proportions just as well as it cuts standard proportions (see Appendix B.12). However, if the tooth thicknesses are adjusted for hobbing, it is usually possible to shape them with standard shaping tools and have only a small error in whole depth. With this thought in mind, the following formula for hobbed teeth is a good one to use to design all kinds of gears*:

$$s = \frac{p_t - j}{2} + \Delta h_a(2\tan\alpha) \quad \text{(metric)}, \tag{5.5}$$

where $p_t = \pi m$, backlash is denoted as j, and $\Delta h_a = h_a - 0.5\,h'$, or

$$t = \frac{p_t - B}{2} + \Delta a(2\tan\phi) \quad \text{(English)}, \tag{5.6}$$

where $p_t = \pi/P_d$, backlash is denoted as B, and $\Delta a = a - 0.5\,h_k$.

The tooth thickness values obtained from Equations 5.5 and 5.6 are arc tooth thickness. See Section 5.1.7 for chordal tooth thickness values.

5.1.6.1 Backlash

The design amount of backlash j (B) should be chosen to meet the requirements of the application. In power gearing, it is usually a good policy to use relatively generous backlash. The amount should be at least enough to let the gears freely turn when they are mounted on the shortest center distance and are subject to the worst condition of temperature and tooth error. In control gearing, the design can usually have but very little backlash. Some binding of the teeth as a result of eccentricity and tooth error may be preferable to designing so that there is appreciable lost motion due to backlash.

Table 5.10 shows amounts of backlash as a guide for gear design with spur, helical, bevel, and spiral bevel gears. When the gears are not running fast and there are relatively small temperature changes, the table is useful for determining the minimum amount of backlash to use—assuming that extra backlash represents lost motion and may be somewhat undesirable from a performance standpoint. (A reversing gear drive, for instance, should not have any more backlash than necessary.)

TABLE 5.10
Tolerances on Tooth Thickness

Method of Cutting	Degree of Care: Very Best		Close Work		Easy to Meet	
	mm	in.	mm	in.	mm	in.
Grinding	0.005	0.0002	0.013	0.0005	0.05	0.002
Shaving	0.005	0.0002	0.013	0.0005	0.05	0.002
Hobbing	0.013	0.0005	0.050	0.0020	0.10	0.004
Shaping	0.018	0.0007	0.050	0.0020	0.10	0.004

In epicyclic gears, it may be necessary to use a low backlash to keep oating suns or ring gears from oating too far and causing high vibration at light load. Table 5.10 is again useful as a guide.

Although Table 5.10 is based on general practice, it should be kept in mind that the values are suggested values and must be used with discretion. Where gears have a large center distance or where the casing material has a different expansion from that of the gears, some of the values shown are risky. For instance, 2.5-module high-speed gears used for either marine or aircraft applications will need a minimum backlash of the order of 0.15 or 0.20 mm (0.006 or 0.008 in.) to satisfactorily operate under all temperature conditions.

5.1.6.2 Tolerances and Tooth Thickness

The variation in backlash will considerably depend on the tooth-thickness tolerance. In hobbing or shaping teeth, a tolerance of 0.05 mm (0.002 in.) on thickness represents close work. Adding tolerances for the pinion and the gear, a backlash variation of 0.10 mm (0.004 in.) is obtained from cutting alone! Obviously, it would not be a good policy to design gear teeth so that the backlash tolerances made the tooth-cutting costs prohibitive. Table 5.11 shows a study of tooth-thickness tolerances for different cutting methods for teeth in the 2- to 5-module ranges. These values should be considered when setting ranges of backlash and tooth-thickness tolerances.

5.1.7 Chordal Dimensions

The tooth thickness of a spur or helical gear is often measured with calipers. This instrument is set for a depth corresponding to the *chordal addendum*, and it measures a width corresponding to the *chordal tooth thickness*. Figure 5.6 shows the tooth thickness being measured. Figure 5.7 de nes the chordal dimensions.

The chordal addendum is obtained by adding the *rise of arc* to the addendum. The equations generally used are the following:

$$\bar{h}_a = h_a + \frac{s^2\cos^2\beta}{4d_p} \quad \text{(metric)}, \tag{5.7}$$

* Unfortunately, s is used for both tooth thickness and stress in the metric system. The reader needs to be aware that some symbols can have double meanings.

For involute helical teeth, use normal module, normal circular pitch, and normal pressure angle. The tooth thickness obtained is in the normal section. (For English calculation, use similar normal-section values and the result will be normal tooth thickness.) The normal circular pitch is $p_n = p_t\,cos\psi$ for English.

TABLE 5.11
Calculation Sheet for the Meshing-Zone Dimensions for a Pair of Involute Gear Teeth

Basic Data		
1. Pressure angle	20	
2. Cosine, pressure angle	0.939693	
3. Tangent, pressure angle	0.363970	
4. Circular pitch	9.424778	
5. Base pitch, (2) × (4)	8.856394	
Calculations	**Pinion**	**Gear**
6. Number of teeth	25	96
7. Addendum	3.54	2.46
8. Pitch diameter, (4) × (6) ÷ π	75.00	288.00
9. Outside diameter, (8) + 2 × (7)	82.08	292.92
10. Base diameter, (8) × (2)	70.47695	270.63148
11. (9) ÷ (10)	1.164636	1.082357
12. (11) × (11)	1.356377	1.171498
13. $[(12) - 1.000]^{0.5}$	0.596973	0.414123
14. (13) − (3)	0.233003	0.050153
15. (6) ÷ 6.283185	3.978873	15.27887
16. Contact ratio, addendum, (14) × (15)	0.92709	0.76628
17. Contact ratio, pair, $(16)_1 + (16)_2$	1.69337	
18. Line of action, addendum, (16) × (5)	8.21067	6.78647
19. Zone of action, $(18)_1 + (18)_2$	14.99715	
20. 0.5 × (10) × (3)	12.82575	49.25090
21. Line of action, total, (18) + (20)	21.03642	56.03737
22. (21) − (19)	6.03927	41.04022
23. $[2 \times (22)]^2$	145.89113	6737.1986
24. Limit diameter, $[(23) + (10)^2]^{0.5}$	71.50449	282.80487
25. (10) ÷ (9)	0.858637	0.923909
26. Pressure angle, outside diameter, $\cos^{-1}(25)$	30.836078	22.495559
27. Roll angle, outside diameter, 57.29578 × tan(26)	34.204057	23.727487
28. (10) ÷ (24)	0.985630	0.956955
29. Pressure angle, limit diameter, $\cos^{-1}(28)$	9.725034	16.872143
30. Roll angle, limit diameter, 57.29578 × tan(29)	9.819514	17.377365

Note: See Figure 5.12 for identi cation of items in this table. This table can be calculated in the metric system by using millimeters for all dimensions. Columns 1 and 2 show a metric example of 25-tooth pinion meshing with 96-tooth gear. This table can be calculated in the English system by using inches for all dimensions. All angles are in degrees.

$$a_c = a + \frac{t^2 \cos^2 \psi}{4d} \quad \text{(English)}. \tag{5.8}$$

Equations 5.7 and 5.8 work for both helical and spur gears. In helical gears, the tooth thickness used should be the normal tooth thickness.

Figure 5.8 shows the rise of arc for a range of diameters and tooth thicknesses. For helical gears, curve readings are multiplied by the cosine squared of the helix angle.

Chordal tooth thicknesses are obtained by subtracting a small amount from the arc tooth thickness. This arc-to-chord correction is so small that it can be neglected in many gear designs. The calculation should always be made, though, for coarse-pitch pinions, master gears, and gear tooth gauges. The approximate equations which are usually used are the following:

$$\bar{s} = s - \frac{s^3 \cos^4 \beta}{6 d_p^2} \quad \text{(metric)}, \tag{5.9}$$

$$t_c = t - \frac{t^3 \cos^4 \psi}{6 d^2} \quad \text{(English)}. \tag{5.10}$$

For helical gears, the normal tooth thickness should be used in Equations 5.9 and 5.10. The arc-to-chord correction s_c is plotted in Figure 5.9 for a range of diameters and tooth thicknesses. Multiplying constants are shown to take care of helical gears.

FIGURE 5.6 Measuring tooth thickness with gear tooth calipers. (Courtesy of Vi-Star Gear Co., Inc., Paramount, California.)

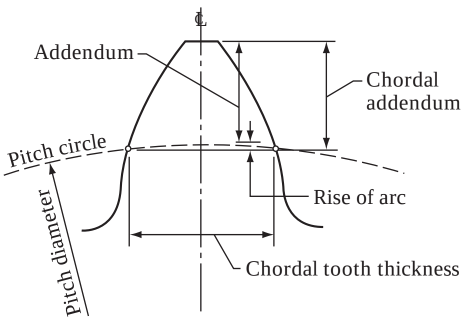


FIGURE 5.7 Chordal dimensions of a gear or a pinion tooth.

5.1.8 Degrees Roll and Limit Diameter

The checking machines used to measure the involute pro- le of spur and helical gears usually record error in involute against degrees of roll. This makes it necessary in many cases to determine the degrees roll at the start of active pro le and at the end of active pro le. The start of active pro le—so far as checking goes—is at the form diameter. The end of active pro le is at the outside diameter.

Figure 4.10 shows the basic involute relations in the English system. Figure 5.10 is a similar diagram showing basic involute relations in the metric system. In Figure 3.10, the roll angle to the pitch circle (or to the pitch point) is the angle designated θ_r. The roll angle to any point on the involute curve is θ_{r1}.

The solution of the roll angle problem lies on the fact that the length of the line unwrapped from the base cylinder is equal to the arc length from the origin of the involute to the tangency point of the involute action line to the base cylinder. An angle in radians is equal to the arc length of the angle divided by the radius. These relations plus other obvious triangular relations led to the basic relations of the involute shown in Figure 5.10 (or in Figure 4.10).

The degrees roll at the outside diameter may be calculated by solving the following equations:

Metric	English	
$\varepsilon_a = \dfrac{d_a}{d_p}$	$m_o = \dfrac{d_o}{d}$	(5.11)
$\cos\alpha_a = \dfrac{\cos\alpha_t}{\varepsilon_a}$	$\cos\phi_o = \dfrac{\cos\phi_t}{m_o}$	(5.12)
$\theta_{ra} = \dfrac{180° \tan\alpha_a}{\pi}$	$\varepsilon_{ro} = \dfrac{180° \tan\phi_o}{\pi}$	(5.13)

The angle θ_{ra} (ε_{ro}) is the degrees roll at the outside diameter. The roll angle at the limit diameter is calculated in a similar manner.

The *limit diameter* is the diameter at which the outside diameter of the mating gear crosses the line of action. See Figure 5.11. Roll angles for limit diameter or form diameter can be obtained by the following:

Metric	English	
$\varepsilon_l = \dfrac{d_l}{d_p}$	$m_l = \dfrac{d_l}{d}$	(5.14)
$\cos\alpha_l = \dfrac{\cos\alpha_t}{\varepsilon_l}$	$\cos\phi_l = \dfrac{\cos\phi_t}{m_l}$	(5.15)
$\theta_{rl} = \dfrac{180° \tan\alpha_l}{\pi}$	$\varepsilon_{rl} = \dfrac{180° \tan\phi_l}{\pi}$	(5.16)
$\varepsilon_f = \dfrac{d_f'}{d_p}$	$m_f = \dfrac{d_f}{d}$	(5.17)
$\cos\alpha_f = \dfrac{\cos\alpha_t}{\varepsilon_f}$	$\cos\phi_f = \dfrac{\cos\phi_t}{m_f}$	(5.18)
$\theta_{rf} = \dfrac{180° \tan\alpha_f}{\pi}$	$\varepsilon_{rf} = \dfrac{180° \tan\phi_f}{\pi}$	(5.19)

The calculations in Table 5.12 give the step-by-step procedure for calculating all the dimensions in the zone of action. The following items are of particular interest:

- Line of action for the addendum (18)
- Zone of action (19)
- Line of action, total (21)
- Limit diameter (24)
- Roll angle, outside diameter (27)
- Roll angle, limit diameter (30)

This calculation sheet is illustrated by an example worked out for 25 pinion teeth meshing with 96 gear teeth with a pressure angle of 20°. The teeth are of 3 module and spur. The constant 57.29578 is 180° divided by pi (3.14159265).

Figure 5.12 illustrates the dimensions that are calculated in Table 5.12. This table works for either metric or English dimensions.

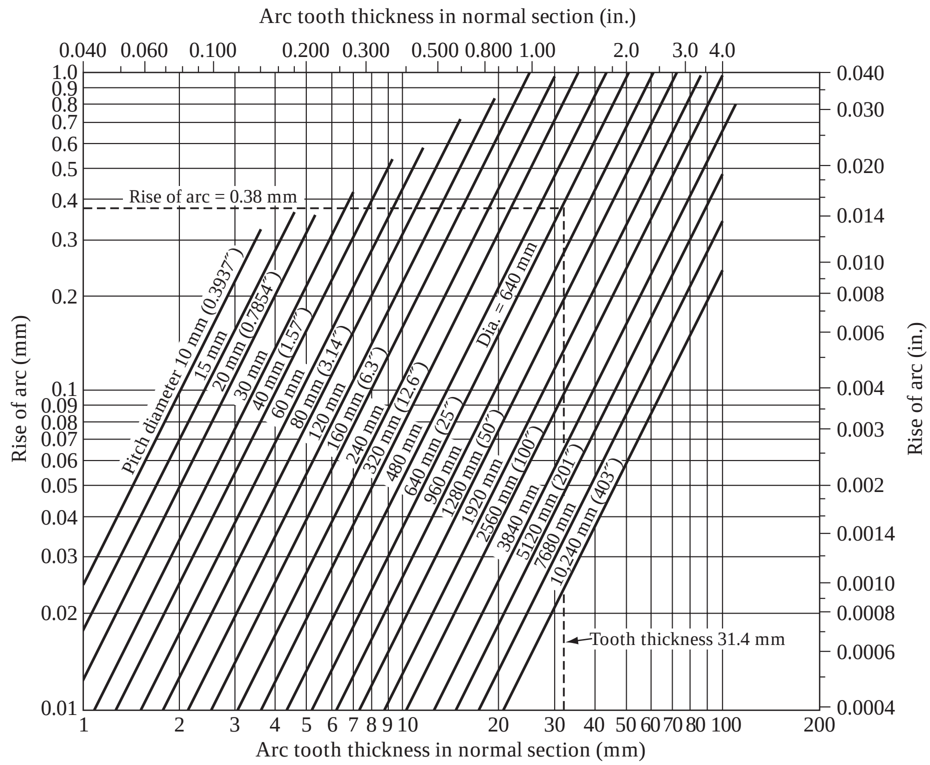


FIGURE 5.8 Rise of arc graph. Read the rise of arc as shown, and then correct for helix angle.

5.1.9 Form Diameter and Contact Ratio

Form diameter is an important design parameter of a gear and of a pinion. Torque capacity of a gear pair, as well as other output capabilities of it, depends on contact ratio. The actual value of contact ratio is tightly connected with the actual value of the form diameter of a gear and of a mating pinion.

5.1.9.1 Form Diameter

The *form diameter* of a gear is the diameter which represents the design limit of involute action. A theoretical limit diameter d_l is first obtained, and then the practical value is obtained by subtracting an allowance Δd_l for outside-diameter runout and center-distance tolerance.

Limit diameter may be graphically calculated or by solving a series of equations. Equations 5.20 through 5.22 are the necessary equations, and Table 5.12 shows a sample calculation.

After d_l is obtained, the form diameter is

$$d_f' = d_l - \Delta d_l. \tag{5.20}$$

The allowance Δd_l should be large enough to accommodate for the effect of various tooth errors in extending the contact below the theoretical point plus the effect of tooth bending under load in extending the contact deeper. For general applications, the value of $0.05m_t$ has worked out quite well. If the pinion has a small number of teeth, so that the limit diameter is close to the base circle, a form diameter that is below the base circle should never be specified. With a standard-addendum pinion of 20° pressure angle, an allowance for excess involute as large as $0.05m_t$ should be used only when there are 25 or more teeth in the pinion.

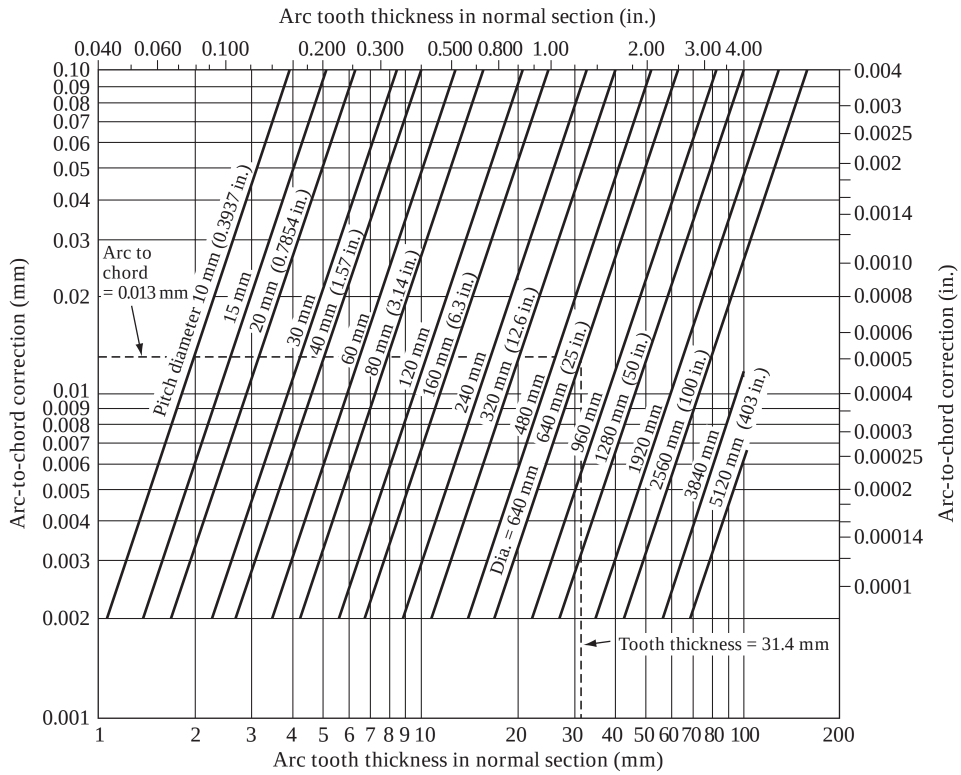
The value of $0.05m_t$ (transverse module is denoted by m_t) has an English equivalent of 0.050 in./P_d. Some sample values are the following:

Tooth Size	Δd_l
1 module	0.05 mm
5 module	0.25 mm
20 module	1.00 mm
20 pitch	0.0025 in.
5 pitch	0.01000 in.
1 pitch	0.0500 in.

For calculation purposes, it is handy to make the excess involute a function of the circular pitch (instead of a function of the module or an inverse function of the diametral pitch). On this basis, it works out that

$$\Delta d_l = 0.016 \times \text{circular pitch} = 0.016 \times \pi m_t \quad \text{(metric)}, \tag{5.21}$$

$$\Delta d_l = 0.016 \times \text{circular pitch} = 0.016 \times \frac{\pi}{P_t} \quad \text{(English)}. \tag{5.22}$$

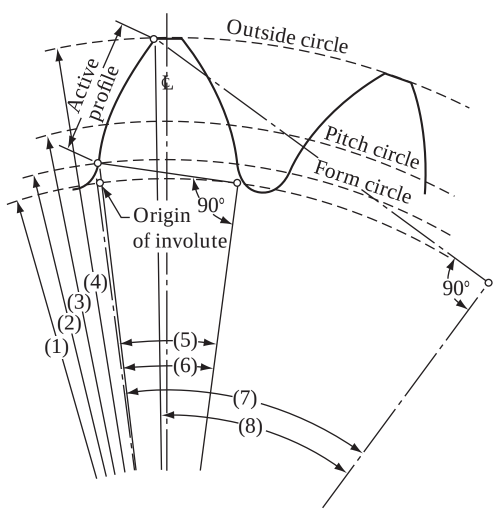


Helix correction:

Multiply curve reading by the following constants to correct for helix.

Helix	45°	40°	35°	30°	25°	20°	15°	10°	5°	0°
Constant	0.25	0.34	0.45	0.56	0.67	0.78	0.87	0.94	0.98	1.00

FIGURE 5.9 Arc-to-chord correction graph. Read the arc-to-chord correction as shown, and then correct for helix angle.



Item	Name	Symbol Metric	Symbol English
(1)	Base diameter	d_b	d_b
(2)	Form diameter	d_f'	d_f
(3)	Pitch diameter	d_p	d
(4)	Outside diameter	d_a	d_o
(5)	Pressure angle, form diameter	α_f	φ_f
(6)	Roll angle, form diameter	θ_{rf}	ε_{rf}
(7)	Roll angle, outside diameter	θ_{ra}	ε_{ro}
(8)	Pressure angle, outside diameter	α_a	φ_o

FIGURE 5.10 Roll angles which de ne the active pro le of the gear.

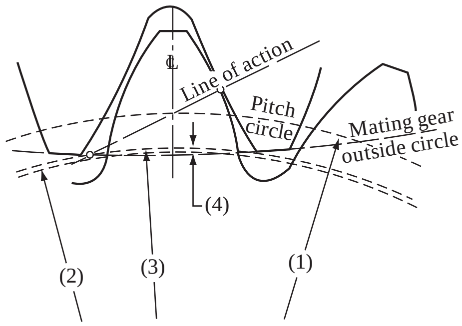
5.1.9.2 Contact Ratio

The contact ratio of a pair of spur or helical gears represents the length of the zone of action divided by the base pitch. The basic equation is the following:

$$\text{Contact ratio} = \frac{\text{zone of action}}{\text{base pitch}}, \tag{5.23}$$

$$\varepsilon_\alpha = \frac{g_\alpha}{p_b} \quad \text{(metric)}, \tag{5.24}$$

$$m_p = \frac{Z}{p_b} \quad \text{(English)}, \tag{5.25}$$



Item	Name	Symbol: Metric	Symbol: English
(1)	Pitch diameter	d_p	d
(2)	Form diameter	d'_f	d_f
(3)	Limit diameter	d_l	d_l
(4)	Extra involute	$\frac{d_l - d'_f}{2}$	$\frac{d_l - d_f}{2}$

FIGURE 5.11 Limit diameter and form diameter. The allowance for extra involute should be large enough to allow for center-distance variations and variations in outside diameter of the mating gear.

TABLE 5.12
Spur Gear Dimensions

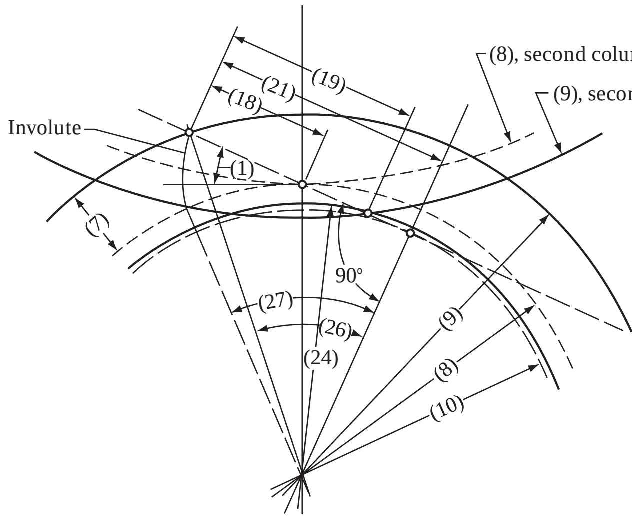
Item	Reference	Metric: Pinion	Metric: Gear	English: Pinion	English: Gear
1. Center distance	Sections 4.2.2 through 4.2.4	181.50		7.145655	
2. Circular pitch	Equation 1.2	9.42478		0.37105	
3. Pressure angle	Section 5.3	20		20	
4. Working depth	Table 5.6	6.00		0.2362	
5. Module, transverse	Equation 1.3	3		–	
6. Diametral pitch, transverse	Equation 1.3	–		8.66667	
		Pinion	**Gear**	**Pinion**	**Gear**
7. Number of teeth	Sections 5.1 and 5.2	25	96	25	96
8. Pitch diameter	Equation 1.4	75.00	288.00	2.9527	11.3386
9. Addendum	Section 5.5	3.54	2.46	0.1394	0.0968
10. Whole depth	Table 5.6	7.05	7.05	0.2775	0.2775
11. Outside diameter	(8) + [2 × (9)]	82.08	292.92	3.2315	11.5322
12. Root diameter	(11) − [2 × (10)]	67.98	278.82	2.6765	10.9772
13. Arc tooth thickness	Section 5.6	4.991	4.204	0.1965	0.1655
14. Chordal tooth thickness	Section 5.7	5.026	4.243	0.1979	0.1671
15. Chordal addendum	Section 5.7	3.62	2.47	0.1397	0.0969
16. Pin size	Table B.1	5.334	5.334	0.210	0.210
17. Diameter over pins	Table B.2	83.1905	304.2113	3.2752	11.9768
18. Face width	Section 4.8	37.0	35.0	1.45	1.38
19. Form diameter	Table B.3	71.35	282.65	2.8090	11.1279
20. Roll angle, outside diameter	Table B.3	34.20	23.73	34.20	23.73
21. Roll angle, form diameter	Table B.3	9.07	12.27	9.07	17.27
22. Base circle, diameter	Table 5.1	70.477	270.631	2.77468	10.6548
23. Minimum root radius	Section 5.4	0.84	0.79	0.033	0.031
24. Accuracy	Section 7.4.1	See Note 4			

Note: 1. See Chapter 6 for gear materials and speci cations.
2. See Chapter 13, Problem 10.4, for an example of spread center design. (This table is for standard center design.)
3. Metric dimensions are in millimeters; English dimensions are in inches. Angles are in degrees (and decimals of degree).
4. After the designer has done the load rating and studied Section 10.4.1, accuracy needs may be satis ed by an ISO or an AGMA quality level, or the designer may need to write a special recipe for accuracy limits.

where g_α (Z) designates zone of action (item 19 of Table 5.12); $p_b = (\pi m_t)\cos\alpha_t$ for metric; and $p_b = \pi/P_t \cos\phi_t$ for English.

Figure 5.13 shows plotted contact ratio values for involute spur gear teeth of 20°, 22.5°, and 25° pressure angles. Note that the 25% long-addendum teeth have somewhat lower contact ratios than standard-addendum teeth. Also note that the 25° pressure-angle teeth have a much lower contact ratio than 20° teeth.

The term *contact ratio* should be thought of as “average number of teeth in contact.” As spur gears roll through the mesh, there are either two pairs of teeth in mesh or one pair.



Item	Name	Symbol Metric	Symbol English
(1)	Pressure angle	α	φ
(7)	Addendum	h_a	a
(8)	Pitch diameter	d_p	d
(9)	Outside diameter	d_a	d_o
(10)	Base diameter	d_b	d_b
(18)	Line of action, addendum	g_a	Z_r
(19)	Zone of action	g_α	Z
(21)	Line of action, total	g_t	Z_t
(24)	Limit diameter	d_l	d_l
(26)	Pressure angle at outside diameter	α_a	φ_o
(27)	Roll angle to outside diameter	θ_{ra}	ε_{ro}

FIGURE 5.12 Identi cation of dimensions used in calculations for the meshing of a pair of involute gear teeth. The numbers correspond to the line number in Table 5.12.

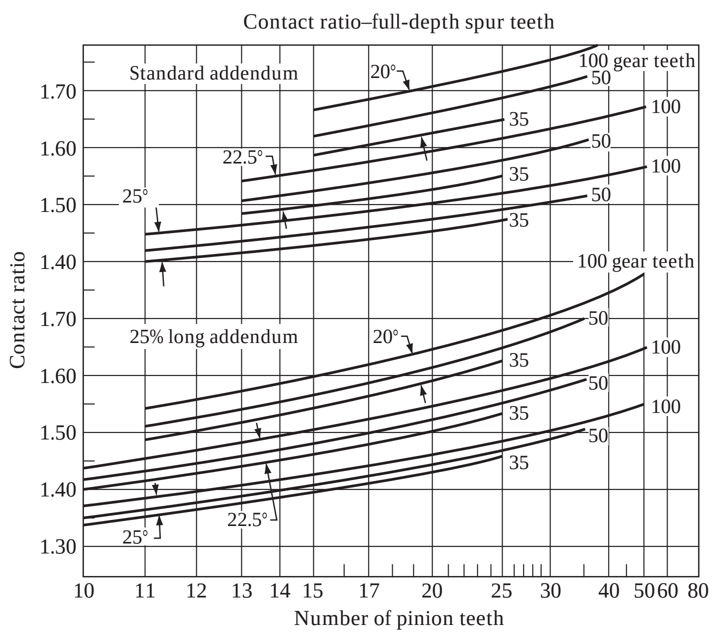


FIGURE 5.13 Transverse contact ratios for standard-addendum and 25% long-addendum spur gear teeth at different pressure angles.

(A 20° pair with a 1.7 contact ratio never really has 1.7 pair of teeth in contact!)

5.1.10 Spur Gear Dimension Sheet

In calculating gear dimensions, it is helpful to use a *dimension sheet* which lists all the items to be calculated. This ensures that all necessary values are calculated, and it is a convenient sheet to le in a design book as a record of different jobs. Table 5.13 shows a recommended dimension sheet for spur gears. The sheet shows how to obtain design values for each item. To illustrate the use of the sheet, numerical values are given for a sample problem.

Several of the dimensions on Table 5.13 require tolerances. In power gearing of small size, a center-distance tolerance of 0.05 mm (0.002 in.) is often used. This affects the backlash about 0.038 mm (0.0015 in.). On control gearing, a closer value would be needed.

The face width of the pinion is usually made a little wider than that of the gear. It is easier to provide the extra width for error in axial positioning on the pinion than on the gear.

The tolerance on outside diameter is based on whatever tolerance is reasonable to hold in lathe or cylindrical grinding operations. This dimension does not need to be held too close unless accuracy is needed in order to set calipers for close tooth-thickness checking.

TABLE 5.13
Addendum Proportions and Limiting Numbers of Teeth for Internal Spur Gears of 20° Pressure Angle

		Min. No. of Gear Teeth		Gear Addendum			
No. of Pinion Teeth z_1 (N_P)	Pinion Addendum h_{a1} (a_P)	Axial Assembly, z_2 (N_G)	Radial Assembly, z_2 (N_G)	u = min* (m_G = min), h_{a2} (a_G)	u = 2 (m_G = min), h_{a2} (a_G)	u = 4 (m_G = min), h_{a2} (a_G)	u = 8 (m_G = min), h_{a2} (a_G)
12	1.350	19	26	0.472	0.510	0.582	0.616
	1.510	19	26	0.390	0.412	0.451	0.471
13	1.290	20	27	0.507	0.556	0.635	0.673
	1.470	20	27	0.419	0.445	0.488	0.509
14	1.230	21	28	0.543	0.601	0.688	0.729
	1.430	21	28	0.447	0.479	0.525	0.548
15	1.180	22	30	0.574	0.642	0.733	0.777
	1.400	22	30	0.470	0.506	0.554	0.577
16	1.120	23	32	0.608	0.688	0.786	0.834
	1.380	23	32	0.487	0.526	0.574	0.597
17	1.060	24	33	0.642	0.734	0.839	0.890
	1.360	24	33	0.505	0.546	0.594	0.617
18	1.000	25	34	0.676	0.779	0.892	0.947
	1.350	25	34	0.516	0.558	0.605	0.628
19	1.000	27	35	0.702	0.792	0.808	0.950
	1.330	27	35	0.539	0.578	0.625	0.648
20	1.000	28	36	0.713	0.802	0.903	0.952
	1.320	28	36	0.550	0.590	0.636	0.658
22	1.000	30	39	0.733	0.821	0.912	0.957
	1.290	30	39	0.577	0.621	0.666	0.688
24	1.000	32	41	0.750	0.836	0.920	0.960
	1.270	32	41	0.599	0.644	0.687	0.709
26	1.000	34	43	0.766	0.849	0.926	0.963
	1.250	34	43	0.620	0.666	0.709	0.729
30	1.000	38	47	0.792	0.870	0.936	0.968
	1.220	38	47	0.654	0.702	0.741	0.761
40	1.000	48	57	0.836	0.903	0.952	0.976
	1.170	48	57	0.718	0.764	0.797	0.814

* Minimum $u = z_2/z_1$ is minimum for axial assembly.

The tolerance on root diameter should allow for variation in tool thickness as well as for the worker's error in setting the gear-cutting machine.

Tooth-thickness tolerances should be governed by backlash requirements of the application as well as by Table 5.11 considerations.

The calculation of diameter over pins may be done by the procedure given in Appendix B, Table B.2.

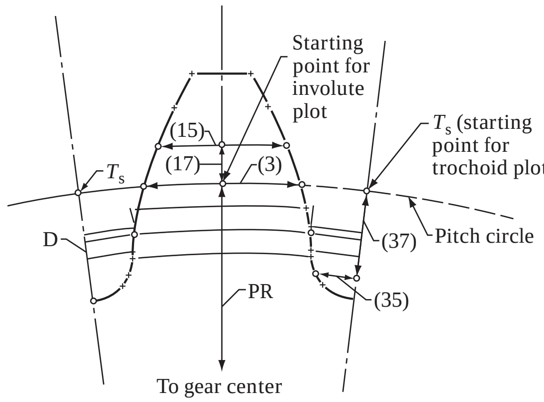
Tolerances on tooth accuracy and data on material and heat treatment will be needed (see Sections 6.2.4 and 10.4.1). Usually, these data are not shown on the speci cation sheet unless they can be given by referring to a standard speci cation. Frequently, they are shown only on the part drawing.

In many cases, it is desired to make layouts of pinion or gear teeth to see what they will look like. This can be done by calculating the involute part of the tooth pro le as a rst step, then calculating the root llet trochoid as a second step. The calculations in Tables B.4 and B.5 in Appendix B show how to do this. The 25-tooth pinion of Table 5.13 is as an example. Figure 5.14 shows how this tooth was plotted.

The 25-tooth pinion in Figure 5.14 looks good. The tip of the tooth is generously wide (should be no trouble in carburizing). The root llet is generous in curvature, and the base of the tooth is wide. This tooth can be expected to have good beam strength.

5.1.11 Internal Gear Dimension Sheet

The dimensions for internal gears may be calculated in a manner quite similar to that of just described for spur gears. However, internal gears are subject to two kinds of trouble that do not affect external gears. If the internal gearset is designed for full working depth, the gear may contact the pinion at a lower point on the tooth ank than the cutting tool generated on the involute pro le. When internal gearsets have too little difference between the number of teeth in



Item	Label	Reference
Pitch radius	PR	Half of item 1 in Table 9.4
Arc tooth thickness at pitch line	(3)	Item 3 in Table 9.4
Dedendum	D	Item 3 in Table 9.5
Circular pitch	T_s to T_s arc	Item 4 in Table 9.5
Distance from pitch circle	(17)	Item 17 in Table 9.4
Arc tooth thickness, any radius	(15)	Item 15 in Table 9.4
x coordinate for trochoid plot	(35)	Item 35 in Table 9.5
y coordinate for trochoid plot	(37)	Item 37 in Table 9.5

FIGURE 5.14 Oversize layout of 25-tooth pinion.

the pinion and the number of teeth in the gear, there may be *interference between the tips* of the teeth. The interference is most apt to occur as the pinion is radially moved into mesh with the gear.

It is possible to get around the radial-interference dif culty by assembling the set by an axial movement of the pinion. The difference between numbers of teeth can be less when this method of assembly is used, but there will still be interference if the difference is too small. Figures 5.15 and 5.16 show radial interference and axial assembly of 19/26 teeth gearset having 20° pressure angle.

The calculations (or layout procedures) that avoid these troubles with internal gears are rather complicated. Instead of presenting the method here, Tables 5.14 and 5.15 will be given to show what dimensions can be safely used with internal gears.

In Tables 5.14 and 5.15, the addendum of the internal gear has been shortened so that the internal gear will contact the pinion no deeper than it would be contacted by a rack tooth with an addendum equal to $2.0m - h_{a1}$ (or $2.0/P_d - a_P$). This makes it possible to manufacture the pinion of an internal gearset in the same way that a pinion would be manufactured to go with an external gear with a large number of teeth. The minimum numbers of teeth for the different methods of assembly are calculated so that the tips clear each other by at least $0.02m$ mm (or $0.020/P_d$ in.).

Tables 5.14 and 5.15 are worked out so as to get the most that is possible out of internal gears. Many designers have used internal gear data which did not use quite so much working depth and quite so close tooth numbers as are shown in this Table 5.14. If the design is not critical and center-distance accuracy is not close, it may be desirable not to use quite so low a ratio as would be permitted by the table.

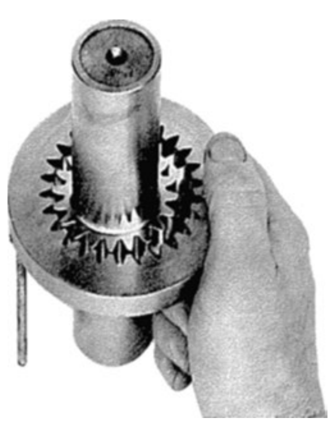


FIGURE 5.15 Internal gear of 26 teeth *will not radially* assemble with a 19-tooth pinion. Both parts cut to Table 5.14 proportions.

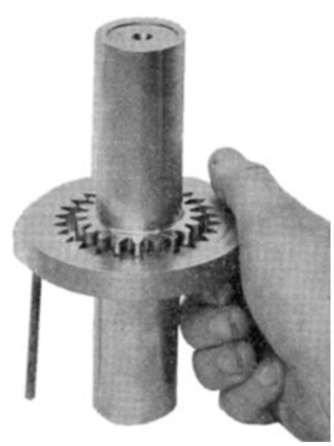


FIGURE 5.16 Internal gear of 26 teeth *will axially* assemble with a 19-tooth pinion. Both parts cut to Table 5.14 proportions.

TABLE 5.14
Addendum Proportions and Limiting Numbers of Teeth for Internal Spur Gears of 25° Pressure Angle

		Min. No. of Gear Teeth		Gear Addendum			
No. of Pinion Teeth z_1 (N_P)	Pinion Addendum h_{a1} (a_P)	Axial Assembly, z_2 (N_G)	Radial Assembly, z_2 (N_G)	u = min* (m_G = min), h_{a2} (a_G)	u = 2 (m_G = min), h_{a2} (a_G)	u = 4 (m_G = min), h_{a2} (a_G)	u = 8 (m_G = min), h_{a2} (a_G)
12	1.000	17	20	0.699	0.793	0.900	0.934
	1.220	17	21	0.601	0.656	0.720	0.740
13	1.000	18	22	0.718	0.810	0.908	0.940
	1.200	18	22	0.622	0.680	0.742	0.761
14	1.000	19	24	0.734	0.824	0.915	0.944
	1.180	19	24	0.644	0.703	0.763	0.782
15	1.000	20	25	0.748	0.837	0.921	0.948
	1.170	20	25	0.659	0.719	0.776	0.794
16	1.000	21	26	0.761	0.847	0.926	0.951
	1.150	21	26	0.679	0.741	0.797	0.815
17	1.000	22	27	0.773	0.857	0.930	0.954
	1.140	22	27	0.694	0.755	0.809	0.826
18	1.000	23	28	0.783	0.865	0.934	0.957
	1.130	23	28	0.708	0.769	0.820	0.837
19	1.000	24	29	0.793	0.873	0.938	0.959
	1.120	24	29	0.721	0.782	0.832	0.848
20	1.000	25	30	0.802	0.879	0.941	0.961
	1.110	26	29	0.741	0.795	0.843	0.859
22	1.000	28	32	0.824	0.891	0.947	0.965
	1.090	28	32	0.765	0.820	0.866	0.881
24	1.000	30	34	0.837	0.900	0.951	0.968
	1.080	30	34	0.782	0.836	0.879	0.893
26	1.000	32	37	0.847	0.908	0.955	0.970
	1.060	32	37	0.806	0.859	0.900	0.914
30	1.000	36	40	0.865	0.921	0.961	0.974
	1.050	36	40	0.829	0.879	0.915	0.927
40	1.000	46	51	0.896	0.941	0.971	0.981
	1.020	46	51	0.880	0.923	0.952	0.961

* Minimum $u = z_2/z_1$ is minimum for axial assembly.

It will be noted that the tables show two addendum values for each number of teeth. The rst one is the minimum addendum that might be used for the pinion. This addendum has been lengthened only in those cases in which there is danger of undercut. The second addendum has been lengthened to balance tooth strength. The lengthening is not quite so much as layouts of the teeth might indicate, but it is about the right amount to use considering the fact that the quality of material in the gear may not be quite as good as that in the pinion. Also, the stress concentration at the root of the internal gear tooth is apt to be high. In general, the minimum addendum should be used *when the gear drives the pinion*, and the maximum addendum should be used *when the pinion drives the gear.*

The internal gear usually cannot have a clearance of much over $0.250m$ (or $0.259/P_d$) because of the effect of the concave (aring) sides of the teeth.

The outside diameter of an internal gear is really the inside diameter. The inside diameter is obtained by subtracting twice the gear addendum from the pitch diameter.

The tooth thickness of the pinion is usually adjusted for the amount of long addendum which the pinion has. The gear tooth thickness is obtained by subtracting the pinion thickness and the backlash from the circular pitch. No consideration is given to gear addendum in obtaining gear tooth thickness. This is a result of the fact that the gear addendum has been arbitrarily stubbed to avoid tip interference. The pinion thickness for a full-depth pinion is as follows:

$$s_1 = \frac{p - j}{2} + (h_{a1} - 1.000m)(2\tan\alpha) \quad \text{(metric)}, \quad (5.26)$$

$$t_p = \frac{p - B}{2} + \left(a_p - \frac{1.000}{P_d}\right)(2\tan\phi) \quad \text{(English)}. \quad (5.27)$$

The chordal addendum of the gear is not ordinarily needed unless the gear has a large enough inside diameter to permit

TABLE 5.15
Internal Gear Dimensions

Item	Reference	Metric		English	
1. Center distance	Section 1.14	15.000		0.590550	
2. Circular pitch	(8) × (7) ÷ π	9.42478		0.371054	
3. Pressure angle	Section 5.1.3	25		25	
4. Working depth	Section 5.1.11	5.51		0.217	
5. Module, transverse	(8) ÷ (7)	3		–	
6. Diametral pitch, transverse	(7) ÷ (8)	–		8.466667	
		Pinion	**Gear**	**Pinion**	**Gear**
7. Number of teeth	Tables 5.14 and 5.15	18	28	18	28
8. Pitch diameter	Section 1.14	54.00	84.00	2.12598	3.30708
9. Addendum	Section 5.1.11, Tables 5.14 and 5.15	3.39	2.12	0.133	0.083
10. Whole depth	Section 5.1.11	7.05	6.26	0.2775	0.2465
11. Outside diameter	(8) + 2 × (9)	60.78	–	2.392	–
12. Inside diameter	(8) – 2 × (9)	–	79.76	–	3.141
13. Root diameter	(12) + 2 (10), gear	46.68	92.28	1.837	3.634
14. Arc tooth thickness	Section 5.1.11	4.980	4.240	0.1960	0.1670
15. Chordal tooth thickness	Equations 5.9 and 5.10	4.973	4.238	0.1958	0.1669
16. Chordal addendum	Section 5.1.11	3.34	2.12	0.1315	0.083
17. Pin size	Table B.1	5.3340	5.3340	0.2100	0.2100
18. Diameter over pins	Table B.2	61.912	–	2.4375	–
19. Diameter between pins	Table B.6	–	77.092	–	3.0351
20. Face width	Section 4.2.2	27.0	25.0	1.06	1.00
21. Form diameter	Equations 5.28 and 5.29	50.035	90.536	1.970	3.564
22. Roll angle, outside diameter	Table B.7	42.195	17.904	42.195	17.904
23. Roll angle, form diameter	Table B.7	12.185	36.877	12.185	36.877
24. Base circle diameter	(8) × cos(3)	48.941	76.130	1.9268	2.9972
25. Minimum root radius	Section 5.1.11	0.90	0.77	0.035	0.030
26. Accuracy	Section 7.18	See Note 3			

Note: 1. Metric dimensions are in millimeters, English dimensions are in inches, and angles are in degrees.
2. This sheet shows a standard center-distance design.
3. After the designer has done the load rating and studied Section 10.4.1, accuracy needs may be satis ed by an ISO or an AGMA quality level, or the designer may need to write a special recipe for accuracy limits.

tooth caliper to be used. It takes at least a 130 mm (5 in.) inside diameter to allow a person's hand inside the gear to measure tooth thickness. Usually, small internal gears are checked for thickness by using measuring pins between the teeth. In an internal gear, the rise of arc must be subtracted from the addendum. Because of the concave nature of the gear tip, only about two-thirds of the pitch line rise of arc is effective.

The form diameter of the gear is obtained by adding the allowance for excess involute instead of subtracting it. Thus,

$$d'_{f2} = d_l + \Delta d_l \quad \text{(metric)}, \tag{5.28}$$

$$D_f = D_l + \Delta D_l \quad \text{(English)}. \tag{5.29}$$

The limit diameter for internal sets is obtained by the same general procedure as that shown for external gears in Table 5.12. However, it is necessary to add instead of subtract. For the internal gear, the following equations should be used:

$$\theta_{rl} = \theta_r + (\theta_{ra} - \theta_r)\frac{z_1}{z_2} \quad \text{(metric)}, \tag{5.30}$$

where $\theta_r = 180° \tan \alpha/\pi$,

$$\varepsilon_{rl} = \varepsilon_r + (\varepsilon_{ro} - \varepsilon_r)\frac{N_P}{N_G} \quad \text{(English)}, \tag{5.31}$$

where $\varepsilon_r = 180° \tan \phi/\pi$.

Once the roll angle at the limit diameter is known, the limit diameter is easily calculated by the method shown in Section 5.1.8.

The cutter that generates an internal gear will usually make a llet that is almost the same radius of curvature as itself. Generally, a cutter for an internal gear will have only about two-thirds of the radius given in Table 5.6.

TABLE 5.16
Helical Gear Basic Tooth Data

	Normal Section Data				Transverse Section Data				Pitch	
Typical Use	**Pressure Angle, α_n (ϕ_n)**	**Working Depth, h' (h_k)**	**Whole Depth, h (h_t)**	**Edge Radius, r_{a0} (r_T)**	**Helix Angle, β (ψ)**	**Pressure Angle, α_t (ϕ_t)**	**Circular Pitch, p_t (p_t)**	**Axial Pitch, p_x (p_x)**	**Module, Transverse m_t**	**Diametral Pitch, P_t**
General purpose	20	2.000	2.350	0.350	10	20.2836	3.1901	18.092	1.015427	0.984808
	20	2.000	2.350	0.350	15	20.6469	3.2524	12.138	1.035276	0.965926
	20	2.000	2.350	0.350	30	22.7959	3.6276	6.283	1.154700	0.866025
	20	2.000	2.350	0.350	35	23.9568	3.8352	5.477	1.220776	0.819152
Extra load capacity	22.50	2.000	2.300	0.325	10	22.8118	3.1901	18.092	1.015427	0.984808
	22.50	2.000	2.300	0.325	15	23.2109	3.2524	12.138	1.035276	0.965926
	22.50	2.000	2.300	0.325	30	25.5614	3.6276	6.283	1.154700	0.866025
	22.50	2.000	2.300	0.325	35	26.8240	3.8352	5.477	1.220776	0.819152
High load capacity	25	2.000	2.250	0.300	10	25.3376	3.1901	18.092	1.015427	0.984808
	25	2.000	2.250	0.300	15	25.7693	3.2524	12.138	1.035276	0.965926
	25	2.000	2.250	0.300	30	28.3000	3.6276	6.283	1.154700	0.866025
	25	2.000	2.250	0.300	35	29.6510	3.8352	5.477	1.220776	0.819152
Special for low noise	18.2377	2.000	2.400	0.400	10	18.5000	3.1901	18.092	1.015427	0.984808
	17.9105	2.000	2.400	0.400	15	18.5000	3.2524	12.138	1.035276	0.965926
	16.1599	2.000	2.400	0.400	30	18.5000	3.6276	6.283	1.154700	0.866025
	15.3275	2.000	2.400	0.400	35	18.5000	3.8352	5.477	1.220776	0.819152
Special for very low noise	17.2500	2.200	2.575	0.375	10	17.5000	3.1901	18.092	1.015427	0.984808
	16.9384	2.200	2.575	0.375	15	17.5000	3.2524	12.138	1.035276	0.965926
	15.2727	2.200	2.575	0.375	30	17.5000	3.6276	6.283	1.154700	0.866025
	14.4817	2.200	2.575	0.375	35	17.5000	3.8352	5.477	1.220776	0.819152

Note: All angles are in degrees. For the metric system, all dimensions are in millimeters. Multiply by normal module for dimensions of other than 1 module. For English system, all dimensions are in inches. Divide by normal diametral pitch for dimensions of other than 1 normal diametral pitch. Module in normal section is 1.000 for metric system. Diametral pitch is 1.000 in normal section for English system.

Table 5.16 shows a dimension sheet for internal gears. Data for a sample problem of an 18-tooth pinion driving a 28-tooth gear are also shown.

The calculation of the diameter *between pins* for the internal gear is somewhat different from the calculation of diameter over pins for an external gear. Table B.6 in Appendix B shows how this is done.

The calculation of the tooth pro le of an internal gear is shown in Table B.8. Figure B.16 shows a layout of the pro le of the 28-tooth internal gear on an enlarged scale. Note that the root llet area is quite small. (The designer may not want to use this design for this reason—this shows that a design layout is important in the design process.)

The layout of the 18-tooth mating pinion is shown in Figure B.17. The pinion looks good.

5.1.12 Helical Gear Tooth Proportions

A wide variety of tooth proportions has been used for helical gears. In spite of efforts to standardize helical gear tooth proportions, there is still no recognized standard. In this section, we will consider typical kinds of helical gear designs and discuss why each kind is used.

In most cases, a gear maker will have gear-cutting tools on hand, and so a new design is very often based on existing tools. Hobs can be used to cut different helix angles merely by changing settings and gear ratios in the hobbing machine. For instance, spur gear hobs are very often used to cut helical gears up to about 30°. If the gear teeth are to be shaped, a shaper-cutter is generally needed to each helix angle, and so spur gear shaper-cutters are not usable for cutting helical gears. The guides used on a gear-shaping machine cut a constant lead rather than cut a constant helix angle. As tooth numbers change, the helix angle that is produced by a given guide will change.

A designer of helical gears generally needs certain kinds of tooth proportions to make the gear unit properly function, and this is usually the chief reason for picking certain proportions. The consideration of available cutting tools then becomes secondary. The designer, of course, would hope that a gear shop cutting the design would have tools on hand, but the most important thing would be to make a good helical gear design regardless of the availability* of cutting tools.

Table 5.17 shows ve kinds of helical gear teeth, with examples shown for 10°, 15°, 30°, and 35° helix angles. When hobs are available, the normal section data of the helical gear must agree with those of the hob. After a helix angle is picked,

* See Sections 11.1 and 11.2 for more details on cutting tools like gear shaper-cutters and hobs.

TABLE 5.17
Helical Gear Dimensions

Item	Reference	Metric		English	
1. Center distance	Sections 4.2.2 and 4.2.4	187.9026		7.39774	
2. Normal circular pitch	Equation 1.8	9.4248		0.371054	
3. Normal pressure angle	Section 5.1.12, Table 5.17	20		20	
4. Working depth	Table 5.17	6.00		0.236	
5. Helix angle	Section 5.1.12	15		15	
6. Transverse pressure angle	Equations 5.32 and 5.33	20.64689		20.64689	
7. Module, normal	Equation 1.9	3		–	
8. Normal diametral pitch	Equation 1.10	–		8.466667	
		Pinion	**Gear**	**Pinion**	**Gear**
9. Number of teeth	Section 5.1.13	25	96	25	96
10. Pitch diameter	Equations 1.4 and 1.5	77.6457	298.1595	3.05692	11.73856
11. Addendum	Section 5.1.13	3.00	3.00	0.118	0.118
12. Whole depth	Table 5.17	7.05	7.05	0.2775	0.2775
13. Outside diameter	(10) + 2 × (11)	83.65	304.16	3.293	11.974
14. Root diameter	(13) − 2 × (12)	69.55	290.06	2.738	11.419
15. Normal arc tooth thickness	Section 5.1.6	4.617	4.617	0.1818	0.1818
16. Chordal tooth thickness	Equations 5.9 and 5.10	4.594	4.609	0.1809	0.1815
17. Chordal addendum	Equations 5.7 and 5.8	3.047	3.014	0.1200	0.1185
18. Ball size	Table B.1	5.33401	5.33401	0.210	0.210
19. Diameter over balls	Table B.9				
20. Face width	Sections 5.1.12 and 5.1.13	78	77	3.07	3.03
21. Form diameter	Table B.3	73.393	293.338	2.8894	11.5487
22. Roll angle, outside diameter	Table B.3	32.68	24.87	3268	24.87
23. Roll angle, form diameter	Table B.3	8.17	18.60	8.17	18.60
24. Base circle diameter	(10) × cos(6)	72.6586	279.0091	2.86057	10.98458
25. Handedness of helix	Section 1.13	RH	LH	RH	LH
26. Lead	(2) × (9) ÷ sin(5)	910.364	3495.796	35.8410	137.625
27. Minimum root radius	Section 5.1.13	0.89	0.78	0.035	0.030
28. Accuracy	Section 10.4.1	See Note 3			

Note: 1. Metric dimensions are in millimeters, English dimensions are in inches, and angles are in degrees.
2. This sheet shows a standard center-distance design. See Section 13.1 for an example of a special spread-center design.
3. After the designer has done the load rating and studied Section 10.4.1, accuracy needs may be satis ed by an ISO or an AGMA quality level, or the designer may need to write a special recipe for accuracy limits.

the hob can be put on a hobbing machine to cut gears at this helix angle, and the transverse-section data are obtained. In a helical gear tooth, the transverse-section data must correlate, of course, with the normal section data, regardless of how the teeth are made.

When the helical gear teeth are to be shaped, the shaping tool design is primarily based on the transverse section. The normal section data for shaped teeth are calculated the same way as for hobbed teeth.

In Table 5.17, the general-purpose design is typical of relatively standard 20° pressure-angle spur gear hobs being used to cut helical gears. The practice of using a spur gear hob for helical gears is OK when the helical gears are small and narrow in face width. Large, wide-face helical gears need a special tapered hob (see Section 11.2). The spur gear hob without taper would break down too quickly.

The extra load-capacity tooth proportions are based on 22.5° normal pressure angle and standard teeth. These teeth are stronger from a beam-strength standpoint and also from the standpoint of surface durability. The contact ratio is still relatively good, and they will tend to run quite smoothly, although not quite as smoothly as the general-purpose 20° normal-pressure-angle design.

The high load capacity helical gear tooth, 25° normal pressure angle, tends to have a maximum load-carrying capacity. These teeth do not run quite as smoothly as the designs just mentioned because of the lower contact ratio. However, if tooth pro le modi cations are properly made, they will run quite well at the design load (teeth modi ed for a heavy load may run roughly and noisily at light loads).

In high horsepower designs running at high speed, it is often desirable to use a special tooth design to get quiet operation. (A good example would be 10,000 kW going through a helical gearset at a pitch line speed of 100 m/s or higher.)

The four groupings in Table 5.17 show examples of special helical gear tooth proportions for low noise.

In a few cases, the helical gear application may be so critical that a very special design is justi ed.

The fifth grouping in Table 5.17 shows some examples in which a relatively low pressure angle is used and the teeth are made deeper than standard depth (the standard working depth for spur or helical gears is usually thought of as 2.000 times the module in the normal section). Helical gear teeth made to the fifth grouping will not have maximum load-carrying capacity, but they will still have relatively good load-carrying capacity. The design, of course, is biased toward low noise rather than toward maximum torque capacity for the size of the gears.

The designs at 10° and 15° helix angles are preferred for single helical gears because the thrust load is relatively low. It is usually possible to use a face width that is at least as much as the axial pitch. (To get the full benefit of helical tooth action, the face width should be equal to at least two axial pitches). When the gears are made double helical, there is no thrust load, because the thrust of one helix is opposed by an equal and opposite thrust from the other helix.

Double-helical gears should be made with at least a 30° helix angle. At helix angles this high, it is possible to get several axial pitches within the face width of each helix. This is beneficial for quiet running. The full benefit of helical tooth action can be quite readily obtained with double-helical gears. Note the double-helical design illustrated in Figure 5.17. There are over five axial pitches per helix.

In a double-helical gear design, the usual design practice is to position the gear axially with thrust bearings, then permit the pinion to axially float so that the load is evenly divided between the two helices. There may be resistance to axial float resulting from friction in coupling devices. If there is enough resistance to axial movement, the division of load in a double-helical gear may be impaired. A 30° helix angle, or higher, is needed in the double-helical design to ensure that resistance to axial movement from coupling can be quite readily overcome (it would be a design mistake—in most cases—to use 15° helix angle for a double-helical set).

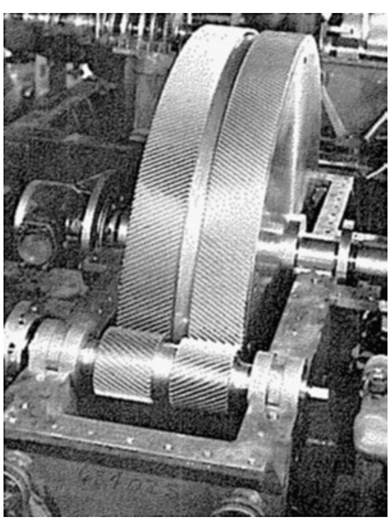


FIGURE 5.17 Helical gearset used to drive a 1750 kW generator at 1200 rpm. Turbine speed is 10,638 rpm. (Courtesy of Transamerica DeLaval, Trenton, New Jersey.)

The relation between the normal and transverse pressure angles of helical gears is given by the following equation:

$$\tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta} \quad \text{(metric)}, \tag{5.32}$$

$$\tan\phi_t = \frac{\tan\phi_n}{\cos\psi} \quad \text{(English)}. \tag{5.33}$$

5.1.13 Helical Gear Dimension Sheet

Table 5.18 shows a helical gear dimension sheet. All the tooth dimensions which will ordinarily be needed are shown on this sheet. In some low-cost applications, items like roll angles, form diameter, and axial pitch may not be required.

The face width dimensions should be based on the load-carrying requirements, as discussed in Section 5.2.4. Also, it should be remembered that the wider the face width, the harder it is to secure accuracy enough to uniformly make the whole face width carry load. In addition to these considerations, the designer should consider the fact that a certain amount of face width is required to get the benefit of helical gear action. General experience indicates that it takes at least two axial pitches of face width to get the full benefit from the overlapping action of helical teeth. In critical high-speed gears, where noise is the problem, the designer should aim to get four or more axial pitches in the face width. If the face width is less than one axial pitch, the tooth action will be in between that of a spur and of a true helical gear.

Helical pinions should have enough addenda to avoid undercut. Equations 5.3 and 5.4 can be used to calculate the number of teeth at which undercut just starts on a helical pinion. Two pressure angles have to be used, though, to get the right answer. In calculating the h_{fx} dimension, the normal pressure angle α_n is used. In calculating z, the transverse pressure angle α_t is used. Helical pinions can go to lower numbers of teeth without undercut than spur pinions can. It is seldom that helical gearing requires long and short addenda to avoid undercut.

In general, there is not so much need to use long and short addenda to balance strength between pinion and gear in helical gearing as in spur gearing. Low-hardness helical gearing usually has strength to spare and is in no danger of scoring trouble. Wear in the form of pitting is the main thing that limits the design. In this situation, a long-addendum pinion meshing with a short-addendum gear offers only a slight advantage over equal addendum on both members. In high-hardness helical gearing where the pinion has a low number of teeth and the ratio is high, it may be desirable to use about the same ratio between pinion addendum and gear addendum as would be used for spur gears. In this case, Figure 5.5 may be used as a guide.

The minimum root fillet radius of a helical gear is not usually specified on the drawing. This is a hard dimension to check because you cannot get a good projection view of the

TABLE 5.18
Bevel Gear Proportions

Kind of Bevel Gear	Size Range: Module, m_t	Size Range: Diametral Pitch, P_t	Pressure Angle, α (or ϕ)	Working Depth, h' (or h_k)	Whole Depth, h (or h_t)
Straight	25.4–0.40	1–64	20°	2.000	$2.188+\dfrac{0.005}{m_t}$ or $0.002P_t$
Spiral	25.4–1.27	1–20	20°	1.700	1.888
	25.4–1.27	1–20	16°	1.700	1.888
Zerol	5.00–0.40	5–64	20°	2.000	$2.188+\dfrac{0.05}{m_t}$
	5.00–0.80	5–32	22.5°	2.000	
	5.00–1.27	5–20	25°	2.000	or $0.002P_t$

Note: For metric design, the values are multiplied by the module to get the working depth and the whole depth. For English design, the values are divided by the diametral pitch to get the whole working depth and the whole depth. To protect ne pitches, a constant value is added to the whole depth. This is done for all pitches, but it is not very signi cant on large pitches. The value is 0.05 mm or 0.002 in.

Examples for straight bevel are as follows:

	5 Module	1 Module
Working depth	10.00 mm	2.000 mm
Whole depth	10.99 mm	2.398 mm

llet without cutting out and mounting a section* of the gear. If beam strength is not critical, then it is not necessary to hold a close control over root llet curvature. It is good judgment, though, to require that all cutting tools have as much radius as possible.

The equations for the minimum radius of curvature in the root llet of a helical gear are the following:

$$\rho_f = \frac{r_{a0}^2 - (h_f - r_{a0})^2}{d_p/(2\cos^2\beta) + (h_f - r_{a0})} \quad \text{(metric)}, \qquad (5.34)$$

where $h_f = h - h_a$, and r_{a0} is edge radius of generating rack,

$$\rho_f = \frac{r_T^2 - (b - r_T)^2}{d/(2\cos^2\psi) + (b - r_T)} \quad \text{(English)}, \qquad (5.35)$$

where $b = h_t - a$, and r_T is edge radius of generating rack.

Although Equations 5.34 and 5.35 are based on the generating action of a rack, the designer can use the equations for any type of helical gear manufacture on the assumption that other methods of tooth cutting can meet the same minimum radius value.

The shape of helical teeth in the normal section is almost exactly the same as that of a spur gear with a larger number of teeth. For instance, the 25-tooth helical pinion on a 77.6457 mm pitch diameter shown on the sample dimension sheet would be matched in tooth contour by a spur pinion of 27.74 teeth on an 83.220 mm pitch diameter with a 20° pressure angle.

The matching number of teeth in a spur gear is called the *virtual* number of teeth. The virtual number of spur teeth is equal to the number of helical teeth divided by the cube of the cosine of the helix angle. See Table B.11.

5.1.14 Bevel Gear Tooth Proportions

The proportions of straight bevel, Zerol† bevel, and spiral bevel gears can quite be logically considered together. Gleason Works of Rochester, NewYork, has done an excellent job of standardizing the designs of these kinds of gears. Their work is generally accepted by the gear industry, and in time, it usually becomes part of the standards of the AGMA. The material in this section and the next three sections are taken from the revised editions of Gleason bevel gear systems for straight, spiral, and Zerol bevel gears.

The pressure angles and the tooth depths generally recommended for bevel gears are given in Table 5.19. The whole depth speci ed is sometimes slightly exceeded by the practice of rough cutting some pitches a small amount deeper than the calculated depth to save wear on the nishing cutters.

The various Gleason systems have the amount of addendum for the gear and the pinion worked out so as to avoid undercut with low number of teeth and balance the strength of gear and pinion teeth. In each case, though, there is a limit to how far the system will go. Table 5.20 shows the minimum numbers or teeth that can be used in different combinations.

In spur and helical gear works, the amount of addendum for the pinion is rst determined. Then, what is left of the working depth is used for the gear addendum. In bevel gear

* The section cut is normal to the helical tooth. The minimum radius speci ed is, of course, in the normal section.

† Trademark registered in the U.S. Patent Of ce by Gleason Works, Rochester, New York, United States.

TABLE 5.19
Minimum Tooth Numbers for Bevel Gears

Kind of Gear	Pressure Angle	Number of Pinion Teeth	Min. Number of Gear Teeth
Straight bevel	20°	13	31
		14	20
		15	17
		16	16
Spiral bevel	20°	12	26
		13	22
		14	20
		15	19
		16	18
		17	17
Zerol bevel	20°	15	25
		16	20
		17	17
	22.5°	13	15
		14	14
	25°	13	14
		13	13

TABLE 5.20
Gear Addendum for Bevel Gears

Kind of Bevel Gear	Metric	English
Straight or Zerol	$0.540m_t + \dfrac{0.460m_t}{u^2}$	$\dfrac{0.540}{P_d} + \dfrac{0.460}{P_d m_G^2}$
Spiral bevel	$0.540m_t + \dfrac{0.390m_t}{u^2}$	$\dfrac{0.540}{P_d} + \dfrac{0.390}{P_d m_G^2}$

Note: The pinion addendum is obtained by subtracting the gear addendum from the working depth.

practice, just the opposite procedure is used. The gear addendum is determined rst.

Table 5.21 shows the amount of gear addendum recommended for bevel gears.

Since bevel gears and bevel pinions usually have different addendums, it is necessary to use different tooth thicknesses for the two members. The standard systems worked out by Gleason Works adjust the tooth thicknesses so that approximately equal strength is obtained for each member. The adjustment of the tooth thickness is accomplished by a factor called k. This k should not be confused with the K *factor* used as an index of tooth load. The k values can be read from Figures 5.18 through 5.21.

The thicknesses of bevel gear teeth are calculated for the large ends of the teeth. It is customary to calculate the circular thicknesses of the teeth *without allowing any backlash*. In straight bevel gearing, chordal tooth thicknesses are calculated *with backlash*. The chordal thicknesses of a straight bevel gear tooth can be measured with tooth calipers.

The formula for the gear circular tooth thickness of any kind of bevel gear is as follows:

$$s_2 = \frac{p_t}{2} - (h_{a1} - h_{a2})\frac{\tan\alpha_t}{\cos\beta} - km_t \quad \text{(metric)}, \tag{5.36}$$

$$t_G = \frac{p_t}{2} - (a_P - a_G)\frac{\tan\phi_t}{\cos\psi} - \frac{k}{P_d} \quad \text{(English)}, \tag{5.37}$$

where k is the circular tooth thickness factor given by the appropriate curve for straight, Zerol, or spiral bevel gears. See Figures 5.18 through 5.21.

The recommended amount of backlash for bevel gearsets when they are assembled and ready to run is given in Table 5.22. In instrument and control gearings, it may be desirable to use values even lower than those shown. Conversely,

TABLE 5.21
Nominal Backlash for Bevel Gears at Tightest Point of Mesh

Tooth Size Range		Backlash for Low Accuracy		Backlash for High Accuracy	
Module, m_t	Diametral Pitch, P_d	mm	in.	mm	in.
25–20	1.00–1.25	1.14–1.65	0.045–0.065	0.51–0.76	0.020–0.030
20–17	1.25–1.50	0.89–1.40	0.035–0.055	0.46–0.66	0.018–0.026
17–14	1.50–1.75	0.63–1.10	0.025–0.045	0.41–0.56	0.016–0.022
14–12	1.75–2.00	0.51–1.00	0.020–0.040	0.36–0.46	0.014–0.018
12–10	2.00–2.50	0.46–0.76	0.018–0.030	0.30–0.41	0.012–0.016
10–8.5	2.50–3.00	0.38–0.63	0.015–0.025	0.25–0.33	0.010–0.013
8.5–7.2	3.00–3.50	0.30–0.56	0.012–0.022	0.20–0.28	0.008–0.011
7.2–6.3	3.50–4.00	0.25–0.51	0.010–0.020	0.18–0.22	0.007–0.009
6.3–5.1	4.00–5.00	0.20–0.41	0.008–0.016	0.15–0.20	0.006–0.008
5.1–4.2	5.00–6.00	0.15–0.33	0.006–0.013	0.13–0.18	0.005–0.007
4.2–3.1	6.00–8.00	0.13–0.25	0.005–0.010	0.10–0.15	0.004–0.006
3.1–2.5	8.00–10.00	0.10–0.20	0.004–0.008	0.076–0.127	0.003–0.005
2.6–1.6	10.00–16.00	0.076–0.127	0.003–0.005	0.051–0.102	0.002–0.004
1.6–1.2	16.00–20.00	0.051–0.102	0.002–0.004	0.025–0.076	0.001–0.003

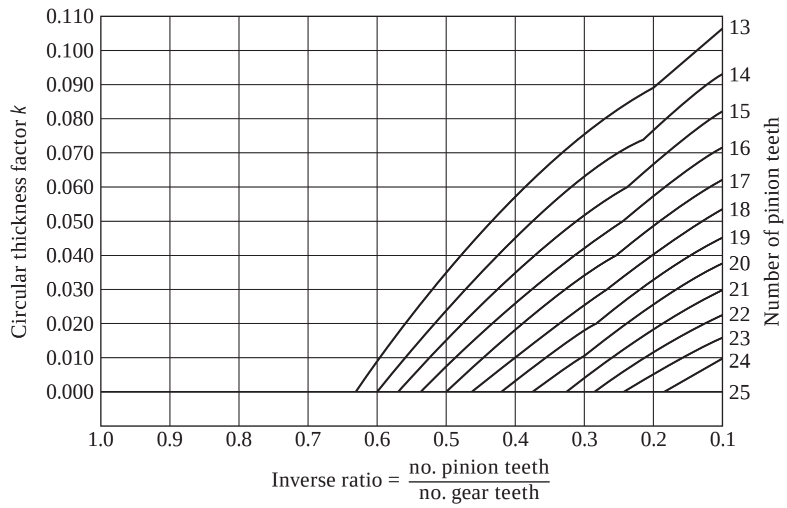


FIGURE 5.18 Circular thickness factor for straight bevel gears with 20° pressure angle.

some high-speed gears and gears mounted in casing with a material different from that of the gear may require more backlashes.

5.1.15 Straight Bevel Gear Dimension Sheet

A dimension sheet for calculating straight bevel gear tooth data is given in Table 5.23. This sheet gives several formulas not previously discussed. These can be quite readily tried out by working through numerical calculations for the sample design that is shown in the table.

The backlash allowance is taken from Table 5.22. The designer should consult the text in Section 5.1.6.1 and then decide whether or not special requirements of the application might take it desirable to depart from standard backlash.

The correction for chordal tooth thickness and chordal addendum are made by these equations:

Chordal tooth thickness

$$\bar{s}_2 = s_2 - \frac{(s_2)^3}{6(d_{p2})^2} - \frac{j}{2} \quad \text{(metric)} \tag{5.38}$$

$$t_{cG} = t_G - \frac{(t_G)^3}{6D^2} - \frac{B}{2} \quad \text{(English)} \tag{5.39}$$

$$\bar{s}_1 = s_1 - \frac{(s_1)^3}{6(d_{p1})^2} - \frac{j}{2} \quad \text{(metric)} \tag{5.40}$$

$$t_{cP} = t_P - \frac{(t_P)^3}{6d^2} - \frac{B}{2} \quad \text{(English)} \tag{5.41}$$

Chordal addendum

$$\bar{h}_{a2} = h_{a2} + \frac{(s_2)^2 \cos\delta_2'}{4d_{p2}} \quad \text{(metric)} \tag{5.42}$$

$$a_{cG} = a_G + \frac{(t_G)^2 \cos\Gamma}{4D} \quad \text{(English)} \tag{5.43}$$

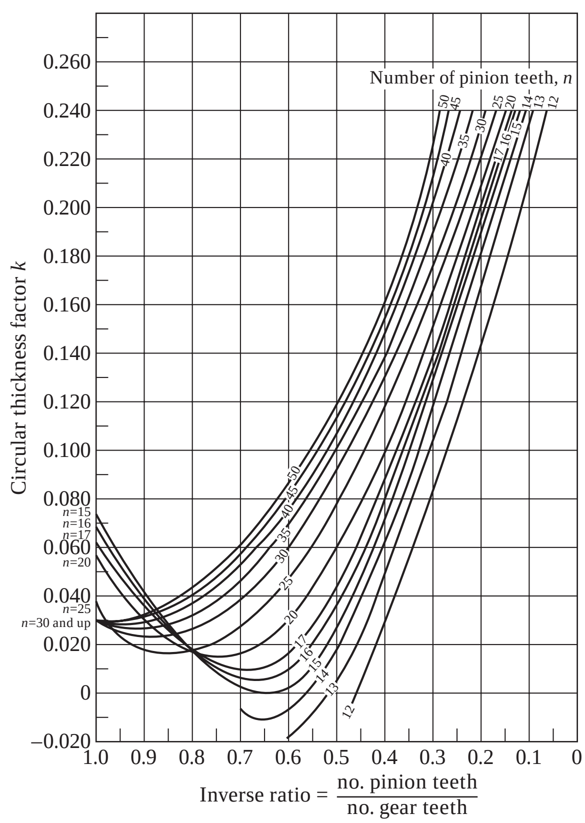


FIGURE 5.19 Circular thickness factor for spiral bevel gears with 20° pressure angle and 35° spiral angle. LH pinion driving clockwise or RH pinion driving counterclockwise.

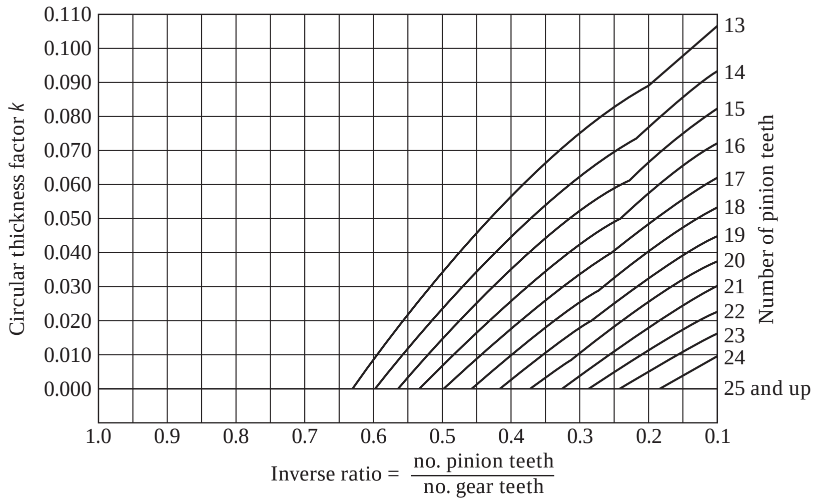


FIGURE 5.20 Circular thickness factor for Zerol bevel gears with 20° pressure angle operating at 90° shaft angle.

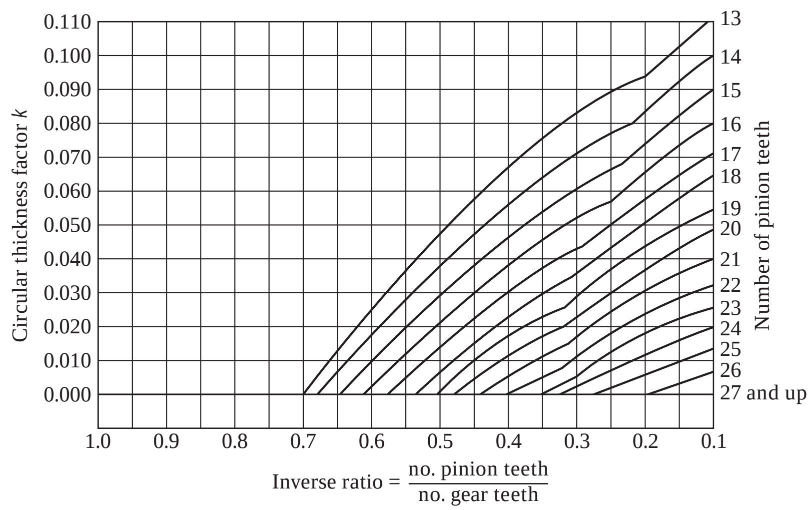


FIGURE 5.21 Circular thickness factor for Zerol bevel gears with 25° pressure angle operating at 90° shaft angle.

$$\bar{h}_{a1} = h_{a1} + \frac{(s_1)^2 \cos \delta_1'}{4d_{p1}} \quad \text{(metric)} \tag{5.44}$$

$$a_{cP} = a_P + \frac{(t_P)^2 \cos \gamma}{4d} \quad \text{(English)} \tag{5.45}$$

where backlash (in millimeters for metric) is designated as j, and backlash (in inches for English) is denoted be B.

The *tooth angle* is a machine setting used on a Gleason two-tool straight-bevel-gear generator. See Table 5.24 for the de nition of this angle.

The *limit-point width* is the maximum width of the point of a straight-sided V-tool which will touch the sides and the bottom of the tooth at the small end. The tool actually used must not be larger than this dimension, but it may be a small amount less.

The *tool advance* corresponds to the 0.05 mm or 0.002 in. in the whole depth formula. (See Table 5.19.) This dimension sets the tool deeper and increases the clearance along the length of the tooth.

Table 5.23 gives the calculations for bevel gears mounted on a 90° shaft angle only. If bevel gears are to be mounted on some other angle, special calculations have to be made. Gleason data references at the end of the book may be consulted to get information beyond the scope of this book. A complete Gleason dimension sheet for the bevel gears in Table 5.24 is given in Table B.12.

The face width of straight bevel gears should not exceed 32% of the outer cone distance nor exceed 3.18 times the circular pitch. These rules may be rounded off to 30% of the face width and 3 times the circular pitch for convenience. If either

TABLE 5.22
Straight Bevel Gear Dimensions

Item	Reference	Metric		English	
1. Circular pitch	(8) × π ÷ (7)	15.7079		0.61842	
2. Pressure angle	Section 1.15	20		20	
3. Working depth	Table 5.19	20.00		0.3937	
4. Shaft angle	Assumed to be 90°	90		90	
5. Module	(8) ÷ (7)	5.000		–	
6. Diametral pitch	(7) ÷ (8)	–		5.080	
		Pinion	**Gear**	**Pinion**	**Gear**
7. Number of teeth	Table 5.20	16	49	16	49
8. Pitch diameter	Section 4.2.5	80.00	245.00	3.1496	9.6457
9. Pitch angle	Section 1.15, Table 5.24	18.08	71.92	18.08	71.92
10. Outer cone distance	Table 5.24	128.87	128.87	5.074	5.074
11. Face width	Sections 4.2.5 and 5.1.15	40.00	40.00	1.575	1.575
12. Addendum	Table 5.21	7.05	2.95	0.278	0.116
13. Whole depth	Table 5.19	10.99	10.99	0.433	0.433
14. Dedendum angle	Table 5.24	1.73	3.55	1.73	3.55
15. Face angle	Table 5.24	21.63	73.65	21.63	73.65
16. Root angle	Table 5.24	16.35	68.37	16.35	68.37
17. Outside diameter	Table 5.24	93.41	246.83	3.678	9.718
18. Pitch apex to crown	Table 5.24	120.31	37.20	4.737	1.465
19. Circular tooth thickness	Equations 5.36 and 5.37	9.514	6.194	0.3745	0.2438
20. Backlash	Table 5.22, Section 5.1.6	0.13–0.18	0.13–0.18	0.005–0.007	0.005–0.007
21. Chordal tooth thickness	Equations 5.38 and 5.41	9.43	6.13	0.3712	0.2413
22. Chordal addendum	Equations 5.42 through 5.45	7.32	2.96	0.2882	0.1165
23. Tooth angle	Table 5.24	2.75	2.67	2.75	2.67
24. Limit point width: large end	Section 5.1.15, Table B.12	3.30	3.63	0.130	0.143
25. Limit point width: small end	Section 5.1.15, Table B.12	2.31	2.54	0.091	0.100
26. Tool point width	Section 5.1.15, Table B.12	1.65	1.78	0.065	0.070
27. Tool edge radius	Section 5.1.15, Table B.12	0.63	0.63	0.025	0.025

Note: Metric dimensions are in millimeters, English dimensions are in inches, and angles are in degrees. For more complete data, see Gleason dimension sheet in Table B.12.

rule is exceeded by much, there is apt to be trouble in properly making and fitting the teeth. Real load-carrying capacity will probably be lost as a result of poor running fits when bevel teeth gears are made wider in face width than the rules just mentioned will allow.

5.1.16 Spiral Bevel Gear Dimension Sheet

Table 5.25 shows the relatively simple design sheet that is used for spiral bevel gears. Control of the spiral bevel gear's geometry is obtained by controlling the machine settings of the machine used to make the gears and by testing each gear with standard test gears.

Table 5.22 is issued as a design guide for backlash of spiral bevel gears. In general, all the backlash is subtracted from the pinion thickness.

Table 5.25 should be used only for spiral bevel gears on 90° shaft angle. The standard spiral angle is 35°.

The mean circular thickness shown in item 20 (see Table 5.55) is the tooth thickness at the midsection (midway from the face width). The dimensions in the midsection are all smaller (due to the conical shape of a bevel gear) than they are at the large end. The relation of midsection to large end is

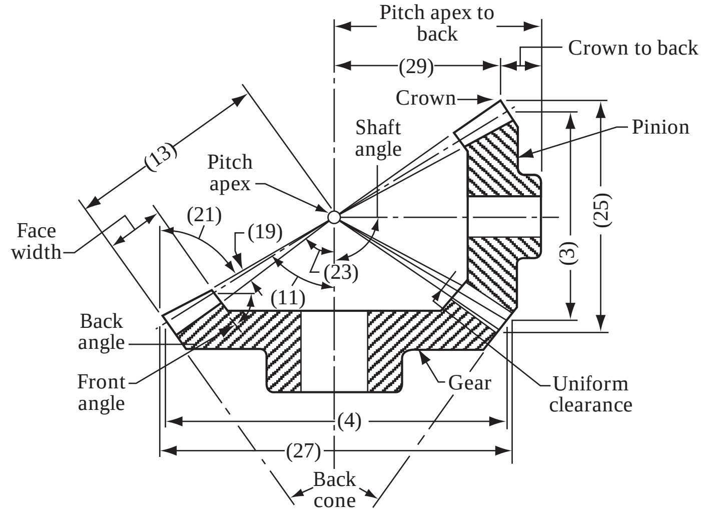
$$\frac{\text{Midsection}}{\text{Large end}} = \frac{2.0 \times \text{outer cone distance} - \text{face width}}{2.0 \times \text{outer cone distance}}. \tag{5.46}$$

If the face width is 30% of the cone distance, this ratio will come out to be 0.850. This means the mean tooth thicknesses will be about 85% of the thicknesses at the large end.

The tooth thicknesses shown in Table 5.25 for the 16/49 spiral bevel set have been reduced from the theoretical to allow for a minimum backlash for precision spiral bevel teeth. The sum of the tooth thicknesses is less than the mean circular pitch by the backlash. (The mean circular pitch for the example is 13.2700 mm or 0.5224 in.)

The data shown in Tables 5.23 and 5.26 have been calculated by computer at Gleason Works. They do not exactly

TABLE 5.23
Calculation of Bevel Gear Body Dimensions



1. No. pinion teeth	16	22. Pinion RA, (10) − (16)	16.3322
2. No. gear teeth	19	23. Gear RA, (11) − (19)	68.3464
3. Pinion, pitch diameter	80.00	24. 2.0 (5) × cos(10)	13.4035
4. Gear, pitch diameter	245.00	25. Pinion OD, (3) + (24)	93.4035
5. Pinion addendum	7.05	26. 2.0 (6) × cos(11)	1.8314
6. Gear addendum	2.95	27. Gear OD, (4) + (26)	246.8314
7. Working depth	10.00	28. 0.50 × (4) − (5)sin(10)	120.3117
8. Whole depth	10.99	29. Pinion pitch apex to crown	(28)
9. (1) ÷ (2)	0.326531	30. 0.50 × (3) − (6)sin(11)	37.1957
10. Pinion PA, $\tan^{-1}(9)$	18.0834	31. Gear pitch apex to crown	(30)
11. Gear PA, 90° − (10)	71.9165	For straight bevels only:	
12. 2 sin(10)	0.620804	32. Pinion circular thickness	9.514
13. Outer cone distance, (3) ÷ (12)	128.8652	33. Pressure angle	20°
14. (8) − (5)	3.94	34. 0.50 × (32) + (14)tan(33)	6.19104
15. (14) ÷ (13)	0.030575	35. 57.30 ÷ (13)	0.44465
16. Pinion ded. angle, $\tan^{-1}(15)$	1.7512	36. Pinion tooth angle, (34) × (35)	2.7528
17. (8) − (6)	8.04	37. Gear circular thickness	6.194
18. (17) ÷ (13)	0.062391	38. 0.50 × (37) + (17)tan 33)	6.02332
19. Gear ded. angle, $\tan^{-1}(18)$	3.5701	39. Gear tooth angle, (35) × (38)	2.6782
20. Pinion FA, (19) + (10)	21.6535		
21. Gear FA, (16) + (11)	73.6677		

Note: These calculations are for straight, spiral, or Zerol bevel gears on a 90° shaft angle. The abbreviations used are as follows: FA = face angle; OD = outside diameter; PA = pitch angle; RA = root angle.

agree with the data given in Gleason references at the end of the book. With somewhat more complex calculations—handled easily in a large computer program—the proportions can be modi ed to give best results for the planned method of manufacture. A slightly tilted root-line taper, for instance, is often used to allow for maximum tooling-point widths. This achieves the best tool life and the largest llet radii (for decreased bending stresses).

On large production jobs, using Gleason machine tools, it is available to obtain a complete summary of design and manufacturing dimensions from Gleason Works. These dimension sheets show considerably more data than the basic engineering data. Also, computations at Gleason Works are carried out on a large computer. Small variations to favor a particular job can be made very readily. As examples, the following dimension sheets are given in Appendix B:

Table B.12 Straight Bevel	Matches Table 5.23
Table B.13 Spiral Bevel	Matches Table 5.25
Table B.14 Zerol Bevel	Matches Table 5.26

TABLE 5.24
Spiral Bevel Gear Dimensions

Item	Reference	Metric		English	
1. Circular pitch	(9) × π ÷ (8)	15.7079		0.61842	
2. Pressure angle	Section 1.17	20		20	
3. Spiral angle	Section 5.1.16	35		35	
4. Working depth	Table 5.19	8.30		0.327	
5. Shaft angle	Assumed to be 90°	90		90	
6. Module	(9) ÷ (8)	5.00		–	
7. Diametral pitch	(8) ÷ (9)	–		5.080	
		Pinion	**Gear**	**Pinion**	**Gear**
8. Number of teeth	Table 3.19	16	49	16	49
9. Pitch diameter	Section 4.2.5	80.00	245.00	3.1496	9.6457
10. Pitch angle	Table 5.24	18.08	71.92	18.08	71.92
11. Outer cone distance	Table 5.24	128.87	128.87	5.074	5.074
12. Face width	Section 5.1.16	38	38	1.496	1.496
13. Addendum	Table 5.21	5.91	2.38	0.233	0.094
14. Whole depth	Table 5.19	9.22	9.22	0.363	0.363
15. Dedendum angle	Table 5.24	1.13	2.83	1.13	2.83
16. Face angle	Table 5.24	20.92	73.05	20.92	73.05
17. Root angle	Table 5.24	16.95	69.08	16.95	69.08
18. Outside diameter	Table 5.24	91.26	246.48	3.593	9.704
19. Pitch apex to crown	Table 5.24	120.67	37.74	4.751	1.485
20. Mean circular tooth thickness	Section 5.1.16	8.13	5.04	0.320	1.1984
21. Backlash	Section 5.1.16, Table 5.22	0.13–0.18	0.13–0.18	0.005–9.007	0.005–9.007
22. Handedness of spiral	One LH, other RH	LH	RH	LH	RH
23. Function	Design choice	Driver	Driven	Driver	Driven
24. Direction of rotation	Design choice	CCW	CW	CCW	CW
25. Cutter diameter	Table B.13	190.5	190.5	7.50	7.50
26. Cutter edge radius	Table B.13	0.63	0.63	0.025	0.025

Note: Metric dimensions are in millimeters, English dimensions are in inches, and angles are in degrees. For more complete data, see Gleason dimension sheet in Table B.13.

Spiral bevel gears should not have a face width exceeding 30% of the outer cone distance.

5.1.17 Zerol Bevel Gear Dimension Sheet

The Zerol bevel gear has curved teeth like the spiral bevel gear, but its spiral angle is 0°. The dimension sheet (Table 5.26) is similar in form to the one used for spiral bevel gears, but the dimensions have numerical values more like those used for straight bevel teeth.

The mean tooth-thickness values shown in Table 5.26 are similar to those just discussed in Section 5.1.16 for spiral bevel gears. They are for the middle of the face width, and they have been reduced to allow for an amount of backlash for precision gears.

The dedendum angle for Zerol bevel gears has a small amount added to the angle that would be obtained using the method shown in Table 5.24 for bevel gear body dimensions. This added amount, in degrees, can be obtained from the following:

$$\Delta\text{dedendum angle} = A - B - C, \tag{5.47}$$

where A can be one of the following:

	Value of A	
Pressure Angle	**Metric**	**English**
20°	$111.13 \div z$	$111.13 \div N_c$
22.5°	$81.13 \div z$	$81.13 \div N_c$
25°	$56.87 \div z$	$56.87 \div N_c$

Number of crown teeth

$$z_a = 2.0(R_a \div m_t) \quad \text{(metric)} \tag{5.48}$$

$$N_c = 2.0\, P_d A_o \quad \text{(English)} \tag{5.49}$$

The value of B is the same regardless of pressure angle:

$$B = \frac{25.2\sqrt{d_{p1} \sin \delta_2'}}{zb} \quad \text{(metric)}, \tag{5.50}$$

TABLE 5.25
Zerol Bevel Gear Dimensions

Item	Reference	Metric		English	
1. Circular pitch	(8) × π ÷ (7)	7.8540		0.30921	
2. Pressure angle	Section 1.16	20		20	
3. Working depth	Table 5.19	5.00		0.197	
4. Shaft angle	Assumed to be 90°	90		90	
5. Module	(8) ÷ (7)	2.500		–	
6. Diametral pitch	(7) ÷ (8)	–		10.160	
		Pinion	**Gear**	**Pinion**	**Gear**
7. Number of teeth	Table 5.20	32	98	32	98
8. Pitch diameter	Section 4.2.5	80	245.00	3.1496	9.6456
9. Pitch angle	Table 5.24	18.08	71.92	18.08	71.92
10. Outer cone distance	Table 5.24	128.87	128.87	5.074	5.074
11. Face width	Section 5.1.17	32	32	1.260	1.260
12. Addendum	Table 5.21	3.53	1.47	1.1390	0.0579
13. Whole depth	Table 5.19	5.47	5.47	0.2154	0.2154
14. Dedendum angle	Table 5.24	1.37	3.28	1.37	3.28
15. Face angle	Table 5.24	21.37	73.28	21.37	73.28
16. Root angle	Table 5.24	16.72	68.63	16.72	68.63
17. Outside diameter	Table 5.24	86.71	245.91	3.414	9.681
18. Pitch apex to crown	Table 5.24	121.41	38.60	4.780	1.520
19. Mean circular tooth thickness	Section 5.1.17	3.92	2.86	0.1543	0.1126
20. Backlash	Section 5.1.17, Table 5.22	0.05–0.10	0.05–0.10	0.002–004	0.002–004
21. Cutter diameter	Table B.14	152.4	152.4	6.00	6.00

Note: Metric dimensions are in millimeters, English dimensions are in inches, and angles are in degrees. For more complete data, see the Gleason dimension sheet in Table B.14.

$$B = \frac{5\sqrt{d \sin \Gamma}}{N_c F} \quad \text{(English).} \tag{5.51}$$

The value of C is also the same regardless of pressure angle:

$$C = 5.918 \div (z \times m_t) \quad \text{(metric),} \tag{5.52}$$

$$C = (0.2333\, P_d) \div N_c \quad \text{(English).} \tag{5.53}$$

Zerol bevel gears are quite frequently used in critical instrument work. For this kind of work, it is frequently necessary to use almost no backlash. If the gears are to be used for general-purpose work, Table 5.22 may be used as a design guide for backlash.

The dedendum angles shown in Table 5.26 are made to suit the *Duplex* method of cutting Zerol bevel gears. This is a rapid method of cutting which allows both pinion and gear to be cut spread blade (both sides of a tooth space are simultaneously nished). Present machine capacity limits the use of Duplex method to cut gears approximately 2.5 module (10 pitch) and ner and to ground gears approximately 4 module (6 pitch) and ner. The minimum number of pinion teeth is 13.

Table 5.26 is for use only when the shaft angle is 90°. Angular Zerol bevel gears require special calculations. The face width of Zerol bevel gears should not exceed 25% of the outer cone distance.

5.1.18 Hypoid Gear Calculations

The hypoid gear quite closely resembles a spiral bevel gear in appearance. The major difference is that the pinion axis is offset above or below the gear axis. In the regular bevel gear family, the axis of the pinion and that of the gear always intersect. The hypoid type of gear does not have intersecting axes. Since this kind of gear is basically different from the bevel gear, hypoid gears are not ordinarily called *hypoid bevel gears*. Instead, they are just called *hypoid gears*. From a manufacturing standpoint, though, they are cut or ground with the same kinds of machinery that are used to make spiral bevel gears.

Hypoid gears can be more readily designed for low numbers of pinion teeth and high ratios than can spiral bevel gears. Gleason Works recommends the following minimum numbers of teeth for Formate* pinions:

Ratio	Minimum Number of Pinion Teeth
2½	15
3	12
4	9
5	7
6	6
10	5

* Formate pinions are pinions which are matched to run with formate gears. Formate gears are cut without any generating action.

TABLE 5.26
Tooth Proportions and Diameter Constants for Face Gears, 20° Pressure Angle

No. of Pinion Teeth z_1 (N_P)	Pinion Addendum h_{a1} (a_P)	Pinion Tooth Thickness s_1 (t_P)	Gear Addendum h_{a2} (a_G)	Gear Diameter Constants for Gear Ratios: Ratio = 1.5		Ratio = 2.0		Ratio = 4.0		Ratio = 8.0	
				ε_o (m_o)	ε_i (m_i)	ε_o (m_o)	ε_i (m_i)	ε_o (m_o)	ε_i (m_i)	ε_o (m_o)	ε_i (m_i)
12	1.120	1.790	0.700	–	–	1.221	1.020	1.221	0.960	1.221	0.945
13	1.100	1.745	0.760	1.202	1.064	1.202	1.015	1.202	0.959	1.202	0.945
14	1.080	1.700	0.820	1.187	1.062	1.187	1.011	1.187	0.958	1.187	0.944
15	1.060	1.660	0.880	1.174	1.052	1.174	1.007	1.174	0.957	1.174	0.944
16	1.040	1.620	0.940	1.161	1.051	1.161	1.004	1.161	0.956	1.161	0.944
17	1.020	1.580	0.980	1.156	1.041	1.156	1.000	1.156	0.955	1.156	0.944
18	1.000	1.570	1.000	1.150	1.039	1.150	0.997	1.150	0.954	1.150	0.943
	1.250	1.752	0.750	1.176	1.042	1.176	0.999	1.176	0.955	1.176	0.943
20	1.000	1.570	1.000	1.144	1.030	1.144	0.991	1.144	0.953	1.144	0.943
	1.250	1.752	0.750	1.166	1.032	1.166	0.993	1.166	0.953	1.166	0.943
22	1.000	1.570	1.000	1.140	1.022	1.140	0.987	1.140	0.952	1.140	0.943
	1.200	1.715	0.800	1.156	1.024	1.156	0.988	1.156	0.952	1.156	0.943
24	1.000	1.570	1.000	1.133	1.015	1.133	0.983	1.133	0.951	1.133	0.942
	1.200	1.715	0.800	1.150	1.017	1.150	0.984	1.150	0.951	1.150	0.943
30	1.000	1.570	1.000	1.121	1.001	1.121	0.975	1.121	0.949	1.121	0.942
	1.150	1.680	0.850	1.131	1.001	1.131	0.975	1.131	0.949	1.131	0.942
40	1.000	1.570	1.000	1.109	0.986	1.109	0.966	1.109	0.946	1.109	0.941
	1.100	1.640	0.900	1.113	0.986	1.113	0.966	1.113	0.946	1.113	0.941

Note: For metric design, the dimensions are in millimeters and are for 1 module. For English design, the dimensions are in inches and are for 1 diametral pitch.

The calculations involved in designing a set of hypoid gears are quite long. A calculation sheet of about 150 items must be calculated to get all the needed answers. About 45 items on the sheet must be worked through on a trial basis, then repeated two or three times until the assumed and calculated values check. With the advent of computers, hypoid gear calculations became much easier, since they could be programmed.

In view of the large amount of material needed to explain hypoid gear calculations, it is not possible to present the method here. Those interested in obtaining hypoid gear designs should either consult a manufacturer of hypoid gears or study the hypoid gear design information published by Gleason Works. A practical approach for most jobs is to get a hypoid dimension sheet made by Gleason Works.

5.1.19 Face Gear Calculations

The pinion of a face gearset can be designed in just the same manner as a spur or a helical pinion would be designed. The pinion should have enough addenda to avoid being undercut. Equations 5.3 and 5.4 may be used to check a design to see if it is in danger of undercutting. The meshing action of the face gear with the pinion is somewhat similar to that of a rack meshing with the pinion. The involute profile of the pinion should be accurately finished to a deep enough depth to permit the pinion to mesh with a rack.

One of the principal design problems in face gears is to calculate the face width. Since the face gear teeth run across the end of a cylindrical blank instead of across the outside diameter, the length of the tooth sets an inside diameter and an outside diameter. The face gear tooth changes its shape as you move lengthwise along the tooth. The minimum inside diameter is determined by the point at which the undercut portion of the gear profile extends to about the middle of the tooth height. The maximum outside diameter is established by the point at which the top land of the tooth narrows to a knife-edge. In general, it is a good design practice to make the face width of the gear somewhat shorter than these two extremes.

Table 5.27 shows some recommended proportions for spur face gears of 20° pressure angle. A ratio of less than 1.5 is not generally recommended. The diameter constants are used to calculate the limiting outside and inside diameters of the gear as follows:

$$d_{a2} = \varepsilon_o z_2 m_t \quad \text{(metric)}, \tag{5.54}$$

where $\varepsilon_o = m_o$

$$D_o = \frac{m_o N_G}{P_d} \quad \text{(English)}, \tag{5.55}$$

$$d_{i2} = \varepsilon_i z_2 m_t \quad \text{(metric)}, \tag{5.56}$$

where $\varepsilon_i = m_i$

TABLE 5.27
Tooth Proportions for Crossed-Helical Gears

Helix Angle		Normal Pressure			
Driver, β_1 (ψ_1)	Driven, β_2 (ψ_2)	Angle, α_n (ϕ_n)	Addendum, h_a (a)	Working Depth h' (h_k)	Whole Depth h (h_t)
45°	45°	14°30′	1.200	2.400	2.650
60°	30°	17°30′	1.200	2.400	2.650
75°	15°	19°30′	1.200	2.400	2.650
86°	4°	20°	1.200	2.400	2.650

Note: The addendum, working depth, and whole depth values are for 1 normal module (metric) in millimeters or for 1 diametral pitch (English) in inches. Normal module $m_n = 1$ and normal circular pitch $p_n = 3.14159$ mm (normal diametral pitch $P_{nd} = 1$; normal circular pitch $p_n = 3.14159$ in.).

$$D_i = \frac{m_i N_G}{P_d} \quad \text{(English).} \tag{5.57}$$

In Equations 5.54 through 5.57, z_2 and N_G are the number of face gear teeth. The constants m_o and m_i may be read from Table 5.27.

The maximum usable face width of the gear is as follows:

$$b_2 = \frac{d_{a2} - d_{i2}}{2} \quad \text{(metric),} \tag{5.58}$$

$$F_G = \frac{D_o - D_i}{2} \quad \text{(English).} \tag{5.59}$$

The face width of the pinion is usually made a little greater than that of the gear to allow for error in axial positioning of the pinion.

The face gear is cut with a pinion-shaped cutter that has either the same number of teeth as the pinion or just slightly more. If the cutter has slightly more teeth, the gear tooth will be slightly crowned, and the contact will be heavy in the center of the face width, with little or no contact at the ends of the teeth. The face gear is never cut with a cutter that has a smaller number of teeth than that of the pinion. If it were, contact would be heavy at the ends and hollow in the center—a very unsatisfactory condition.

When the pinion of a face gearset has less than 17 teeth, it is necessary to reduce the cutter outside diameter to keep the top land of the cutter from becoming too narrow. If the land is too narrow, the cutter will wear too fast at the tip. It should be noted that the combinations shown in Table 5.27 have a working depth of less than 2.000 when the pinion has less than 17 teeth. Also, the pinion tooth thickness is increased more than what might be expected for the amount of long addendum. These things are done to make it possible to design cutters for the gear with a reasonable top land.

The whole depth of the face gear is

$$h = h_{a1} + h_{a2} + c \quad \text{(metric),} \tag{5.60}$$

$$h_t = a_P + a_g + c \quad \text{(English),} \tag{5.61}$$

where clearance is designated by c. A reasonable value for gear clearance is $0.25m_t$ (or $0.25/P_d$).

The pinion may be made to the same whole depth as that given in Table 5.6, or the depth may be adjusted to agree with the gear. The choice usually depends on what tools are on hand to cut the pinion.

The face gear tooth thickness can be obtained by subtracting the pinion thickness plus the backlash from the circular pitch. Since the gear tooth is tapered, this value does not mean much. The size of the face gear teeth is usually controlled by measuring backlash when the face gear is assembled in a test fixture with a master pinion.

The profile of the face gear cannot be measured in presently available checking machines. The shape of the tooth can be checked only by contacting the tooth with the tooth of a mating pinion. The shape of the pinion tooth *can* be checked in an involute machine. In figuring the roll angle of the pinion at the form diameter, Table 5.12 does not apply. Instead, the roll angle at the limit diameter is

$$\theta_{rl} = \theta_{r1} - \frac{360\, h_{a2}}{\pi\, d_{pl} \sin\alpha_t \cos\alpha_t}, \quad \text{(degrees; metric),} \tag{5.62}$$

$$\varepsilon_{rl} = \varepsilon_{rP} - \frac{360\, a_G}{\pi\, d \sin\phi_t \cos\phi_t} \quad \text{(degrees; English).} \tag{5.63}$$

After the pinion roll angle of the limit diameter is obtained, the pinion form diameter and the roll angle of the form diameter may be obtained in the usual manner.

The top and bottom of the face gear tooth are defined by axial dimensions. The addendum and the whole-depth dimensions of the gear are used in figuring these axial dimensions.

5.1.20 Crossed-Helical Gear Proportions

The elements of a crossed-helical gear are the same as those of a parallel-helical gear. The bigger difference in the calculations is that the crossed-helical gears meshing together may not have the same pitch, the same pressure angle, or the same helix angle. The only dimensions that are necessarily the same

for both members are those in the normal section. By comparison, a mating parallel-helical gear and pinion have the same dimensions in both the transverse and normal planes.

Most designers favor making crossed-helical gears with deeper teeth and lower pressure angles than parallel-helical gears. This is done to get a contact ratio equal to or greater than 2. Since the crossed-helical gear has only point contact instead of contact across a face width, its load-carrying capacity and its smoothness of running are much improved by having a design, which gives a contact ratio of 2 or better. When the ratio is this high, there will be either two or three pairs of teeth in contact at every instant of time as the teeth roll through mesh. If the contact ratio were less than 2, there would be one interval of time when only a single pair of teeth would be carrying the load.

There is no trade standard for crossed-helical gear- tooth proportions. Many manufacturers use crossed-helical gears only occasionally. To save tool cost, they frequently design and cut crossed-helical gears with the same hobs or other tools used for parallel-helical gears. This practice will usually give a workable design, but it will not have the load-carrying capacity and the smoothness of running that can be obtained with tooth proportions designed for crossed-helical gears.

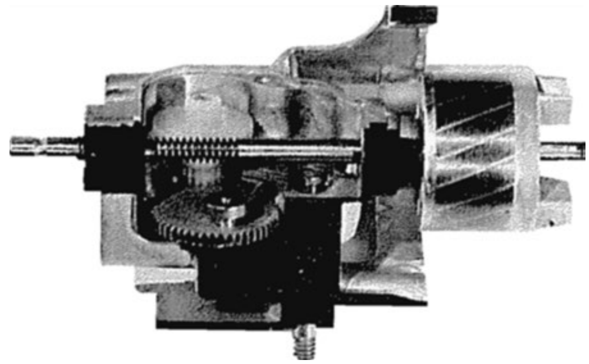


FIGURE 5.22 Crossed-helical gears in a small mechanism. The driver is steel and the driven gear is nylon.

Table 5.28 shows a recommended set of proportions for the design of crossed-helical gears. These proportions have been used with good results in many high-speed applications.

Figure 5.22 shows an example of crossed-helical gears in a small mechanism. Since the crossed-helical gear is not critical on center-distance accuracy or on axial position, it is very handy to use when the power is low and the costs are critical.

The proportions shown in Table 5.28 will give a contact ratio greater than 2 for all combinations of teeth where

TABLE 5.28
Crossed Helical Gear Dimensions

Item	Reference	Metric		English	
1. Center distance	$[(9)_{driver} + (9)_{driven}]/2$	100.725		3.9655	
2. Normal circular pitch	Design choice (Note 4)	7.853982		0.309212	
3. Normal pressure angle	Table 5.28	17.50		17.50	
4. Working depth	Table 5.28	6.00		0.236	
5. Module, normal	(2) ÷ 3.14159265	2.500		–	
6. Normal diametral pitch	3.14159265 ÷ (2)	–		10.160	
		Driver	**Driven**	**Driver**	**Driven**
7. Number of teeth	Section 5.1.20	12	49	12	49
8. Helix angle	Equations 1.23 and 1.28	60° RH	30° RH	60° RH	30° RH
9. Pitch diameter	(7) × (2) ÷ [π × cos(8)]	60.00	141.451	2.3622	5.5689
10. Addendum	Table 5.28	3.00	3.00	0.118	0.118
11. Whole depth	Table 5.28	6.63	6.63	0.261	0.261
12. Outside diameter	(9) + 2.0 × (10)	66.00	147.45	2.598	5.805
13. Root diameter	(12) − 2.0 × (11)	52.74	134.19	2.076	5.283
14. Normal tooth thickness	Sec. 5.1.6	3.88	3.88	0.1525	0.1525
15. Chordal tooth thickness	Equations 5.9 and 5.10	3.88	3.88	0.1525	0.1525
16. Chordal addendum	Equations 5.7 and 5.8	3.02	3.02	0.1188	0.1188
17. Face width	Section 5.1.20	27	19	1.0625	0.75
18. Lead	(2) × (7) ÷ sin(8)	108.828	769.690	4.2846	30.3028
19. Base circle diameter	See Note 2	48.0508	130.4024	1.9098	5.1339
20. Accuracy	Section 5.1.18	See Note 5			

Note: 1. Metric dimensions are in millimeters, English dimensions are in inches, and angles are in degrees.
2. Base circle diameter = cos(transverse pressure angle) × (9), and tan(transverse pressure angle) = tan (3) ÷ cos(8).
3. The shaft angle is assumed to be 90° for this table, and the helix angles are assumed to be of the same hand for both members of the set.
4. After a preliminary size is chosen, a rating estimate for the set may be made with the help of Section. 5.2.7 in this chapter.
5. After the designer has done the load rating and studied Section 10.4.1, accuracy needs may be satis ed by an ISO or an AGMA quality level, or the designer may need to write a special recipe for accuracy limits.

undercut is not present. The minimum number of driven teeth needed to avoid undercut is 20. This same number holds for all four combinations shown in the table.

The minimum numbers of driver teeth needed to avoid undercut are as follows:

Helix Angle of Driver	Minimum Number of Teeth
45°	20
60°	9
75°	4
86°	1

One of the first two combinations shown in Table 5.28 should be used when both members are to be either shaved or hobbed. Both these operations are difficult to perform when the helix angle is above 60°. When very high ratios are needed or there is need for a large-diameter driver, the last two combinations become attractive.

The whole depth shown does not allow for a very large root fillet radius. In crossed-helical gears, root stresses are not high, and so a full fillet radius is not needed. Long and short addenda are sometimes used in crossed-helical gear design, but there is not much need for it. Since the strength of the teeth is not a problem in most cases, there is no need to juggle addenda either to balance strength between two members or to avoid undercut and permit the use of a smaller number of driver teeth and a resultant coarser pitch.

Table 5.29 shows a dimension sheet and a sample problem for calculating crossed-helical gears. It is based on a 90° shaft angle, and helix angles of like hand for each member.

The exact face width needed for crossed-helical gears depends on how far the point of contact moves as the teeth roll through the mesh. When the set is new, the arc of action will be a little longer than two normal circular pitches. After the set has worn in, there will be meshing action over about three or slightly more normal circular pitches.

Table 5.29 shows the pinion face width to be equivalent to a projection of three normal circular pitches plus an increment. The added increment keeps contact away from the ends of the tooth and allows for error in axial positioning and variation in the amount of wear. For 2.5 normal module (10 pitch), this increment should be at least 6.3 mm (¼ in.).

5.1.21 Single-Enveloping Worm Gear Proportions

Like crossed-helical gearing, worm gearing seldom has two mating members with the same module (diametral pitch). The axial pitch of the worm is equal to the circular pitch of the gear. Also, both members have the *same* normal circular pitch.

In the past, it has been quite common practice to make the tooth proportions a function of the circular pitch (or axial pitch). An addendum of one m_n (P_d)—which is standard in many kinds of gears—is equivalent to 0.3183 p_n, where p_n is the normal circular pitch in either millimeters or inches.

With low lead angles and only one or two worm threads, it has been quite customary to use an addendum of 0.3183 p_n, a working depth of 0.6366 p_n, and a whole depth equal to the working depth plus 0.050 p_n.

With multiple threads and high lead angles, it is necessary to use quite high pressure angles. It is necessary to shorten the addendum and the working depth to avoid getting sharp-pointed teeth and to keep out of undercut trouble. If the tooth proportions are based on the circular pitch of the gear, a high lead angle like 45° may require that the addendum and the working depth be cut back to about 70% of the values mentioned earlier. However, if the tooth proportions are based on the normal circular pitch, the addendum will stay in about the right proportion to the lead angle. For instance, 0.3183 p_n at a 45° lead angle is equal to 0.2250 p.

In the fine-pitch field, fine-pitch worm gearing is discussed in considerable detail in AGMA standards. The standards base the proportions of fine-pitch worm gears on the normal circular pitch. There are generally no accepted trade standards for the proportions of medium-pitch worm gears. From a practical standpoint, it is best to base the proportions of all sizes of worm gears on the normal circular pitch.

Table 5.30 shows recommended proportions for three general kinds of applications. The fine-pitch design is the same as that given in AGMA standards. The design for index and holding mechanisms represents average shop practice.

TABLE 5.29
Tooth Proportions for Single-Enveloping Worm Gears

Item	No. of Worm Threads, z_1 (N_w)	Cutter Pressure Angle, α_0 (ϕ_c)	Addendum, h_a (a)	Working Depth, h' (h_k)	Whole Depth, h (h_t)
Index or holding mechanism	1 or 2	14°30′	0.3183 p_n	0.6366 p_n	0.7000 p_n
Power gearing	1 or 2	20°	0.3183 p_n	0.6366 p_n	0.7000 p_n
	3 or more	25°	0.2860 p_n	0.5720 p_n	0.6350 p_n
Fine-pitch (instrument)	1–10	20°	0.3183 p_n	0.6366 p_n	0.7003 p_n + 0.05[a]

Note: The addendum, the working depth, and the whole depth values are for 1 normal module (metric) in millimeters or for 1 normal diametral pitch (English) in inches.

[a] 0.05 is for metric system. Add 0.002 in. for English system.

TABLE 5.30
Single-Enveloping Worm Gear Dimensions

Item	Reference	Metric		English	
1. Center distance	Section 4.2.6	249.565		9.82537	
2. Axial pitch of worm	$\pi \times (10)_G \div (9)_G$	25.00		0.98425	
3. Cutter pressure angle	Table 5.30	25°		25°	
4. Worm lead angle	Equations 5.70 through 5.75	25°		25°	
5. Working depth	Table 5.30	14.42		0.568	
6. Module, transverse	(2) ÷ 3.141593	7.95775		–	
7. Diametral pitch, transverse	3.141593 ÷ (2)	–		3.191858	
8. Normal circular pitch	(2) × cos(4)	22.6577		0.892035	
		Worm	**Gear**	**Worm**	**Gear**
9. Number of teeth	Section 5.1.22	5	52	5	52
10. Pitch diameter	Section 5.1.22	85.327	413.803	3.3593	16.2914
11. Addendum	Table 5.30	7.21	7.21	0.284	0.284
12. Whole depth	Table 5.30	15.86	15.86	0.625	0.625
13. Outside diameter	(10) + 2.0 × (11)	99.75	428.22	3.927	16.859
14. Maximum outside diameter, gear	Section 5.1.22	–	435.35	–	17.140
15. Root diameter	(13) − 2.0 × (12)	68.03	396.50	2.677	15.609
16. Normal tooth thickness	Section 5.1.22	10.88	11.32	0.428	0.446
17. Chordal addendum	Equations 5.7 and 5.8	7.27	7.27	0.2865	0.2865
18. Face width	Section 5.1.22	115.0	54.20	4.52	2.13
19. Pressure angle change	Equations 5.66 and 5.67	0.5559	–	0.5559	–
20. Normal pressure angle	Equations 5.64 and 5.65	24.4441	–	24.4441	–
21. Tool to produce the worm. Straight-sided milling cutter. 150 mm (5.905 in.) diameter					
22. Accuracy		See Note 3			

Note: 1. The helix angle of the worm gear is the same as the lead angle of the worm when the shaft angle is 90°.
2. If the worm is to be hardened and ground, then the tool de nition in (21) should state the diameter of the grinding wheel and the form of grinding wheel.
3. There are some individual company standards for the accuracy of worm gearing, but there are no generally recognized trade standards.
4. Metric data is in millimeters, English data is in inches, and all angles are in degrees.

5.1.22 Single-Enveloping Worm Gears

Low-lead-angle worm gearsets are frequently self-locking. This means that the worm cannot be driven by the gear. The set will hold when the worm has no power applied to it.

The exact lead angle at which a worm will be self-locking depends on variables like the surface nish, the kind of lubrications, and the amount of vibration where the drive is installed. Generally speaking, though, self-locking occurs if the lead angle is below 6°, and it may occur with as much as 10° lead angle.

The pressure angle of the tool shown in Table 5.30 is the pressure angle of a straight-sided conical milling or grinding wheel used to nish the worm threads. The normal pressure angle of the worm is a small amount less than this value. The equations for the normal pressure angle are as follows:

$$\alpha_n = \alpha_0 - \Delta\alpha \quad \text{(metric)}, \tag{5.64}$$

$$\phi_n = \phi_c - \Delta\phi \quad \text{(English)}, \tag{5.65}$$

where

$$\Delta\alpha = \frac{90 d_{p1} \sin^3 \gamma}{z_1 (d_{p0} \cos^2 \gamma + d_{p1})}, \tag{5.66}$$

$$\Delta\phi = \frac{90 d \sin^3 \lambda}{N_w (d_c \cos^2 \lambda + d)}, \tag{5.67}$$

where
γ (λ)—lead angle
d_{p1} (d)—the worm pitch diameter
d_{p0} (d_c)—the cutter pitch diameter

The angle $\Delta\alpha$ ($\Delta\phi$) is in degrees.

Table 5.30 allows more clearance than the old gure of 0.050 *p*. This is in line with the practice of putting more generous tip radii on tools.

The pitch diameter of the worm usually represents a compromise among several considerations. If the worm is small

compared with the gear, the lead angle will be high and the ef ciency will be good. However, the face width of the gear will be small, and there may be trouble getting the bearing close enough together to prevent bending of the small worm. A large worm compared with the gear gives lower ef ciency, but it may be possible to use a large enough bore in the worm to permit keying it on its shaft instead of making it integral with the shaft.

An approximate value for the worm mean diameter is as follows:

$$d_{p1} = \frac{a^{0.875}}{2.2} \quad \text{(metric)}, \tag{5.68}$$

$$d = \frac{C^{0.875}}{2.2} \quad \text{(English)}. \tag{5.69}$$

It is believed that this value gives a good practical size for the worm, considering all the factors mentioned. Of course, if the designer is not primarily interested in the ef -cient transmission of power, it is possible to widely depart from Equations 5.68 and 5.69 and still get satisfactory operation. The worm pitch diameter may range from $a^{0.875}/1.7$ to $a^{0.875}/3.0$ ($C^{0.875}/1.7$ to $C^{0.875}/3.0$) without substantial effect on the power capacity.

If the worm addendum is equal to the gear addendum, the mean worm diameter is actually the pitch diameter.

In the ne-pitch eld, the ef ciency and the power-transmitting ability of a given size set are usually not too important. For this reason, no particular effort has been made to proportion the worm pitch diameter to equations like Equations 5.68 and 5.69. A series of standard axial pitches and a series of standard lead angles are listed in AGMA standards. Indirectly, this results in a series of standard pitch diameters. The axial pitches range from 0.75 to 4.0 mm (0.03 to 0.16 in.), and the lead angles range from 30′ to 30°. A considerable amount of tabulated data is given in the standard for each of the combinations shown.

The lead angle of the worm is equal to the helix angle of the worm gear when the shaft angle is 90°. The lead angle may be calculated from any of the following relations:

$$\tan\gamma = \frac{d_{p2}}{u d_{p1}} \quad \text{(metric)}, \tag{5.70}$$

$$\tan\gamma = \frac{p_x z_1}{\pi d_{p1}} \quad \text{(metric)}, \tag{5.71}$$

$$\sin\gamma = \frac{p_n z_1}{\pi d_{p1}} \quad \text{(metric)}, \tag{5.72}$$

$$\tan\lambda = \frac{D}{m_G d} \quad \text{(English)}, \tag{5.73}$$

$$\tan\lambda = \frac{p_x N_W}{\pi d} \quad \text{(English)}, \tag{5.74}$$

$$\sin\lambda = \frac{p_n N_W}{\pi d} \quad \text{(English)}. \tag{5.75}$$

In Equations 5.70 through 5.75, p_x is the axial pitch and $u(m_G)$ is the number of gear teeth divided by the number of worm threads.

It is wise to not use more than 6° lead angle per thread. For instance, if the lead angle is 30°, there should be at least ve threads of the worm. If too few threads are used, the problem of designing tools and producing accurate curvatures on the worm threads and on the gear teeth becomes too critical for good manufacturing practice.

In general, the number of teeth on the gear should be less than 29. As an exception, the number can be reduced to 20 when the cutter pressure angle is 25° and the lead angle does not exceed 15°. The number of teeth on the gear and the number of threads on the worm should be picked to get a hunting ratio. This is particularly important in worm gearing, because the hob for generating the gear should have the same number of threads as the worm.

Different designers use several different means to get the worm gear face width and the worm gear outside diameter. A simple method which is always safe and which will practically utilize all the worm gear tooth that is worth using is to make the face width equal to (or just slightly greater than) the length of a tangent to the worm pitch circle between the points at which it is intersected by the worm outside diameter. The worm gear maximum diameter is made just large enough to have about 60% of the face width throated. This design is shown as design *A* in Figure 5.23. Design *B* shows an alternative that has been widely used for power gearing. Design *C* shows a nonthroated design which may be used for instrument or other applications in which maximum power transmission is not an important design consideration.

The pro le of the worm will have a slight convex curvature when it is produced by a straight-sided milling cutter or grinding wheel. Section 11.4 shows how to calculate the normal section curvature of the worm.

In worm gearing, it has been customary to obtain backlash by thinning the worm threads only. The worm gear teeth are given a design thickness equal to half the normal circular pitch (when equal addendums are used for the worm and the worm gear). Generally speaking, worm gears require more backlash than spur or helical gears do. Quite often, a steel worm and a bronze gear are housed in a cast-iron casing. The temperature of the set may change quite considerably during operation because of the high sliding velocity of the worm threads. Different expansions can easily cause an appreciable change in backlash. Except for slow-speed control gears, the design should provide enough backlash to keep the gears from binding at any speed or temperature condition under which the set may have to operate.

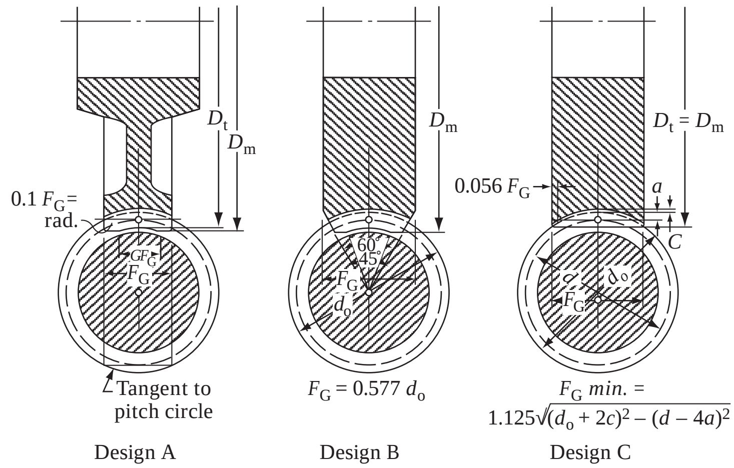


FIGURE 5.23 Worm gear design examples. (Dimensions shown have English system symbols.)

TABLE 5.31
Tooth Proportions for Double-Enveloping Worm Gears

Normal Pressure Angle α_n (ϕ_n)	Addendum h_a (a)	Working Depth h' (h_k)	Whole Depth h (h_t)
20°	0.225 p_n	0.450 p_n	0.500 p_n

Table 5.31 shows a dimension sheet for single-enveloping worm gears and the solution of a sample problem.

5.1.23 Double-Enveloping Worm Gears

Although several types of double-enveloping worm gears are mentioned in gear literature, one type is presently much more widely used in the industry than any other type. This is the Cone Drive gear, produced by Cone Drive Operations, a division of Ex-Cell-O Corporation. The material in this section is based on the Cone Drive design of double-enveloping worm gear. Other types of double-enveloping worm gears are in use, but the Cone Drive type is produced in such large quantities in the United States that it seems most appropriate to cover only this type in this book.

Table 5.32 shows the proportions recommended by Cone Drive for double-enveloping worm gears.

The pitch diameter of the worm should be approximately $a^{0.875}/2.2$ ($C^{0.875}/2.2$). This is the same value as that given by Equations 5.68 and 5.69. The equation for worm root diameter is as follows:

$$d_{f1} = d_{p1} + 2h_{a1} - 2h_1 \quad \text{(metric)}, \tag{5.76}$$

$$d_R = d + 2a_P - 2h_{tP} \quad \text{(English)}. \tag{5.77}$$

If this value comes out less to be than $a^{0.875}/3$ ($C^{0.875}/3$), the worm pitch diameter should be increased so that the root diameter is not less than this value.

In picking numbers of threads, the designer should not have more than about 6° per thread lead angle for sets of 125 mm (5 in.) or less center distance.

The number of gear teeth should be picked so as to give a hunting ratio if possible. This is not quite as important as with single-enveloping worm gears. It is further recommended that the relation of the number of gear teeth to the center distance be as follows:

Center Distance		
mm	**in.**	**Number of Gear Teeth**
50	2	24–40
150	6	30–50
300	12	40–60
600	24	60–80

The axial pressure angle of any kind or worm is determined by the following relation:

$$\tan \alpha_x = \frac{\tan \alpha_n}{\cos \gamma} \quad \text{(metric)}, \tag{5.78}$$

$$\tan \phi_x = \frac{\tan \phi_n}{\cos \lambda} \quad \text{(English)}. \tag{5.79}$$

For double-enveloping worm gears, the lead angle which is used is at *the center* of the worm.

The base-circle diameter of the double-enveloping worm gear can be determined by layout, or it can be calculated by

$$d_{b2} = d_{p2} \sin(\alpha_x + \Delta\alpha) \quad \text{(metric)}, \tag{5.80}$$

$$D_b = D \sin(\phi_x + \beta) \quad \text{(English)}, \tag{5.81}$$

where $\sin \Delta\alpha = p_x/2d_{p2}$ (metric) or $\sin \beta = p_x/2D$ (English).

TABLE 5.32
Double-Enveloping Worm Gear Dimension

Item	Reference	Metric		English	
1. Center distance	Section 4.2.6	253.558		9.9826	
2. Axial pitch of worm[a]	$\pi \times (10)_G \div (9)_G$	25.40		1.000	
3. Normal pressure angle	Table 5.32	20		20	
4. Worm lead angle[a]	Equations 5.70 through 5.75	25		25	
5. Working depth	Table 5.32	10.36		0.406	
6. Module, transverse	(2) ÷ π	8.0851		–	
7. Diametral pitch, transverse	π ÷ (2)	–		3.141593	
8. Normal circular pitch	(2) × cos(4)	23.020		0.906308	
		Worm	**Gear**	**Worm**	**Gear**
9. Number of teeth	Section 5.1.23	5	52	5	52
10. Pitch diameter	Section 5.1.23	86.69	420.42	3.413	16.552
11. Addendum	Table 5.32	5.18	5.18	0.203	0.203
12. Whole depth	Table 5.32	11.51	11.51	0.453	0.453
13. Outside diameter	From layout	115.57	445.64	4.550	17.545
14. Throat diameter	(10) + 2 × (11)	97.05	430.78	3.819	16.958
15. Root diameter	Equations 5.76 and 5.77	74.03	–	2.913	–
16. Normal tooth thickness	Section 5.1.23	8.99	13.82	0.354	0.544
17. Axial pressure angle	Equations 5.78 and 5.79	21.88020	–	21.88020	–
18. Base circle diameter	Equations 5.80 and 5.81	–	168.39	–	6.63
19. Face width	Section 5.1.23	160.3	66.7	6.3125	2.625
20. Face angle	From layout	45	87.5	45	87.5
21. Accuracy		See Note 3			

Note: 1. The helix angle of the worm gear is the same as the lead angle of the worm when the shaft angle is 90°.
2. Metric data are in millimeters, English data are in inches, and all angles are in degrees.
3. There are no trade standards for accuracy of double-enveloping worm gears. Cone Drive can be consulted for appropriate accuracy limits.

[a] At the center of the worm.

The active part of the worm face width should almost be equal to the base-circle diameter. Usually, the face width of the worm is made a little shorter. Thus,

$$b_1 = d_{b2} - 0.03a \quad \text{(metric)}, \tag{5.82}$$

$$F_w = D_b - 0.03C \quad \text{(English)}. \tag{5.83}$$

In general, the face width of the gear is made slightly less than the root diameter of the worm.

The thicknesses of the worm threads and gear teeth are controlled by side-feeding operations. It is customary to make the worm thread be equal to 45% of the axial circular pitch and the gear tooth be equal to 55% of the normal circular pitch, provided that the tool design will permit and gear strength is critical. Backlash is subtracted from the worm thread thickness. For average applications, the following amounts of backlash are reasonable:

Center Distance		Backlash	
mm	**in.**	**mm**	**in.**
50	2	0.08–0.20	0.003–0.008
150	6	0.15–0.30	0.006–0.012
300	12	0.30–0.50	0.012–0.020
600	24	0.45–0.75	0.018–0.030

Table 5.33 is a dimension sheet for calculating double-enveloping worm gears. The solution of a sample problem is shown on the dimension sheet.

5.2 GEAR-RATING PRACTICE

After the gear tooth data have been calculated, it is necessary to calculate the capacity of the gearset. Since the design was started from an estimate (see Chapter 4), it may happen that the first design which is worked out in detail is too small or too large.

Once all the gear tooth data have been calculated, it is possible to use design formulas to determine a *rated* capacity of the gearset. This rated capacity should be larger than the *actual* load which will be applied to the gearset.

In some field of gearing, there are well-established trade standards in regard to the rating of gearsets. Sometimes these are quite conservative. In a few unusual cases, a trade standard may rate a gearset as able to carry more load than it will really carry. It must be recognized that a trade standard is based on the general level of quality that representative members of the industry can produce. This quality includes the accuracy of the gear teeth, the accuracy with which the gears are positioned in their casings, and the quality of the materials from which the gears are made. Manufacturers who

TABLE 5.33
Gear Terms, Symbols, and Units Used in Load Rating of Gears

	Metric		English		
Term	**Symbol**	**Units**	**Symbol**	**Units**	**Reference or Formula**
Module, transverse	m or m_t	mm	–	–	Metric tooth size
Module, normal	m_n	mm	–	–	$m_n = 25.5/P_{nd}$
Diametral pitch, transverse	–	–	P_d or P_t	in.$^{-1}$	English tooth size
Diametral pitch, normal	–	–	P_{nd}	in.$^{-1}$	$P_{nd} = 25.5/m_n$
Center distance	a	mm	C	in.	Figure 4.14
Face width	b	mm	F	in.	Figure 1.18
Face width, effective	b'	mm	F_e	in.	Equations 5.123 and 5.124
Pitch diameter	d_p	mm	d	in.	Figure 4.10
of pinion	d_{p1}	mm	d	in.	Equation 1.3
of gear	d_{p2}	mm	D	in.	Equations 1.4 and 1.13
of driver (crossed-helical)	d_{p1}	mm	D_1	in.	Equation 5.118
of driven (crossed-helical)	d_{p2}	mm	D_2	in.	
Throat diameter, worm	d_{t1}	mm	d_1	in.	Equations 5.133 and 5.134
Ratio (tooth)	u	–	m_G	–	No. gear teeth ÷ No. pinion teeth
Pressure angle, normal	α_n	deg	ϕ_n	deg	Equation 5.118
Pressure angle, transverse	α_t	deg	ϕ_t	deg	Figure 1.18
Helix or spiral angle	β	deg	ψ	deg	Figure 1.19
Lead angle	γ	deg	λ	deg	Figure 1.19
Rotational speed, pinion or worm	n_1	rpm	n_P or n_W	rpm	Equations 5.84 through 5.86
Rotational speed, gear	n_2	rpm	n_G	rpm	Equations 5.127 and 5.128
Sliding velocity	v_s	m/s	v_s	fpm, fps	Equations 5.122, 5.125, 5.126, 5.133, and 5.134
Power	P	kW	P	hp	Equations 5.84 through 5.86
Torque	T	N · m	T	in. lb	Equations 5.84 through 5.86
Torque on pinion	T_1	N · m	T_P	in. lb	Equations 5.84 through 5.86
Tangential load or force	W_t	N	W_t	lb	Equations 5.87, 5.88, 5.125, and 5.126
Dynamic load	W_d	N	W_d	lb	Equation 5.113
in normal plane	W_n	N	W_n	lb	Equation 5.121
Unit load	U_l	N/mm²	U_l	psi	Equations 5.93, 5.94, 5.129, and 5.130
Number of load cycles	n_c	–	n_c	–	
for pinion	n_{c1}	–	n_{cP}	–	Equation 5.89
for gear	n_{c2}	–	n_{cG}	–	Equation 5.90
Reliability level	L	–	L	–	End of Section 5.2.1
K factor, pitting index	K	N/mm²	K	psi	Equations 5.96, 5.98, and 5.99
Stress on tooth surface	s	N/mm²	s	psi	Equation 5.66
Bending stress (strength)	s_t	N/mm²	s_t	psi	Equation 5.91
Contact stress (surface)	s_c	N/mm²	s_c	psi	Equation 5.96
Overall derating factor:					
for bending strength	K_d	–	K_d	–	Section 5.2.2, Equation 5.105
for surface durability	C_d	–	C_d	–	Section 5.2.6, Equation 5.116
Geometry factor:					
for bending strength	K_t	–	K_t	–	Sections 5.2.2 and 5.2.3
for surface durability	C_k	–	C_k	–	Sections 5.2.2 and 5.2.5
Application factor:					
for bending strength	K_a	–	K_a	–	Section 5.2.4
for surface durability	C_a	–	C_a	–	Section 5.2.6
Size factor:					
for bending strength	K_s	–	K_s	–	Section 5.2.4
for surface durability	C_s	–	C_s	–	Section 5.2.6
Dynamic load factor:					
for bending strength	K_v	–	K_v	–	Sections 5.2.4 and 5.2.7
for surface durability	C_v	–	C_v	–	Sections 5.2.4 and 5.2.6
Load-distribution factor:					

(*Continued*)

TABLE 5.33 (CONTINUED)
Gear Terms, Symbols, and Units Used in Load Rating of Gears

	Metric		English		
Term	**Symbol**	**Units**	**Symbol**	**Units**	**Reference or Formula**
for bending strength	K_m	–	K_m	–	Section 5.2.4
(face width effects)	K_{mf}	–	K_{mf}	–	Equations 5.106, 5.107, 5.108, 5.109, and 5.110
(transverse effects)	K_{mt}	–	K_{mt}	–	Equation 5.106
for surface durability	C_m	–	C_m	–	Section 5.2.6
(face width effects)	C_{mf}	–	C_{mf}	–	Section 5.2.4
Modulus of elasticity	x_E	N/mm²	E	psi	Equation 5.119
Wear-in amount	e_w	mm	e_w	in.	Table 5.39
Mismatch error	e_t	mm	e_t	in.	Equations 5.107 and 5.108
Material and type constant	C_p	–	C_p	–	Equations 5.114 and 5.115
Wear load	–	–	W_w	lb	Equation 5.117
Aspect ratio	m_a	–	m_a	–	Section. 5.2.4
J factor (geometric strength)	J	–	J	–	Alt. to K_t, Section 5.2.3
I factor (geometric pitting)	I	–	I	–	Alt. to C_k, Section 5.2.5
Contact ratio	ε_α	–	m_p	–	Section 5.2.5
Flash temperature index	T_f	°C	T_f	°F	Equation 5.136
Gear body temperature	T_b	°C	T_b	°F	Equation 5.136
Geometry constant	Z_t	–	Z_t	–	Equation 5.136
Surface finish constant	Z_s	–	Z_s	–	Equation 5.136
Scoring-criterion number	Z_c	°C[a]	Z_c	°F[a]	Equation 5.136
Oil-film thickness (EHD)	h_{min}	μm	h_{min}	μin.	Equation 5.144
Effective surface finish	S'	μm	S'	μin.	Equation 5.149
Lambda ratio	Λ	–	Λ	–	Equation 5.150

Note: Abbreviations for units are as follows:

Metric System		**English System**	
mm	millimeters	in.	inches
deg	degrees	deg	degrees
rpm	revolutions per minute	rpm	revolutions per minute
m/s	meters per second	fpm	feet per minute
		fps	feet per second
kW	kilowatts	hp	horsepower
N	newtons	lb	pounds
N m	newton-meters	in. lb	inch-pounds
N/mm²	newtons per square millimeter	psi	pounds per square inch
°C	degrees Celsius	°F	degrees Fahrenheit
μm	micrometers (10^{-6} m)	μin.	microinches (10^{-6} in.)

See Table 5.2 for terms, symbols, and units in the calculation of gear dimensional data.

[a] Z_c is not a temperature, but becomes a change of temperature when multiplied by Z_t and Z_s.

cannot live up to the normal level of quality for a particular kind of gears cannot expect their gears to safely carry the full ratings allowed by a trade standard. Likewise, manufacturers who can build gearing of appreciably better quality than the general level of the trade might expect that their gears would be able to carry somewhat more load than that allowed by a trade standard.

As a general rule, in checking gear capacity, it is wise to first check the design using general formulas for tooth strength and durability. Then, if the application falls into the field of a particular trade standard, it should be checked against the trade standard. If the design meets both tests, it is probably all right. In this section, we shall consider both general formulas and trade standards for rating gears.

5.2.1 General Considerations in Rating Calculations

Gear designers make rating calculations to establish that a given gear design is suitable in size and in quality to meet the specified requirements of a gear application. (The buyer

of a gear unit or the user of a gear unit may also make rating calculations to check the design being bought or used.)

Rating calculations are concerned with more than the direct load-carrying capacity. All three of these somewhat different concerns tend to apply:

- Has the rating been correctly calculated with a *good* formula, and have all the *right* assumptions and decisions been made in regard to materials, material quality, and geometric quality?
- Does the design calculate a suitable rated power, length of life, and degree of reliability to satisfy a *speci ed formula* in the business contract or one established and *recognized* in normal trade practice? For instance, AGMA formulas cover many gear applications. There is a legal obligation in many projects, both in the United States and worldwide,* to meet all applicable AGMA standards.
- Will the gear unit meet its *normal rating capacity* in the power package using the gears? In a power system, there are often situations in which gears are prematurely damaged by unde ned overloads, misalignments, unexpected temperature excursions, or contamination of the lubricant by some foreign material (water, ash, chemical vapors at the site, and so forth).

The designer needs to rate the gears to meet the application. In a new application with unknown hazards, an appropriate gear rating may not be possible until several prototype units have been in service for a suitable period. A reliable gear rating may not be practical until enough development work has been done and enough eld experience has been acquired to prove that a gear design is suitable for the hazards of the application. (See Chapter 12 for a discussion of things that may be problems in power breakages.)

Somewhat aside from the mechanics of making gear-rating calculations and the concerns of contract requirements and application hazards is the fact that there is a shortage of good technical data and speci cations to closely control all the things that go into a rating.

In earlier times, the mechanical designer tried to calculate a safe stress. It was thought that all parts with less than the safe stress limit would perform without failure. Gears, bearings, and many other mechanical components have a probability of failure. A safe stress is in reality only a stress at which the probability of failure is low.

Theoretical gear design has now shifted to using stress values primarily based on some level of probability of failure. Unfortunately, the data available to set the probability of failure are still rather limited. This problem is compounded by the fact that a system of probability of failure versus stress needs to be tied into a system of material quality grades.

Material quality grades are still not very clearly de ned. AGMA established two grades of quality for aircraft gears in the 1960s. In the 1970s, two grades were established for vehicle gears. As this is being written, work is under way to establish grades for high-speed gearing used with marine and land turbine units.

Generally speaking, there is about a 20% to 25% change in load-carrying capacity as you go from grade 1 to 2 material. In a fully developed gear grading system, there should probably be more than two grades of material (at least three), and the material speci cations of each grade should be more closely de ned than is now the case. The designer who is going to shift a rating up or down 20% for material grade certainly needs some rather clear-cut de nitions of what must be achieved to qualify the material for its designated grade. (See Section 6.2.6 for detailed information on material quality.)

In earlier times, failure by tooth breakage meant a broken tooth, and failure by pitting meant a substantial number of destructive pits. Recent work shows that both of these criteria of failure are inadequate. An aircraft gear with some small cracks in the root found at overhaul has usually failed. It is not safe to use the part if the cracks are above a certain size in a critical location, so the gear is scrapped—even if it *might* run another 1000 hours.

At the other extreme, an industrial gear may have a whole layer of material pitted away from the lower ank and still be able to run for several more years. In this case, failure by pitting only comes when so much metal has worn away and load sharing between teeth has become so bad that the teeth break.

A further design complication comes from the fact that gear teeth may work harden in service and may polish up rough tooth surfaces. The designer used to design gears based on the hardness of the part and the nish and t of the part when it left the gear shop. Now the nal nishing of the gear teeth and the nal metallurgical character of the tooth surface are often established in service.

The data on how to allow for improvements in nish and t and for work hardening are still rather limited. Technical papers† have shown quantitative data that indicate a 2-to-1 improvement in nish and a change in hardness of 100 points Vickers or 100 BHN is not uncommon in lower-hardness gears running at medium to low pitch-line velocities.

5.2.1.1 Calculation Procedure

The calculation begins with a requirement that a gearset must handle a speci ed power at a given input speed. In a simple rating calculation, the preliminary sizing of the unit has established the following things:

* In an international project, the gears might be built in Europe and installed in South America. If the project was insured or nanced by U.S. interests, there would probably be a contract requirement that the gears meet AGMA standards, even though they were not built in the United States.

† Material presented by P. M. Nityanandan at the June 1982 International Federation for the Theory of Machinery and Mechanisms gearing committee meeting in Eindhoven, Holland, was very revealing in regard to the performance of low-hardness gears in India.

- Power to be transmitted, in kilowatts (kW) or horsepower (hp)
- Rotational speed of pinion
- Number of pinion teeth and number of gear teeth
- Face width
- Pitch diameters of pinion and gear
- Pressure angle (normal pressure angle for helical gears)
- Size of teeth by module or diametral pitch
- Helix angle
- Hours of life needed at *rated load*

The rst calculation step is to get the *tangential driving load*. The pinion torque is calculated rst, then the torque is converted to a tangential force acting at the pitch diameter.

The torque is

$$\text{Pinion torque} = \frac{\text{power} \times \text{constant}}{\text{pinion rpm}}, \tag{5.84}$$

$$T_1 = \frac{P \times 9549.3}{n_1} \quad \text{(metric)}, \tag{5.85}$$

$$T_\text{P} = \frac{P \times 63,025}{n_\text{P}} \quad \text{(English)}. \tag{5.86}$$

In metric equation, the power P is in kilowatts, and the calculated torque will be in newton-meters (N · m N m). For English equation, the power is in horsepower, and the answer will be in inch pounds (in. lb).

The tangential driving force is as follows:

$$W_\text{t} = \frac{T_1 \times 2000}{d_\text{p1}} \quad \text{(N; metric)}, \tag{5.87}$$

$$W_\text{t} = \frac{T_\text{P} \times 2.0}{d} \quad \text{(lb; English)}. \tag{5.88}$$

The geometry factor for strength is usually read from a table or a curve sheet. Section 5.2.3 gives typical values. The overall derating factor may be read from a table or calculated. Section 5.2.4 covers overall derating for strength. The geometry factor durability is covered in Section 5.2.5, while overall derating for durability is in Section 5.2.6.

The bending and the contact stresses can now be calculated. Before checking whether or not these stresses are permissible, the designer must make some further determinations. The life cycles required for the pinion and for the gear need to be known, and an estimate of which quality grade of material will be used needs to be made. Also, the level of reliability needed for the application should be determined.

The loading cycles are the following:

$$n_{c1} = \text{pinion rpm} \times \text{hours life} \times 60 \times \text{no. contacts}, \tag{5.89}$$

$$n_{c2} = \text{gear rpm} \times \text{hours life} \times 60 \times \text{no. contacts}. \tag{5.90}$$

In the above equations, a pinion driving three gears would have three contacts per revolution. Likewise, if ve planet pinions drive a single gear, there would be ve contacts per revolution of the gear. This kind of situation is common in epicyclic gears. A single pinion driving a single gear has one contact per revolution. (The gear also has one contact for each revolution it makes.)

5.2.1.2 Grades of Material Quality

In general, two quality grades can be considered possible. These are the following:

Grade 2: Grade 2 is the best quality obtainable with an approximate choice of material composition and processing that is close to optimum. Usually, there is extra cost for the best quality.

Grade 1: Grade 1 is a quality of material that is good, but not optimum. This could be considered a typical quality from gear makers doing good industrial work at competitive costs.

For highly critical work in space vehicles, or highly sensitive applications, a grade 3 quality can be considered. This might be thought of as essentially perfect material made under such rigid controls that there is relatively absolute assurance that the highest possible perfection is obtained. Obviously, the very high–cost grade 3 gears will carry more stress—or will have higher reliability at the same stress—than grade 2 gears. Grade 3 gear data will not be given in this book. It is too complex and limited in usage.

5.2.1.3 Reliability of Gears

In regard to reliability, these general concepts should be kept in mind:

Reliability level:

L.1	Fewer than 1 failure in 1000	Seldom used
L1	Fewer than 1 failure in 100	Typical gear design
L10	Fewer than 1 failure in 10	May be used in vehicle gears
L20	Fewer than 1 failure in 5	Expendable gearing
L30	Fewer than 1 failure in 2	Highly expendable gearing

The L.1 level has been used in some highly critical aerospace work (particularly space vehicles). L1 is typical of industrial turbine work, helicopter work, and high-grade electric motor gearing. Vehicle gears for land use have tended to be in the L1 to L10 range. Home tools, toys, gadgets, etc., may be in the L20 area, or even up to L50.

There is a limited amount of laboratory test data, and if an average curve is drawn through failure points, the curve will be around L50. (It takes much test data and eld experience to

determine where a stress level that is equivalent to an L1 level of reliability can be located.)

5.2.2 General Formulas for Tooth Bending Strength and Tooth Surface Durability

The rating formulas for gear tooth strength and gear surface durability have become very long and very difficult to handle in many of the rating standards of the AGMA, the International Standards Organization (ISO), and other trade groups. In this work, short and simple formulas will be used. In Appendix B, the complete formulas are given in Section B.9.

The short formulas group all variables into essentially three factors:

- An index of load intensity, with dimensions
- A geometry factor, dimensionless
- A derating factor, dimensionless

The load-intensity evaluations is based on all real numbers that relate the size of the gears to the power being carried by the gears.

The geometry factor evaluates the shape of the tooth. This involves pressure angle, helix angle, depth of tooth, root fillet radius, and proportion of addendum to dedendum.

The derating factor handles all the things that tend to reduce the load-carrying capacity. (Concentration of too much load at spots on the surfaces of some teeth reduces the average allowable load, since failure can be avoided only when the most overloaded spots are still within safe limits.)

The derating factor handles nonuniform load distribution across the face width and in a circumferential direction. It also handles the dynamic overloads due to spacing error and the masses of the pinion and gear meshing together, and other things—quality related—such as surface finish effects, overload effects due to nonsteady power, and variations in metal quality between very large gears and small gears.

5.2.2.1 Strength Formula

The simplified general formula for tooth bending stress of spur, helical, and bevel* gears is

$$s_{\mathrm{t}} = K_{\mathrm{t}} U_{\mathrm{l}} K_{\mathrm{d}}. \tag{5.91}$$

The bending stress s_{t} is measured in newtons per square millimeter when the metric system is applied, and in pounds per square inch for English system.

In Equation 5.91, the geometry factor for bending strength is designated as K_{t} (see Section 5.2.3):

$$K_{\mathrm{t}} = \frac{\text{constant} \times \cos(\text{helix angle})}{J - \text{factor}}. \tag{5.92}$$

Unit load, index for tooth breakage, is denoted by U_{l}:

$$U_{\mathrm{l}} = \frac{W_{\mathrm{t}}}{b m_{\mathrm{n}}} \quad (\text{N/mm}^2;\text{metric}), \tag{5.93}$$

$$U_{\mathrm{l}} = \frac{W_{\mathrm{t}} P_{\mathrm{nd}}}{F} \quad (\text{psi; English}). \tag{5.94}$$

The overall derating for bending strength is designated as K_{d} (see Section 5.2.4):

$$K_{\mathrm{d}} = \frac{K_{\mathrm{a}} K_{\mathrm{m}} K_{\mathrm{s}}}{K_{\mathrm{v}}} \quad (\text{metric or English}). \tag{5.95}$$

The individual items in the previous equations are defined in Table 5.7, along with many others used in load-rating equations.

5.2.2.2 Durability Formula

The simplified general formula for tooth surface durability of spur, helical, and bevel† gears give the contact stress:

$$s_{\mathrm{c}} = C_{\mathrm{k}} \sqrt{K C_{\mathrm{d}}}, \tag{5.96}$$

where geometry factor for durability C_{k} is equal (see Section 5.2.5) to

$$C_{\mathrm{k}} = \text{constant} \times \sqrt{\frac{1}{I} \cdot \frac{\text{ratio}}{(\text{ratio}+1)}}. \tag{5.97}$$

K factor (index for pitting) is calculated from

$$K = \frac{W_{\mathrm{t}}}{b d_{\mathrm{p1}}} \times \frac{u+1}{u} \quad (\text{metric}), \tag{5.98}$$

$$K = \frac{W_{\mathrm{t}}}{F d} \times \frac{m_{\mathrm{G}}+1}{m_{\mathrm{G}}} \quad (\text{English}), \tag{5.99}$$

and, finally, overall derating for durability C_{d} is calculated by (see Section 5.2.6)

$$C_{\mathrm{d}} = \frac{C_{\mathrm{a}} C_{\mathrm{m}} C_{\mathrm{s}}}{C_{\mathrm{v}}}. \tag{5.100}$$

In metric system, the contact stress s_{c} (see Equation 5.96) is measured in newtons per square millimeter, and for English

* See Section 5.2.2.4. Also, see Figure 4.21 for the method of determining the unit load on a bevel gear.

† See Section 5.2.2.4. Also, see Figure 4.21 for the method of determining the K factor on a bevel gear.

system, the contact stress s_c is measured in pounds per square inch.

In most long-life power gearing, the size of the gear drive (center distance and face width) is determined by the durability formula. The calculated contact stress must not exceed the allowable stress for the number of cycles of life required and the quality (grade) of material used. The allowed stress is adjusted for the permissible risk of failure (L1, L10, etc.).

The gear-strength calculations are primarily used to determine what size of tooth (module or pitch) is needed to keep the bending stress within allowable limits for the number of cycles of life required, the quality of the material, and the permissible risk of failure.

An exception to this occurs when gears with a very high torque rating are needed for a low number of cycles. (Final drive vehicle gears are often in this situation.)

The gear size may primarily beset by gear-strength considerations. The gears may suffer some degree of surface failure and still satisfactorily run for the need life. Generally speaking, gear teeth that suffer surface damage in less than 1 million (10^6) cycles of high torque cannot keep on running for more than about 100 million (10^8) cycles, even if the continuous torque is rather light. Turbine gearing, for instance, generally has to run more than a billion (10^9) cycles. Turbine gears cannot have surface damage due to a high torque at low cycles and be expected to last for many years of operation.

Much gear research work is being done in the 1980s to set accurate values for all the variables needed to accurately rate gears. The 1980 ASME paper by Winter and Weiss (1980), for instance, gives a good summary of German work at the Technical University of Munich. Some very good comparisons of proposed ISO rating formulas and AGMA standards are given in the 1980 ASME papers by Imwalle et al. (1980) of the United States and by Castelliani (1981) of Italy.

In writing this section on rating formulas and the next five sections on factors in the rating formulas, the following strategy has been accepted:

- The general formulas and their principal terms will be given in a manner that should remain unchanged for many years.
- Some items that are being changed—and perhaps will continue to change for many years—will be given in numerical values that are conservative for the gearing design practice. The reader will be given practical guidance in what to use in rating gears, but may obviously be able to find better numerical values to use, as worldwide gear research and field experience give more information about how to evaluate all the many things that enter into gear rating.
- This book will endeavor to give the reader a means of determining the approximate size of gear teeth needed to handle a desired rating.* Whenever recognized trade standards or contract specifications are involved, the final sizing and design of the gears should be adjusted to meet these obligations. (In general, gears sized by the principles in this book should not need much adjustment to fit contract obligations. If a contract would allow significantly smaller or weaker gears, the designer should review the field experience available and decide whether or not going beyond the recommendations given here is reasonable.)
- In many areas, not enough is known to rate the gears with any great confidence that the rating is really right. This is particularly true in regime I lubrication conditions. (See Section 4.1.4 for a decision of lubrication regimes.) If extensive field experience is not available to verify a gear-rating procedure, it is generally necessary to do initial factory and field testing of prototype units to prove their specified ratings before going into production of the units.

* See Chapter 12 for a discussion of gear ratings and of possible factors other than the basic rating which may cause failures.

5.2.2.3 Rating Curves of Stress versus Cycles

The following rating curves are presented as general guides to the gear designer:

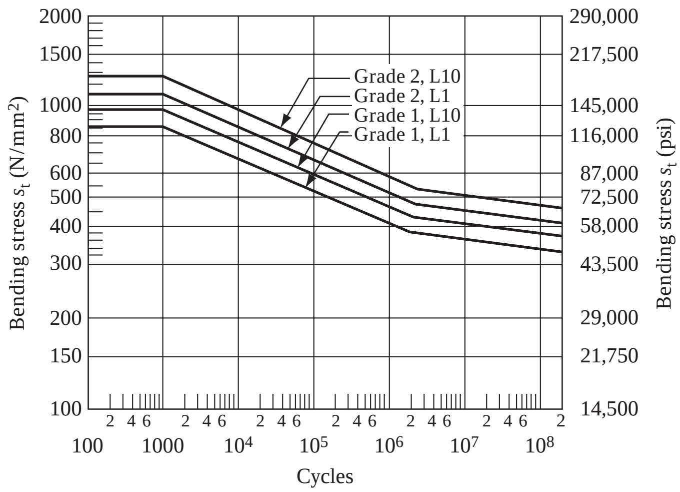
Figure 5.24	Covers allowable bending stress[a] for short cycles
Figure 5.25	Covers allowable bending stress[a] for long cycles
Figure 5.26	Covers contact stress for regime II lubrication and short cycles
Figure 5.27	Covers contact stress for regime III lubrication and long cycles

[a] The bending stress that is considered in Figures 5.24 and 5.25 is an unidirectional stress for normal gear action. When gear teeth are subject to loading in two directions, such as a planet in an epicyclic set that is loaded on both sides of the teeth, the rating must be reduced for reverse bending. An approximately correct rating can be obtained by multiplying the allowable bending stress by 0.70 before comparing it to the calculated stress.

These curves are somewhat different from the presently published curves used in gear standards. They reflect the concept that high loads in fewer than 10^7 cycles can cause microscopic metal damage that makes the gear tooth unable to run for 10^9 cycles or longer. They also take into account the regimes of lubrication and the *change in slope* of the stress versus cycle curves for durability.

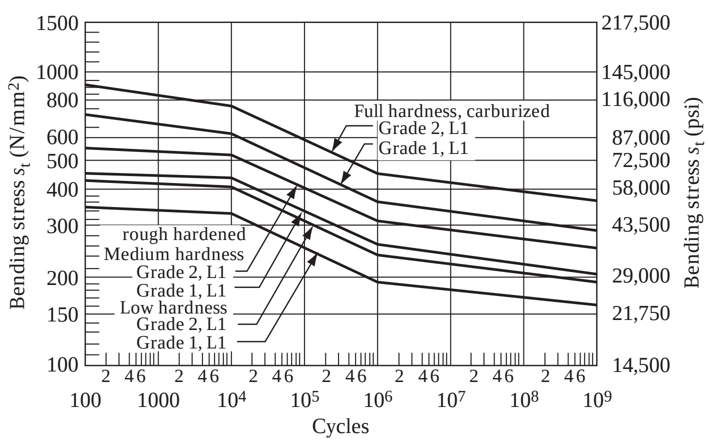
There are not enough good data in the gear trade to plot a set of curves like those in Figures 5.24 through 5.27 with a high degree of confidence. The data shown are intended to represent as good a judgment as can be determined, given the present state of the gear art. Hopefully, more research work and field experience will make it possible to have better data.

These curves are intended to be used with the load histogram manner of design and an obligation to control the scoring problem. See Section 5.2.8 for more on the load histogram



Note: ese curves are drawn for full-hardness carburized gears at 60 HRC minimum hardness—700 HV.
See Section 6.2.2 for comments on the load-carrying capacity of nitrided gears.
ese curves are for unidirectional tooth loading.
If teeth are subject to reverse bending (idlers, planets, etc.), multiply allowable bending stress by 0.70.

FIGURE 5.24 Bending stress for a short-life design.



Note: Full hardness is 60 HRC–700 HV minimum.
Medium hardness is 300 HB–320 HV minimum.
Low hardness is 210 HB–220 HV minimum.
ese curves are for unidirectional tooth loading.
If teeth are subject to reverse bending (idlers, planets, etc.), multiply allowable bending stress by 0.70.

FIGURE 5.25 Bending stress for a long-life design.

method, and for a discussion on how to handle the scoring hazard.

5.2.2.4 Rating Bevel Gears

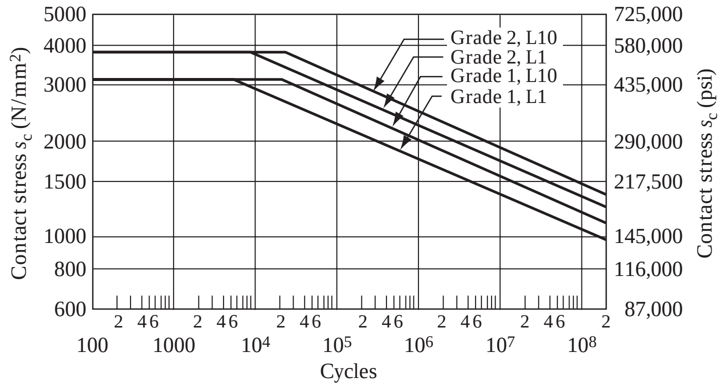
When the intensities of tooth loading for bevel gears (K factor and unit load) are calculated by the method shown in Figure 4.31, the values obtained are comparable to those of spur and helical gears. This is very helpful in preliminary design work when a decision as to whether the body shape of the gears should be cylindrical or conical has not yet been made.

As a general rule, a reasonable K factor on a bevel gear tends to be about equal to that on a matching spur or helical gear with the same hardness, quality, and pitch line speed. The unit load may be somewhat less, since it is harder to get a large root llet radius on a bevel gear than on a matching spur or helical gear.

Size is another factor. Large bevel gears need more derating for size than cocylindrical gears.

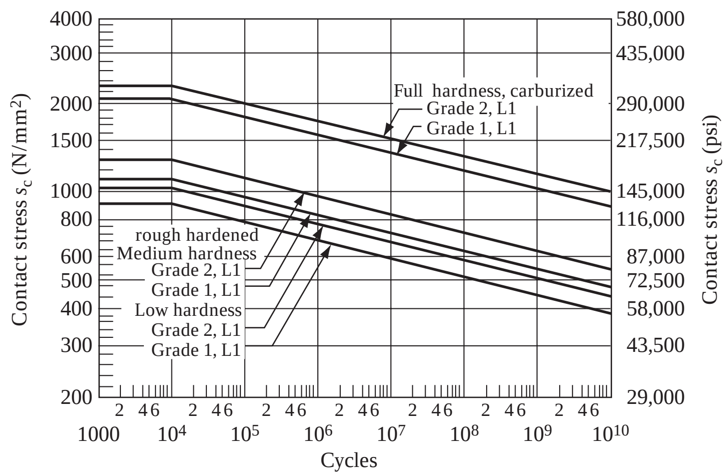
With these factors in mind, this book was written with the plan that the basic calculations for surface compressive stress and bending stress of bevel gears would be handled the same way and use the same *allowable stress* as those for spur and helical gears.

For a long while, bevel gear rating equations were put together differently than those for spur and helical gears. Furthermore, the allowable stress values were different. The



Note: ese curves are drawn for full-hardness carburized gears at 60 HRC minimum hardness—700 HV. See Section 6.2.2 for comments on the load-carrying capacity of nitrided gears.

FIGURE 5.26 Contact stress for regime I and a short-life design.



Note: Full hardness is 60 HRC–700 HV minimum.
Medium hardness is 300 HB–320 HV minimum.
Low hardness is 210 HB–220 HV minimum.

FIGURE 5.27 Contact stress for regime III and a long-life design.

trend now is to use similar formulas and to bring the stress values together. (Seemingly, the stress values would be the same if they were true stresses. Of course, as Section 4.1 points out, the stresses used in gear rating are not really true values. They are stress numbers that have been established by experience in developing gear ratings.)

The geometry factors and the derating factors for bevel gears in this book quite closely follow the general practice developed in the United States over last years. Using Equation 5.91 for strength and Equation 5.96 for surface durability and the de nitions of all the factors in Sections 5.2.2 through 5.2.6 result in calculation stress for bevel gears that generally come out high compared with the allowable stresses in Figures 5.24 through 5.27. Sample calculations show that for bevel gears known to be satisfactory in service, calculated stresses predict quick failure. This is a serious problem. *How should bevel gears be treated*?

In time, there will be adjustment in the geometry and the derating factors for both bevel and cylindrical gears. Quite likely, these changes, plus some adjustments in the stress curves, will make it possible to use the same stresses for both bevel and cylindrical gears. It would seem, though, that it will take certain time for this process to run its course.

For those using this book, the best plan is the following:

- Establish initial designs by the methods shown here, but cross-check these with applicable AGMA standards, proven experience in bevel gear work, and recommendations by Gleason Machine Division in Rochester, New York, United States.
- Keep in mind that this book does not cover ratings of bevel gears for aerospace work or for vehicle gears. Only industrial bevel gears are covered.
- Keep in mind that strength of bevel gears is more critical than that of cylindrical gears. Lower numbers of pinion teeth are advisable, particularly on fully hardened gears. The strength rating is often the controlling rating for bevel gears; whereas the

durability rating is generally the controlling rating for cylindrical gears.

- Remember that the bending stress of a bevel gear calculated by Equation 5.91 can generally be divided by an *adjustment factor* of 1.4 before it is compared with the strength rating curves of Figure 5.25.
- Keep in mind that the surface compressive stress of a bevel gear calculated by Equation 5.96 can generally be divided by an adjustment factor of 1.25 before it is compared with the durability rating curves of Figure 5.27.

5.2.3 Geometry Factors for Strength

The geometry factor for strength is a dimensionless factor that evaluates the shape of the tooth, the amount of load sharing between teeth, and the stress concentration in the root area. AGMA has standard procedure for handling this factor. The ISO standards now being developed have somewhat different procedures. Section 4.3 gave a general background on how gear tooth strength has been handled in the past.

The geometry factors for strength presented in this section are based on AGMA standards. The standards show the procedures for determining J factors and, in addition, show main graphs of J factor values for different gear designs. Although the ISO is developing a somewhat different method for determining geometry factors for strength, the AGMA system has worked quite well in practice and is well accepted in the gear trade.

In this book, simpli ed calculation methods are used, and a K_t factor—rather than a J factor—is used in the stress formulas. The relation of K_t to J is as follows:

$$K_t = \frac{1.0}{J} \quad \text{for spur gears,} \tag{5.101}$$

$$K_t = \frac{\cos(\text{helix angle})}{J} \quad \text{for helical gears,} \tag{5.102}$$

$$K_t = \frac{1.0}{J} \quad \text{for straight Zerol bevel gears,} \tag{5.103}$$

$$K_t = \frac{\cos(\text{spiral angle})}{J} \quad \text{for spiral bevel gears.} \tag{5.104}$$

Since the strength factor is dimensionless, it can be used equally well in metric and English calculations.

5.2.3.1 Lack of Load Sharing

Normally, the strength of spur gear teeth is calculated on the basis of the teeth sharing load at the rst point of contact and at the last point of contact. This is why the critical load is taken at the highest point of single-tooth contact. If the teeth are not cut accurately enough to share load, they may still wear in enough so that load sharing exists before there have been many stress cycles.

In some cases, the accuracy may be poor enough or the metal hard enough so that essentially no useful load sharing is achieved. If this happens, then the geometry factor for strength should be determined with full load taken at the tip of the tooth. Table 5.34 gives some typical strength factors for full load taken at the tip. Table 5.35 gives some guideline information as to how much error it takes to cause a failure of load sharing.

Calculations for helical gears and bevel gears are not often made for the condition where load sharing does not exist. In general, these gears are either made accurately enough to share load or lapped until a satisfactory contact pattern is achieved. (If they cannot be tted so that they properly contact, they may be rejected.) Inaccurate low-hardness gears may wear and cold- ow enough to develop relatively good contact patterns—and good load sharing—early in their service life.

5.2.3.2 Helical Gears with Narrow Face Width

A good helical gear should have enough face width so that contact ratio in the axial plane is at least 2. If the face width

TABLE 5.34
Geometry Factors K_t for Strength of Spur Gears Loaded at the Tip

	20° Pressure Angle			25° Pressure Angle		
No. of Teeth	1.25 Addendum	1.00 Addendum	0.75 Addendum	1.25 Addendum	1.00 Addendum	0.75 Addendum
12	3.95	–	–	3.10	3.70	–
15	3.77	4.50	–	2.97	3.51	–
18	3.66	4.29	–	2.88	3.34	–
25	3.52	3.97	–	2.77	3.09	–
35	3.40	3.70	4.12	2.67	2.92	3.20
50	–	3.57	3.85	–	2.79	2.99
100	–	3.39	3.51	–	2.66	2.75
275	–	3.26	3.29	–	2.58	2.61

Note: It is assumed that a pinion with 1.25 basic addendum meshes with a gear of 0.75 addendum and that the tooth thicknesses are adjusted for the change in addendum. It is assumed that a 1.00 addendum pinion meshes with a 1.00 addendum gear and that the tooth thicknesses are standard. These data are for extra-depth teeth cut with a relatively full radius llet (whole depth from 2.35 to 2.40).

TABLE 5.35
Limiting Error in Action for Steel Spur Gears

Load Intensity per Unit of Face Width		Dimensional Error on Line of Action between Contact Points			
		Teeth Share Load		Teeth Failed to Share Load	
Metric, W_t/b (N/mm)	English, W_t/F (lb/in.)	Metric; mm	English; in.	Metric; mm	English; in.
100	571	0.005	0.0002	0.013	0.0005
200	1142	0.008	0.0003	0.020	0.0008
300	1713	0.010	0.0004	0.030	0.0012
400	2284	0.013	0.0005	0.041	0.0016
600	3426	0.020	0.0008	0.061	0.0024
1000	5710	0.033	0.0013	0.076	0.0030
1600	9140	0.050	0.0020	0.122	0.0048

Note: The error values are useful only as rough guides. Perfect load sharing, of course, can only occur with perfect accuracy. The meaning of *teeth share load* is that reasonably good load sharing can be expected. The meaning of *teeth failed to share load* is that very little useful load sharing can be expected unless the teeth wear enough to remove most of the error. The error on the line of action may be either pro le error or spacing error, or some combination of the two. Adjacent spacing error is usually the most troublesome.

is narrow relative to the pitch diameter of the pinion or if the helix angle is quite low (like 8° or less), the axial contact ratio may be less than 1. How should the strength of a helical pinion or gear be calculated when the axial contact ratio is only 0.50?

When the axial contact ratio is less than 1.0, the helical gear can best be thought of as in an intermediate zone between a helical gear and a spur gear. At an axial contact ratio of 0.50, a close approximation of the geometry factor for strength can be taken midway between the value for a spur gear and the value for a helical gear. It is shown in AGMA standards how to derive the J factor for helical gears, either wide face width or narrow face width. This standard can be used to get data beyond those given in this book.

5.2.3.3 Geometry Factors for Strength for Some Standard Designs

The geometry factors for strength have been calculated and plotted on curve sheets for several standard designs of spur and helical gears. See Figures 5.28 and 5.29. The method used was that proposed by AGMA. The data shown agree with tabulated data by AGMA.

Although much design practice is based on AGMA standards, it is not known with certainty that the geometry factors are just right. The work by Drago and Luthans (1981) has shown evidence that rim thickness is important and that the true stress may not exactly agree with a stress determined by AGMA standards.

The ISO method does not agree with the AGMA method. In particular, the relation of spur teeth to helical teeth is different. Works by Castellani and Castelli (1980) and by Imwalle and LaBath (1981) has brought this out.

Figures 5.28 and 5.29 give results that are reliable as long as gear rims are reasonably thick and centrifugal forces are low. When gear research work provides better values, they will probably not be very different from those shown.

Geometry factors for the strength of bevel gears are given in Table 5.36. These are used when the unit load is calculated for the middle of the bevel tooth. See Figure 4.21.

5.2.4 Overall Derating Factor for Strength

The overall derating factor evaluates all the things that tend to make the load higher than it should be for the load being transmitted. Speci cally, the factor is

$$K_d = \frac{K_a K_m K_s}{K_v}, \tag{5.105}$$

where

K_a—application factor
K_m—load-distribution factor
K_s—size factor
K_v—dynamic factor

Each of these factors is explained in the following sections. The numerical values given should be considered as somewhat typical of what is being used, but not necessarily precise values for any given situation. Most of them are based on experience as much as on theoretical logic. Experience, of course, tends to be an average of known data. Anything that is an average represents the mean of a range of actual values. Also, the known data may be somewhat limited compared with the unknown data—which would have given a different value, had they been known.

Another problem is that much data have been accumulated in the past on gears that were not too high in power and had

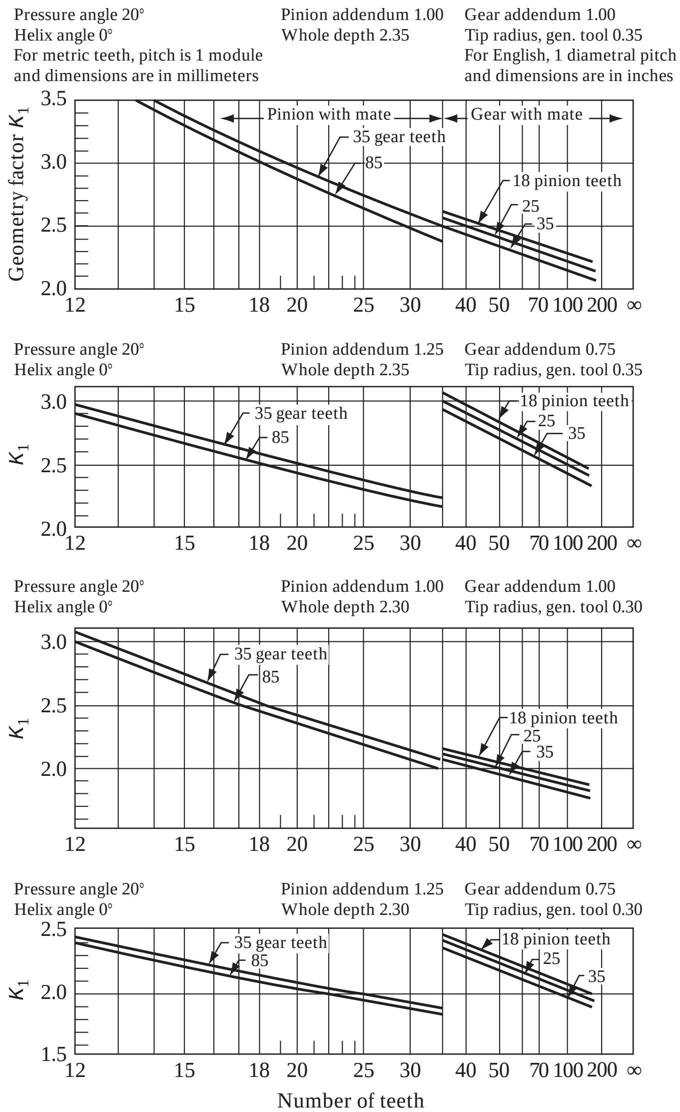


FIGURE 5.28 Geometry factors for strength of spur gears based on the highest point of single tooth loading.

low-hardness gear teeth. These parts were not too large in size, and the lower hardness was helpful in letting the teeth wear in to accommodate manufacturing errors. As gears get larger in size, they are more dif cult to make with good quality; if they are much harder, the teeth have very little tendency to wear in during service.

A good example of this kind of changes has occurred in mill gearing. Certain mill gearing formerly transmitted around 1000 kW and used gearing around 300 HV (280 HB). In recent years, the power has gone up to 3000 kW and much gearing is being built with surface hardness over 600 HV (550 HB). In this case, past derating practices need modi cation to handle the new situation, but the bulk of the experience in the gear trade was accumulated in earlier years on the smaller, lower-hardness gears. How can the gear trade quickly obtain the experience needed to design new products out of the range of older products?

The derating values given in this section must be considered as general guides. AGMA and other organizations will write new standards. In a particular contract job, the buyer may specify that the gearing must meet certain design standards of AGMA (or others). Since derating is so critical and so potentially variable, the gear designer should endeavor to meet these objectives:

- Any contract design speci cations should either *be met* or variations negotiated.
- The design should *look reasonable* by the logic of this book and other books applicable to the gear job at hand.
- If the organization building the gears has no good depth of experience in making gears of the size, the hardness, or the kind required, the job should be *considered developmental* until adequate experience is obtained. This involves things like bench testing of components, factory testing of whole units at full load and full speed for some millions of contact cycles, and then a eld evaluation of a few prototype

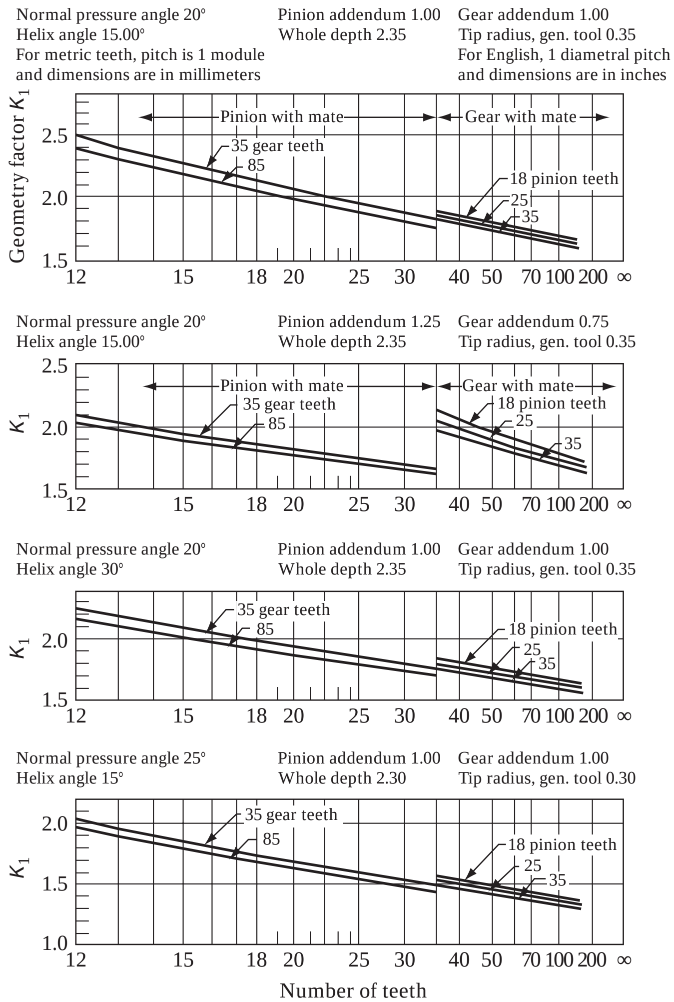


FIGURE 5.29 Geometry factors for strength of helical gears based on the highest point of single tooth loading and sharing of load between teeth in the zone of contact.

production units under actual field service conditions. (A design standard is useful, but it does not remove the need for gear builders to obtain a good depth of experience in the product they are making.)

5.2.4.1 Application Factor K_a

The K_a factor evaluates external factors that tend to apply more load to the gear teeth than the applied load W_t. A rough-running prime mover and/or a rough-running piece of driven equipment can seriously increase the effect of the applied load. On an instantaneous basis, the torque being transmitted may be considerably fluctuating. The transmitted torque is then just the mean value of the torque fluctuations.

Other things like accelerations and decelerations, rudder turns or ship, engine misfiring, power-system vibrations, enter into the application factor.

In the past, the *service factor* was somewhat similar to the application factor. The service-factor concept, though, involved life and reliability as well as overloads. The best present practice treats life on the basis of cycles and reliability on a probability-of-failure basis. The application factor is used, and service factor is not used.

The application factor is determined by experience. An application factor of 1.0 is best thought of as a perfectly smooth turbine driving a perfectly smooth generator at an always constant load and speed. If another application has to have the load reduced from 2 to 1 so that the same gears will last as long as in the ideal turbine/generator application, then the application factor is 2.0.

Table 5.37 gives application factors for a range of gear applications.

5.2.4.2 Load-Distribution Factor K_m

K_m factor evaluates nonuniform load distribution across the face width and nonuniform load distribution in the meshing direction (transverse plane).

The general formula is

$$K_m = K_{mf} K_{mt}. \tag{5.106}$$

The transverse effect is evaluated in some of the ISO standards that are in process of approval at this time. If the gear teeth fit reasonably well, the effect of K_m is relatively small and will be taken care of by making K_{mf} appropriately large.

TABLE 5.36
Geometry Factors K_t for the Strength of Bevel Gears

Number of Pinion Teeth	Number of Gear Teeth	Straight Bevel Gears		Spiral Bevel Gears	
		Pinion	Gear	Pinion	Gear
15	20	5.00	5.75	4.93	4.93
	35	4.48	5.56	3.79	3.79
	50	4.35	5.49	3.21	3.21
	100	3.79	4.90	2.57	2.57
20	20	5.04	5.04	4.45	4.45
	35	4.25	5.00	3.62	3.62
	50	4.15	4.90	3.05	3.05
	100	3.57	4.35	2.48	2.48
25	20	4.61	4.91	3.80	3.80
	35	4.11	4.67	3.48	3.48
	50	3.95	4.61	2.94	2.94
	100	3.42	4.00	2.42	2.42
35	35	4.14	4.14	3.13	3.13
	50	3.75	4.20	2.78	2.78
	100	3.16	3.66	2.35	2.35
50	50	3.71	3.71	2.64	2.64
	100	2.95	3.33	2.29	2.29

Note: The values in this table are subject to change as re nements are made in the procedure of determining stress intensity due to bending load on gear teeth. This table is based on the normal design and manufacturing procedure recommended by Gleason Works. In many cases, there will be reason to modify the actual design, and this will tend to cause changes in the geometry factors from the guideline values given. The straight bevel data are from Coni ex bevel gears having 20° pressure angle. The geometry factors for Zerol bevel teeth tend to be the same as those for straight bevels.

The data given in this book anticipated that a value of 1.0 will be used for K_{mt}.

The factor K_{mf} should evaluate all the effects which may be due to the following:

- Helical spiral of pinion does not match helical spiral of mating gear (helix error effect).
- The pinion body bends and twists under load so that there is a mismatch between the pinion and the gear teeth (de ection effects).
- The pinion axis, under load, is not parallel to the gear axis. Or, in bevel gears, the pinion axis is not at a 90° axis angle (position error effects, under load).
- Centrifugal forces distort the shape of the pinion or the gear and mismatch the teeth (centrifugal effects).
- Thermal gradients distort the shape of the pinion or the gear and mismatch the teeth (thermal effect).
- Deliberate design modi cations, such as crowning, end easement, or helix correction, concentrate the load in one area and relieve the load in another area. This is usually done to lessen the effect of one of the preceding items, but it is an effect in itself (design effects).

The listing just given shows why load distribution is one of the most complex subjects in gear design. The later proposed standard method of handling all these variables had about 150 pages—and it was found that it did not adequately cover all the possible situations!

5.2.4.3 Effect of Helix Error and Shaft Misalignment

In this section, some limited data will be given on misalignment effects that are due to error in matching the helical spirals and on de ection errors that are due to the aspect ratio of the pinion. Table 5.38 gives some target values that the designer should try to meet. (Instead of guring how high K_m is when all tolerances and de ections are allowed for, gure close to the tolerances and the de ections must be held to get a reasonable value of K_m.)

Figure 5.30 shows the approximate face load–distribution factor as a function of mismatch error e_t and intensity of load. The curves were plotted up to K_{mf} = 2.0 from the following relations:

$$K_{mf} = \frac{10,000b}{W_t} \times e_t + 1 \quad \text{(metric)}, \tag{5.107}$$

$$K_{mf} = \frac{1,450,000F}{W_t} \times e_t + 1 \quad \text{(English)}. \tag{5.108}$$

In Equations 5.107 and 5.108, W_t is in newtons for the metric calculation and in pounds for the English calculation. The

TABLE 5.37
Typical Application Factors K_a for Power Gearing

Prime Mover			
Turbine	**Motor**	**Internal Combustion Engine**	**Driven Equipment**
			Generators and Exciters
1.3	1.3	1.7	Based load or continuous
1.1	1.1	1.3	Pack duty cycle
			Compressors
1.7	1.5	1.8	Centrifugal
1.7	1.5	1.8	Axial
1.8	1.7	2.0	Rotary lobe (radial, axial, screw, and so forth)
2.2	2.0	2.5	Reciprocating
			Pumps
1.5	1.3	1.7	Centrifugal (all service except as listed below)
2.0	1.7	–	Centrifugal–boiler feed
2.0	1.7	–	High-speed centrifugal (over 3600 rpm)
1.7	1.5	2.0	Centrifugal–water supply
1.5	1.5	1.8	Rotary–axial flow–all types
2.0	2.0	2.3	Reciprocating
			Blowers
1.7	1.5	1.8	Centrifugal
			Fans
1.7	1.4	1.8	Centrifugal
1.7	1.4	1.8	Forced draft
2.0	1.7	2.2	Induced draft
			Paper Industry
1.5	1.5	–	Jordan or refiner
1.3	1.3	–	Paper machine, line shaft
–	1.5	–	Pulp beater
			Sugar Industry
1.5	1.5	1.8	Cane knife
1.7	1.7	2.0	Centrifugal
1.7	1.7	2.0	Mill
			Processing Mills
–	1.75	–	Autogenous, ball
–	1.75	–	Pulverizers
–	1.75	–	Cement mills
			Metal Rolling or Drawing
–	1.4	–	Rod mills
–	2.0	–	Plate mills, roughing
–	2.75	–	Hot blooming or slabbing

Note: The values given are illustrative. As more experience is gained, new applications factors will be established in the gear trade. The values given may vary in a multistage drive. Experience and study will often show that the first stage needs a different application factor than that needed for the last stage. The power rating and the kind of gear arrangement affect the application factor. The values given here represent somewhat average situations. (Be wary of new gear designs of high power. The old experience on application factors may be wrong for the new situation.)

TABLE 5.38
Some Target Values of K_m for Spur and Helical Gears

Hardness of Gearset and Load per Unit of Face Width	Target Values of K_m for Gears with Face Width			
	50 mm (2 in.)	100 mm (4 in.)	250 mm (10 in.)	750 mm (30 in.)
High Hardness (675 HV)				
$W_t/b = 100$ ($W_t/F = 571$)	1.7	1.9	–	–
$W_t/b = 300$ ($W_t/F = 1713$)	1.4	1.6	1.8	–
$W_t/b = 800$ ($W_t/F = 4568$)	1.2	1.3	1.6	1.8
Medium Hardness (300 HV)			**1.8**	
$W_t/b = 100$ ($W_t/F = 571$)	1.5	1.6	1.8	–
$W_t/b = 300$ ($W_t/F = 1713$)	1.2	1.3	1.6	1.8
$W_t/b = 800$ ($W_t/F = 4568$)	1.1	1.2	1.4	1.5
Low Hardness (6210 HV)				
$W_t/b = 100$ ($W_t/F = 571$)	1.3	1.4	1.5	1.6
$W_t/b = 300$ ($W_t/F = 1713$)	1.1	1.2	1.4	1.5
$W_t/b = 800$ ($W_t/F = 4568$)	1.0	1.1	1.2	1.3

Note: These values assume that the accuracy is adjusted, with high-hardness gears being more accurate than low-hardness gear. For gears over 100 mm face width, it is assumed that each set is matched or tted to get an acceptable contact pattern at full load. It is assumed that the low-hardness gears wear in (or cold- ow the metal) to improve the contact. This happens to a lesser extent with medium-hard gears, and almost no useful wear-in occurs with high-hardness gears.

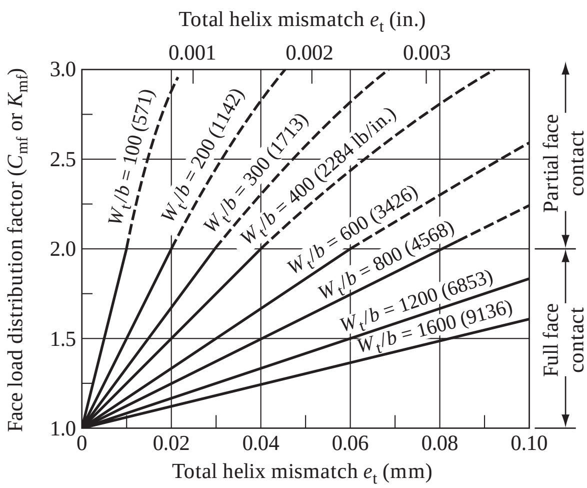


FIGURE 5.30 Approximate load-distribution factors K_{mf} from combined effects of helix mismatch and shaft misalignment.

face width and the error are in millimeters for the metric calculation and in inches for the English calculation. The answer is dimensionless.

The basics for Equations 5.107 and 5.108 is that the total mesh de ection is

$$\text{Mesh deflection} = \frac{\text{load}}{\text{stiffness constant} \times \text{face width}}$$

$$= \frac{W_t}{20{,}000b} \quad \text{(metric)} \tag{5.109}$$

$$= \frac{W_t}{2{,}900{,}000F}. \tag{5.110}$$

When K_{mf} is over 2.0, the error is great enough to make the face width in contact *less than* the total face width. The equation for the face load–distribution factor then becomes as follows:

$$K_{mf} = \sqrt{\frac{40{,}000be_t}{W_t}} \quad \text{(metric)}, \tag{5.111}$$

$$K_{mf} = \sqrt{\frac{5{,}800{,}000Fe_t}{W_t}} \quad \text{(English)}. \tag{5.112}$$

The choice of stiffness constant of 20,000 N/mm^2 (2,900,000 psi) deserves some comment. Tests of gear teeth show that this is a good average value for a typical gear design. If it is an error, it is more apt to be a little low than a little high. In a limited amount of testing, it is found that 23,000 N/mm^2 is typical for high-strength tooth design.

In earlier years, values like 2,000,000 psi and even 1,000,000 psi have been used. (2,000,000 psi is equivalent to about 14,000 N/mm^2.) These values seemed to give reasonable correlation with results observed in practice. The reason they did was that either the teeth tend to wear in to develop a good t or the gears were exible enough in their mountings to shift so that the contact was better than it should have been, based on errors and true stiffness of teeth.

The problem with this design approach is that large gears and very hard gears scarcely shift at all, and they do not wear in, either. Therefore, it is best to make a relatively true calculation of K_{mf}. If there is wear-in or gear-body shifting, this can more appropriately be allowed for by directly determining the compensating amounts and then subtracting them from the helix error.

Table 5.39 shows amounts of wear-in e_w that might be expected. There are relatively little good data on wear-in of gears, and so this table should be considered a good opinion on what is likely to happen rather than the results of any study in depth.

5.2.4.4 Aspect Ratio Effects

The relative slenderness of the pinion is called the *aspect ratio*. The aspect ratio m_a is the contacting face width of the pinion divided by the pitch diameter. For double-helical pinions, the best practice is to use the total face width (the width of the two helices plus the gap between the helices).

When the aspect ratio approaches 2.0, both single-helical and double-helical pinions bend and twist enough to tend to develop relatively high K_{mf} values. Figure 5.31 shows a plot of K_{mf} against the aspect ratio for both single-enveloping and double-enveloping pinions. The curve sheet was drawn to t these assumptions:

- The teeth are cut to true and exactly matching helix angles.

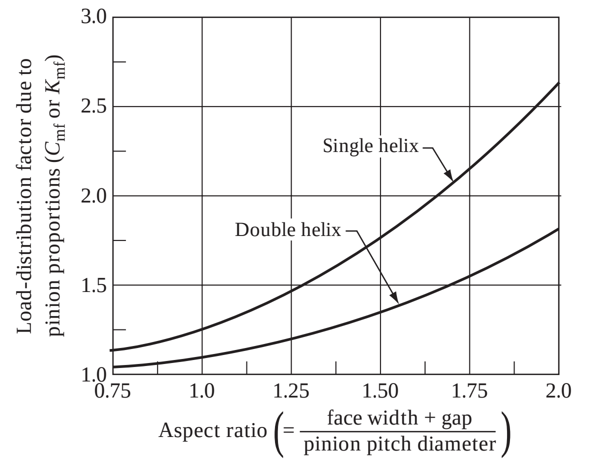


FIGURE 5.31 Effect of pinion bending and twisting on load distribution across the face width.

- The pinions and gears are straddle mounted on bearings, and the bearings are at a reasonable distance from the tooth ends.
- There is no wear-in to compensate for de ection.
- An approximation formula was used to get the de ection. The de ection was taken as an error and converted to K_{mf} by the formulas just discussed. Since de ection is proportional to load intensity, the answer in K_{mf} is the same regardless of the load intensity.

TABLE 5.39
Approximate Wear-In Amounts e_w That May Be Realized with Approximate Lubricants and Good Break-In Procedure

Material Hardness		Initial Misfit in Helix Width: 0.023 mm (0.0009 in.)		0.038 mm (0.0015 in.)		0.064 mm (0.0025 in.)		0.010 mm (0.0040 in.)	
HV	HB	mm	in.	mm	in.	mm	in.	mm	in.
Regime I—Less Than 1 m/s (200 fpm) Pitch-Line Velocity									
210	200	0.0150	0.0006	0.0230	0.0009	0.0380	0.0015	0.0500	0.0020
320	300	0.0125	0.0005	0.0200	0.0008	0.0300	0.0012	0.0380	0.0015
415	400	0.0100	0.0004	0.0180	0.0007	0.0230	0.0009	0.0300	0.0012
530	500	0.0075	0.0003	0.0125	0.0005	0.0150	0.0006	0.0250	0.0010
675	600	0.0050	0.0002	0.0075	0.0003	0.0125	0.0005	0.0200	0.0008
Regime II—Less Than 5 m/s (1000 fpm) Pitch-Line Velocity									
210	200	0.0125	0.0005	0.0180	0.0007	0.0250	0.0010	0.0300	0.0012
320	300	0.0100	0.0004	0.0150	0.0006	0.0200	0.0008	0.0250	0.0010
415	400	0.0075	0.0003	0.0125	0.0005	0.0150	0.0006	0.0200	0.0008
530	500	0.0500	0.0002	0.0075	0.0003	0.0125	0.0005	0.0150	0.0006
675	600	0.0025	0.0001	0.0050	0.0002	0.0100	0.0004	0.0125	0.0005
Regime III—Less Than 20 m/s (4000 fpm) Pitch-Line Velocity									
210	200	0.0075	0.0003	0.0100	0.0004	0.0125	0.0005	0.0150	0.0006
320	300	0.0050	0.0002	0.0075	0.0003	0.0100	0.0004	0.0125	0.0005
415	400	0.0050	0.0002	0.0050	0.0002	0.0075	0.0003	0.0100	0.0004
530	500	0.0025	0.0001	0.0025	0.0001	0.0050	0.0002	0.0075	0.0003
675	600	0.0025	0.0001	0.0025	0.0001	0.0025	0.0001	0.0050	0.0002

Figure 5.31 makes it look like the double helix does better than the single helix. This is really not so, because of the gap. When a single-helical pinion and a double-helical pinion have the same working face width, the K_{mf} value tends to be about the same.

The high K_{mf} value for an aspect ratio around 2.0 can be considerably reduced by helix modification. In general, helix modification (also called *helix correction*) should be given strong consideration when the aspect ratio of single-helical pinions exceeds 1.15 or that of double-helical pinions exceeds 1.60.

The problem of K_{mf} begins to look rather formidable from all the foregoing. Some things can be done, though, to improve the situation for spur and helical gears. (Bevel gears are discussed next.)

- If the aspect ratio is under 1.15 and the pinion and gear are straddle mounted in their bearings, there should be no serious effect from bending and twisting. Of course, if the pinion is overhung, special calculations will be needed to find the overhung deflection, even with an aspect ratio of 1.0.

  The main problem when the aspect ratio is under 1.15 is manufacturing error.
- If the aspect ratio is over 1.5, the bending and twisting effects need to be calculated. Helix correction is apt to be needed. When helix correction is made, the pinions are generally fitted to their gears so that under no load (or very light load), the contact pattern is open where the teeth have been relieved. Using special measuring techniques, the amount the teeth are open can be measured.

  If the pinion does not properly match its gear, it can be reshaved or reground. (In some cases, bearings may be slightly shifted to fix the fit.) When the fitting job is done, the desired mismatch to compensate for deflection has been achieved, and the manufacturing errors in cutting teeth or boring cases have been taken care of.
- When the aspect ratio is quite low—like 0.25 or less—there is often a tendency for the gear (or both pinion and gear) to deflect or shift so as to distribute the load relatively evenly, even when an appreciable helix mismatch exists. Tests and observations may reveal that a pair of gears that would have a K_{mf} value of around 2.5 may actually run with a *real* K_{mf} value of only 1.4! It is beyond the scope of this book to represent engineering analysis data on how gears can deflect sideways or shift on their mountings to achieve a substantial reduction in K_{mf} (or C_{mf}). The designer should be alert for this possibility and thoroughly test new designs as that any favorable reduction in K_{mf} can be noted and used.

  As a matter of interest, it is quite common for vehicle transmission gears to have a low aspect ratio, and a favorable tendency to run at much lower K_{mf} values than a simple calculation based on helix errors and tooth load per unit of face width would indicate.
- Helical gears are loaded on an inclined line that runs from the top of the tooth to the limit diameter. Under misaligned conditions, the area of high loading is localized, and the developed field of stress in the tooth root is not as severe as it would be for a spur gear.

  It is often practical to use a somewhat lower K_{mf} for helical gears than for spur gears when the aspect ratio is low. For high–aspect ratio gears, K_{mf} is generally considered to be equal to C_{mf}, the durability factor for misalignment effects.

5.2.4.5 Load-Distribution Factor K_m for Bevel Gears

Bevel gears are generally made with the following design controls:

- The face width is 0.30 or 0.25 of the outer cone distance (an aspect ratio control).
- The teeth are cut or ground so that the tooth contact is localized. A slight crown is introduced to compensate for small errors in tooth making and in mounting the bevel gears on their axes.
- The bevel gears are tested in special contact-checking machines. If the fit is not right, the bevel gears are either rejected or lightly lapped to make the fit acceptable. If the misfit is more than what lapping can correct, the pinion or gear may be recut or reground to bring the tooth fit on the tester under control.

The result of these special practices and controls is to make it much easier to establish reasonable load-distribution factors for bevel gears than for spur or helical gears.

Table 5.40 shows the general pattern of load-distribution factors for bevel gears. The *same* load-distribution factor is generally used for both strength and durability rating calculations ($K_m = C_m$).

Industrial bevel gears are often made in large sizes. In general, both members are straddle mounted. Figure 5.32 shows the general trend for the increase in mounting factor as the parts get larger and there is more difficulty in controlling the accuracy.

The mounting factors for bevel gears are right for the rated load. The reason for this is that the localized contact is developed to suit the rated torque. If the bevels are operated at lower-than-rated torque, the mounting factors tend to increase because the fit is not right. This is not too critical, though, because the torque generally decreases faster than the load-distribution factor increases.

A serious problem can occur when a bevel set has to handle very high overload torques for a small number of cycles (like 10^5) and then must run for a large number of cycles (like 10^9) at rated torque. At rated torque, the load distribution will not be as good as it should be, and a higher than normal load-distribution factor will be needed.

TABLE 5.40
Load-Distribution Factors for Bevel Gears, K_m and C_m

Application	Both Members Straddle Mounted	One Member Straddle Mounted	Neither Member Straddle Mounted
General industrial	1.00–1.10	1.10–1.25	1.25–1.40
Vehicle	1.00–1.10	1.10–1.25	–
Aerospace	1.00–1.25	1.10–1.40	1.25–1.50

Note: These values are based on setting the position of the cone element very close to the right position. For instance, if a bevel gear with 25 mm (1 in.) face width was out of position about 0.15 mm (0.006 in.), a K_m value of 1.05 would increase to about 1.8! These values are based on the face width not exceeding 25 mm. Industrial bevel gears are made in rather large sizes. Figure 5.32 shows how the load-distribution factor tends to increase as the face width increases.

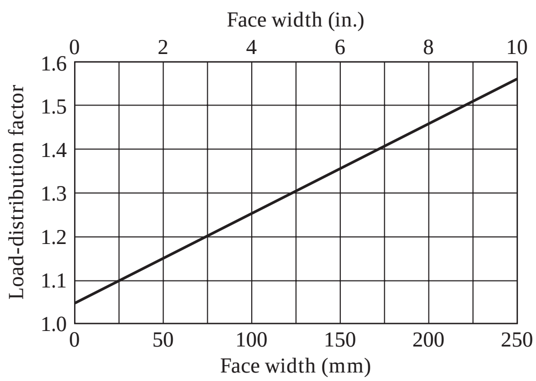


FIGURE 5.32 Load-distribution factor for bevel gears, K_m and C_m. This curve is applicable for straddle mounting only and industrial units.

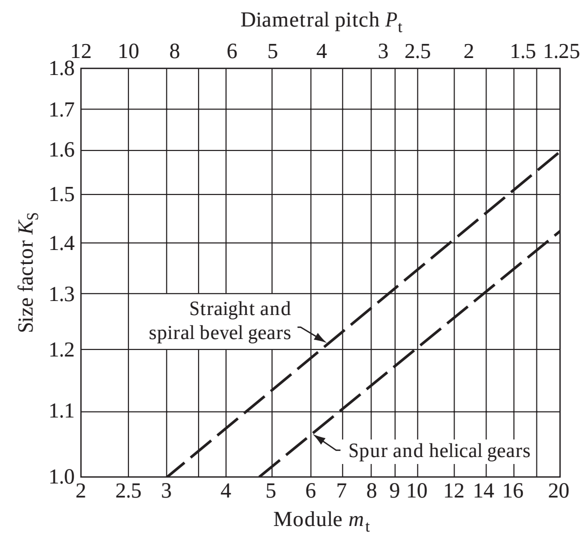


FIGURE 5.33 Size factor for strength of spur, helical, and bevel gears.

5.2.4.6 Size Factor K_s

The size factor derates the gear design for the adverse effect of size on material properties. Large gears may have the same hardness as small gears, but the material may not be as strong or fatigue resistant. Inclusions or other aws in the steel tend to be more numerous in a large stressed area than in a small stressed area.

The making of gears involves pouring ingots, forging ingots, quenching and normalizing of forgings, hardening and tempering of gears, etc. All these operations can be done with better control on small parts than on very large parts.

The size factor tends to go up to around 2 in going from rather small gears to very large gears—when all the adverse affects of size are considered. Fortunately, some things can be done to decrease the effect of size.

If the steel used is chosen to have enough alloy content, a much better heat-treating response can be obtained. Also, if temperatures or other variables are somewhat out of control, the richer-alloy steels are more *forgiving* of less than optimum conditions.

Figure 5.33 shows size factors for the strength of spur, helical, and bevel gears. The bevel curve is not the one normally used in older standards. The old curve had a unity value for a very large tooth and a 0.5 value for a small tooth. This was compensated for by using about one-half the allowed stress in bending for bevel teeth as for spur and helical teeth. The new trend is to use the same allowed stresses for spur, helical, and bevel teeth. This is done by making the size factor greater than unity as size becomes detrimental.

5.2.4.7 Dynamic Load Factors K_v and C_v

The dynamic factor makes allowance for overload effects generated by a pair of meshing gears. If both members of the gear pair were perfect, a uniform angular rotation of one member would result in a uniform angular rotation of the other member. The ratio of the number of gear teeth to the number of pinion teeth would be the exact ratio of the two angular velocities.

Gear teeth are never perfect, although a high-precision pair of gears is much more perfect than a low-precision or commercial accuracy pair. The tooth error result in a *transmission error* which makes the ratio of input speed to output speed tend to uctuate. On an instantaneous basis, each member of the gear pair is constantly going through slight accelerations

and decelerations. This results, in turn, in dynamic forces being developed because of the mass of the pinion and its shaft and the gear and its shaft.

Tooth errors in spacing, runout, and profile cause transmission error. Helix errors influence transmission error because of their indirect effect on load sharing between teeth and their effect on the stiffness constant for the mesh.

The dynamic factor is used as a derating factor to compensate for the adverse effect of the dynamic overloads caused by the driving or the driven machinery. The application factors K_v and C_v are used to handle dynamic loads unrelated to gear tooth accuracy.

The equation for dynamic factor is

$$K_v = C_v = \frac{W_t}{W_t + W_d} \quad \text{(metric or English)}, \qquad (5.113)$$

where

C_v—dynamic factor for durability
W_t—transmitted load, newtons or pounds
W_d—dynamic load, newtons or pounds

Equation 5.113 makes the dynamic factor for strength the same as the dynamic factor for durability. The definition used for dynamic load is unrelated to the kind of trouble that the load can cause when it is too great. Excessive dynamic loads are a hazard to gear tooth strength, gear tooth surface durability, and gear tooth scoring resistance.

For AGMA calculations of bending stress or contact stress, it has been customary to put the dynamic factor in the *denominator*, whereas the other derating factors are put in the *numerator*. This makes a high dynamic factor come out to some value like 0.50 instead of 2.0. The ISO work puts the dynamic factor in the numerator. In this book, AGMA practice will be followed.

Figure 5.34 shows dynamic factors for spur and helical gears. A range of values is shown for four general levels of gear accuracy. These levels can be described as follows:

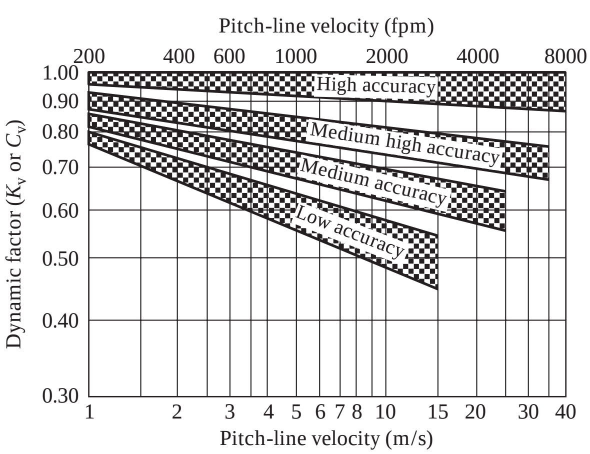


FIGURE 5.34 Dynamic factor for strength and durability for spur and helical gears.

1. *High accuracy*
   Generally ground gears may be hobbed or shaped to very close limits and then precision shaved. The tooth design provides good profile modification and helix modification (across the face width). Tooth spacing (both tooth-to-tooth and accumulated), profile (both slope and modification), helix (crown, end easement, and/or allowance for bending and twisting), gear runout, and gear surface finish (smoothness and lack of waviness) are held to very close limits. The modifications in profile and helix are based on a single load that is reasoned to be the significant load determining the rating.
2. *Medium-high accuracy*
   Gears are ground or shaved, but quality is a significant step lower than that outlined earlier.
3. *Medium accuracy*
   Gears are precision finished by hobbing or shaping. Modifications in profile or helix are either not made or made without close control. This is the best that skilled gear people can do by taking extra time for finishing and using the most suitable machine tools.
4. *Low accuracy*
   Gears are hobbed or shaped to normal practice. Profile modifications are either not made or not made with close control. Machine tools used are good, but may not have the accuracy capability of the newest types.

These levels of accuracy cannot be closely tied to the limits recommended by AGMA. Some requisite quality items are not specified in AGMA standards, and some are not set to match the latest machine tool capability. As a rough guide, though, high accuracy is like AGMA quality level 12 to 13, and medium accuracy is like AGMA quality level 8. Deutsches Institute für Normung (DIN) quality grade 5 is in the high-accuracy range. (See Section 10.4.1 for more on gear accuracy.)

The range shown in Figure 5.34 is typical of the practice that has been successfully followed for many years. A gear designer can judge how closely every accuracy item is apt to be held in gears that will be made to a given design. If the gears are going to be extra good for the way they are made, then there is reason to go to the top of the range.

Figure 5.34 stops low-accuracy gears at a pitch-line velocity of 15 m/s (3000 fpm) and medium-high accuracy gears at 35 m/s (7000 fpm). Generally speaking, these kinds of gears are not apt to be used at higher speeds, although they might give satisfactory service. (If a low-accuracy gear runs too fast, there is apt to be trouble with noise and vibration, besides the uncertainty as to whether or not the gear unit will perform as well as its rating would anticipate.)

High-accuracy gears (and superhigh-accuracy gears) are being used with relatively good success at pitch-line velocities of up to 200 m/s (40,000 fpm). What dynamic load is appropriate for high-precision gears in the 40 to 200 m/s speed range?

High-speed gears must be made very precisely. In general, the dynamic load factor will be quite low—from 0.85 to 0.95. To get the best results, the teeth need to be helical, with enough face width and axial contact ratio to get two axial crossovers.

The following values show what may be expected in high-speed gearing:

- Helical, axial contact ratio over 2.0, truly high precision in all details:

$$K_v = C_v = 0.95.$$

- Helical, axial contact ratio of 1.0, truly precision:

$$K_v = C_v = 0.90.$$

- Helical, high precision, but pro le and helix corrections not made:

$$K_v = C_v = 0.85.$$

- Spur truly high precision in all details (The high-speed spur gear needs to be narrow in face width to run above 40 m/s. Generally, the upper limit for spur gears is around 20 m/s.):

$$K_v = C_v = 0.85.$$

The dynamic load factors for bevel gears are shown in Figure 5.35. The curves represent the practices that were successfully followed for a long while. The values plotted are the same as those shown in AGMA bevel gear standards and in publications by Gleason Works.

Straight bevel gears are not often used above 10 m/s (2000 fpm) pitch-line velocity. If they are not cut to precision accuracy and tted for the best contact pattern, the dynamic factor should be read from the second or the third curve.

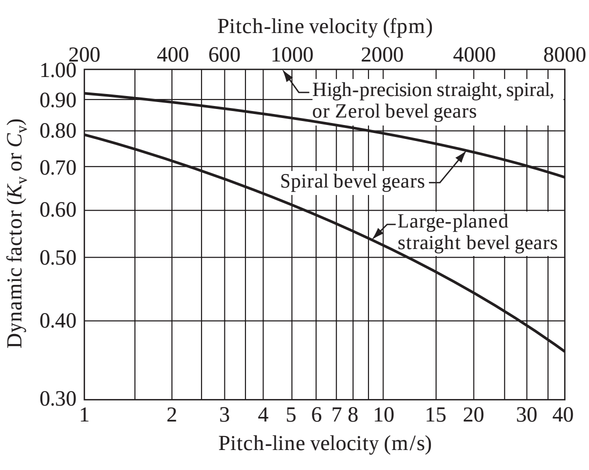


FIGURE 5.35 Dynamic factor for strength and durability for bevel gears.

Spiral bevel gears that are nished by cutting and tted for a good contact pattern should use the dynamic factor given by the second curve.

Some further discussion of dynamic load is given in Appendix B. See Section B.2.

5.2.5 Geometry Factors for Durability

The geometry factors for durability evaluate the shape of the tooth and the amount of load sharing between teeth. AGMA and ISO have procedures for evaluating these factors. The material in this section is based on AGMA methods.

The I factors shown in AGMA standard are geometry factors, but not the same kind as those in this book, with one exception: The C_k factor given by AGMA for spur and helical vehicle gears is the same kind.

The relation of C_k to I is as follows:

$$C_k = \frac{C_p}{12.043}\sqrt{\frac{1}{I} \times \frac{u}{u+1}} \quad \text{(metric)}, \tag{5.114}$$

$$C_k = \frac{C_p}{1.00}\sqrt{\frac{1}{I} \times \frac{m_G}{m_G+1}} \quad \text{(English)}, \tag{5.115}$$

where

C_p—constant for material and gear type (C_p = 2300 for steel spur or helical gears, and C_p = 2800 for steel straight, spiral, or Zerol bevel gears)
I—dimensionless constant de ned by AGMA standards
u—metric symbol for tooth ratio
m_G—English symbol for tooth ratio

Figure 5.36 shows geometry factors for the durability of spur gears of standard-addendum and 25% long-addendum pinion designs for both 20° and 25° pressure angles. These geometry values are based on the most critical stress being taken at the lowest point of single tooth contact (LPSTC).

Choosing the lowest point is felt to be more conservative than determining the stress at the pitch line. If the number of pinion teeth is over 25 and the contact ratio is 1.7 or higher, there is not much practical difference between stress taken at the LPSTC and those taken at the pitch line.

Figure 5.37 shows geometry factors for helical gear teeth of 15° and 30° helix angles and normal pressure angles of 20° and 25°. The critical stress is determined by an AGMA method that allows for load sharing in the zone of action. It is assumed that the axial contact ratio is 2.0 or more (minimum of two axial crossovers). If the axial contact ratio is only 1.0, the geometry factors will increase by a small amount. AGMA standards have provision for calculating this special case. If the axial contact ratio is less than 1.0, the helical gear approaches the spur gear. (At an axial contact ratio of 0.5, the character of the gearset is about halfway between the character of a spur gear and that of a helical gear.)

Table 5.41 shows geometry factors for the durability of bevel gears. These values are based on I factors that were

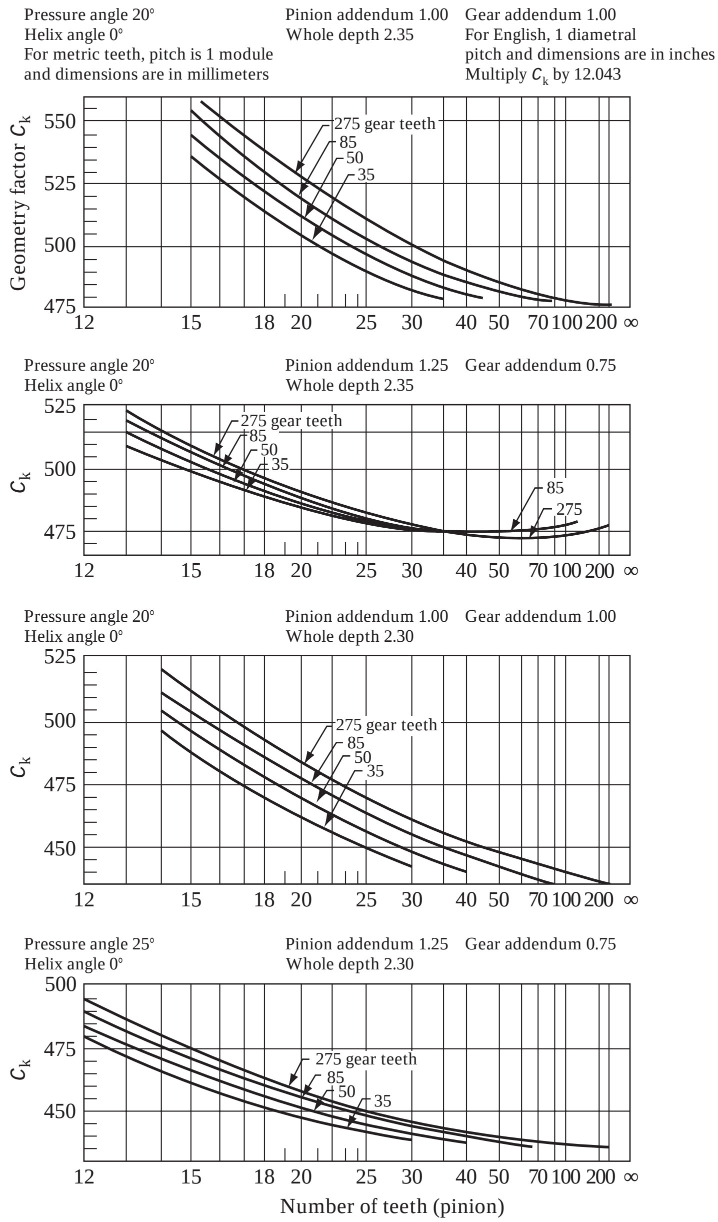


FIGURE 5.36 Geometry factors for durability of spur gears based on the LPSTC.

recommended in AGMA standards and in publications of Gleason Works.

It seems likely that gear research will develop values for the durability geometry factors that are somewhat more complex and therefore more accurate than those given in Figures 5.36 and 5.37 and in Table 5.41. There is not much chance, though, that they will change by any large amount. Experience in the gear trade shows that the geometry constants for surface durability determined by AGMA methods are quite good.

5.2.6 Overall Derating Factor for Surface Durability

The overall derating factor evaluates all the things that tend to make the load higher than it should be for the torque being transmitted. The factor is as follows:

$$C_d = \frac{C_a C_m C_s}{C_v} \quad \text{(metric or English)}, \tag{5.116}$$

where

C_a—application factor. Generally C_a is taken to be the same as K_a. See Section 5.2.4.

C_m—load-distribution factor. This factor is generally taken to be the same as K_m. See Section 5.2.4. In some special cases, K_m may be justifiably lower than C_m.

C_s—size factor. This factor is not the same as K_s.

C_v—dynamic factor. This factor is the same as K_v. See Section 5.2.4.

Since the overall derating for surface durability is rather close to being the same as the overall derating factor for

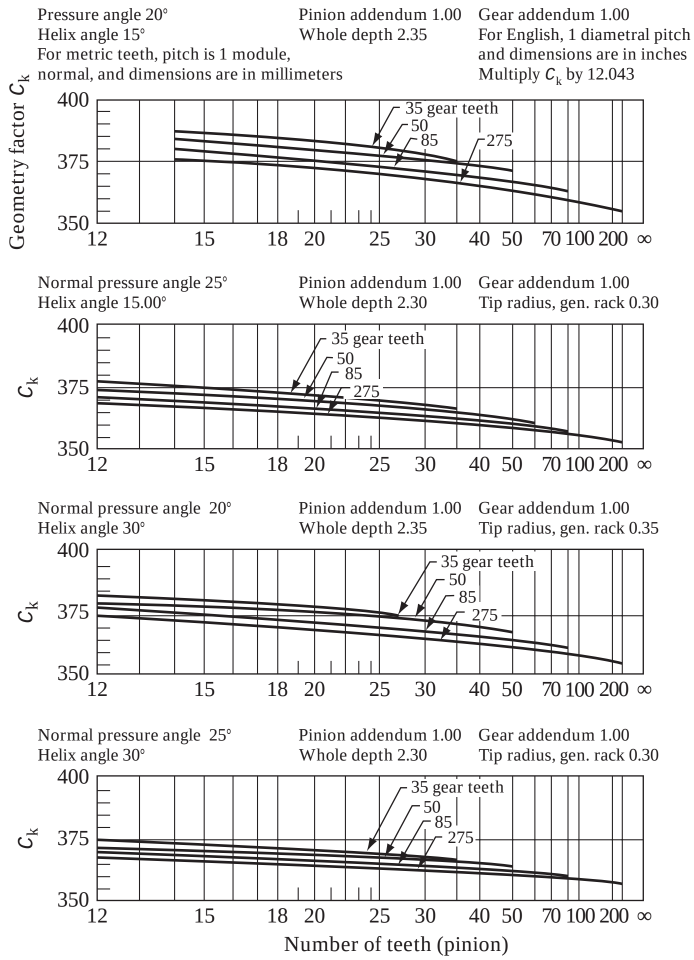


FIGURE 5.37 Geometry factors for durability of helical gears based on tip loading and sharing of load between teeth in the zone of contact.

strength, Section 5.2.4 should be read for general information and for speci c data on the factors that are the same in derating for strength and for durability.

5.2.6.1 Size Factor C_s

From a surface durability standpoint, face width is probably the best way to evaluate the detrimental effect of size. (For tooth breakage, tooth size seems to be the best measure of the size effect.)

The size factor for durability (like that for strength discussed in Section 5.2.4) is primarily intended for derating gears for the fact that large pieces of steel tend to have more aws and are more dif cult to forge and heat treat for ideal metallurgical properties than small pieces of steel.

Since 1966, AGMA standards have acknowledged the need for a size factor in the rating formula for durability of spur and helical gears. However, no numerical values have yet been agreed upon and written into standards. It is left to each designer to assign an appropriate value when large gears cannot be made with metal quality comparable with that of the small gears that were used to set basic allowable stresses in the standards.

For bevel gears, AGMA did set a size factor C_s, with numerical values. This standard, though, deviates from spur and helical practices in that the detrimental effect of size is not handled by a factor greater than unity. Instead, a factor near unity is given to a large gear and a factor of 0.5 to a small gear. This, in effect, says that the large gear is the standard of reference and gives the small gear credit for being better than the large gear. The more usual AGMA philosophy is that the most is known and made right with the small gear, and it should be the standard of reference. Large gears are then derated for the fact that their size tends to give imperfection that reduces the amount of stress that can be carried.

AGMA standards were issued to cover spur, helical, and spiral bevel gears. These standards show a minimum C_s value of 1.0 for spur and helical gears, but it does show C_s values less than 1.0 for spiral bevel gears.

Figure 5.38 shows a plot of size factors for durability versus face width. The spiral bevel curve is worked out to place the unity value on the small gear and derate the large gear (rather than uprate the small gear). This change was made for this book by shifting the allowable contact stress so that the value used is right for the small gear at $C_s = 1.0$.

The *size factor*—by AGMA de nition—does include "area of stress pattern." This variable shifts more in going from small to large bevel gears than it does in spur or helical

TABLE 5.41
Geometry Factors C_k for Durability of Bevel Gears

No. Pinion Teeth	No. of Gear Teeth	Straight Bevel Gears	Spiral Bevel Gears
15	20	8433	7780
	35	8731	7281
	50	8967	6837
	100	8752	6260
20	20	7952	7537
	35	8102	7285
	50	8315	6803
	100	8144	6283
25	20	7766	7379
	35	7808	7251
	50	7935	6742
	100	7766	6271
35	35	7565	6956
	50	7593	6627
	100	7317	6222
50	50	7353	6458
	100	6989	6121

Note: These values are subject to change as re nements are made in the procedure to determine stress intensity due to bending load on gear teeth. This table is based on the normal design and manufacturing process recommended by Gleason Works. In many cases, there will be reason to modify the actual design, and this will tend to cause changes in the geometry factors from the guideline values given here. The straight bevel data are for Coni ex bevel gears with a 20° pressure angle. The spiral bevel data are for 20° pressure angle, 35° spiral angle bevel gears. The geometry factors for Zerol bevel teeth tend to be the same as those for straight bevel gears.

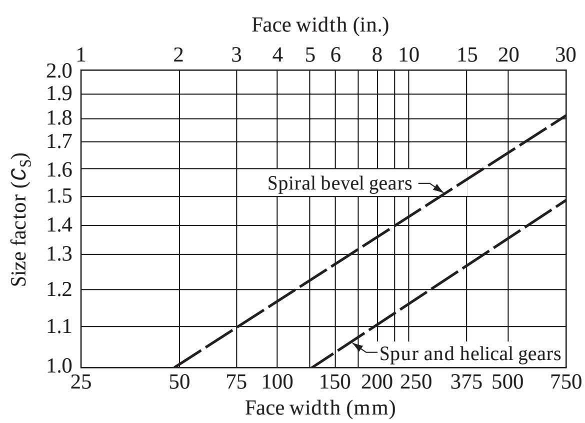


FIGURE 5.38 Size factor for durability of spur, helical, and spiral bevel gears.

gears. This has to do with the way the teeth are made and t-ted. (It must be kept in mind that bevel teeth are on the surface of a *cone*, while spur and helical teeth are on a *cylinder*.)

The size factor for spiral bevel gears changes from about 2 to 1 in going from a small gear to a large gear. The dashed curve for spur and helical gears changes from about 1.5 to 1.0 in going from a small gear to a very large gear. The relative difference is essentially due to the geometric difference between bevel gears and helical gears.

For spur and helical gears, the curve for C_s in Figure 5.38 is shown dashed. It is not based on any AGMA standard. Instead, the curve represents what can normally be expected in good industrial gear manufacture.

5.2.6.2 Complementary Considerations

Here are the considerations that relate to recommended values of the size factor C_s for gears of various applications:

- At up to 125 mm (5 in.) face width, it should be possible to pick steel and process it well enough to have essentially no size effect. $C_s = 1.00$.
- At above 125 mm face width, it becomes increasingly dif cult to avoid size effects. A gearset with about 400 mm (16 in.) face width is apt to have a pinion around 400 mm pitch diameter mating with a gear in the range of 1600 mm (64 in.) pitch diameter. Metallurgical studies* of gears of this size generally show quality degradations ranging from small to very serious. A size derating of 1.3 for a gear with 400 mm face width seems to be about the average for industrial gear manufacture around the world by those skilled in the gear art. (Those unskilled may, of course, do much worse.)
- In the aerospace eld, great skill and effort are used to keep gear metallurgical quality under close control. Aerospace people could probably make gears up to 250 mm face width without any noticeable size effects. (Most aerospace gears have less than 125 mm face width.)
- Marine gears are made in face widths up to 1000 mm (40 in.). Marine practice for ocean-going ships is highly specialized. An appropriate size derating for a 1000 mm marine drive might be around $C_s = 1.20$.

5.2.7 Load Rating of Worm Gearing

The complete worm-gear family are the following:

- Crossed-helical gears (nonenveloping)
- Cylindrical worm gearing (single-enveloping)
- Double-enveloping worm gears

This section will cover methods for estimating the load-carrying capacity of each of the three family members. The method for crossed-helical gears will follow the work of Buckingham (1931). This method gives reasonable results and is a good guide. There are no trade standards on crossed-helical gears. Since Buckingham's work is all in English units, the synopsis given here will be in English symbols and units only.

* See Section 6.2.4.

(Readers can probably best check the reference by sticking to English units.)

The rating for cylindrical worm gearing will follow AGMA standards. This is a good guide developed over many years. Not much change can be anticipated in the immediate future.

The rating for double-enveloping worm gears will follow AGMA standards. This again gives good guidance and will probably not change much in the immediate future.

The rating of all kinds of worm gearing is primarily based on surface durability. Tooth strength is usually not much of risk—unless abnormally small threads or teeth are used. For this reason, strength is handled on a very approximate basis. The set is *sized* from the durability estimates.

5.2.7.1 Crossed-Helical Gear Durability

Crossed-helical gears are generally rated only by a surface durability formula. With point contact, not enough load can be carried to cause much danger of tooth breakage. The method presented by Buckingham (1931) is a handy general formula for crossed-helical gears, which gives very reasonable results.

The rst step is to calculate a wear load W_w:

$$W_w = A^6 B^3 KQ. \tag{5.117}$$

The factors A and B depend on the ratio of the radii of curvature of the pro les of the driver and the driven. The radius of curvature of the driver is

$$R_{c1} = \frac{D_1 \sin\phi_n}{2\cos^2\psi_1}. \tag{5.118}$$

The radius of curvature of the driven member is determined by a similar equation. The pitch diameter of the driver is D_1, and the normal pressure angle is ϕ_n. The helix angle is ψ_1.

Table 5.42 shows the constants A^6 and B^3.

The constant K is *not* the same as the K factor discussed in Chapter 4. This K is

$$K = 29.7662 s^3 \left(\frac{1}{E_1} + \frac{1}{E_2}\right)^2. \tag{5.119}$$

TABLE 5.42
Factors for Rating Crossed-Helical Gears

Curvature Ratio, R_{c2}/R_{c1}	A^6	B^3	A^6B^3
1.000	0.560	1.0000	0.560
1.500	1.302	0.4490	0.583
2.000	2.411	0.2520	0.609
3.000	6.053	0.1120	0.678
4.000	11.620	0.0640	0.744
6.000	30.437	0.0292	0.889
10.000	106.069	0.0108	1.141

TABLE 5.43
Load Stress Factors for Crossed-Helical Gears

Pinion (Driver)	Gear (Driven)	s (psi)	K (lb)
	With Initial Point Contact		
Hardened steel	Hardened steel	150,000	446
Hardened steel	Bronze, phosphor	83,000	170
Cast iron	Bronze, phosphor	83,000	302
Cast iron	Cast iron	90,000	385
	With Short Running-In Period		
Hardened steel	Hardened steel	–	446
Hardened steel	Bronze, phosphor	–	230
Cast iron	Bronze, phosphor	–	600
Cast iron	Cast iron	–	770
	With Extensive Running-In Period		
Hardened steel	Hardened steel	–	446
Hardened steel	Bronze, phosphor	–	300
Cast iron	Bronze, phosphor	–	1200
Cast iron	Cast iron	–	1500

The value s is the stress on the tooth surface, and E is the modulus of elasticity.

The value Q is a ratio factor:

$$Q = \left(\frac{R_{c1}R_{c2}}{R_{c1} + R_{c2}}\right)^2. \tag{5.120}$$

The allowable K for crossed-helical gears that Buckingham recommends is given in Table 5.43.

It should be noted that Table 5.43 shows quite a difference in capacity with running in. A little wear at light load tends to considerably broaden the point of contact.

The calculated wear load in Equation 5.117 should be compared with the dynamic load in the normal plane. If the torque on the driver is T_1, the transmitted load in the normal plane is

$$W_n = \frac{2T_1}{D_1 \cos\psi_1 \cos\phi_n}. \tag{5.121}$$

If a high degree of accuracy is obtained, it is usually possible to keep the dynamic load on crossed-helical gears within about 150% of the transmitted load in the normal plane. The calculated wear load should be enough less than the transmitted load to allow for dynamic load effects and to allow for any application factor that may be appropriate for the job.

There is little conclusive information to go on to judge the limiting rubbing velocity that can be handled by spiral gears of different materials. Hardened steel on bronze will handle the most speed. With ordinary materials and good commercial accuracy, the rubbing speed should be probably not exceed 30 m/s (6000 fpm). The use of special bronze material in combination with a case-hardened and ground driver makes it possible to handle rubbing speed up to 50 m/s (10,000 fpm)

in aircraft and steam turbine applications. The other material combinations shown in Table 5.43 should probably not be used above a rubbing speed of 20 m/s (4000 fpm) in ordinary applications.

The sliding or rubbing velocity may be calculated by

$$v_s = 0.262 \frac{n_1 D_1}{\sin \psi_1} \quad \text{fpm}, \tag{5.122}$$

where n_1 is the rotational speed of the driver in rpm.

A special word of caution should be added. The crosswise rubbing in crossed-helical sets tends to destroy the EHD oil lm. This means that these gears can get into more trouble than other worm gears, bevel gears, or regular helical gears when the oil is thin or the surface nishes are poor. Those using crossed-helical gears need past experience or test stand results to make sure that the choice of lubricant and the quality of the tooth surface nishes are appropriate for the application.

5.2.7.2 Cylindrical Worm Gear Durability

The load-carrying capacity of worm gearsets may be estimated from the general formula given. Since Equations 5.123 and 5.124 have been developed more by experience than by rational derivation, it is more reasonable to calculate tangential load capacity than it is to calculate stress and compare the calculated stress to allowable stress. The general formula for cylindrical worm gearsets is

$$W_t = \frac{K_s d_{p.2}^{0.8} b' K_m K_v}{75.948} \quad (\text{N; metric}), \tag{5.123}$$

$$W_t = K_s D^{0.8} F_e K_m K_v \quad (\text{lb; English}), \tag{5.124}$$

where

K_s—materials factor (see Table 5.44; this is different from the K_s size factor)

d_{2p} or D—worm gear pitch diameter (mm or in.)

b' or F_e—effective face width, mm or in. (not exceed 2/3 of the worm pitch diameter)

K_m—ratio correction factor, dimensionless (see Table 5.45)

K_v—dynamic factor, dimensionless (see Table 5.46)

The sliding velocity is

$$v_s = \frac{d_{p1} n_1}{19{,}098 \cos \gamma} \quad (\text{m/s; metric}), \tag{5.125}$$

$$v_s = 0.2618 \frac{d n_w}{\cos \lambda} \quad (\text{fpm; English}), \tag{5.126}$$

where

n_1 or n_w—rotational speed of worm (rpm)

γ or λ—lead angle of threads at mean worm diameter (°)

TABLE 5.44
Material Factor K_s for Cylindrical Worm Gears

For Units of 75 mm (3 in.) Center Distance to about 1 m (40 in.) Center Distance

Gear Pitch Diameter mm	in.	Sand Cast	Static Chill Cast	Centrifugal Cast
65	2.5	1000	–	–
75	3	960	–	–
100	4	900	–	–
125	5	855	–	–
150	6	820	–	–
175	7	790	–	–
200	8	760	1000	–
250	10	715	955	–
375	15	630	875	–
505	20	570	815	–
635	25	525	770	1000
760	30	–	740	985
1015	40	–	680	960
1270	50	–	635	945
1775	70	–	570	920

For Units with Less Than 75 mm (3 in.) Center Distance

Center Distance mm	in.	Maximum K_s Value
12	0.5	725
25	1.0	735
38	1.5	760
50	2.0	800
63	2.5	880
75	3.0	1000

Source: AGMA, Standard practice for single and double reduction cylindrical worm and helical worm speed reducers (AGMA 6034-B92), 1959. With permission.

Note: For bronze worm gear and steel worm with at least HV 655 (HRC 58) surface hardness. See Chapter 6 for worm gear material data. Sliding velocity not to exceed 30 m/s (6000 fpm); worm speed is not more than 3600 rpm.

After the allowable value of W_t is determined, an output power capacity (at an application factor of 1) can be determined from

$$P = \frac{W_t d_{p2} n_2}{19{,}090{,}800} \quad (\text{kW; metric}), \tag{5.127}$$

$$P = \frac{W_t D n_G}{126{,}800} \quad (\text{hp; English}), \tag{5.128}$$

where n_2 (or n_G) is gear speed (rpm).

The available output power from the worm gearset alone has to be divided by a service factor to make allowance for the smoothness or roughness of the driving and driven equipment. In worm gear practice, this factor is still a service factor, since

TABLE 5.45
Ratio Correction Factor K_m for Cylindrical Worm Gears

Ratio, u (m_G)	K_m	Ratio, u (m_G)	K_m
3.0	0.500	14.0	0.799
3.5	0.554	16.0	0.809
4.0	0.593	20.0	0.820
4.5	0.620	30.0	0.825
5.0	0.645	40.0	0.815
6.0	0.679	50.0	0.785
7.0	0.706	60.0	0.745
8.0	0.724	70.0	0.687
9.0	0.744	80.0	0.622
10.0	0.760	90.0	0.555
12.0	0.783	100.0	0.490

Source: AGMA, Standard practice for single and double reduction cylindrical worm and helical worm speed reducers (AGMA 6034-B92), 1959. With permission.

TABLE 5.46
Velocity Factor K_v for Cylindrical Worm

Sliding Velocity		Velocity	Sliding Velocity		Velocity
m/s	fpm	Factor, K_v	m/s	fpm	Factor, K_v
0.005	1	0.649	3.0	600	0.340
0.025	5	0.647	3.5	700	0.310
0.050	10	0.644	4.0	800	0.289
0.100	20	0.638	4.5	900	0.272
0.150	30	0.631	5.0	1000	0.258
0.200	40	0.625	6.0	1200	0.235
0.300	60	0.613	7.0	1400	0.216
0.400	80	0.600	8.0	1600	0.200
0.500	100	0.588	9.0	1800	0.187
0.750	150	0.588	10.0	2000	0.175
1.000	200	0.588	11.0	2200	0.165
1.250	250	0.500	12.0	2400	0.156
1.500	300	0.472	13.0	2600	0.148
1.750	350	0.446	14.0	2800	0.140
2.000	400	0.421	15.0	3000	0.134
2.250	450	0.398	20.0	4000	0.106
2.500	500	0.378	25.0	5000	0.089
2.750	550	0.358	30.0	6000	0.079

Source: AGMA, Standard practice for single and double reduction cylindrical worm and helical worm speed reducers (AGMA 6034-B92), 1959. With permission.

it also allows for the amount of time the unit is to be operated. As was explained in Section 5.2.4, the latest practice for spur and helical gears is to use an application factor instead of a service factor. The application factor considers roughness of connected equipment but does not evaluate life cycles or hours of operation.

Table 5.47 shows typical service factors.

If input power is needed, the output power must have added to it the losses in the gearbox. These losses come from friction on the gear teeth, the bearing losses, the seal losses, and the losses due to oil churning. The relation is

Input power = power used by driven machine
+ sum of all losses.

The calculation of worm gear ef ciencies is too complex to be included in this book.

Worm gears are used with materials other than a bronze gear and a steel worm. There are no trade standards for these other materials. Some rough guidance as to what these other materials might be expected to do is given in Table 5.48.

The strength of worm gear teeth is not well established. Since strength is usually not critical, there has not been any extensive research activity to establish calculation means and allowable bending stress. (Some worm gearing builders have done a considerable amount of testing to establish data for their own products.)

A unit load can be calculated for worm gears:

$$U_l = \frac{W_t}{b' m_n} \quad (\text{N/mm}^2;\ \text{metric}), \tag{5.129}$$

$$U_l = \frac{W_t P_{nd}}{F_e} \quad (\text{psi};\ \text{English}), \tag{5.130}$$

where
W_t—tangential load (N [metric] or lb [English])
b'—effective face width (mm [metric])
F_e—effective face width (in. [English])

$$m_n = \frac{p_x \cos\gamma}{3.14159} \quad (\text{metric})$$

$$P_{nd} = \frac{3.14159}{p_x \cos\lambda} \quad (\text{English})$$

Table 5.49 gives some rather approximate values for unit load. Generally, there is no problem if the unit load is less than the values shown (providing no unusual shock loads are present).

5.2.7.3 Double-Enveloping Worm Gear Durability

The load-carrying capacity of double-enveloping worm gears is calculated as the input horsepower by a simpli ed empirical formula:

$$P = \frac{0.7457 n_1 K_s K_m K_a K_v}{u} \quad (\text{kW};\ \text{metric}), \tag{5.131}$$

$$P = \frac{n_W}{m_G} K_s K_m K_a K_v \quad (\text{hp};\ \text{English}), \tag{5.132}$$

TABLE 5.47
Service Factors for Cylindrical Worm Gear Units

Prime Mover	Duration of Service per Day	Driven Machine Load Classification: Uniform	Moderate Shock	Heavy Shock
Electric motor	Occasional ½ hour	0.80	0.90	1.00
	Intermittent 2 hours	0.90	1.0	1.25
	10 hours	1.00	1.25	1.50
	24 hours	1.25	1.50	1.75
Multicylinder internal combustion engine	Occasional ½ hour	0.90	1.00	1.25
	Intermittent 2 hours	1.00	1.25	1.50
	10 hours	1.25	1.50	1.75
	24 hours	1.50	1.75	2.00
Single-cylinder internal combustion engine	Occasional ½ hour	1.00	1.25	1.50
	Intermittent 2 hours	1.25	1.50	1.75
	10 hours	1.50	1.75	2.00
	24 hours	1.75	2.00	2.25
Following Service Factors Apply for Applications Involving Frequent Starts and Stops				
Electric motor	Occasional ½ hour	0.90	1.00	1.25
	Intermittent 2 hours	1.00	1.25	1.50
	10 hours	1.25	1.50	1.75
	24 hours	1.50	1.75	2.00

Source: AGMA, Standard practice for single and double reduction cylindrical worm and helical worm speed reducers (AGMA 6034-B92), 1959. With permission.
Note: Time speci ed for intermittent and occasional services refers to total operating time per day. The term *frequent starts and stops* refers to more than 10 starts per hour.

TABLE 5.48
Guide to Approximate Material Capacity for a Variety of Worm Gear Material Combinations

Material: Worm	Worm Gear	Durability Constant, K_s	Speed Range, Rubbing Velocity
Steel, 53 HRC minimum	Bronze, phosphor	600	Up to 30 m/s (6000 fpm)
Steel, 35 HRC minimum	Bronze, phosphor	600	Up to 10 m/s (2000 fpm)
Steel, 53 HRC minimum	Bronze, super manganese	1000	Up to 2 m/s (400 fpm)
Steel, 53 HRC minimum	Bronze, forged manganese	700	Up to 10 m/s (2000 fpm)
Cast iron	Cast iron	700	Up to 2 m/s (400 fpm)
Cast iron	Bronze, phosphor	600	Up to 10 m/s (2000 fpm)

where
n_1 or n_W—worm speed (rpm)
u or m_G—tooth ratio $\left(= \dfrac{\text{no. gear teeth}}{\text{no. worm threads}}\right)$
K_s—pressure constant based on center distance (see Figure 5.39)
K_m—ratio correction factor (see Table 5.50)
K_a—face width and materials factor based on center distance (see Table 5.51)
K_v—velocity factor based on rubbing or sliding speed (see Figure 5.40)

The sliding speed of double-enveloping worm gear is

$$v_s = \frac{d_{t1}n_1}{19,089\cos\beta} \quad \text{(m/s; metric)}, \tag{5.133}$$

$$v_s = \frac{0.2618 d_t n_W}{\cos\psi} \quad \text{(fpm; English)}, \tag{5.134}$$

where
d_{t1} or d_t—worm throat diameter (Figure 1.31)
n_1 or n_W—worm (rpm)
β or ψ—worm gear helix angle

TABLE 5.49
Approximate Values of Unit Load for Cylindrical Worm Gears

Material Combination		U_l (psi)	
Worm	**Worm Gear**	**Running**	**Static**
Steel, hardened	Bronze, phosphor	1350	5500
Steel, hardened	Bronze, super manganese	3200	12,500
Steel, hardened	Bronze, forged manganese	1800	7000
Cast iron	Cast iron	1000	5500
Cast iron	Bronze, phosphor	1350	5500

Note: The hardened steel worm will usually be HV 655 or HRC 58 to get surface durability. Some slow-speed worm gears may use through-hardened steel worms. These have less surface durability. It is assumed that the hardened steel worm will at least have HV 354 or HRC 35 hardness. These values assume an average contact ratio of only 1.5. Many worm gear designs will have a higher contact ratio and therefore somewhat higher unit load capability.

TABLE 5.50
Ratio Correction Factor K_m for Double-Enveloping Worm Gears

Ratio, u (m_G)	K_m	Ratio, u (m_G)	K_m
3.0	0.380	14.0	0.720
3.5	0.435	16.0	0.727
4.0	0.490	20.0	0.737
4.5	0.520	30.0	0.746
5.0	0.550	40.0	0.748
6.0	0.604	50.0	0.750
7.0	0.632	60.0	0.751
8.0	0.665	70.0	0.752
9.0	0.675	80.0	0.752
10.0	0.690	90.0	0.753
12.0	0.706	100.0	0.753

Source: AGMA, Standard practice for single-, double-, and triple-reduction worm and helical worm speed reducers (AGMA 6035-A02), 1978. With permission.

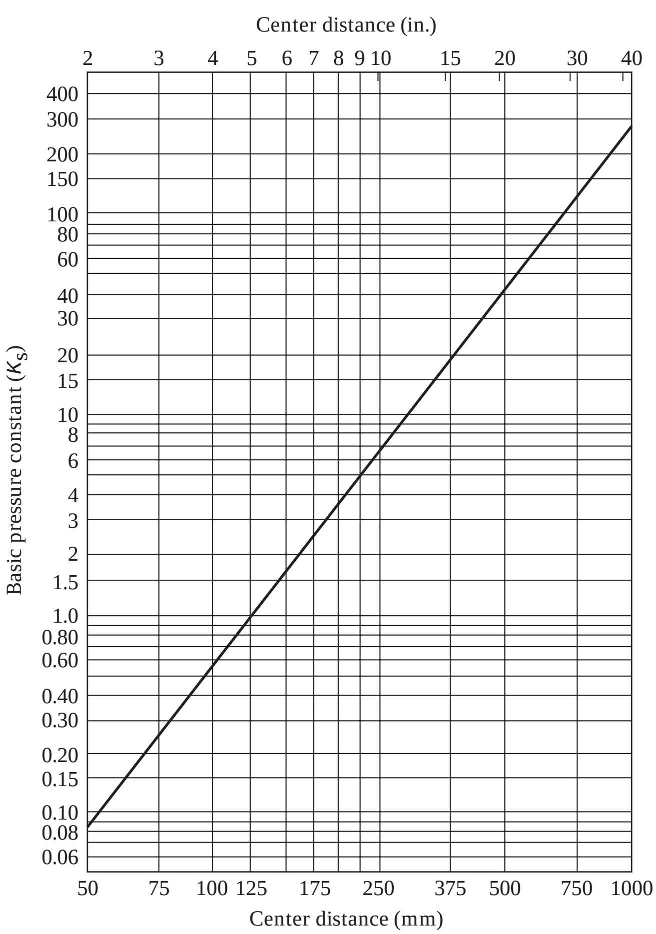


FIGURE 5.39 Basic pressure constant based on center distance for standard-design double-enveloping worm gears.

TABLE 5.51
Face Width and Materials Factors for Standard Design Double-Enveloping Worm Gears

Center Distance		
mm	**in.**	**Materials Factor, K_a**
50.08	2.000	0.620
63.5	2.500	0.684
76.2	3.000	0.755
88.9	3.500	0.780
101.9	4.000	0.855
127.0	5.000	0.934
152.4	6.000	1.014
177.8	7.000	1.073
203.2	8.000	1.113
254.0	10.000	1.175
304.8	12.000	1.250
381.0	15.000	1.281
457.2	18.000	1.328
533.4	21.000	1.368
609.6	24.000	1.398
660.4	26.000	1.411
711.2	28.000	1.425
762.0	30.000	1.438
812.8	32.000	1.445
863.6	34.000	1.453
914.4	36.000	1.460
965.2	38.000	1.469
1016.0	40.000	1.476

Source: AGMA, Standard practice for single-, double-, and triple-reduction worm and helical worm speed reducers (AGMA 6035-A02), 1978. With permission.

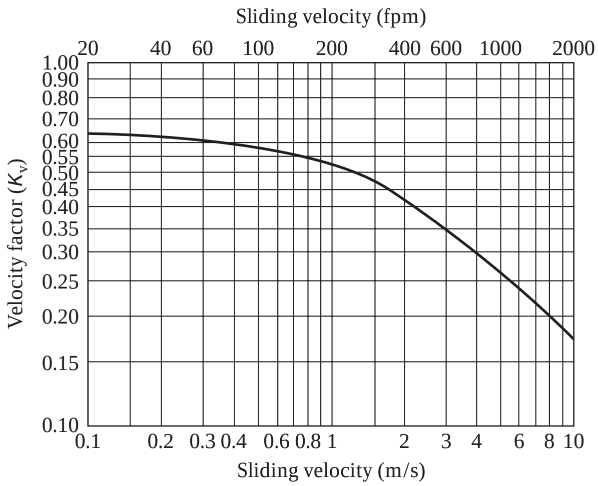


FIGURE 5.40 Velocity factor for double-enveloping worm gears.

The output power can be obtained by subtracting all the losses from the input power. Thus,

$$\text{Output power} = \text{input power} - \text{sum of all losses.} \tag{5.135}$$

The standard method of calculating the rating for double-enveloping worm gears is shown in AGMA standards. In this standard, overall ef ciency values are given for a wide range of standard designs. The rating data shown above are extracted from this standard. Those designing double-enveloping worm gears should use this standard to obtain more data than can be covered in this book.

In double-enveloping worm gear practice, the service factors are based on the shock in the system. (There is no particular difference between an electric motor drive and a turbine.) The usable power is the power obtained from Equations 5.131 and 5.132) divided by the service factor. Table 5.52 shows service factors extracted from AGMA.

TABLE 5.52
Service Factors for Double-Enveloping Worm Gears

Hours/Day	Uniform	Moderate Shock	Heavy Shock	Extreme Shock
½	0.6	0.8	0.9	1.1
1	0.7	0.9	1.0	1.2
2	0.9	1.0	1.2	1.3
10	1.0	1.2	1.3	1.5
24	1.2	1.3	1.5	1.75

Source: AGMA, Standard practice for single-, double-, and triple-reduction worm and helical worm speed reducers (AGMA 6035-A02), 1978. With permission.

5.2.7.4 Comparison of Double-Enveloping and Cylindrical Worm Gear Rating Procedures

There are several signi cant differences in practice. A wider variety of design practices are used for cylindrical worm gears. This has led to the use of more general-purpose formulas rather than highly specialized formulas.

To help keep the reader from getting confused over the data presented, these comparisons are worth noting:

Item	Comparison
Power	Cylindrical ratings are based on output power. Double-enveloping ratings are based on input power.
Size	The worm gear pitch diameter is the primary size quantity for cylindrical worm gear units. The double-enveloping units use center distance as the primary size quantity.
Materials	Several materials are in general use for the worms or the worm gears of cylindrical worm units. Double-enveloping worm gear units usually use through-hardened steel worms and chill-cast or centrifugally cast bronze gears. For each center distance and speed, there is essentially only one choice of material. (This explains why the material factor is handled indirectly as a function of center distance and rubbing speed.)
Design exibility	Cylindrical worm gear practice allows some exibility in regard to worm pitch diameter for a given center distance and some exibility on other design variables. For double-enveloping worm gears, all variables are tied quite closely to the center distance and the desired ratio.

5.2.8 Design Formulas for Scoring

The problem of designing gears to resist scoring is still not completely solved. Section 4.1.5 gave the historical background of PVT and ash-temperature calculations.

Much work has been done on elastohydrodynamic calculations of oil- lm thickness and the possible relation of gear tooth surface roughness to EHD lm.

The best that can be done is to estimate the hazard of scoring. There is no positive assurance that gears that are calculated to be quite good will not score, and gears that are calculated to be somewhat poor may still perform satisfactorily. The best procedure is to calculate the scoring risk and then plan to handle it by either design changes to lessen the scoring risk or special development of the gearset and its lubrication system to handle a latent scoring hazard.

This section will give two design approaches. One is based on ash temperature and *hot scoring*; the other is based on oil- lm thickness and *cold scoring*.

5.2.8.1 Hot Scoring

A general design formula for spur and helical gears is

$$T_f = T_b + Z_t Z_s Z_c, \tag{5.136}$$

where

T_f— ash temperature index (°C or °F)
T_b—gear body temperature (°C or °F)

Z_t—geometry constant (dimensionless)
Z_s—surface finish constant (dimensionless)
Z_c—scoring criterion number (°C factor or °F factor)

The body temperature is hard to measure, but it can be measured with thermocouples and a means of getting the reading out of the rotating part (by slip rings or by miniature radio). In well-designed gears with oil nozzles delivering enough oil, the temperature rise of the gear body (over the incoming temperature) ought not to exceed the following values:

- 25°C (45°F) for aerospace gears
- 15°C (27°F) for turbine gears

The geometry constant Z_t was defined in Section 4.1.5, Equation 4.52. If the teeth have no tip relief, Z_t should be calculated for the tips of the pinion and the gear, since the hazard of scoring will be highest at the tips. Table 5.53 gives some representative values of Z_t for tip conditions.

High-performance gears with teeth 2.5 module (10 diametral pitch) or larger are generally given a standard profile modification. With profile modification, the most critical point for scoring is generally at the start of modification.

Table 5.54 shows a design guide for profile modification worked out for general-purpose gearing with moderately heavy loads. (This guide may not be appropriate for highly critical aerospace power gears or the most critical vehicle gears—more on this see Section 13.2.)

The values of Z_t that match the modification depth shown in Table 5.54 are plotted in Figures 5.41 through 5.44. See Appendix B.14 for a general method to determine Z_t at any point on the profile.

A standard amount of modification for general design can be set:

$$\text{Gear tip modification} = \frac{6.5 C_m W_t}{10^5 b} \quad (\text{mm; metric}), \tag{5.137}$$

$$\text{Gear tip modification} = \frac{4.5 C_m W_t}{10^7 F} \quad (\text{in.; English}), \tag{5.138}$$

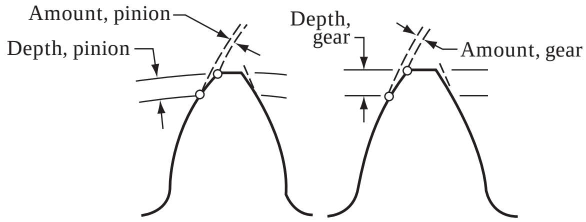
$$\text{Pinion tip modification} = \frac{4.1 C_m W_t}{10^5 b} \quad (\text{mm; metric}), \tag{5.139}$$

TABLE 5.53
Geometry Constant for Scoring Z_t at the Tip of the Tooth

Pressure Angle, α_t or ϕ_t	No. of Pinion Teeth, z_1 or N_P	No. of Gear Teeth, z_2 or N_G	Pinion Addendum, h_{a1} or a_P (for m = 1.0 or P_d = 1.0)	Gear Addendum, h_{a2} or a_G (for m = 1.0 or P_d = 1.0)	Z_t At Pinion Tip	Z_t At Gear Tip
20°	18	25	1.00	1.00	0.0184	−0.0278
	18	35	1.00	1.00	0.0139	−0.0281
	18	85	1.00	1.00	0.0092	−0.0307
	25	25	1.00	1.00	0.0200	−0.0200
	25	35	1.00	1.00	0.0144	−0.0187
	25	85	1.00	1.00	0.0088	−0.0167
	12	35	1.25	0.75	0.0161	−0.0402
	18	85	1.25	0.75	0.0107	−0.0161
	25	85	1.25	0.75	0.0104	−0.0112
	35	85	1.25	0.75	0.0101	−0.0087
	35	275	1.25	0.75	0.0070	−0.0072
25°	18	25	1.00	1.00	0.0135	−0.0169
	18	35	1.00	1.00	0.0107	−0.0168
	18	85	1.00	1.00	0.0074	−0.0141
	25	25	1.00	1.00	0.0141	−0.0141
	25	35	1.00	1.00	0.0107	−0.0126
	25	85	1.00	1.00	0.0069	−0.0103
	12	35	1.25	0.75	0.0328	−0.0160
	12	85	1.25	0.75	0.0500	−0.0151
	18	85	1.25	0.75	0.0056	−0.0095
	25	85	1.25	0.75	0.0082	−0.0073
	35	85	1.25	0.75	0.0078	−0.0060
	35	275	1.25	0.75	0.0056	−0.0048

Note: When proper profile modification is made, the risk of scoring is probably more critical at the start of modification than at the tip of the tooth. Values of Z_t at the approximate start of modification are given in Figures 5.41 through 5.44.

TABLE 5.54
Depth to Start of Profile Modification for General-Purpose Gears



Schematic diagram to show locations of amounts of profile modification and depth of profile modification on a pinion and on a gear.

Normal Pressure Angle	Pinion (Driver)	Gear (Driven)
20°	0.400	0.450
22.5°	0.365	0.415
25°	0.325	0.375

Note: The depth shown is in millimeters for 1 module, normal, or in inches for 1 normal diametral pitch. For other teeth size, multiply depth by normal module for metric, or divide depth by normal diametral pitch for English.

In some cases, manufacturing considerations lead to putting all the modification on the pinion. If this is done, the modification that might have been put at the gear tip is put at the pinion form diameter. In this case, a diameter for start modification can be calculated for the gear. Then as a next step, Table B.16 formulas can be used to find the diameter on the pinion that matches this diameter. This matching pinion diameter is then used as a start of modification diameter for the lower flank of the pinion.

$$\text{Pinion tip modification} = \frac{2.8 C_m W_t}{10^7 F} \quad \text{(in.; English)}, \tag{5.140}$$

where C_m is the load-distribution factor for surface durability.

The factor Z_s allows for surface finish conditions. This is a most difficult variable to determine. With much test-stand experience, a manufacturer can determine reasonable values for calculation purposes. On a new product, one must consider the finish achieved in the shop, the initial wear-in (which may considerably improve the effective finish, from a scoring standpoint), and the beneficial effects of lubricant additives. (Special oils and additives may be used to condition the surfaces of gears to avoid critical scoring.)

The following values represent a rough guide for Z_s:

Initial Finish, AA	Z_s	Comment
0.3 μm (12 μin.)	1.2	Usually honed; after finish ground
0.5 μm (20 μin.)	1.5	Fine finish; some break-in needed
0.75 μm (30 μin.)	1.7	Good finish; special break-in needed (for Z_s to be equal to 1.7)
1 μm (40 μin.)	2.0	Nominal finish; extensive break-in needed (for Z_s to be equal to 2.0)
1.5 μm (60 μin.)	2.5	Poor finish; special break-in procedure should be used (then Z_s = 2.5 is possible)

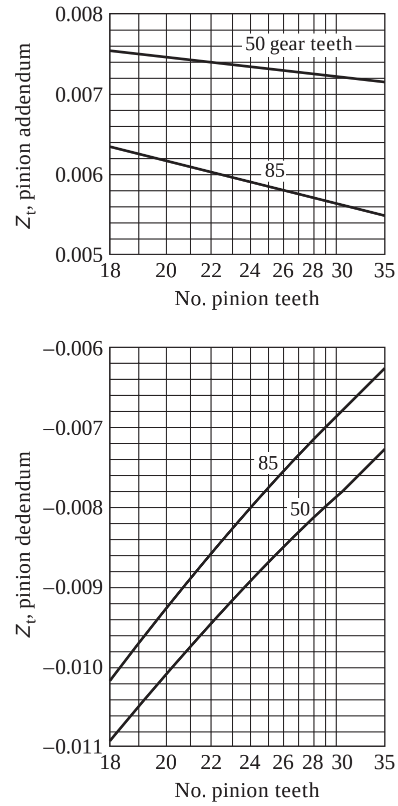


FIGURE 5.41 Geometry factor for scoring Z_t at the start of standard profile modification. Full depth, 20° pressure angle spur gears. Pinion addendum 1.00, gear addendum 1.00.

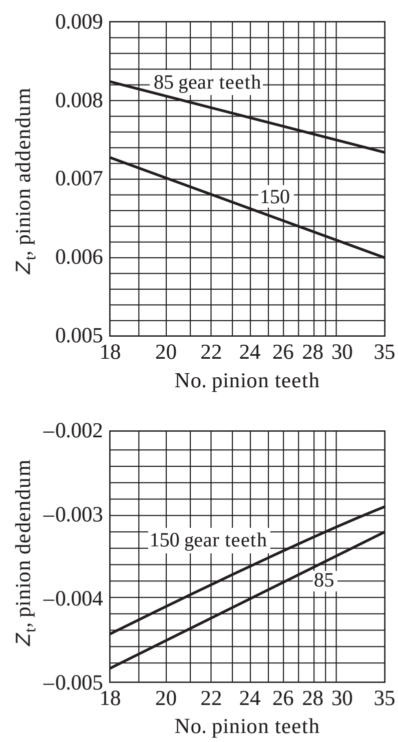


FIGURE 5.42 Geometry factor for scoring Z_t at the start of standard profile modification. Full depth, 20° pressure angle spur gears. Pinion addendum 1.5, gear addendum 0.75.

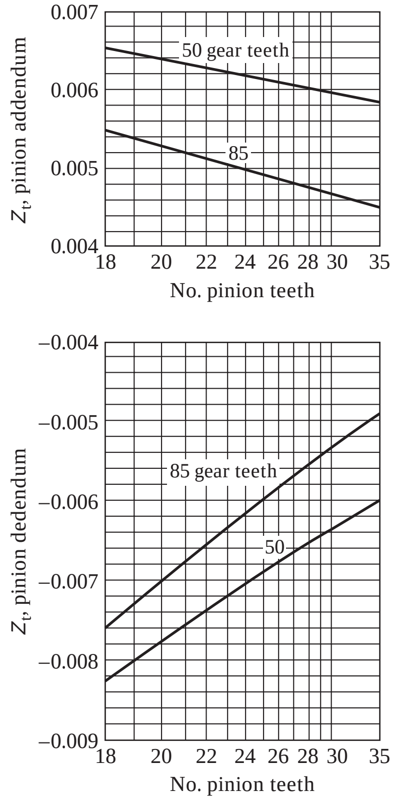


FIGURE 5.43 Geometry factor for scoring Z_t at the start of standard pro le modi cation. Full depth, 25° pressure angle spur gears. Pinion addendum 1.00, gear addendum 1.00.

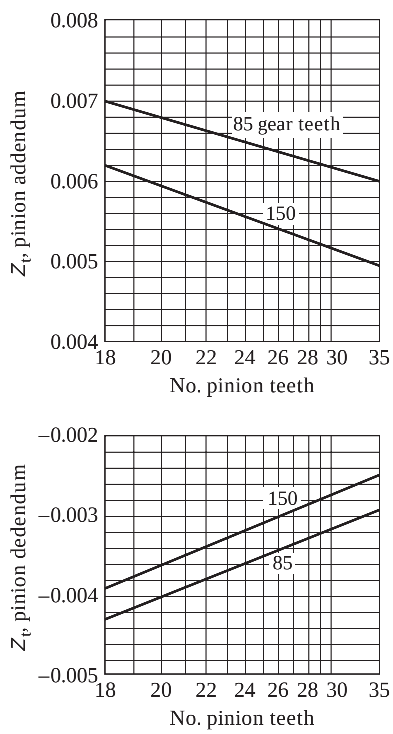


FIGURE 5.44 Geometry factor for scoring Z_t at the start of standard pro le modi cation. Full depth, 25° pressure angle spur gears. Pinion addendum 1.25, gear addendum 0.75.

The factor Z_c is the scoring criterion number. This index of scoring risk was presented in Section 4.1.5, Equation 4.50. This factor will be given in somewhat more detail as

$$Z_c = \left(\frac{W_{te}}{b}\right)^{0.75} \frac{(m_t)^{0.25}}{1.094}\sqrt{n_1} \quad (\text{°C factor, metric}), \tag{5.141}$$

$$Z_c = \left(\frac{W_{te}}{F}\right)^{0.75} \frac{1}{(P_t)^{0.25}}\sqrt{n_P} \quad (\text{°F factor; English}), \tag{5.142}$$

where

W_{te}—tangential load that is applied to a point on the pro le in danger of scoring (newtons [metric] or pounds [English])
b and F—face width (mm or in.)
n_1 and n_P—pinion (rpm)
m_t—module, transverse
P_t– transverse diametral pitch

The load applied at the start of pro le modi cation should generally be the full load. For spur gears with modi cation, the design is generally worked out so that the start of modi cation becomes a real highest point of single-tooth loading. For helical gears, the design may have a high enough transverse contact ratio to keep the maximum effective load somewhat below the full load. (Usually, doing an extensive analysis of load sharing to nd out if the helical tooth merited less than the full load at the start of modi cation is not worth the trouble.)

If the teeth are not modi ed, close to full load may be applied at the tooth tips. The spur gear, unmodi ed, would only get about 50% load at the tip—since two pairs of teeth are sharing the load. However, scoring is a hazard at the position of the worst tooth-to-tooth spacing error. At this position, the tooth tip may get full load because the spacing error prevented load sharing. Then the few worst teeth may score, and the scoring may wear away enough metal to restore load sharing. If the scoring is not too bad, it may heal up, and then the unit may run without further scoring.

For both spur and helical gears that are unmodi ed, a simple and conservative design practice is to take full load at the tooth tips when guring the scoring risk.

The load distribution across the face width also enters into the choice of load. If C_m = 1.5, then the load W_{te} should be 50% higher. Scoring, of course, is most apt to happen at the end of the face width that is overloaded.

To sum it up:

$$W_{te} = W_t \times C_m \times \text{load percentage for pro le position.} \tag{5.143}$$

Some values of the ash temperature and a rough guess as to the probability of scoring are given in Table 5.55.

TABLE 5.55
Flash Temperature Limits T_f and Scoring Probability

Kind of Lubricant	Risk of Scoring: Low °C	Low °F	High °C	High °F
Synthetic Oil				
Mil-L-7808	135	275	175	350
Mil-L-23699	150	300	190	375
Mineral Oil				
Mil-O-6081. grade 1005	65	150	120	250
Mil-L-6086, grade medium	160	325	200	400
SAE 50 motor oil with mild EP	200	400	260	500
Mil-L-2105, Grade 90				
SAE 90 gear oil	260	500	315	600

5.2.8.2 Cold Scoring

Cold scoring occurs when EHD oil lm is small compared with the surface roughness, and the lubricant does not have enough additives to prevent scoring as the asperities on the contacting gear tooth surfaces abrasively wear.

The rst design step is to calculate an approximate EHD oil- lm thickness h_{min}. A relatively exact calculation of h_{min} is quite complicated and requires special data about the oil. (Paraf nic oils have somewhat different data from napthenic oils.)

A simple, but approximate, calculation for the minimum oil lm at the pitch line will be given. The calculation will be given in English units only. The answer, of course, can easily be changed into metric units.

$$h_{min} = \frac{44.6 r_e \text{(lubricant factor)(velocity factor)}}{\text{(loading factor)}}, \quad (5.144)$$

where the effective radius of curvature at pitch diameter r_e is

$$r_e = \frac{C \sin \phi_n}{\cos^2 \psi} \times \frac{m_G}{(m_G + 1)^2}, \quad (5.145)$$

$$\text{Lubricant factor} = (\alpha E')^{0.54}, \quad (5.146)$$

where

α—lubricant pressure–viscosity coef cient (in.^{2}/lb) (see Table 5.56)

E'—effective elastic modulus for a steel gearset $E' = (\pi/2[E/(1-\nu)^2] = 51.7 \times 10^6$ psi, since $E = 30{,}000{,}000$ psi and Poisson's ratio $\nu = 0.3$)

$$\text{Velocity factor} = \left(\frac{\mu_0 u}{E' r_e}\right)^{0.7}, \quad (5.147)$$

where

μ_0—lubricant viscosity at operating temperatures (cP) (see Table 5.56)

TABLE 5.56
Nominal Lubricant Properties

Kind of Oil	Temperature °C	Temperature °F	Viscosity (cP)	Lubricant Pressure-Viscosity Coefficient (in.^{2}/lb)
Mil-L-7808	100	212	3.6 min.	0.000105
	71	160	5.5 min.	0.000115
Mil-L-23699	100	212	5.0 min.	0.000105
	71	160	8.0 min.	0.000115
Mil-O-6081, grade 1005	100	212	5.0 min.	0.000075
Mil-L-6086, grade medium	100	212	8.0 min.	0.000096
	71	160	17.0 min.	0.000110
SAE 30 motor oil	100	212	11.5 min.	0.000096
SAE 50 motor oil	100	212	17.0 min.	0.000096
SAE 90 gear oil	100	212	16.5 min.	0.000096
	71	160	45.0 min.	0.000110
SAE 140 gear oil	100	212	35.0 min.	0.000096

u—rolling velocity (at the pitch line) (in./s) ($u = \pi n_P d \sin \phi_t/60$)

d—pinion pitch diameter

n_P—pinion (rpm)

ϕ_t—transverse pressure angle

$$\text{Loading factor} = \left(\frac{W_t}{FE' r_e}\right)^{0.13}, \quad (5.148)$$

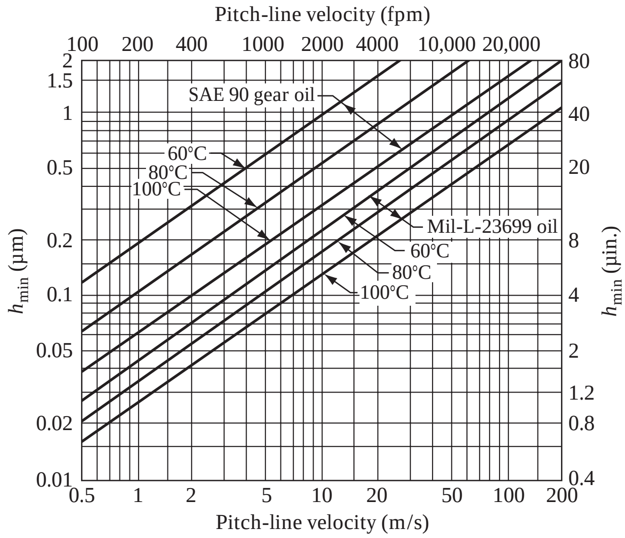
where

W_t—tangential load (lb)

F—face width (in.)

The answer h_{min} comes out in microinches (μin.). This value can be changed to micrometers (μm) by dividing by 39.37.

Figure 5.45 shows some plotted values of h_{min} for a heavy vehicle oil, Society of Automotive Engineers (SAE) 90 gear



Note: 25/85 teeth
α or φ = 20°
Center dist. = 139.7 mm (5.50 in.)
r_e = 8.39 mm (0.3304 in.)

FIGURE 5.45 Approximate EHD oil–film thickness for heavy mineral oil used in vehicles and light synthetic oil used in aircraft. Note size of gearset.

oil, and for a light synthetic oil, Mil-L-23699, that are used in the aerospace field. Note that the cold synthetic oil gives less oil-film thickness than the hot vehicle oil.

Table 5.57 shows a general study of how oil-film thickness changes with gear size, gear tooth load, and pitch-line velocity. Note that pitch-line velocity is by far the most important variable. This table was made for oil that might be used with medium-speed industrial gears.

After the minimum EHD oil–film thickness is determined, it can be compared with the surface finish of the gear teeth by Figure 5.46. The surface finish to be used is not the finish before the gears have operated, but the effective finish after they have been given whatever break-in or wear-in treatment is intended.

The effective finish for a pair of gears running together is

$$S' = \sqrt{S_1^2 + S_2^2}, \tag{5.149}$$

where

S_1—finish of one gear after break-in, arithmetic average
S_2—finish of mating gear after break-in, arithmetic average

The ratio of film thickness to surface finish is

$$\Lambda = \frac{h_{min}}{S'}. \tag{5.150}$$

This ratio, which can be called the *lambda ratio*, is usually over 1.0 for regime III conditions. In regime II, it will tend to be less than 1.0, with usual values around 0.4 to 0.8. In regime I, the lambda ratio is often around 0.1 to 0.3.

Since lambda ratio considerably varies, the best way to estimate probable gear running conditions is to use a diagram like Figure 5.46. (Gear builders can plot their own Figure 5.46 once they gain considerable experience in the application of their gears.) The secret, of course, to successful gear application over all regimes of lubrication is to make up for small lambda ratio values by appropriately strong additives in the oil.

In general, these conclusions hold:

Regime	Conclusion
I	High hazard of cold scoring; needs a strong EP oil
II	Moderate hazard of cold scoring; needs an antiwear oil or an EP oil
III	A mild antiwear oil or a straight mineral oil will probably be OK; no serious hazard of cold scoring

5.2.8.3 Design Practice to Handle Scoring

In most cases, gears are sized to meet durability requirements. Then the tooth size is made large enough to meet tooth strength requirements. Gear sizing is seldom done to meet scoring requirements.

The usual procedure is to establish the gear design and then *check the design* for scoring hazard. What can the designer do if

1. There is a serious risk of hot scoring?
2. There is a serious risk of cold scoring?

In the case of hot scoring, the most effective things to do are to improve the surface finish and to use more score-resistant oil.

Using smaller gear teeth and changing the addendum proportions may be of some help. (Note in Figures 5.41 through 5.44 how the Z_t value changes as more or less long addendum is put on the pinion.)

A higher pressure angle or a special profile modification can also help (note Problem 10.6 in Chapter 13), as can better tooth accuracy (note data on tooth accuracy in Section 10.4.1).

If a design is marginal and all the obvious things have been done, it may be necessary to copper plate or silver plate the teeth. A very thin deposit of copper or silver is quite effective in preventing scoring. Large turbine gears, running at very high speeds, are routinely plated with copper or silver to control the scoring hazard. Some gear drives for large propeller-driven aircraft have had to use silver-plated teeth.

If cold scoring is the hazard and the gears are in regime I or in the transition zone to regime II, the situation may be rather critical. The most obvious thing to do is to use stronger EP oil. (The chemical additives in the extreme pressure oil tend to make a chemical film that will substitute for the EHD film that is missing.)

TABLE 5.57
Approximate EHD Minimum Oil-Film Thickness for Mil-L-6086 Oil (Similar to AGMA 2, SAE 30 Motor Oil or ASTM 315 Oil)

Gear Mesh										
Temperature		*K* Factor[a]	h_{min}, μm for Pitch-Line Speed (m/s)				h_{min}, μin. for Pitch-Line Speed (fpm)			
°C	°F	(N/mm²)	0.5	2.5	10	50	100	500	2000	10,000
					Small Gear Unit[b]					
60	140	1.38	0.053	0.163	0.44	1.33	2.08	6.41	17.50	52.40
		4.14	0.045	0.141	0.38	1.15	1.80	5.56	15.17	45.40
		13.80	0.039	0.120	0.33	0.98	1.54	4.75	12.97	38.80
80	176	1.38	0.031	0.097	0.26	0.79	1.24	3.82	10.42	31.20
		4.14	0.027	0.084	0.23	0.69	1.08	3.31	9.03	27.10
		13.80	0.023	0.072	0.20	0.58	0.92	2.83	7.72	23.10
100	212	1.38	0.021	0.065	0.18	0.53	0.83	2.56	6.98	20.90
		4.14	0.018	0.056	0.15	0.46	0.72	2.22	6.05	18.10
		13.80	0.015	0.048	0.13	0.39	0.61	1.89	5.17	15.40
					Large Gear Unit[c]					
60	140	1.38	0.086	0.265	0.72	2.15	3.40	10.43	28.40	85.10
		4.14	0.073	0.229	0.62	1.87	2.90	9.04	24.60	73.80
		13.80	0.063	0.196	0.53	1.60	2.50	7.73	21.00	63.00
80	176	1.38	0.051	0.157	0.43	1.28	2.01	6.20	16.90	50.61
		4.14	0.044	0.136	0.37	1.11	1.74	5.38	14.60	43.89
		13.80	0.038	0.116	0.32	0.95	1.49	4.59	12.50	37.50
100	212	1.38	0.034	0.105	0.29	0.86	1.35	4.16	11.30	33.97
		4.14	0.029	0.092	0.25	0.75	1.117	3.61	9.80	29.50
		13.80	0.025	0.078	0.21	0.64	1.00	3.08	8.37	25.18

[a] 1.38 N/mm² = 200 psi; 4.14 N/mm² = 600 psi; 13.8 N/mm² = 2000 psi.
[b] r_e = 8.39 mm = 0.3304 in.
[c] r_e = 41.96 mm = 1.652 in.

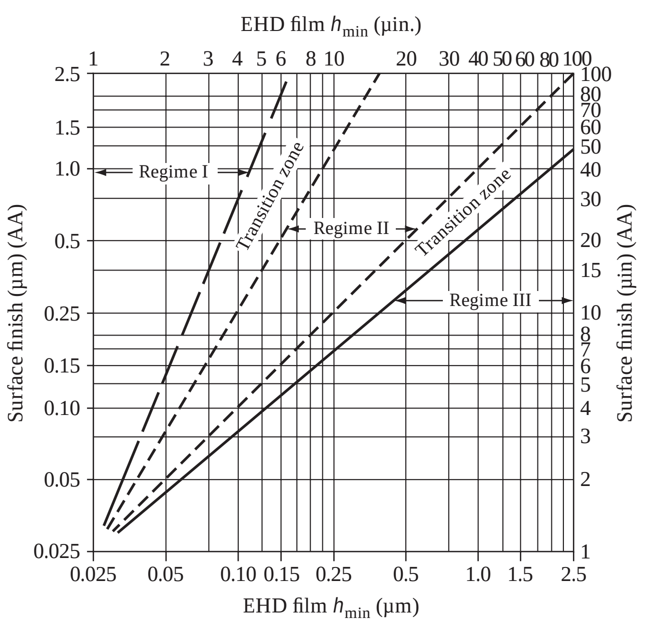


FIGURE 5.46 Regimes of lubrication are determined by minimum EHD oil– lm thickness and effective surface nish.

In regime I, and to a lesser extent in regime II, the gears tend to machine themselves in service. A gear pair with a 2 μm nish when it left the factory, may, in a few days of running, wear in a new surface that is as smooth as 0.5 μm (in the direction of sliding).

This means that it is critical to use an extrastrong EP oil during the break-in period. Also, the gears should not be operated at maximum torque or maximum temperature while they are wearing in. If the wear-in is accomplished without serious damage to the pro les, then the gears can often run for years without the need for special EP oil.

When cold scoring is a hazard, it is helpful to use as heavy an oil as possible and to use the right amount of pro le modi cation.

In concluding to this section, it should be said that experience is *most important* in handling scoring problems. If gears score on the test stand or in the eld, it is usually possible to try different improvements in lubrication and in the way that the gear teeth are nished. Sometimes the pattern of grinding or cutting marks can be changed, with much improvement in scoring resistance. Sometimes an oil specialist can recommend a change in lubricant or a lubricant additive that will clear up the scoring problem. Sometimes a change in the heat-treatment

procedure used in making the gear teeth will result in a more score-resistant tooth surface. Each builder of gears for applications with high scoring hazard needs to learn a recipe for making gears that will survive without undue scoring trouble.

5.2.9 Trade Standards for Rating Gears

Sections 5.2.1 to 5.2.7 gave general methods for determining gear load-carrying capacity. The reader was probably surprised to note that the load-carrying capacity is quite elastic. If one assumes that a high degree of accuracy is obtained in the gearing and that the driving and the driven apparatus do not impose shock, one obtains a large amount of capacity for a given size of gearset. On the other hand, if poor-accuracy and rough-running apparatuses are assumed, the calculated capacity becomes quite low. In a sense, the general rating formulas require the designer to use considerable judgment in evaluating all the factors that affect the capacity of a given design.

In contrast, standard rating formulas are not usually elastic. They give a definite answer.

Standards are established by a technical group which meets to discuss the intensity of loading that their field experience has shown to be safe and conservative. In some cases, the degree of quality on which a standard is based is spelled out in the standard. In many cases, though, not all the things that contribute to the quality of a gear are precisely defined. A newcomer in a given gear field may have to scout around to find out just how good the gears that are being made by established manufacturers in the field are.

The principal groups writing general trade standards for gears are the following:

American Gear Manufacturers Association (AGMA)
Suite 1000, 1901 North Fort Myer Drive
Arlington, VA 22209, U.S.A.

International Standards Organization (ISO)
DIN * Kamestrasse 2–8
D-5000 Köln, Germany

Deutsches Institute für Normung (DIN)
Berlin 30 and Köln 1, Germany

The standards for gear design or for gear rating may be written to cover a broad field, or they may be product standards for a limited area of usage. For instance, a "mother" standard on gear rating will tend to cover the principles and the practice for rating the surface durability and the strength of all kinds of spur and helical gears. This kind of standard will often give more than one method for determining a variable. A "product" standard will cover only a limited field of usage. The product standard will be quite specific on how to rate the gears. The principle is that the product standard is derived from the mother standard, and so it can be much shorter and more specific. In a limited field, those who are building and using the gears should have rather good experience to judge what is correct for good technical practice.

Product standards on gear units cover a number of things beyond the capacity of the gear teeth. Frequently, the capacity of a set has to be limited by the capacity of the casing to radiate and conduct heat away. The kind of lubrication, the kind of material, the kind of bearings, and the amount of the service factor are some of the other things that may be specified in gear standards.

The material in this book covers many of the things that are in trade standards. However, a designer who wishes to design a gear in accordance with a particular standard should get a copy of the standard and study every detail of it. Standards are like legal documents. All the fine print must be read and followed before one can claim to be acting in accordance with the standard.

Purchase contracts frequently specify standards or other details of gear unit design. A builder of gears under contract is, of course, obligated to meet the contract. What should be done if some requirement relating to the gears seems unreasonable or impractical, or if it allows gears that have too much risk or failure to be built? (This book is intended to be conservative and to alert the reader the risks in gear making—hence—a gear contract may allow things that do not agree with this book.)

If this book is more conservative than some gear standard or some contract provision, the gear builder or gear buyer would be well advised to consider whether or not there is enough proven experience to justify going beyond the recommendations given here. In the last analysis, it is proven experience that sets the design. Certainly, progress is being made in all aspects of the gear art. It is to be expected that there will be many situations in which there is enough experience and technical know-how to go beyond things in this book.

5.2.10 Vehicle Gear Rating Practice

Vehicle gear designers are always under great pressure to make their gears both small and inexpensive. A gear failure is not too serious, because there is not much risk to the safety of the vehicle driver. Expensive equipment is not tied up by a failure, as it is when gears in a power plant or a ship fail. However, failure cannot be allowed to happen very often, or the vehicle will get a reputation for being poorly designed. Also, the customer complaint charges may become serious.

The vehicle designer needs to know quite precisely where the danger point is in loading the gears. There are two good sources of information. Since vehicles are built in very large quantities, a large amount of statistical data is available. These can be collected and plotted to give design curves which are quite reliable for the limited field of gear work for which they apply.

The second source of information is the product experience of individual manufacturers. After many thousands of transmissions and rear ends have been built and put into service, the builder finds out which parts are weak and which parts are stronger than they need to be. If the weak spots are doctored, the rating of the gears can usually be increased. In many cases, automotive manufacturers have been able to improve their gear drives at about the same rate as engine horsepower has been increased. Present-day vehicle engines

are much more powerful than they were 20 years ago. In general, though, the gear drives are not larger. They are just made with better accuracy and better materials.

The AGMA standard for spur and helical vehicle gears defines two quality grades, grades 1 and 2. Grade 2 steel is specified to be cleaner, harder, and having better metallurgical structure than grade 1. The standard shows composition of low-alloy, medium-alloy, and high-alloy steel. The AGMA standard explains that that the higher-alloy steels are required as the gears become larger and the vehicle is subjected to more severe duty requirements.

The standard gives typical derating factors for different vehicle applications. Table 5.58 shows a sampling of these factors.

The design stress limits are adjusted for grade 2 or 1 material, and also adjusted for a probability of failure of L10 or L1. An L10 designation means that 10 out of 100 gears might prematurely fail by the mode of failure for which the stress level is given.

Table 5.59 shows the design stress limits in tabular form. (The standard presents the values in curve form.)

Vehicle gear design is characterized by two things:

- Very severe loads are permitted at low numbers of cycles (less than 10^7). This means that the teeth are apt to have some surface or subsurface damage from the high loading. The damage may take the form of microcracking, surface cold work, or small amounts of pitting. This kind of damage is not expected to result in failure before 10^8 cycles, provided the gear meets the design limits. At 10^8 cycles, the gear may be definitely damaged, and it may be quite unfit to run for 10^9 or 10^{10} cycles.
- In general, vehicle gears operate under regime II conditions. To meet their design ratings, they must have appropriate lubricants that have the right viscosity and appropriate additives. Because of regime II conditions, the load that can be carried on the surface of the tooth drops rather rapidly as the number of cycles increases.

TABLE 5.58
A Sampling of Overall Derating Factors

Application		Overall Derating Factor: Medium Precision	Overall Derating Factor: Medium-High Precision
Final drive	Off-highway	1.2	1.0
	truck	1.4	1.2
Shifting	Off-highway	1.5	1.3
transmission	truck	1.6	1.4
	Passenger car	1.7	1.5
Epicyclic	Off-highway	1.7	1.6
	passenger car	–	2.0

Source: AGMA 170.01, Design guide for vehicle spur and helical gears, AGMA, 1976. With permission.

In general, vehicles are built in large quantities. This makes it practical (on new design) to build a few gear drives that are made just as small as possible by vehicle criteria. The first few drives are then extensively tested. The test results will probably show some failures. In general, design modifications are made to improve the design without changing the overall size. With improvements, the design will satisfactorily pass the test, and it is then ready to go into initial production. After a unit has been in production for some time, field results may show a need for further design improvements.

The design improvements might include any one or all of the following things:

- Change to a higher-alloy steel or improve the heat-treating practice to get better hardness and metallurgy.
- Change the profile modification or the helix modification to get better fit.
- Change the pitch of the teeth, the pressure angle, or the tooth proportions to get better geometric conditions.
- Improve the accuracy of the teeth, or improve the bearing and casing structures that support the gears.
- Use a better lubricant and/or a better means of cooling the gears when they are running under the worst heavy-loading, extreme temperature conditions.

More could be added.

5.2.11 Marine Gear Rating Practice

The gears used to drive large ships are almost all helical. The gears are so large that it is difficult to case-harden them and retain enough dimensional accuracy to permit finish grinding. The high speed and long life requirements of marine gears make it necessary to grind the gears after case-carburizing. In some cases, the smaller marine gears (epicyclic marine gears, for instance) are cut, shaved, or ground before nitriding, then nitrided with such overall skill and control of the manufacturing process that they can be put into service after final hardening without any grinding in the final hard condition.

There has been considerable use of through-hardened gears for marine service that are precision cut and shaved or ground. These gears have no case and are essentially uniform in hardness throughout the tooth area.

The gear drives on a large ship are very important piece of machinery. Ship owners want the gearing to be good enough to stay in service for something like 20 to 30 years. The gears run at fairly high speeds, so the lifetime cycles may get up to 10^{10} or even higher. If a gear drive on a ship were to fail, there would be a chance that a storm might be in progress, and the ship might be blown onto the rocks and wrecked. There is also the risk that a ship with disabled gears might be so long in getting into port that a perishable cargo would be lost.

The design of gearing for ships tends to be very conservative. Those who purchase gear drives frequently control the

TABLE 5.59
Design Stress Limits

No. of Stress Cycles	Bending Stress[a]				Contact Stress			
	Grade 2		Grade 1		Grade 2		Grade 1	
	L10	L1	L10	L1	L10	L1	L10	L1
Metric (N/mm^2)								
10^3	1270	1170	980	880	–	–	–	–
10^4	960	880	750	690	–	3270	3450	2860
10^5	760	690	570	520	3100	2620	2690	2240
10^6	580	520	450	410	2480	2140	2140	1790
2×10^6	530	480	410	370	–	–	–	–
10^7	530	480	410	370	2000	1720	1690	1400
10^8	530	480	410	370	1650	1380	1310	1100
English (psi) (Multiply by 1000)								
10^3	185	170	142	128	–	–	–	–
10^4	140	128	109	100	–	475	500	415
10^5	110	100	83	76	450	380	390	325
10^6	84	75	65	59	360	310	310	260
2×10^6	77	70	60	54	–	–	–	–
10^7	77	70	60	54	290	250	245	203
10^8	77	70	60	54	240	200	190	160

Note: These limits are for short life (2×10^8 maximum) and or operation in regime II. Some microdamage can be tolerated at cycles below 10^6.

[a] The bending stress data show no slope after 2 × 106 cycles. However, the standard concedes that a shallow slope may be needed and suggests a drop of 34 N/mm^2 (5000 psi) in going from 2×10^6 cycles to 10^8 cycles.

conservatism of the design by specifying maximum *K* factor and unit load values. As an example, a large ship using medium-hard gears might have gears designed to meet values like the following:

First stage	
High-speed pinions	Hardness 262–311 HB
High-speed gears	Hardness 241–285 HB
K factor not to exceed 0.86 N/mm^2 (125 psi)	
Second stage	
Low-speed pinions	Hardness 241–285 HB
Low-speed gears	Hardness 223–269 HB
K factor not to exceed 0.69 N/mm^2 (100 psi)	

Those who operate large ships almost always insure the ship. Those who write insurance for ships or for machinery on the ship need independent certi cation of the capability and the condition of the ship or the piece of equipment under consideration. This kind of work is ordinarily handled by organizations that are commonly called *classi cation societies*. These groups are concerned with the power rating of gears, the quality of gears, and the character of the drive system associated with the gears. The latter concern involves things like propeller shaft bearings, mountings for turbines, and hull de ections under different sea conditions.

The classi cation societies are involved in approving new designs, as well as evaluating machinery in service. Each of them tends to have rules pertaining to gear rating. A marine gear designer will often nd that a design is required by contract to meet the rules of a designated classi cation society.

The names and addresses of the principal classi cation societies are as follows:

American Bureau of Shipping
45 Broad St.
New York, NY 10004, U.S.A.

Bureau Veritas
31 Rue Henri-Rochefort
Paris 17e, France

Det Norske Veritas
Veritasveien 1, Høvik
Oslo, Norway

Germanischer Lloyd
Postfach 30 20 60
2000 Hamburg 36, Germany

Lloyds Register of Shipping
71 Fenchurch St.
London, EC3M 4BS, England

5.2.12 Oil and Gas Industry Gear Rating

The gears used in the oil and gas industry need high reliability. Power gearing often handles power from 1000 kW to as much as 30,000 kW. A compressor or a generator drive system will often be required to run for 20,000 to 30,000 hours before overhauling, and have a total life requirement of 50,000 to 100,000 hours. At an overhaul, seals and couples, and sometimes bearings, might be replaced. It is generally expected that the gears themselves will last for the total design life. (Sometimes a gear or pinion will last for the total design life, but some rework of teeth or journal bearing surfaces may be required during the life.)

Very often, fast-running turbines drive the gear units in this led. The total life cycles of gear parts are often in the area of 10^{10} to 10^{11} cycles.

The high-speed-gear standard that has been in use for a considerable period is AGMA 6011-I03 (and its predecessor back to 421.01). The large power gears all have pitch-line speeds that make them come under a high-speed-gear standard.

In October 1969, Dudley presented a paper (AGMA Paper No. 159.02) recommending some changes in the factors used in this standard. This paper made a particular point of the higher reliabilities needed in gears for the oil industry and advocated a more conservative design approach. A general vision of AGMA standard has been in process for the last few years, and is now quite close to being approved and ready to issue.

The American Petroleum Institute (API) is concerned with the reliability of power gears. API issued a short standard 613 of their own, in which special rating controls were put along with other materials relating to the design and manufacture of gears.

Table 5.60 shows some material extracted from API Standard 613. The material index number is really the *K* factor when the application factor is equal to 1. In most cases, thought, API recommends an application factor greater than 1, and so the real *K* factor is obtained by dividing the material index number by the application factor.

TABLE 5.60
Some Design Limits

Kind of Material	Material Index Number		Bending Stress Number	
	N/mm²	psi	N/mm²	psi
	Carburized			
HRC 59	3.10	450	269	39,000
HRC 55	2.83	410	248	36,000
	Through-Hardened			
HB 300	1.38	200	179	26,000
HB 200	0.90	130	145	21,000

Source: API 613: Special-Purpose Units for Re nery Services. With permission.

Note: When the material index is divided by the application factor, it becomes a design *K* factor. The bending stress equation used by API has the application factor in it and an overall derating factor of 1.8.

API uses the term *service factor*. The values used, though, are appropriate for the new *AGMA* de nition of application factor. (API sets their limits low enough to handle about 10^{10} cycles, and so their so-called SF values are really application factors.)

In some cases, large power gears used in the oil and gas industry are heated to the point where the t of the teeth is considerably affected by thermal distortions and the body temperature of the pinion may be over 50° hotter than the incoming oil. Thermal problems are being handled, but no trade standards have yet been developed. Section 9.3.4 of this book has further material on this subject. The 1972 ASME paper by Martinaglia gives some very worthwhile engineering data relative to thermal behavior of high-speed gears.

5.2.13 Aerospace Gear Rating Practice

Aerospace gears must be very light in weight for the loads carried, and yet they must be very reliable. Many of the gears are so important to the operation of the aircraft that a broken tooth would cause an aircraft to crash.

The life of an aircraft gear at full torque may vary anywhere from about 10^6 to 10^9 cycles.

Laboratory and bench tests are extensively used to provide design data for aircraft gears. New aircraft engines are so expensive to design and build that it is worth doing a lot of work to develop component parts before they are put into an engine. The stakes are high. A small reduction in the volume and the weight of a gearbox could result in one manufacturer's engine being accepted for production contracts and another's being rejected. Gear failures in the eld can cause all aircraft using the particular design to be grounded until the questionable gearing is replaced.

Under laboratory conditions, gears will perform best of all. Usually, the quality is better than that obtained in large-quantity production. Test stands usually provide a gear with better lubrication and alignment than the worst of the engines that the gears will have to run in.

When the load-carrying capacity of a gear has been determined in a test stand, it is necessary to reduce that capacity for use in an actual engine. Allowances must be made so that even the poorest gear operating in the worst engine will not fail.

New engines are given extensive testing in engine test cells to demonstrate that they will meet design requirements. Then, if they appear to be satisfactory, they are ight tested. After this, the engine may be approved for use in commercial or military aircraft. Aircraft gear designers are usually free to use any design formula that looks good to them, but their gears—to be successful—must go through the development and testing procedures just described.

In propeller aircraft, the gear unit is generally considered to be a part of the engine package. Many small propeller-driven aircraft are used for pleasure, but relatively few are now used to carry passengers. Jet-engine aircraft usually use no main power gearing, but have a considerable amount of gearing in accessory drives. The smaller military cargo planes do use propellers and propulsion gears.

A large new use of aerospace gearing is in helicopters. Helicopters are used for commercial purposes, and they have a major use in the army. A typical army helicopter will have one or two main rotor drives, and each main rotor drive will have about three stages of gearing. The first stage of gearing is often built into the turbine engine package and furnished with the engine. The second and third stages are generally in a gear unit furnished with the helicopter. The helicopter gears work rather hard most of the time. Even when the helicopter is loitering, the helicopter rotor must provide lift to support the weight of the helicopter.

The widely used design guide for aerospace gearing was issued by AGMA. This standard recognizes two grades of material and lists derating factors for several kinds of applications. Stress levels are given for both grades of material.

The grade 2 material is a high-quality steel of the American Iron and Steel Institute 9310 type. It is specified to have very good cleanness, high hardness, and very good metallurgical structure. The grade 1 steel may be carburized 9310, or it may be a somewhat similar steel if appropriate hardness and quality can be obtained. It may be air melt rather than vacuum melt. Not so quite high hardness is specified.

Nitriding steel of the Nitralloy 135 type is permitted for grade 1, provided that the teeth are not larger than 2.5 module (10 pitch). The nitride case is thin, even with rather long nitriding time. The standard reflects concern that the case depth may not be adequate for grade 1 levels of tooth loading if the teeth are larger than 2.5 module.

Table 5.61 shows some of the overall derating factors. Table 5.62 shows the design stress limits for different numbers of cycles. These stress limits are intended to go with an L1 probability of failure. At the time this standard was written, the gear trade had not recognized the different regimes of lubrication. It is now acknowledged that these data were developed from test and field experience that was essentially all in regime III. Recent work in the aerospace field has shown that some situations get into regime II—even when the gears are relatively fast-running power gears. The gears subject to intermittent duty in controlling flaps and tail rudders on large aircraft are quite often in regime II.

TABLE 5.61
A Sampling of Overall Derating Factors

Application		Overall Derating Factor: Medium-High Precision	Overall Derating Factor: High Precision
Main propulsion drive gears	Continuous	1.8	1.2
	Take-off and early climb	1.5	1.0
Power take-off accessory gears		2.1	1.5
Auxiliary power units		2.1	1.5

TABLE 5.62
Design Stress Limits

No. of Stress Cycles	Bending Stress		Contact Stress	
	Grade 2	Grade 1	Grade 2	Grade 1
		Metric (N/mm²)		
1	1103	965	2296	2041
10^4	827	710	2296	2041
10^5	627	538	2034	1793
10^6	489	414	1779	1586
10^7	448	379	1551	1379
10^8	414	352	1358	1207
10^9	379	324	1193	1069
10^{10}	345	303	1041	938
		English (psi)		
1	160,000	140,000	333,000	296,000
10^4	120,000	103,000	333,000	296,000
10^5	91,000	78,000	295,000	260,000
10^6	71,000	60,000	258,000	230,000
10^7	65,000	55,000	225,000	200,000
10^8	60,000	51,000	197,000	175,000
10^9	55,000	47,000	173,000	155,000
10^{10}	50,000	44,000	151,000	136,000

Note: These stress limits are based on a probability of failure of 1 in 100 (L1). These limits are intended for long life and for operating in regime III; no microdamage is expected at cycles below 10^6.

The aerospace field is somewhat like the vehicle field. The industry standards, in both cases, are really intended as design guides rather than rigid rules. They show what is being achieved by those who are relatively skilled in the gear art and serve as a guide for new designs. The new design, though, is usually made as small as the designer dares to make it.

As was said earlier in this section, the new design has to be proven by considerable amount of testing on the ground and on air. Frequently, design modifications are made to improve the fit of the gears or the quality of the material in the gears. Profile modifications are very important, as is surface finish. The lubrication system must work well. Generally, the oil is specified, and it is not possible to solve gear problems by using heavy oil or one with more additives. Aircraft are apt to travel all over the world, and so they have to use the lubricants stocked in airfields. Also, an aircraft may start in the Arctic and a few hours later operate under hot desert conditions. Usually, the oil for the gears has to be the same oil used by the turbine engine. These oils stand hot temperatures quite well, and are thin enough for the equipment to operate in most arctic conditions. The additives in the oil are mild, but they are sufficient for regime III operation under favorable conditions.

6 Gear Materials

A wide variety of steels, cast irons, bronzes, and phenolic resins have been used for gears. New materials such as nylon, titanium, and sintered iron have also become important in gear work. Designers might well become hopelessly confused when faced with so many different gear materials, except that there are good and specific reasons for using each of the materials that have been adopted for gears. As their outstanding characteristics, steels have the greatest strength per unit volume and the lowest cost per pound. In many fields of gear work, some kind of steel is the only material to consider.

Cast irons have long been popular because of their good wearing characteristics, their excellent machinability, and the ease with which complicated shapes may be produced by the casting method.

Bronzes are very important in worm-gear work because of their ability to withstand high sliding velocity and their ability to wear in to fit hardened-steel worms. They are also very useful in applications in which corrosion is a problem. The ease with which bronze can be worked makes it a good choice where small gear teeth are produced by stamping or by drawing rod through dies.

Phenolic resins are used in various combinations to produce laminated gears with remarkably good load-carrying capacity despite the low physical strength of the material. In general, these materials are about 30 times as elastic as steel. (Their deformation per unit load is about 30 times that to steel.) Their “rubbery” nature makes them operate with less shock due to tooth errors and allows their teeth to bend enough to make more teeth contact and share load than in metal designs. Some of these materials can stand quite high sliding velocities.

In this chapter we shall study all the kinds of gear materials just mentioned. Condensed information will be given to define the composition, heat treatment, and mechanical properties of these materials. To help the reader with the language of materials, a short glossary is given in Table 6.1.

6.1 STEELS FOR GEARS

There are a number of steels used for gears, ranging from plain carbon steels through the highly alloyed steels and from low– to high–carbon content steels. The choice will depend on a number of factors, including size, service, and design. The following discussions should serve as a guide in selecting steels.

6.1.1 Mechanical Properties

In designing gears, the mechanical properties of the material are of some interest to the designer from a comparative standpoint, but they cannot be used directly in calculating the load-carrying capacity. As was pointed out in Section 4.1.1, the calculated stress on gear teeth are not necessarily the true stresses in the material. Moreover, tensile properties—the most commonly published mechanical properties of a material—are determined by loading small bars in simple tension. The gear tooth has a different geometric shape with a more complex stress pattern; therefore, the actual properties of the material as a gear can best be determined by testing the material in the *shape of a gear tooth*. The allowable stresses for rating gears (see Sections 5.2.1 and 5.2.2) are based on tests and field experience with *gears* rather than on the mechanical properties of the material as determined by routine laboratory tests. Gear materials and treatments can be most readily discussed, however, on the basis of simple mechanical properties. The mechanical properties are valuable indirectly in that they indicate how gears made of a particular material might be expected to perform. Figure 6.1 is a rough guide to the tensile properties of steel.

In gear terminology, some steels are considered to be *alloy* steels and some are considered to be *plain carbon* steels. The steels used for gears tend to vary from those with a small amount of alloys to steels rich in alloys. From a practical standpoint, the steels that are considered to be plain carbon usually have some alloy content, such as manganese and silicon. The steels that are normally thought of as alloy steels usually have chromium, nickel, and molybdenum as well as manganese and silicon.

There is a general feeling in the gear trade that alloy steels are inherently stronger and more fatigue resistant than the so-called plain carbon steels. The real situation is rather complex. If two steels have equal hardness and each has the same tempered martensitic structure, then the ultimate strength, the yield strength, and the endurance strength will be essentially the same—in a small piece of the metal.

The alloy content helps in several very important ways:

- The cooling rate in quenching can be considerably slower. This makes it possible to get a god metallurgical structure in the large gears. (A large gear with low alloy content does tend to be weaker because a good metallurgical structure is not obtained.)
- Gears with high alloy content can be carburized with too rich or too lean a carburizing atmosphere and still come out fairly good. Nickel in particular makes heat-treating operations less sensitive to precise control.
- Certain alloy combinations are helpful in developing fracture toughness. (With these combinations, a small crack grows very slowly, or perhaps ceases to grow. This is important in gears that suffer some surface damage but could run a long time if the surface damage did not lead to tooth breakage.)

TABLE 6.1
Glossary of Metallurgical and Heat Treatment Nomenclature

Term	Definition
Aging	A change in an alloy by which the structure recovers from an unstable or metastable condition produced by quenching or by cold working. The degree of stable equilibrium obtained for any given grade of steel is a function of time and temperature. The change in structure consists of precipitation and is marked by change in physical and mechanical properties. Aging which takes place slowly at room temperature may be accelerated by an increase in temperature.
Annealing	A broad term used to describe the heating and cooling of steel in the solid form. The term *annealing* usually implies relatively slow cooling. In annealing, the size, shape, and composition of steel produced and the purpose of the treatment determine the temperature of the operation, the rate of temperature change, and the time at heat. Annealing is used to induce softness, to remove stress, to alter physical and mechanical properties, to remove gases, to change the crystalline structure, and to produce a desired microstructure.
Austenite	In steels, the gamma form of iron with carbon in solid solution. Austenite is tough and nonmagnetic and tends to harden rapidly when worked below the critical temperature.
Austenitic steels	Steels which are austenitic at room temperature.
Brinell hardness	A hardness number determined by applying a known load to the surface of the material to be tested through a hardened-steel ball of known diameter. The diameter of the resulting permanent indentation is measured. This method is unsuitable for measuring the hardness of sheet or strip metal.
Carburizing	Diffusing carbon into the surface of iron-base alloys by heating such alloys in the presence of carbonaceous materials at high temperatures. Such treatments followed by appropriate quenching and tempering, harden the surface of the metal to a depth proportional to the time of carburizing.
Case hardening	Hardening the outer layer of an iron-base alloy by a process that changes the surface chemical composition, followed by an appropriate thermal treatment. Carburizing and nitriding are typical case-hardening techniques. Induction heating and the hardening of the outer layer of metal is another form of case hardening.
Cold working	Permanent deformation of a metal below its recrystallization temperature. Some metals can be hot-worked at room temperature, while others can be cold-worked at temperatures in excess of 1000°F.
Ductility	The property of metal that allows it to be permanently deformed before final rupture. It is commonly evaluated by tensile testing in which the amount of elongation or reduction of area of the broken test specimen, compared with the original, is measured.
Elastic limit	Maximum stress to which a metal can be subjected without permanent deformation.
Endurance limit	The maximum stress to which material may be subjected an infinite number of times without failure.
Ferrite	Iron in the alpha form in which alloying constituents may be dissolved. Ferrite is magnetic and soft and acts as a solvent for manganese, nickel, and silicon.
Flame hardening	Hardening an iron-based alloy by using a high-temperature flame to heat the surface layer above the transformation temperature range at which austenite begins to form, then cooling the surface quickly by quenching. This process is usually followed by tempering. An oxyacetylene torch is often used in flame hardening.
Grain size	The grain-size number is determined by a count of a definite microscopic area, usually at ×100 magnification. The larger the grain-size number, the smaller the grains.
Grains	Individual crystals in metals.
Hardenability	The property that determines the depth and distribution of hardness induced by quenching. The higher the hardenability value, the greater the depth to which the material can be hardened and the slower the quench that can be used.
Hardness	The property of materials which is measured by resistance to indentation.
Heat treatment	A general term which refers to operations involving the heating and cooling of a metal in the solid state for the purpose of obtaining certain desired conditions or properties. Heating and cooling for the prime purpose of mechanical working are not included in the definition. Gear metal heat treatments include heating in a furnace followed by quenching, flame hardening, and induction hardening.
Inclusions	Particles of nonmetallic materials, usually silicates, oxides, or sulfides, which are mechanically entrapped or are formed during solidification or by subsequent reaction within the solid metal; impurities in metals.
Induction hardening	Hardening of steel by using an alternating current to induce heating, followed by quenching. This process can be used to hardened only the surface by using high-frequency current, or it can through-harden the steel if low frequency is used. Induction hardening is a fast process which can be controlled to produce the desired depth and hardness in a localized area and with low distortion.
Jominy test	The Jominy test is used to determine the end-quench hardenability of steel. It consists of water-quenching one end of a 1 in. diameter bar under closely controlled conditions and measuring the degree of hardness at regular intervals along the side of the bar from the quenched end up.
Martensite	A microconstituent in quenched steel characterized by an acicular, or needle-type, structure. It has the maximum hardness of any structure obtained from the decomposition of austenite.
Modulus of elasticity	The ratio, within the elastic limit, of the stress to the corresponding strain. The stress in pounds per square inch is divided by the elongation of the original gage length of the specimen in inches per inch. Also known as *Young's modulus*.

(*Continued*)

TABLE 6.1 (CONTINUED)
Glossary of Metallurgical and Heat Treatment Nomenclature

Term	Definition
Nitriding	Adding nitrogen to solid iron-base alloys by heating the steel in contact with ammonia gas or other suitable nitrogenous material. This process is used to harden the surface of gears.
Normalizing	A process in which steel is heated to a temperature above the transformation range and subsequently cooled in still air to room temperature.
Pearlite	The lamellar aggregate of ferrite and cementite resulting from the direct transformation of austenite at the lower critical point.
Quenching	Rapid cooling by contact with liquids, gases, or solids, such as oil, air, water, brine, or molten salt.
Residual stress	Stress remaining in a part after the completion of working, heat treating, welding, etc., due to phase changes, expansion, contraction, and other phenomena.
Rockwell hardness	A hardness number determined by a Rockwell hardness tester, a direct-reading machine which may use a steel ball or diamond penetrator.
Secondary hardening	An increase in hardness developed by tempering high-alloy steels after quenching, usually associated with precipitation reactions.
Stress relief	A process of reducing internal residual stresses in a metal part by heating the part to a suitable temperature and holding it at that temperature for a proper time. Stress relieving may be applied to parts which have been welded, machined, cast, heat-treated or worked.
Tempering	A process that reduces brittleness and internal strains by the reheating of quench-hardened or normalized steels to a temperature below the transformation range. A *draw* treatment is a tempering treatment that is hot enough to reduce hardness.
Tensile strength	The maximum load per unit of original cross-sectional area carried by a material during a tensile test.
Through hardening	Increasing the hardness of a metallic part by a process that hardens the core material as well as the surface layers, with the hardness uniform throughout the whole gear tooth and the metal immediately adjacent to the gear tooth.
Transformation range	The temperature range within which austenite forms in ferrous alloys.
Yield point	The load per unit of original cross section at which a marked increase in deformation occurs without any increase in load.

- In general, the impact properties are considerably improved by alloy content. Nickel and molybdenum are particularly valuable for impact strength. (Many gears suffer occasional heavy shock loads and therefore need good impact strength.)

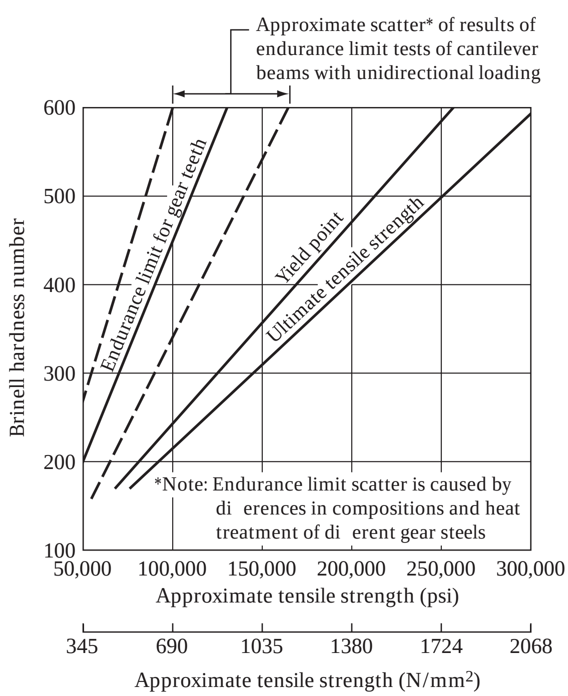


FIGURE 6.1 Approximate tensile properties of steel.

The overall situation is that the composition of the steel used in gears is very important. A *poor choice of alloy content* had often led to early failure in gears built with good precision and sized large enough to meet appropriate design standards.

6.1.2 Heat Treatment Techniques

Gear steels are heat-treated for two general purposes. First, they must be put in condition for proper machinability. Second, the necessary hardness, strength, and wear resistance for the intended use must be developed. Steel in the as-rolled or as-forged condition may be coarser grained in structure and nonuniform in hardness as a result of uncontrolled cooling after the forging or rolling operations. Therefore, a heat treatment followed by controlled cooling is used to develop the type of metallurgical structure most suited to the subsequent machining operations. Gear forgings of carburizing steels are usually *normalized* or *normalized and tempered* to develop a uniform microstructure and reduce their tendency to distort during later hardening operations. Normalizing consists of heating the steel to well above the critical temperature and somewhat above the final hardening temperature, followed by air cooling.

Another treatment similar to normalizing is *annealing*. The part to be annealed is heated above the critical range, just as in normalizing, but it is cooled at a slow rate either by controlling the furnace cooling rate or by allowing the furnace and load to cool off together with the doors closed. Slow cooling in the furnace usually produces a pearlitic or lamellar structure which provides good surface finishes after machining.

An interrupted or *cycle annealing* produces a spheroidized structure which often has maximum machinability.

Steel parts are hardened by quenching and tempering to develop the combination of strength, toughness, hardness, and wear resistance that may be needed to make the part function properly in use. Proper hardening consists of cooling the steel from above the critical temperature quickly enough to form a fully hardened structure and to prevent the formation of undesired structures, which could occur at intermediate temperatures if the cooling rate is too slow. Table 6.2 shows the important temperatures that must be observed with different kinds of steel. Table 6.3 shows the critical cooling rate for each kind of steel made to the minimum composition range. The sizes of round that will through-harden in a mild quench and the hardnesses obtained with 60% martensite formation are also shown in Table 6.3. Table 6.4 shows the approximate cooling rates needed to get a high level of metallurgical quality in gears.

The steels listed in Tables 6.2 and 6.3 are those which are commonly used for gears. The compositions of these steels that have been established by the *American Iron and Steel Institute* (AISI) are given in Table 6.5. (Modi ed compositions of steels are often used as well. When buying gear steel, note the exact composition being speci ed or offered.)

The exact composition of steel may vary somewhat from one steel company to another or from steels used in vehicles to those used in aircraft. For instance, a designer who wants to use 4340-type steel in aircraft gears should specify an *aircraft composition*. The aircraft composition tends to have closer ranges for each alloy element, and there is more control of *trace elements*.

International users of this book should be able to nd German, Japanese, French, or English steel speci cations that are very similar to the American types of steels that are shown. In this chapter, the heat treatment temperatures are given in degrees Fahrenheit rather than degrees Celsius.

TABLE 6.2
Heat Treatment Data for Typical Gear Steels

AISI No.	Normalizing Temp. (°F)	Annealing Temp. (°F)	Hardening Temp. (°F)	Carburizing Temp. (°F)	Reheating Temp. (°F)	M_s Temp.[a] (°F)
1015	1700	1600	–	1650–1700		
1025	1650–1750	1600	1575–1650	1500–1650		
1040	1650–1750	1450	1525–1575			
1045	1600–1700	1450	1450–1550			
1060	1550–1650	1400–1500	1450–1550	–	–	555
1118	1700	1450	–	1650–1700	1650–1700	
1320	1600–1650	1500–1700	–	1650–1700	1450–1500	740
1335	1600–1700	1500–1600	1500–1550	–	–	640
2317	1650–1750	1575	–	1650–1700	1450–1500	725
2340	1600–1700	1400–1500	1425–1475	–	–	555
3310	1650–1750	1575	–	1650–1700	1450–1500	655
3140	1600–1700	1450–1550	1500–1550	–	–	590
4028	1600–1700	1525–1575	–	1600–1700	1450–1500	750
4047	1550–1750	1525–1575	1475–1550			
4130	1600–1700	1450–1550	1550–1650	–	–	685
4140	1600–1700	1450–1550	1525–1625	–	–	595
4320	1600–1800	1575	–	1650–1700	1425–1475	720
4340	1600–1700	1100–1225	1475–1525	–	–	545
4620	1700–1800	1575	–	1650–1700	1475–1525	555
4640	1600–1700	1450–1550	1450–1550	–	–	605
4820	1650–1750	1575	–	1650–1700	1450–1500	685
5145	1600–1700	1450–1550	1475–1525			
5210	–	1350–1450	1425–1600	–	–	485
6120	1700–1800	1600	–	1700	1475–1550	760
6150	1650–1750	1550–1650	1550–1650	–	–	545
8620	1600–1800	1575	–	1700	1425–1550	745
9310	1650–1750	1575	–	1650–1700	1425–1550	650
EX 24	1650–1750	1600	1660	1650–1700	1500–1550	830
EX 29	1650–1750	1600	1600	1650–1700	1500–1550	830
EX 30	1650–1750	1550	1600	1650–1700	1500–1550	830
EX 55	1650–1750	1525	1600	1650–1700	1500–1550	790

[a] The M_s temperature is the temperature at which martensite forms.

TABLE 6.3
Hardenability Data for Typical Gear Steels

AISI No.	Hardness of 60% Martensite (HRC)	Critical Cooling Rate at 1300°F (°F/s)	Size of Round That Will Through-Harden (in.)	Mildly Agitated Quenching Medium
		Through-Hardening Steels		
1045	50.5	400[a]	0.50[b]	Water
1060	54	125[a]	1.20[b]	Water
1335	46	195	1.00	Water
2340	49	125	0.60	Oil
3140	49	125	0.60	Oil
4047	52	195	1.00	Water
			0.40	Oil
4130	44	305	0.70	Water
4140	49	56	1.00	Oil
4340	49	10	2.80	Oil
5145	51	125	0.60	Oil
5210	60	30	1.30	Oil
6150	53	77	0.80	Oil
AISI No.	**Minimum Core Hardness**	**Critical Cooling Rate at 1300°F (°F/s)**	**Size of Round That Will Through-Harden (in.)**	**Mildly Agitated Quenching Medium**
		Data for Core of Case-Carburizing Steels		
1015	15	400[b]	0.50[b]	Water
1025	18	400[b]	0.50[b]	Water
1118	20	400[b]	0.50[b]	Water
1320	20	305	0.70	Water
3310	25	3	3.00	Oil
4028	24	300[a]	0.80[b]	Water
4320	32	195	0.40	Oil
4620	30	305	0.20	Oil
			0.70	Water
4820	35	77	0.80	Oil
8620	25	250[a]	0.80	Water
			0.30	Oil
9310	25	21	1.70	Oil
EX 24	26	195[a]	0.80	Water
EX 29	32	175[a]	1.00[b]	Water
EX 30	35	150[a]	1.20	Water
EX 55	40	42[b]	2.00	Oil

Note: Data based on minimum composition range except where noted.
[a] Estimated values.
[b] Obtained from nominal composition rather than from minimum of speci cation range.

(There is no room to show both in tables.) The conversion to degrees Celsius is

$$\text{Degrees Celsius} = \frac{\text{degrees Fahrenheit} - 32}{1.8}. \qquad (6.1)$$

The hardening temperature of a steel must be slightly above the *critical temperature*. The critical temperature is the temperature at which the steel is completely austenized and ready to undergo the hardening reaction upon quenching. The *critical cooling* rate is the rate of cooling that is just fast enough to produce a fully hardened structure in a particular steel composition.

The M_s *temperature* is the temperature at which the fully hard structure (martensite) rst starts to form during the quench. If the cooling rate is slower than the critical rate, other structures (ferrite, pearlite, and upper bainite) may form at intermediate temperatures between the critical temperature and the M_s temperature.

After quenching, steels are usually *tempered* or *stress relieved*. The tempering operation may be used to reduce the hardness of a part and increase the toughness. By properly adjusting the tempering temperature, a wide range of

TABLE 6.4
Approximate Cooling Rates Needed to Achieve Appropriate Metallurgical Quality for Long-Life, Highly Loaded Gears

		Time[a] (s)		
Carburizing Steel	Core Hardness HV10	To 600°C (1112°F)	To 400°C (752°F)	To 200°C (392°F)
4028	339[b]	3	8	10
8620	339	5	12	28
EX 24	339	6	20	35
4620	339	3	8	10
4320	339	8	22	60
EX 29	339	8	22	60
9310	339	10	35	90
4815	339	6	21	58
EX 30	339	6	21	58
EX 35	390	13	50	125

[a] Time is from start of quench to the temperature listed.
[b] 339 HV10 is approximately 34 HRC. 390 HV10 is approximately 40 HRC.

TABLE 6.5
Compositions of Typical Gear Steels

	Chemical Composition Limits (%)							
AISI No.	C	Mn	P	S	Si	Ni	Cr	Mo
1015	0.13/0.18	0.30/0.60	0.040	0.050	–	–	–	–
1025	0.22/0.28	0.30/0.60	0.040	0.050	–	–	–	–
1045	0.43/0.50	0.60/0.90	0.040	0.050	–	–	–	–
1060	0.55/0.65	0.60/0.90	0.040	0.050	–	–	–	–
1118	0.14/0.20	1.30/1.60	0.045	0.080/0.13	–	–	–	–
1320	0.18/0.23	1.60/1.90	0.040	0.040	0.20/0.35	–	–	–
1335	0.33/0.38	1.60/1.90	0.040	0.040	0.20/0.35	–	–	–
3140	0.38/0.43	0.70/0.90	0.040	0.040	0.20/0.35	1.10/1.40	0.55/0.75	–
3310	0.08/0.13	0.45/0.60	0.025	0.025	0.20/0.35	3.25/3.75	1.40/1.75	–
4028	0.25/0.30	0.70/0.90	0.040	0.040	0.20/0.35	–	–	0.20/0.30
4047	0.45/0.50	0.70/0.90	0.040	0.040	0.20/0.35	–	–	0.20/0.30
4130	0.28/0.33	0.40/0.60	0.040	0.040	0.20/0.35	–	0.80/1.10	0.15/0.25
4140	0.38/0.43	0.75/1.00	0.040	0.040	0.20/0.35	–	0.80/1.10	0.15/0.25
4320	0.17/0.22	0.45/0.65	0.040	0.040	0.20/0.35	1.65/2.00	0.40/0.60	0.20/0.30
4340	0.38/0.43	0.60/0.80	0.040	0.040	0.20/0.35	1.65/2.00	0.70/0.90	0.20/0.30
4620	0.17/0.22	0.45/0.65	0.040	0.040	0.20/0.35	1.65/2.00	–	0.20/0.30
4640	0.38/0.43	0.60/0.80	0.040	0.040	0.20/0.35	1.65/2.00	–	0.20/0.30
4820	0.18/023	0.50/0.70	0.040	0.040	0.20/0.35	3.25/3.75	–	0.20/0.30
5145	0.43/048	0.70/0.90	0.040	0.040	0.20/0.35	–	0.70/0.90	–
5210	0.95/1.10	0.25/0.45	0.025	0.025	0.20/0.35	–	1.30/1.60	–
6120	0.17/0.22	0.70/0.90	0.040	0.040	0.20/0.35	–	0.70/0.90	V 0.10 min
6150	0.48/053	0.70/0.90	0.040	0.040	0.20/0.35	–	0.80/1.10	V 0.15 min
8620	0.18/0.23	0.70/0.90	0.040	0.040	0.20/0.35	0.40/0.70	0.40/0.60	Mo 0.15/0.25
9310	0.08/0.13	0.45/0.65	0.025	0.025	0.20/0.35	3.00/3.50	1.00/1.40	0.08/0.15
EX 24	0.18/0.23	0.75/1.00	0.040	0.040	0.20/0.35	–	0.45/0.65	0.20/0.30
EX 29	0.18/0.23	0.75/1.00	0.040	0.040	0.20/0.35	0.40/0.70	0.45/0.65	0.30/0.40
EX 30	0.13/0.18	0.70/0.90	0.040	0.040	0.20/0.35	0.70/1.00	0.45/0.65	0.45/0.60
EX 35	0.15/0.20	0.20/1.00	0.040	0.040	0.20/0.35	1.65/2.00	0.45/0.65	0.65/0.80

hardnesses may be obtained. Even when no reduction in hardness is desired, a low-temperature (250°F to 350°F) tempering operation is desirable to reduce stresses in the steel and produce a kind of martensite that is tougher than the kind produced immediately upon quenching.

6.1.3 Heat Treatment Data

The gear designer should remember that, in most cases, the strength and hardness of the steel gear will depend primarily on the *skill* and *intelligence* with which the steel has been heat-treated. *Improper quenching* or *tempering at the wrong temperature* will completely defeat the designer's selection of the best steel for the job. Heat treatment is still a *skilled art*. The best gears are made by those who study carefully the microstructure of their product and alter their heat treatment technique to get the best possible results.

Table 6.2 gives heat treatment data for some typical gear steels. The values shown should be considered nominal and should be varied in actual practice as experience dictates.

Figure 6.2 shows the way in which structures from during the cooling of 4340 steel. Transformation diagrams such as this one can be obtained from several companies that are involved in steelmaking. These diagrams are very helpful in developing a suitable heat treatment procedure for a particular steel.

The carbon content of a steel establishes the maximum hardness that can be reached in the fully hardened condition, while the alloying elements determine the critical cooling rate necessary for full hardening and therefore the section thickness that will harden with the quench that is available. For example, plain carbon steels have such high critical doing rates that they must be water- or brine-quenched to be fully hardened even when the section is relatively small. Alloy steels transform more slowly and can be hardened with an oil quench. In the case of some high-alloy tool and die steels, even cooling in still air will develop high hardness!

End-quench hardenability curves demonstrate how alloys slow down the reaction rates of steels and give the heat treater enough time to develop full hardness with a mild quench. With large parts made of plain carbon or low-alloy steels, there is a distinct danger of cracking the parts if they are quenched drastically enough to harden completely. On the other hand, a milder quench may not develop structures with the required strength. In extreme cases, the part may be so large that even with considerable alloy content, it is not practical to use a fast enough quench to develop full hardness. In this case, the designer has to recognize that compromise is necessary, and design with low enough stresses to get by with whatever properties can be obtained in steel.

Figure 6.3 shows end-quench hardenability curves for several kinds of gear steels. These data together with those shown in Tables 6.3 and 6.4 will help the designer choose a

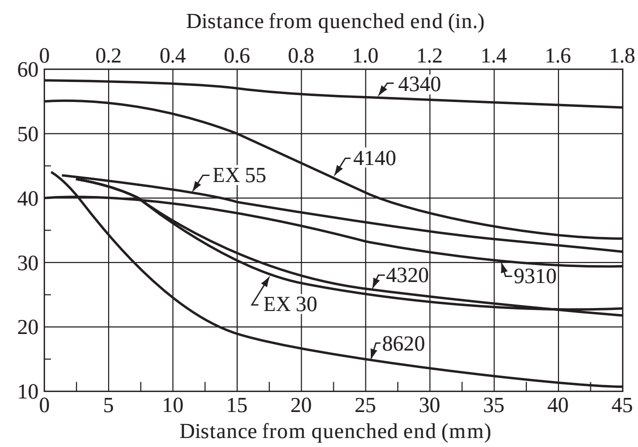


FIGURE 6.3 Some typical Jominy curves showing end-quench hardenability.

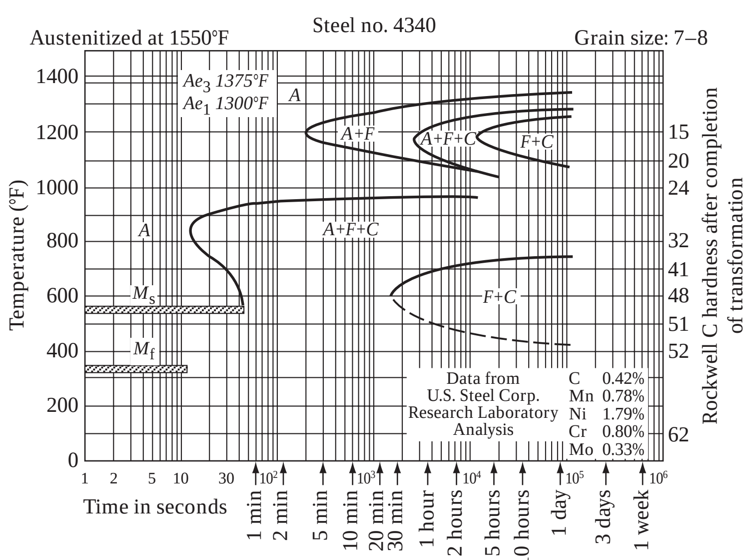


FIGURE 6.2 Transformation diagram of AISI 4340. (Courtesy of International Nickel Co., New York.)

steel that will have suf cient hardenability for a speci c gear. The designer should also note material quality grades in gear-rating speci cations. (See Sections 5.2.1, 5.2.10, and 5.2.13.)

6.1.4 Hardness Tests

The easiest way to determine the approximate tensile strength of a piece of steel is to check its hardness. Gear parts do not always have the same hardness in the teeth as in the rim, web, or hub. One of the best ways for the designer to control the nal condition of the heat-treated gear is to specify the hardness of the teeth. In some cases it is desirable to check both the gear teeth and the gear blank for hardness. Small spur-gear teeth can be checked for hardness right on the working ank of the tooth. Figure 6.4 shows how this is done on small aircraft gears.

Table 6.6 shows some of the commonly used hardness checks and the kinds of gears for which they are most suited. The amount of load used and the kind of ball or point used to indent the piece being tested are also shown.

The approximate relation between the various hardness-test scales is shown in Table 6.7. The values shown are averages of tests on carbon and alloy steels. Because of difference in structure, cold-working tendencies, and other factors, there is no exact mathematical conversion between the scales. For this reason, hardness requirements on a gear drawing should be given *only in the scale* by which they will be checked. This will prevent possible ambiguity of hardness speci cations.

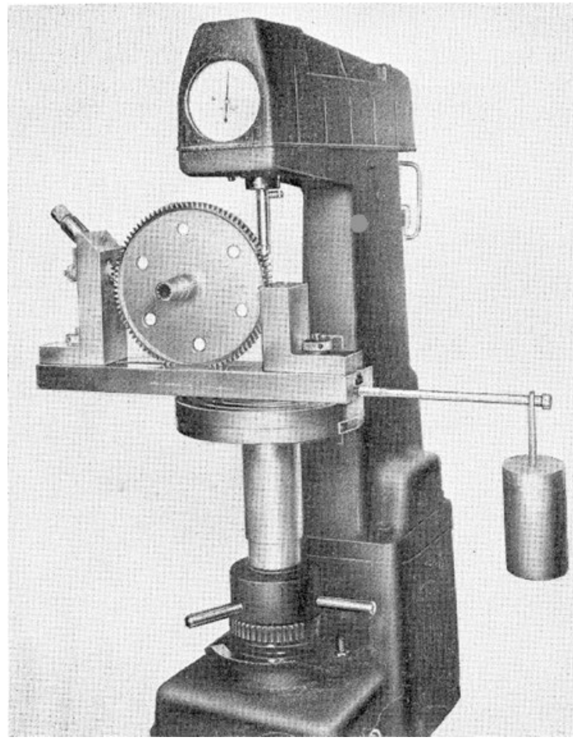


FIGURE 6.4 Hardness-checking an aircraft gear near the pitch line of the tooth. (Courtesy of General Electric Co., Lynn, Massachusetts.)

TABLE 6.6
Hardness-Testing Apparatuses and Applications for Gears

Instrument	Shape and Type of Indenter	Loading	Recommended Uses
Brinell	10 mm steel or tungsten carbide ball	3000 kg	For large gears and shafts in range of hardness from 100 to 400 Brinell. (If gears have a hard case, the case depth must be suf cient and the core strength adequate to support the area under test. Rockwell C tests may be used to reveal an error from such irregular conditions.) Brinell tests, when fairly representative of the general hardness, are a good measure of the ultimate tensile strength of the material.
Rockwell C	Diamond Brale penetrator (120° diamond cone)	150 kg	For gears medium to large in size. Range: approximately 25–68 Rockwell C.
Rockwell A	Diamond Brale penetrator (120° diamond cone)	60 kg	For small gears and tips of large gear teeth. Range: Rockwell A 62 to 85.
Rockwell 30-N	Diamond Brale penetrator (120° diamond cone, special indenter)	30 kg	For small parts and shallow case-hardened parts.
Rockwell 15-N	Diamond Brale penetrator (120° diamond cone, special indenter)	15 kg	Lightest Rockwell load. For testing very small parts and checking the working side of teeth. Check for very thinly case-hardened parts. Rockwell 15-N is used 72-93.
Vickers, pyramid	136° diamond pyramid	50 kg	All applications where piece will not be too heavy for machine. For use in testing hardness of shallow cases, etc.
Scleroscope	Does not indent surface; diamond-tipped tup bounced on specimen	–	For applications permitting no damage or indentation to surfaces (results not always comparable with indentation hardness tests).
Tukon	Knoop indenter	500–1000 g for practical use	A laboratory instrument used only for nding the hardness of material on pieces of cross sections, e.g., hardness from surface inward, every 0.05 mm (0.0002 in.), or hardness of individual microconstituents. All test areas must have at mirror-polished surfaces. An extremely delicate and precise test. Used for any hardness.

TABLE 6.7
Approximate Relation between Hardness-Test Scales

Brinell 3000 kg, 10 mm	Rockwell				Vickers Pyramid	Scleroscope (Shore)	Tukon (Knoop)
	C	A	30-N	15-N			
	70	86.5	86.0	94.0	1076		
	65	84.0	82.0	92.0	820	90	840
	63	83.0	80.0	91.5	763	87	790
614	60	81.0	77.5	90.0	695	81	725
587	58	80.0	75.5	89.3	655	79	680
547	55	78.5	73.0	88.0	598	74	620
522	53	77.5	71.0	87.0	562	71	580
484	50	76.0	68.5	85.5	513	67	530
460	48	74.5	66.5	84.5	485	64	500
426	45	73.0	64.0	83.0	446	61	460
393	42	71.5	61.5	81.5	413	56	425
352	38	69.5	57.5	79.5	373	51	390
301	33	67.0	53.0	76.5	323	45	355
250	24	62.5	45.0	71.5	257	37.5	
230	20	60.5	41.5	69.5	236	34	
	Rockwell						
	B		30-T	15-T			
200	93		78.0	91.0	210	30	
180	89		75.5	89.5	189	28	
150	80		70.0	86.5	158	24	
100	56		54.0	79.0	105		
80[a]	47		47.7	75.7			
70[a]	34		38.5	71.5			

[a] Based on 500 kg load and 10 mm ball.

6.2 LOCALIZED HARDENING OF GEAR TEETH

Several methods are used either to case-harden only the gear teeth or to harden the surfaces of gear teeth and leave the inside part of the tooth at an intermediate hardness. Carburizing, nitriding, induction hardening, and flame hardening can all be used to produce gear teeth which are much harder than the gear blank that supports the teeth. Since the tooth surfaces are much more critically stressed than the gear rim, web, or hub, a high-capacity gear can be obtained by fully hardening the gear teeth only.

6.2.1 Carburizing

Carburizing is the oldest and probably the most widely used process for hardening gear teeth. It consists of heating a 0.10% to 0.25% carbon steel at a temperature above its critical range in a gaseous, solid, or liquid medium capable of giving up carbon to the steel. The surface layer becomes enriched in carbon content and therefore is capable of developing a high degree of hardness after quenching.

The concentration of carbon in the surface layer is determined by many factors; if uncontrolled, it may reach 1.20%. For best strength and toughness, though, the concentration should be kept under 1.0%, preferably around 0.80% to 0.90%. Control of the richness of the carbon case is obtained by controlling the richness of the carburizing atmosphere.

The development of the case depends on the diffusion of carbon into the steel; therefore, time and temperature are the main factors that control the case depth. Steel composition has no great influence on the rate of carbon penetration. Figure 6.5 shows the approximate effect of temperature and time on the depth of case obtained in a gaseous carburizing operation.

Carburized parts may be heat-treated in several ways to achieve a variety of case and core properties. Several of the most important treatments are illustrated in Figure 6.6 and explained in the table in the figure. For heavy-duty gearing, treatment C generally provides both a good case and a strong core. With only one quench, distortion is minimized. In some high-production gear jobs, it has been possible to get satisfactory gears by directly quenching from the carburizing temperature. The designer should hesitate to use this shortcut method unless there is a steel and a procedure that will produce gears which are known (by proper testing) to be good enough for the job.

In the vehicle field, the direct quench method is normally used for carburized gears. The gears are somewhat small, the

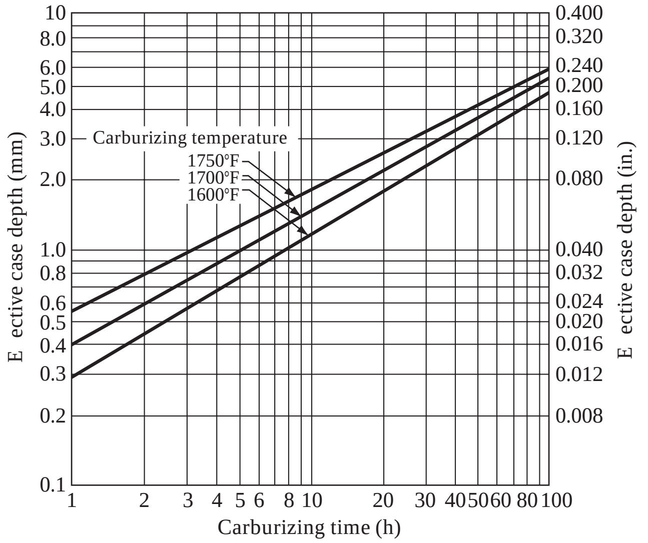


FIGURE 6.5 Nominal time and temperature requirements for different case depths.

volume of the production is high, and the facilities used are picked to be just right for the part. All these things tend to make it practical to use direct quench.

In the turbine field, most carburized gears are given a reheat quench. Here the gear parts tend to be fairly large, general-purpose heat-treating equipment is used, and the volume is low. The reheat method seems to give better metallurgical structure and more assurance that the gears can run satisfactorily for 10^9 or 10^{10} cycles. (In the vehicle field, gears seldom run at high loads for more than 10^8 cycles.)

Commonly used carburized steels are shown in Table 6.3. For best load-carrying capacity, the case should be up to 700 HV (60 HRC), and the core should be in the range of 340 to 415 HV (35 to 42 HRC). Too hard a core promotes brittleness and cuts down the compressive stress which is developed by the slight difference in volume between the case material and the core material. Too soft a core does not provide strength enough to support the high loads that the case can carry. For aircraft work, most designers favor a steel of the AISI 9310 type. The AMS6260* specification quite closely follows the specification of AISI 9310 (AMS means "aircraft material specification"). AISI 9310 is a nickel–chromium–molybdenum type of steel. Other types of steels which are used for making heavy-duty gears for marine, tractor, railroad, and other applications are the nickel, nickel–chromium, and nickel–molybdenum types. In a few gear applications (usually smaller parts), good load-carrying capacity has been obtained using steels with a low alloy content. On a new job, the designer should do some development work to determine the most suitable type of steel for a given application.

The carburizing cycle brings gears up to a red-hot temperature. When the red-hot gear is quenched, it cools rapidly, but somewhat unevenly. The outer surface, of course, cools the fastest because it is in contact with the quenching medium.

As the steel cools, the structure changes and grows stronger. The unequal cooling rates in different parts of the gear body tend to make the gear change size very slightly and to distort. In addition, the carburized case tends to be slightly larger in volume than the core material. This also contributes to the size change and distortion tendency.

The overall result of this is that the pressure angle of the tooth tends to increase slightly and the helix angle tends to unwind. There is also a tendency for the bore to shrink, for the part to develop axial and radial runout, and for the outside diameter to become slightly coned. Because of these things, it is necessary either to make allowances for dimensional changes or to grind the part after carburizing. In either event, quenching dies may be helpful in cutting down distortion and reducing the amount of change that must be either allowed for or ground off.

Carburized gears have to have enough surface hardness to resist surface-initiated pitting. The allowable contact stress is based on the surface hardness. Small gears, with about 2.5-module (10-pitch) teeth and pitch diameters in the range of about 40 to 300 mm, can be carburized to achieve a surface hardness in the range of 700 to 759 HV (60 to 63 HRC). Larger gears, with 5-module (5-pitch) teeth and diameters going up to 1 m (40 in.), are more difficult to carburize with optimum results in hardness and metallurgical structure. The design hardness of the medium large gear should generally be in the range of 675 to 725 HV (58 to 62 HRC).

If there are problems in processing carburized gears, the achievable *minimum* surface hardness may be only 600 HV (55 HRC). The problems may come from the parts being rather small or very large. Parts as small as 1 module (25 pitch) or as large as 25 module (1 pitch) are carburized. Either of these sizes is difficult to handle, and the surface hardness may be low.

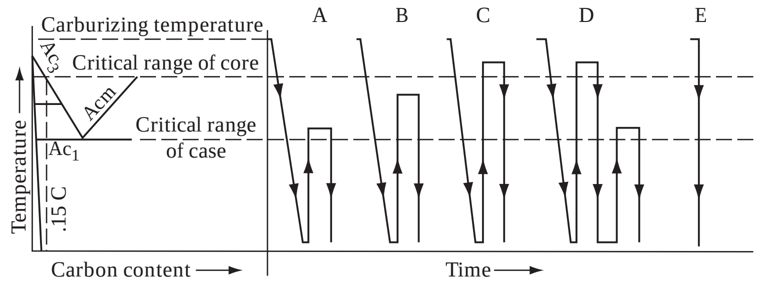
Besides surface hardness, the carburized pinion or gear needs an adequate case depth. There are subsurface stresses that are strong enough to cause cracks in the region of case-to-core interface if the case is too thin.

The carburized case needs to be deep enough to give adequate bending strength and deep enough to resist case crushing or case spalling. The depth needed for bending strength is a function of the normal module (or the normal diametral pitch), but the depth needed to resist subsurface stresses due to contact loads is a function of the pitch diameter of the pinion, the ratio, and the pressure angle (primarily). Figure 6.7 shows an example of a carburized tooth sectioned to study case and core structure—before finish grinding.

The minimum effective case depth in the root fillet for tooth bending strength may be estimated by this relation:

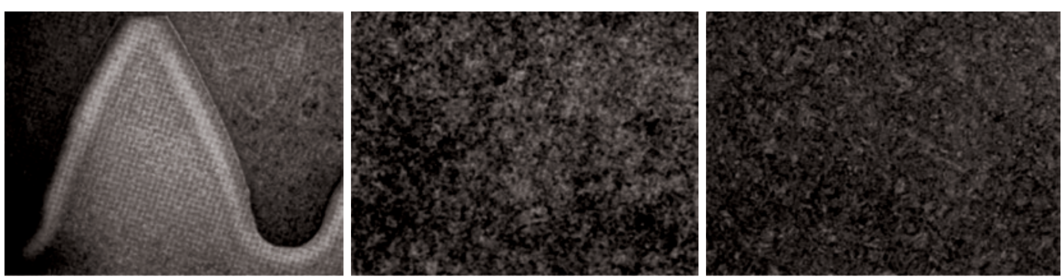
$$h_{\mathrm{et}} = 0.16 m_{\mathrm{n}} \text{ (mm; metric)}, \tag{6.2}$$

* Most of the critical aircraft gears are made of AMS 6265 steel. This is a vacuum-arc remelt steel of unusual cleanness. Both AMS 6260 and 6265 are 9310 types of steel. (AMS 6260 is an air-melt quality of steel.)



Treatment	Case	Core
A (Best adapted to fine-grained steels)	Refined; excess carbide not dissolved	Unrefined; soft and machinable
B (Best adapted to fine-grained steels)	Slightly coarsened; some solution of excess carbide	Partially refined; stronger and tougher than A
C (Best adapted to fine-grained steels)	Somewhat coarsened; solution of excess carbide favored; austenite retention promoted in highly alloyed steels	Refined; maximum core strength and hardness; better combination of strength and ductility than B
D (Best treatment for coarsed-grained steels)	Refined; solution of excess carbide favored; austenite retention minimized	Refined; soft and machineable; maximum toughness and resistance to impact
E (Adapted to fine-grained steel only)	Unrefined with excess carbide dissolved; austenite retained; distortion minimized	Unrefined but hardened

FIGURE 6.6 Diagrammatic representation of different treatments subsequent to carburization and the case and core characteristics obtained. (Courtesy of International Nickel Co., New York.)



(a) (b) (c)

FIGURE 6.7 Carburized gear tooth: (a) section about ×5; (b) case structure about ×1000; and (c) core about ×250.

$$h_{et} = \frac{0.16}{P_{nd}} \text{ (in.; English)}, \tag{6.3}$$

where
m_n—normal module
P_{nd}—normal diametral pitch

The minimum effective case depth on the ank of the tooth for surface durability may be estimated by this relation:

$$h_{ec} = \frac{s_c d_{p1} \sin\alpha_t}{48{,}250 \cos\beta_b} \times \frac{u}{u+1} \text{ (mm; metric)}, \tag{6.4}$$

$$h_{ec} = \frac{s_c d \sin\phi_t}{7\times10^6 \cos\psi_b} \times \frac{m_G}{m_G+1} \text{ (in.; English)}, \tag{6.5}$$

where
s_c—maximum contact stress in the region of 10^6 to 10^7 cycles (N/mm² for metric, psi for English)
d_{p1}—pinion pitch diameter (mm; metric)
d—pinion pitch diameter (in.; English)
α_t, ϕ_t—pressure angle, transverse (metric, English)
β_b, ψ_b—base helix angle (metric, English)
u, m_G—tooth ratio (metric, English)

Figure 6.8 shows a schematic view of the case on a gear tooth. The minimum thickness needed at region A is

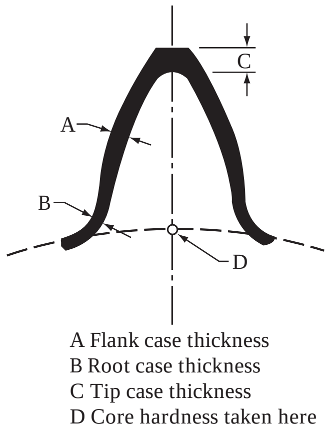


FIGURE 6.8 Carburized case pattern. Shaded area is all 510 HV (50 HRC) or higher in hardness. Unshaded area is less than 510 HV (50 HRC).

determined by Equations 6.4 and 6.5. The minimum thickness needed at region B is determined by Equations 6.2 and 6.3.

If the case is too deep, the depth at region C becomes too great. A deep case at region C is a hazard because the whole top of the tooth may break off. Generally speaking, the depth at region C should not be greater than

$$h_{em} = 0.40 m_n \text{ (mm; metric)}, \quad (6.6)$$

$$h_{em} = \frac{0.40}{P_{nd}} \text{ (in.; English)}. \quad (6.7)$$

In carburizing gears, the case depth is customarily speci- ed for region A, with maximum and minimum limits of effective case depth. The *effective* case depth for parts with a minimum surface hardness of at least 675 HV (58 HRC) is taken at the point at which the case is at least HV 510 (50 HRC). For parts with less surface hardness, it is advisable to put the limit point for effective case depth at about 7 HRC points lower than the minimum surface hardness. Thus, the effective case depth for 600 HV (55 HRC) might be taken at HV 485 (48 HRC).

In specifying case depth, a set of values such as this needs to be used:

- Effective case depth, ank, 0.50 to 0.75 mm (0.020 to 0.030 in.)
- Minimum effective case depth, root, 0.40 mm (0.016 in.)
- Maximum effective case depth, tip, 1.00 mm (0.040 in.)

The above might be about right for a 2.5-module spur pinion, heavily loaded.

The amount of case depth needed for surface durability is illustrated in Figure 6.9. This gure shows an example of spur gears with 22.5° pressure angle and a tooth contact stress of 1800 N/mm² (261,000 psi) for 5 × 10⁶ cycles. This is typical

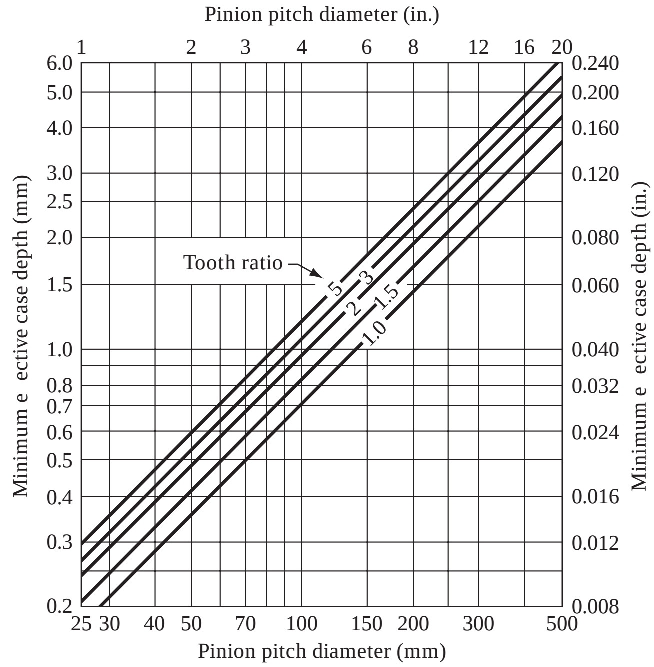


FIGURE 6.9 Minimum effective case depth for carburized spur gears of 22.5° pressure angle and a contact stress of 1800 N/mm² (261,000 psi) for not over 10^7 cycles.

of a very good vehicle gear, with very high torque at relatively slow pitch-line speed. The faster-running turbine gears, for instance, would have lower design stresses in the region of 10^7 cycles and still lower stress levels at a rated load in the region of 10^9 or 10^{10} cycles.

At the present, more research work is going on to determine all the variables that affect the required case depth. Those designing gears should be alert to new requirements that may appear in trade standards.

6.2.2 Nitriding

Nitriding is a case-hardening process in which the hardening agents are nitrides formed in the surface layers of steel through the absorption of nitrogen from a nitrogenous medium, usually dissociated ammonia gas.

6.2.2.1 Features of Nitriding Process

Almost any steel composition will absorb nitrogen, but useful cases can be obtained only on steels that contain appreciable amounts of aluminum, chromium, or molybdenum. Other elements, such as nickel and vanadium, may be needed for their special effects on the properties of the nitrided steel. The gear steels most commonly nitrided are shown in Table 6.8. The temperatures used and the hardnesses obtained are shown in Table 6.9.

A nitride case does not form as fast as a carbon case. Figure 6.10 shows the nominal relation between nitriding time and case depth obtained. Nitriding, like carburizing, is a dif cult process, but because the rate of penetration is slower, the time cycles are quite long.

TABLE 6.8
Nitriding Gear Steels

Steel	Carbon	Manganese	Silicon	Chromium	Aluminum	Molybdenum	Nickel
Nitralloy[a] 135	0.35	0.55	0.30	1.20	1.00	0.20	–
Nitralloy 135 (modified)	0.41	0.55	0.30	1.60	1.00	0.35	–
Nitralloy N	0.23	0.55	0.30	1.15	1.00	0.25	3
AISI 4340	0.40	0.70	0.30	0.80	–	0.25	1
AISI 4140	0.40	0.90	0.30	0.95	–	0.20	–
31 Cr Mo V 9	0.30	0.55	0.30	2.50	–	0.20	–

[a] Nitralloy is trademark of the Nitralloy Corp., New York.

TABLE 6.9
Nominal Temperatures Used in Nitriding and Hardnesses Obtained

Steel	Temperature before Nitriding (°F)	Nitriding (°F)	Hardness (Rockwell C)	
			Case	Core
Nitralloy 135	1150	975	62–65	30–35
Nitralloy 135 modified	1150	975	62–65	32–36
Nitralloy N	1000	975	62–65	40–44
AISI 4340	1100	975	48–53	27–35
AISI 4140	1100	975	49–54	27–35
31 Cr Mo V 9	1100	975	58–62	27–33

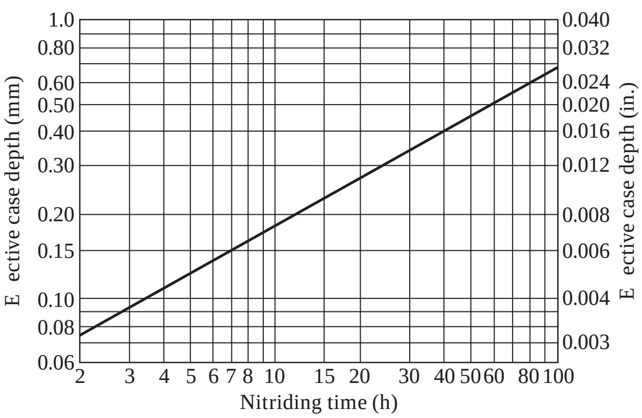


FIGURE 6.10 Nominal times required for different nitride case depths. Note: Under the most favorable conditions, about 40% more case depth is possible.

Nitriding is conducted in sealed retorts in an atmosphere of dissociated ammonia at temperatures between 930°F and 1000°F. The modern practice is to start the process with 30% ammonia dissociation for the first several hours, then to allow dissociation to increase to 85%. At the lower dissociation rate, a weak and brittle layer of overly rich nitrides is formed at a surface layer about 0.05 mm (0.002 in.) deep. This is called a *white layer* because it etches out white in a micrograph. The higher dissociation rate that is used at the end of the cycle allows most of the excess nitrides to diffuse into the metal, leaving only traces of a white layer. Figure 6.11 shows examples of nitride cases on spline and gear teeth.

For best results, parts to be case-hardened by nitriding should be rough-machined and then quenched and tempered. The tempering temperature is usually between 1000°F and 1150°F. After heat treatment, the part is finish-machined, stress relieved at about 1100°F, and then nitrided.

Since the nitriding temperature is lower than the original tempering temperature of the steel, hardening occurs with a minimum of distortion. The method is therefore suitable for complex parts such as gears that can be machined while in the medium-hardness region and then hardened without enough distortion to require grinding. Parts that are too flimsy to stand a quench and draw without serious distortion can often be successfully brought up to full hardness by nitriding.

The formation of nitrides in the case causes the steel to expand, and, as in carburized cases, a favorable compressive stress is developed. This also results in a slight overall growth of the part. In spur gears this is manifested by an increase in outside diameter and in the diameter over pins. As an example, a 2-module (12-pitch) spur gear with a lightweight web showed an average increase in outside diameter of 0.13 mm (0.005 in.) and a 0.20 mm (0.008 in.) growth in diameter over pins. The pressure angle tends to decrease slightly. (This is just the opposite to the change in profile caused by case-carburizing.)

In general, there is no distortion of the gear blank, provided that the blank was thoroughly tempered and stabilized before nitriding. If the blank has residual stresses before nitriding, the many hours at nitriding temperature will relieve these stresses and cause the part to warp.

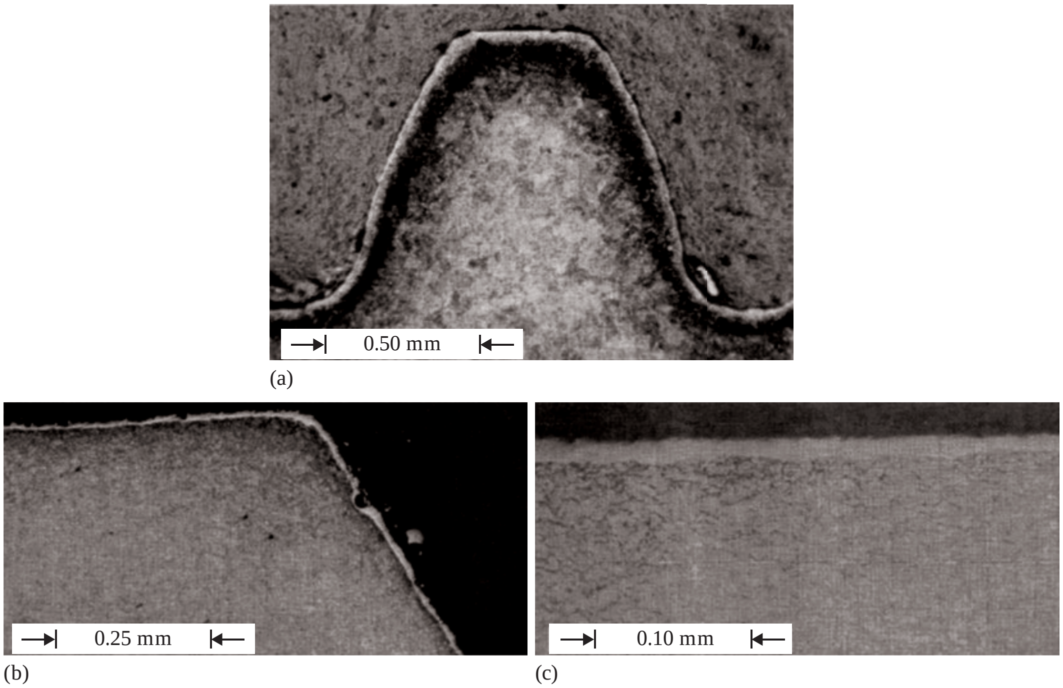


FIGURE 6.11 Examples of nitrided gear tooth structures. (a) Small spline, Nitralloy 135M; white layer 0.03 mm (0.0012 in.); case depth 0.41 mm (0.016 in.). (b) Medium size, internal spline 4340; white layer 0.01 mm (0.0004 in.); case depth 0.38 mm (0.015 in.). (c) Part of 5-module gear tooth, 31 Cr Mo V 9; white layer 0.016 mm (0.0006 in.); case depth 0.65 mm (0.026 in.).

The tendency for the nitride case to grow produces a "cornice" effect, or an overhang of quite brittle material on sharp corners and edges of parts, unless precautions have been taken to break such corners with a small radius before nitriding. If the controlled dissociation method of nitriding is not used, it is usually advisable to grind or lap off the white layer (0.08 mm [0.003 in.] nominally) after nitriding. It is also advisable to take precautions to avoid decarburization of areas to be nitrided. A very weak and brittle case is produced on decarburized steels or on steels that have not been properly quenched and tempered prior to nitriding.

If the steel contains aluminum, a nitrided case is relatively harder and inherently more brittle than a carburized case. Under high pressures, it may crack and spall because of either too little case depth or too weak a core material under the case. In general, nitrided gears need almost as much case as shown in Figure 6.9, but with the coarser pitches, it is not possible to get the required depth. In severe load applications, some help can be obtained by using the Nitralloy N–type material. This material goes through a *precipitation hardening* of the core during the nitriding cycle. This gives the case added support and, to some extent, may compensate for the case being too thin on coarser-pitch gears.

The nitrided gear—because of its high hardness—resists scoring and abrasion better than other types of gears. Tests of 2.5-module (10-pitch) and ner nitrided gears have shown that with proper nitriding technique, just as heavy loads could be carried on a nitrided tooth* as on a good case-carburized tooth. In cases where shock loading is present, or where the pitch is medium to coarse, most designers have found that other types of hardened gears would stand more loading than the nitrided gear.

6.2.2.2 Nitride Case Depth

The case depth needed for nitriding gears seems to be less than that needed for carburized gears. The logic of nitride case depth is still not understood too clearly. The American Society of Mechanical Engineers (ASME) paper by Young† (1980) has a good review of the theory of case depth for nitrided gears and a summary of much practical experience in making large nitrided gears.

From the standpoint of tooth breakage, the nitride case does not generally give a high degree of strength. For spur teeth in the range of 4 module (6 pitch) to 10 module (2.5 pitch), nitriding has some value in increasing the surface load-carrying capacity, but the beam strength has to be based almost entirely on the core hardness of the tooth. Equations 6.2 and 6.3 are a good approximation of the case depth needed if the *nitrided case*—instead of the core—is to govern the beam strength of the nitrided tooth. It can be seen from Figure 6.10 that it takes about 55 hours of nitriding to get a case depth of 0.48 mm (0.019 in.). Equations 6.2 and 6.3 show that a 3-module spur tooth needs a case depth of about 0.48 mm if the case is to control the beam strength. This means that a very long nitriding time is needed if teeth larger than 3 module are going to get much beam-strength help from the nitride case.

* This statement assumes that the nitrided case is the *same* hardness as the carburized case and that the governing load was one for 10^7 cycles or more (not a very high initial load for less than 10^6 cycles).

† Consult the Literature section at the end of this book for complete data on this paper.

From the standpoint of surface durability, the situation is also rather critical on nitride case depth. A pinion of 150 mm (6 in.) pitch diameter running with a gear of 450 mm (18 in.) needs about 1.5 mm (0.060 in.) case depth when carburized and designed to carry a load of 1800 N/mm^2 (261,000 psi). There is some limited experience that says nitride case depth does not need to be quite as deep as carburized depth to handle the same load. If this is true, the nitride case depth would need to be at least 85% of the values given by Equations 6.4 and 6.5.

The net result is that only small pinions (less than 100 mm) will be able to have enough nitride case depth to carry as much load as their surface hardness would indicate.

Many large pinions (and their mating gears) are nitrided and used successfully in turbine-gear applications. How do they get by with a very thin case for their size?

The answer seems to lie in using a surface loading that is *lower* than what the surface hardness could handle, but with *subsurface* stresses that are within the capability of the core material. ASME's *Wear Control Handbook* (Dudley, 1980) shows a simplified method of subsurface stress analysis.

If the nitrided gear has a core hardness of at least HV 335 (34 HRC), the subsurface capability of the material underneath the case should be able to handle a surface loading up to about 1000 N/mm^2 (145,000 psi).

In actual practice, nitrided turbine gears tend to be designed for surface loading in the range of 900 to 1000 N/mm^2 (130,000 to 145,000 psi). This means that they can get by with a much thinner case than Equations 6.4 and 6.5 would indicate—providing they have *good* core hardness.

For nitrided gears, the effective* case depth based on HV 395 (40 HRC) seems to work the best. The low-chromium nitriding steels develop a surface hardness of only about HV 515 (50 HRC). They need a much lower case determination point than do carburized steels. The aluminum types of nitriding steels and the high-chromium nitriding steels develop hardness comparable to that of carburized steels. The shape of their case hardness curves, though, is such that most of the hard metal is above HRC 40. Figure 6.12 shows two actual examples. Note how well the effective case is defined by the HRC 40 point.

To conclude this section, it should be said that more gear research is needed to fully define all the criteria relative to the case depths needed for nitrided and carburized gears. The subject of failure by subsurface stress needs more study and more controlled laboratory tests. The work of Fujita in Japan (private communication) is a good example of the pioneering research needed to understand how cased parts fail. The work of Sharma et al. (1977) has been most helpful in providing initial guidelines for designing gears with a consideration of the subsurface stress. (See the Literature section at the end of the book.)

* It the nitrided gear has a core hardness approaching HV 395 (40 HRC), then a higher value, such as 446 HV (45 HRC), must be used for effective case depth.

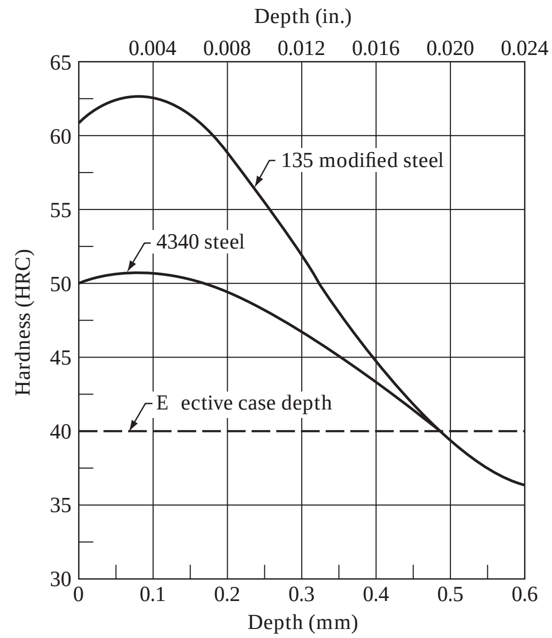


FIGURE 6.12 Examples of two steels nitrided for a total of 45 hours. 4340 is a low-chromium steel. 135 M is a high-chromium and high-aluminum steel.

6.2.3 Induction Hardening of Steel

High-frequency alternating currents can be used to heat locally the surface layers of steel gear teeth. Since the gear body remains relatively cold during the induction hardening, it serves as a fixture to maintain the dimensional accuracy of the induction-hardened teeth. The surface layers of metal undergo some plastic upsetting as they are heated because of the restraint of the cold blank. Upon hardening, the surface layers of metal undergo some expansion due to the volume increase of the hardened metal. There are thus two conflicting tendencies in induction hardening. One is a tendency for the part to shrink, and the other is a tendency for the part to expand. If the right technique is used with the right steel on the right pitch of teeth, good residual compression can be obtained. If the cycle is not just right, damaging residual tension stresses can result. Some gear manufacturers have developed proprietary processes for induction hardening of certain kinds of gears which enable them to build induction-hardened gears similar in strength to case-carburized or nitrided teeth. Many manufacturers have not been able to build induction-hardened gears with as much capacity as case-hardened gears. Designers of induction-hardened gears should plan on running enough experiments to develop just the right kind of cycle for the particular pitch and size of blank they are concerned with.

Induction-hardened gears have some tendency to warp. Usually there is no change in tooth profile, but there may be a "coning" of outside diameter, axial runout, and radial bumps or hollows near holes in the web or near spokes in the wheel. Fortunately, much can be done to control induction-hardened gear distortion by changes in blank design and changes in the induction-heating technique.

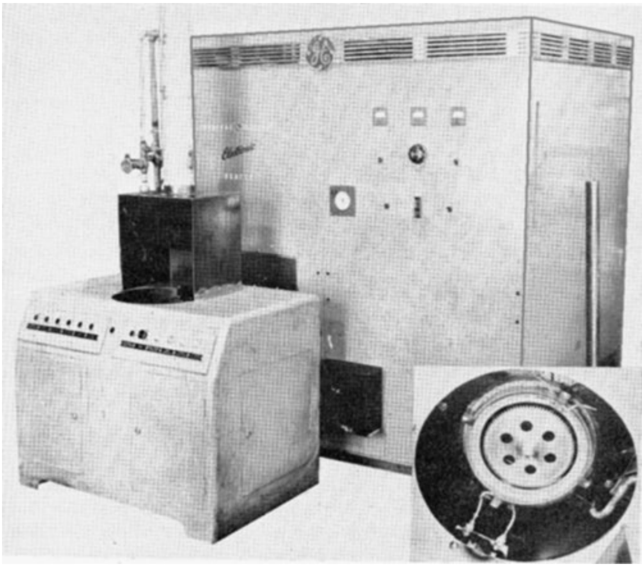


FIGURE 6.13 High-frequency power source and quenching machine for induction-hardening gears. (Courtesy of General Electric Co., Lynn, Massachusetts.)

Both plain carbon and alloy steels are used for induction-hardened gears. Carbon content is usually either 0.40% or 0.50%. If a very fast cycle of heating is used, the choice of alloy will depend on the time that is required for the steel to austenitize. A small gear may be brought to the upper critical temperature in as little as 4 s.

The hardness pattern obtained with induction hardening will depend on the alloy used, the amount of power per square inch of gear surface, the heating time, the frequency, and the pitch of the tooth. Figure 6.13 shows a special machine for induction-hardening gears in a coil. Figure 6.14 shows some examples of hardness pattern obtained in very high-capacity gear teeth.

The induction-hardening current may be obtained from motor-generator sets, spark-gap oscillators, or vacuum-tube oscillators. The power and frequency available from these sources are as follows:

Source	Power (kW)	Frequency (Cycles/s)
Motor generator	5–1000	5000–12,000
Spark-gap oscillator	2–15	10,000–300,000
Vacuum-tube oscillator	2–100	300,000–1,000,000

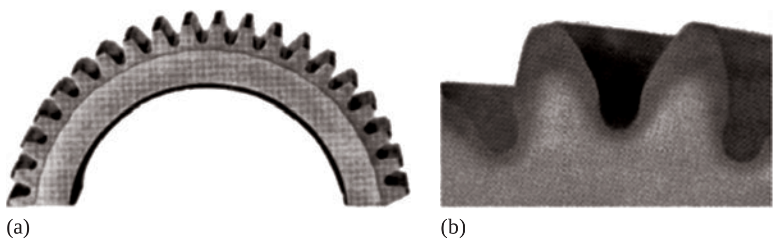


FIGURE 6.14 Hardness patterns obtained with induction-hardened gear teeth. (a) 2-module (12-pitch) pinion and (b) 10-module (2.5-pitch) pinion.

TABLE 6.10
Power and Time Required to Induction-Harden Spur Gears

	Gear Size						
	Diameter		Face Width		Tooth Size		
Power (kW)	mm	in.	mm	in.	m	P_d	Approximate Heating Time (s)
25	12	0.5	12	0.50	1.25	20	5
25	50	2.0	6	0.25	2.50	10	10
25	50	2.0	25	1.00	2.50	10	50
50	25	1.0	6	0.25	1.25	20	4
50	25	1.0	18	0.75	1.25	20	15
50	125	5.0	6	0.25	2.50	10	5
50	125	5.0	18	0.75	2.50	10	7
100	150	6.0	125	5.00	8.00	3	90
100	250	10.0	25	1.00	2.50	10	12
700	150	6.0	50	2.00	3.00	8	6
700	150	6.0	125	5.00	8.00	3	20
700	750	30.0	125	5.00	8.00	3	130

Note: This table is for hardening the whole gear in one operation.

The amount of power required to harden a gear is very hard to estimate. It depends on the efficiency of the coil, the amount of preheat used, and the amount of time that can be spent heating the gear without having too much distortion.

Table 6.10 shows that quite large amounts of power are required to induction-harden gears when a coil is wrapped around the whole gear. Many designs cannot be induction-hardened simply because equipment with power enough to handle then is not available.

Fine-pitch gears require very high frequency for best results, while coarse-pitch gears require low frequency. If the wrong frequency is used, the heat may be developed below the teeth (fine pitch) or only in the tooth tips (coarse pitch). Table 6.11 shows the frequencies generally recommended.

TABLE 6.11
Frequencies Generally Recommended for Induction Hardening

Module	Diametral Pitch	Frequency (Cycles/s)
0.7	32	500,000–1,000,000
2.5	10	300,000–500,000
5	5	10,000–300,000
10	2.5	6000–10,000

FIGURE 6.15 Induction-hardened test piece for case depth. This work was done with a patented submerged inductor by National Automatic Tool Co., Inc., Richmond, Indiana.

6.2.3.1 Induction Hardening by Scanning

During the 1960s and 1970s, methods were developed to induction-harden gear teeth by moving an inductor across the face width of the gear to harden either a single tooth or the space between two teeth. In this way, a large gear could be progressively hardened with a relatively small amount of power.

If a gear had 100 teeth, it would take 100 passes of the inductor across the face width to harden the whole gear. Although this is somewhat time consuming, the method was quite practical. A carburized gear might require 5 to 10 hours of furnace time, while a nitrided gear might require 50 hours of furnace time. If it took 3 hours to harden a gear by the scanning method, this was not bad. Of course, if the gear could have been induction-hardened by a coil wrapped around the gear, it might have been done in something like 5 minutes.

The scanning method offered the ability to achieve quite close control over the contour that was hardened. Figure 6.15 shows some examples of tests for establishing a desired case depth. In the illustration, the S values are rates of coil traverse, and the P values are power settings. Note the deep case at the left with S 10″ and P 4.0 and the thinner case at the right with S 17.5″ and P 5.5. These tests were done with an inductor *between* the teeth.

The scanning method will do large gears with large teeth. It will do internal gears as well as external gears. The inductor can follow a helical spiral to do helical gears. Figure 6.16 shows a machine doing a large internal gear, while Figure 6.17 shows a close-up of the mechanism that scans the teeth. Note in this view the guide pin, the inductor, the power lines, and the lines to pass coolant through the inductor.

In some equipment, the gear and the inductor are submerged in coolant while the inductor hardens the teeth. In other equipment, the gear is kept cool by jets of coolant going along behind the inductor; the inductor is not submerged. In some cases, the setup is quite flexible and can be used in a variety of ways to best suit the part being done.

FIGURE 6.16 Machine induction-hardening a large internal gear. (Courtesy of National Automatic Tool Co., Inc., Richmond, Indiana.)

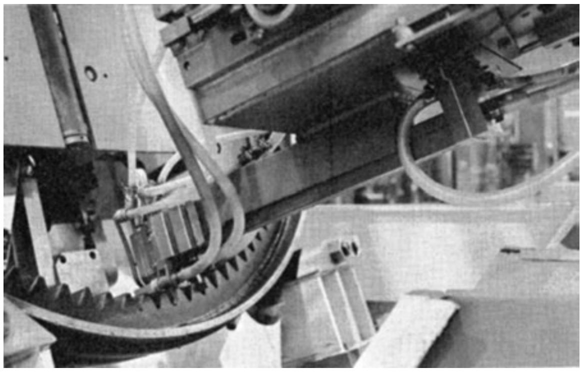


FIGURE 6.17 Close-up view of scanning mechanism on a large induction-hardening machine. Note round guide pin, inductor, electric power cables, coolant lines, and traversing ram. (Courtesy of National Automatic Tool Co., Inc., Richmond, Indiana.)

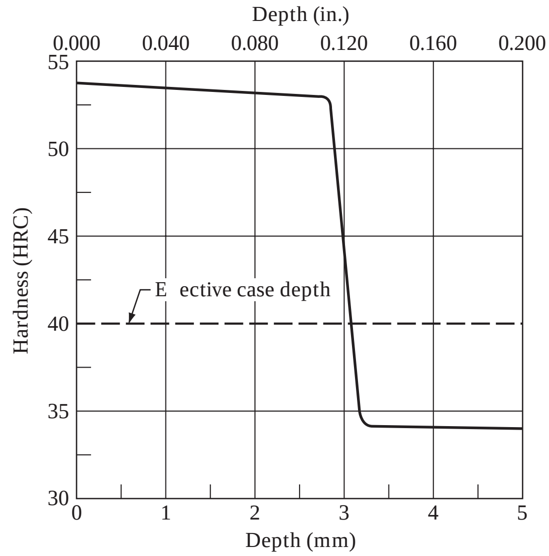


FIGURE 6.19 Example of 4340 gear induction-hardened by the scanning method.

The basics of measuring case depth on teeth hardened by the scanning method are shown in Figure 6.18. As with the carburized tooth shown in Figure 6.8, there are three places to check the case and one location for measuring core hardness.

An example of the hardness pattern that may be obtained with the scanning method is shown in Figure 6.19. Note the rather uniform hardness across the case and then the abrupt drop to core hardness. The effective case depth can be taken at 40 HRC like nitrided gears. (It would not make much difference if it were 45 HRC.)

The scanning method uses frequency values that range from 10 to 300 kHz (a kilohertz is a thousand cycles per second; 10 kHz = 10,000 cycles per second). Table 6.12 shows the nominal ranges of case depths that can be obtained for different tooth sizes and different frequency values. At a given frequency, the case depth is adjusted by the rate of scanning and the power setting. (Figure 6.15 shows test samples done at 200 kHz on 10-module teeth.)

The case depth needed with induction-hardened teeth is somewhat greater than that needed for carburized teeth. Since the transition from case to core is not gradual, the case/core interface may not be able to carry as much load as its hardness would indicate. This makes it desirable to get the interface zone deeper into the tooth, where the subsurface stresses are lower.

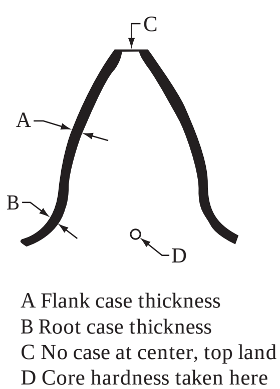


FIGURE 6.18 Induction-hardened case pattern by the scanning method of tooth heating.

6.2.3.2 Load-Carrying Capacity of Induction-Hardened Gear Teeth

The induction-hardened tooth may have about the same load-carrying capacity as a carburized tooth. If the surface hardness is up to about 58 HRC and the pattern of the case is good, a high load-carrying capacity may be achieved by getting an appropriate metallurgical structure and an appropriate residual stress pattern.

The problem, though, is that the *fast* heating and cooling times involved tend to cause considerable variation in both metallurgical structure and the residual stress pattern. The hardness and depth of the case may seem to be right for the job, but the gears may fail prematurely in service because their structure and/or residual stress pattern did not come out right. (*Right* means able to carry the load—not necessarily right from a theory of metals standpoint.)

Pitting is another problem. An induction-hardened tooth may pit, and then a tooth fracture may start at a pit. In many cases, induction-hardened gears have shown less capability to survive under moderately serious pitting than have case-carburized teeth. (It is probable that a good choice of an alloy steel and a biased heat treatment procedure can make an induction-hardened tooth able to stand as much pitting without tooth fracture as a good carburized tooth. Past problems with induction-hardened teeth fracturing after pitting were probably due to a lack of understanding of the metallurgy involved on the part of those making the gears.)

TABLE 6.12
Scanning Induction Heating Frequencies and Case Depth Ranges That Are Practical for Different Sizes of Gear Teeth

Tooth Size		Frequency (kHz)			
Module	Diametral Pitch	300	150	50	10
2.5	10	0.5–1.0 (0.02–0.04)	–	–	–
3	8	0.75–1.25 (0.03–0.05)	–	–	–
4	6	–	0.75–1.50 (0.03–0.06)	0.75–1.50 (0.03–0.06)	–
5	5	–	0.89–1.65 (0.035–0.065)	0.89–1.75 (0.035–0.070)	1.75–2.50 (0.07–0.10)
6	4	–	1.00–1.75 (0.04–0.07)	1.00–2.00 (0.04–0.08)	2.00–2.75 (0.08–0.11)
12	2	–	1.50–2.25 (0.06–0.09)	1.50–2.50 (0.06–0.10)	2.50–3.25 (0.10–0.13)
25	1	–	–	–	2.75–3.50 (0.11–0.14)

Note: The flank case depths shown are ranges that are practical for tooth size and frequency. The tolerance on case depth should be about ±15%. (For 1 mm case, use 0.85–1.15 mm.) The case in the root should be about 70% of the case on the flank. The case depths shown are in millimeters, with equivalent depth in inches shown in parentheses.

In view of the above, the builder of induction-hardened gears needs to develop each design to meet its job requirements. This development involves the following:

- Metallurgical study of sample teeth to verify that the alloy chosen and the induction-hardening cycle will give satisfactory case depth, case pattern, case hardness, and metallurgical structure
- Study of process variable to control uniformity of case from tooth to tooth, from end to end, and from top of tooth to bottom of tooth
- Control study of the induction-hardening machine to maintain its power setting, maintain inductor positioning, avoid over- and undertemperature conditions due to electrical malfunction, etc. (Of particular concern is how to handle a machine stoppage when a gear is partly done without putting a weak spot in the circumference of the gear.)
- Verification that the expected load-carrying capacity is achieved by appropriate full-load testing in the factory, and then follow-up observation of gears in service in the field to verify that the expected life and reliability is being achieved. (Of particular concern is field damage to the teeth by pitting, foreign material going through the mesh, overloads in transient operating conditions, etc.)

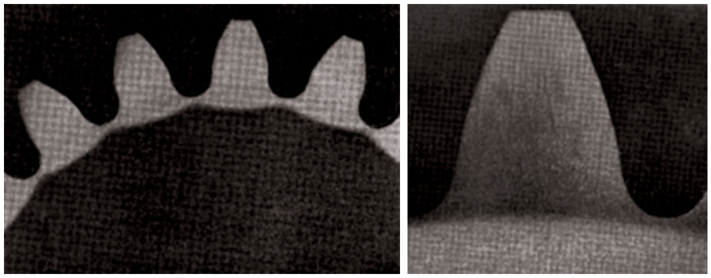
Many experienced builders are producing induction-hardened gears which are very satisfactory for their application. There have been quite a few cases in which inexperienced builders have not understood the complexity of the induction-hardening process and have built gears that have failed prematurely under load ratings that seemed reasonable. With all this in mind, it seems most prudent not to try to establish an industry load rating for induction-hardened gears. Instead, a load rating should become established for a given gearset as the builder acquires enough experience with it to check the manufacturing recipe and to acquire adequate knowledge of how the gears stand up in actual service.

6.2.4 Flame Hardening of Steel

Flame hardening is similar to induction hardening in both results obtained and kinds of steel used. It differs from induction hardening in that the heat is applied to the surface by oxyacetylene flames instead of being generated electrically in a layer extending from the surface to a small distance below the surface. In some cases it is difficult—or impossible—to get the same hardness pattern and fatigue strength by flame as can be obtained by induction hardening.

Special burners have been developed to impinge gas flames simultaneously on all tooth surfaces. Some types of flame-hardening apparatus use electronic control to turn off the gas at precisely the time when the temperature of the part just exceeds the critical temperature.

The flame-hardening process can be used quite handily either to harden the whole tooth or to harden just the working part of the tooth. Where beam strength is critical, the designer



(a) (b)

FIGURE 6.20 Sections through flame-hardened gear teeth. (a) 1.3-module (20-pitch) teeth, ×5 and (b) 4-module (6-pitch) teeth, ×5.

can usually improve the tooth strength by hardening the root fillet as well as the working part of the tooth. If wear is the only consideration, hardening the tooth to the form diameter will do the job and reduces the risk of distorting the gear blank.

Figure 6.20 shows some examples of flame-hardened teeth.

6.2.5 Combined Heat Treatments

In recent years a number of combination heat treatments have been developed. The skilled gear designer can often make use of such heat treatments to get results which are impossible with one procedure alone.

For instance, a double-cycle heat treatment of carburized gears produces a strong, long-wearing case. However, both distortion and time consumed in heat treatment may be more than is desired. In some cases it is practical to carburize, quench from the carburizing heat, and draw the gear to medium hardness. Then the teeth can be finished by shaving or light grinding and, as a last step, induction-hardened. This procedure saves furnace time, since induction heating is very rapid. Also, the distortion is appreciably reduced because the whole gear blank does not undergo a high heat and quench after the teeth are finished.

Induction hardening can be done using a furnace preheat and then a period of both high-frequency and low-frequency heating.

The carburizing and nitriding treatments may be combined by using molten slats (or gases) which liberate both available carbon and nitrogen. The heating temperature does not need to be as high as that required for carburizing, and the case formed requires less time than nitriding alone would require. The case is harder, generally, than that obtained by carburizing alone but softer than nitrided cases on steels containing aluminum. In general, this type of treatment does not achieve quite so high a fatigue strength and quite so much pitting resistance as can be obtained with straight carburizing. In automotive gear work, where the cycles at maximum load are not too great, many types of gears have been produced with a cyanide hardening process, and they appear to perform just as well as carburized gears. In aircraft work, where the cycles at maximum load are usually quite large, very little use has been made so far of gears heat-treated by cyaniding or carbo-nitriding.

The gear designer who wants to know what the best hardening process for a particular gear is should plan on experimenting with the processes just described. Things such as pitch, blank design, steel used, and application requirements all enter into why one heat treatment may work better than others in a particular job.

Hot quenching of gears and other parts may be used to minimize distortions. One such treatment is called *austempering.* It consists of quenching and holding the austenitized gear in a salt bath heated to some temperature below A_{c1},* but below the M_s temperature. A range of hardnesses between 400 and 650 HV (400 and 575 HB) can be developed, depending on the temperature that is used. An extended time in the quench is required, particularly with the more highly alloyed steels, to permit full transformation. The hardness results are similar to those obtained by a quench and medium-temperature draw, but the structures developed are quite different and generally have better toughness at a given hardness.

Another hot-quench type of treatment is *martempering.* The part is quenched in a bath which is either just above or just below the M_s temperature for a time that is just long enough for all sections of the part to cool down to the bath temperature. Then the part is removed and air-cooled to room temperature. This treatment produces full hardness. After martempering, the part should be given a conventional tempering treatment to the desired hardness. Even if no reduction in hardness is desired, tempering is needed to remove stresses and improve the structure. Austempered pieces, however, do not require further tempering.

The two treatments just described are successful only on parts with small sections or with parts of large sections made of highly alloyed steels. Both are relatively slow quenching processes.

6.2.6 Metallurgical Quality of Steel Gears

For reliable performance at the design load rating, the gears in a gear unit must have two *kinds* of quality that are suitable for the application:

- *Geometric accuracy*: This involves profile, spacing, helix, runout, finish, balance, etc.
- *Material quality*: This involves hardness, hardness pattern, grain structure, inclusions, surface defects, residual stress pattern, internal seams or voids, etc.

* A_{c1} is the critical temperature of the case.

In the gear trade, the importance of geometric accuracy has been understood ever since the pioneering work of Earle Buckingham (1931) on dynamic load effects. AGMA and DIN standards set quality classes for geometric accuracy of gears. This work is continuing. More and better geometric accuracy standards for gear teeth may be expected. (See Section 10.4.1 for a discussion of gear-tooth geometric accuracy.)

In the field of gear materials, the quality aspect of the material has not received the attention it deserves. The AGMA aerospace gear standard was the first to recognize gear quality with a definition of grade 1 and grade 2 materials. This standard was first issued in 1955.

The AGMA vehicle gear standard followed this trend in 1976 with definitions of grade 1 and grade 2 materials for vehicle gears.

In the private sector, several major companies building aerospace or turbine gearing for land use have rather detailed specifications for steel gears showing more than one grade of material and several quality control things to check the gear materials. Gears are passed or rejected for material quality just as they are passed or rejected for their geometric quality.

As time goes on, much more work on gear material quality can be expected. Trade standards that cover at least two grades of material for aerospace, vehicle, turbine, industrial, and mill gear fields of usage are needed. The present definitions used need to be expanded to cover more things and to more precisely define what is *acceptable* and *unacceptable* in each area of gear practice.

The possible gradations in gear-material quality have been thought of in these terms:

Grade 0	*Ordinary quality*
	No gross defects, but no close control of quality items
Grade 1	*Good quality*
	Modest level of control on the most important quality items (the normal practice of experienced gear and metals people doing good work)
Grade 2	*Premium quality*
	Close control on all essential critical items (some extra material expense to achieve better load-carrying capacity and/or improved reliability)
Grade 3	*Superquality*
	Essentially absolute control of all critical items (much extra expense to achieve the ultimate; has been used in space vehicle work; the need for this level of quality will be rather rare)

6.2.6.1 Quality Items for Carburized Steel Gears

The principal items to control for material quality of carburized gears are as follows:

- Surface hardness
- Core hardness
- Case structure
- Core structure
- Steel cleanness
- Surface condition, flanks
- Surface condition, root fillet
- Grain size
- Nonuniformity in hardness or structure

The appropriate controls on these items will not be the same for aerospace gears and vehicle gears. For instance, some retained austenite is generally permissible (or even desired) in vehicle gears. In long-life aerospace or turbine gears, almost no retained austenite is permitted. Cleanness is another item. Certain kinds of dirt are tolerable in vehicle gears when the very high stress is applied for less than 10^7 cycles. Aerospace gears with a heavy design load for over 10^9 cycles are often made with vacuum-arc remelt (VAR) steel to remove essentially all the dirt particles in the steel.

Table 6.13 shows some typical data for long-life, high-speed gears, illustrating the possible difference between grade 1 and grade 2 gears. Figure 6.21 shows some metallurgical illustrations of grade 1– and grade 2–permissible structures.

As said earlier, metallurgical quality grades are just being established in the gear industry. Table 6.13 and Figure 6.21 (and Table 6.14 and Figure 6.22) should be considered as examples of what is involved. Much study and work are needed to fully establish material grades in several important fields of gear work.

6.2.6.2 Quality Items for Nitrided Gears

The same list of items for carburized gears applies to nitrided gears. The nitrided gear tends to have a white layer and may have internal cracks as a result of improper nitriding. These things make the actual control of nitrided gear quality somewhat different from that of carburized gears.

Table 6.14 and Figure 6.22 show concepts of quality grade considerations for nitrided gears.

6.2.6.3 Procedure to Get Grade 2 Quality

The premium grade 2 quality requires extra effort and expense. The principal steps that are generally needed are as follows:

1. Choose a steel with enough alloy in it to respond *well* in the heat-treating procedures planned.
2. Check incoming raw material for cleanness. (A certification from the supplier is usually not enough. Take samples and run laboratory tests.)
3. Prove out the heat-treating cycle planned by running one or more test gears. Set time, temperature, location in the furnace, etc., so that sure results can be obtained on the gears.
4. Use a portion of a toothed gear (or a whole gear) as a heat treatment sample when a batch of gears is done.
5. Do laboratory work on the furnace samples after carburizing or nitriding.
6. Inspect finished gears for cracks and possible improper surface condition by nondestructive methods.

TABLE 6.13
General Comparison of Carburized Steel Gear Metallurgical Quality (for Turbine Gears but Not for Vehicle Gears)

Quality Item	Grade 1	Grade 2
Metallurgy of case	Tempered martensite; retained austenite 20% maximum; minor carbide network permitted; some transformation products permitted; acceptable and unacceptable conditions defined by ×1000 microphotos	Tempered martensite; retained austenite 10% maximum; no carbide networks; almost no transformation products such as bainite, pearlite, proeutectoid ferrite, or cementite permitted; acceptable and unacceptable conditions defined by ×1000 microphotos
Metallurgy of core (at tooth root diameter)	Low-carbon martensite; some transformation products permitted; acceptable and unacceptable conditions defined by ×250 microphotos	Low-carbon martensite; almost no transformation products permitted; acceptable and unacceptable conditions defined by ×250 microphotos
Material cleanness	Air-melt (aircraft quality); AMS2301E	VAR or vacuum slag process; AMS2300
Magnetic indications of cracks or flaws on gear teeth	No indications parallel to axis; not more than four surface indications, nonparallel to axis, per tooth, maximum length 4 mm; not more than six subsurface indications, nonparallel to axis, maximum length 5 mm	No indications parallel to axis; not more than two surface indications, nonparallel to axis, per tooth, maximum length 2 mm; not more than three subsurface indications, nonparallel to axis, maximum length 3 mm
Forging grain flow	Forgings required for parts over 200 mm (8 in.) diameter; close grain definition on forging drawing	Forgings required for part over 125 mm (5 in.) in diameter; very close grain flow requirements on forging drawing
Surface defects after carburizing	Surface oxidation not to exceed 0.012 mm (0.0005 in.); decarburization effects not to exceed 60 HKN (3 HRC)	Surface oxidation not to exceed 0.0008 mm (0.0003 in.); decarburization effects not to exceed 40 HKN (2 HRC)
Minimum hardness of case at surface		
Flank (A)	685 HKN (57.8 HRC)	725 HKN (59.6 HRC)
Root (B)	660 HKN (56.5 HRC)	695 HKN (58 HRC)
Minimum hardness of core at root diameter (D)	30 HRC	34 HRC

Note: This table shows a partial amount of the magnetic particle specifications that are needed. Details of acceptable and unacceptable decarburization can be shown by examples of plots of case hardness versus case depth. Maximum case hardness is generally 800 HKN (63 HRC), but it may be specified lower to ensure that the draw treatment after final hardening lowers the hardness slightly and increases the impact strength. The maximum core hardness is generally HRC 41, but it may be made slightly lower to better control the residual stress pattern.

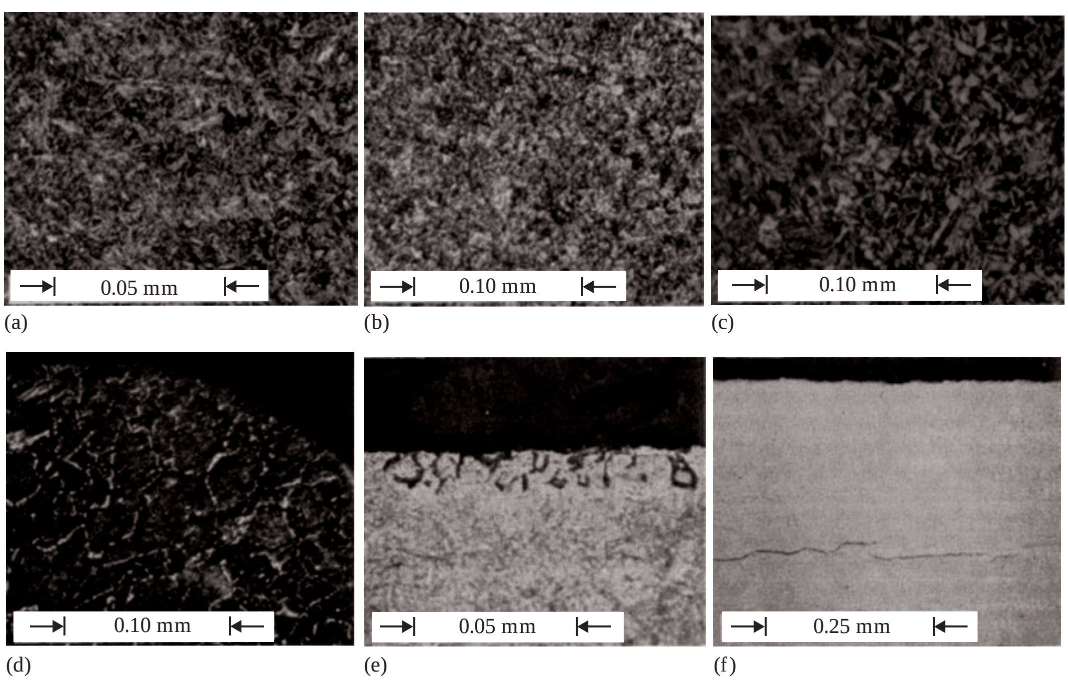


FIGURE 6.21 Metallurgical examples of carburized gears. (a) Case structure; acceptable for grade 2 (and grade 1). (b) Core structure; acceptable for grade 2. (c) Core structure; not acceptable for grade 1. (d) Carbide network at tooth tip; acceptable for grade 1 but not for grade 2. (e) Intergranular oxide layer; acceptable for grade 1 but not for grade 2. (f) Subsurface crack; not acceptable for any grade.

TABLE 6.14
General Comparison of Nitrided Steel Gear Metallurgical Quality (for Small Gears Heavily Loaded or Medium–Large Gears with Low Enough Loading to Permit Nitriding)

Quality Item	Grade 1	Grade 2
Metallurgy of case	White layer permitted up to 0.02 mm (0.0008 in.) maximum; complete grain boundary network acceptable; no microcracks, soft spots, or gross nitrogen penetration areas permitted; acceptable and unacceptable conditions defined in ×500 microphotos	No white layer permitted on working surfaces of the tooth, other surfaces 0.02 mm (0.0008 in.) maximum; no microcracks, soft spots, or heavy nitrogen penetration along grain boundaries permitted; continuous iron nitride network not permitted; acceptable and unacceptable conditions defined in ×500 microphotos
Metallurgy of core (at tooth root diameter)	Medium-carbon tempered martensite; some transformation products permitted; acceptable and unacceptable conditions defined in ×250 microphotos	Medium-carbon tempered martensite; almost no transformation products (bainite and pearlite) or ferrite permitted; acceptable and unacceptable conditions defined in ×250 microphotos
Material cleanness	Air-melt (turbine quality)	Air-melt (aircraft quality); AMS2301E
Minimum hardness of case at surface (A, B):		
Nitralloy 135, 135 M, and N	730 HKN (60 HRC)	740 HKN (60.5 HRC)
31 Cr Mo V 9	700 HKN (58.5 HRC)	725 HKN (59.6 HRC)
AISI 4140, 4340	520 HKN (48.6 HRC)	545 HKN (50.2 HRC)
Minimum hardness of core at root (D):		
Nitralloy 135, 135 M	31 HRC	34 HRC
Nitralloy N	37 HRC	40 HRC
31 Cr Mo V 9	28 HRC	32 HRC
AISI 4140, 4340	30 HRC	34 HRC

Note: This table shows a partial amount of the magnetic particle specifications that are needed. Details of acceptable and unacceptable decarburization can be shown by examples of plots of case hardness versus case depth. Maximum case hardness is generally 800 HKN (63 HRC), but it may be specified lower to ensure that the draw treatment after final hardening lowers the hardness slightly and increases the impact strength. The maximum core hardness is generally HRC 41, but it may be made slightly lower to better control the residual stress pattern.

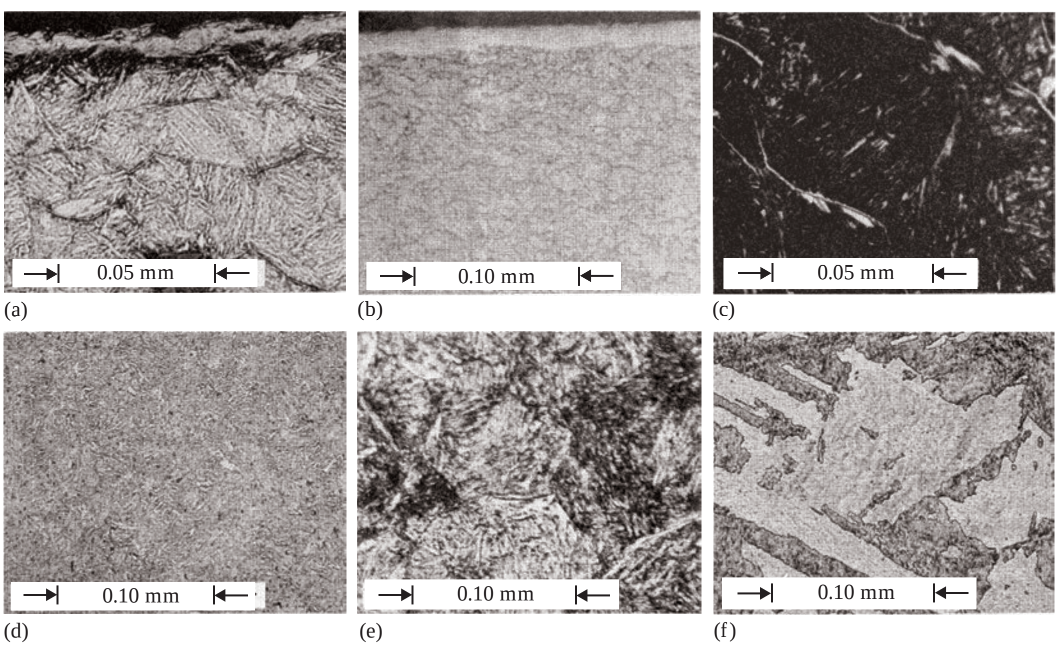


FIGURE 6.22 Metallurgical examples of nitrided gears. (a) Case structure, thin white layer; acceptable for grade 2 by honing off white layer. (b) Case structure, thicker white layer but acceptable for grade 1 without removal. (c) Continuous grain boundary nitrides; not acceptable for grade 1. (d) 4340 core structure; acceptable for grade 2 (or grade 1). (e) Nitralloy core structure; acceptable for grade 1 but not for grade 2. (f) 4340 core structure; not acceptable for grade 1 or 2.

6.3 CAST IRONS FOR GEARS

Cast iron has long been used as a gear material. It has special merit, and some disadvantages, derived from its inherent structure and resulting properties. Gray cast irons, especially the alloyed types commonly used for gears, range in strength up to that of low-hardness steel. A newly developed cast iron with spheroidal graphite extends well into the strength range of heat-treated steels.

Cast irons differ from steel in both composition and structure. Their carbon content usually ranges from 2.5% to 4%, whereas gear steels contain substantially less than 1% carbon. The resulting structural differences, however, are the most important. In cast irons as a class, the carbon is predominantly in the free, or graphitic, state, with only a small proportion in the combined, or pearlitic, form. Steels normally contain only the combined form of carbon.

It is the amount, the form, and the size of the graphite in cast iron which are responsible for its characteristic combination of properties. From the standpoint of mechanical properties, graphite, being weak in itself, reduces ductility, strength, elasticity, and impact resistance; but, on the other hand, it increases the ability of cast iron to damp out vibrations and noise. The graphite also helps cast-iron gear teeth to operate with scanty lubrication.

6.3.1 Gray Cast Iron

Gray cast irons, either plain or alloyed, are characterized by graphite in the ake form. These akes act as a source of stress concentration, breaking up the continuity of the steel-like matrix. Figure 6.23 shows the characteristic structure of gray cast iron in comparison with those of other cast irons.

As cast iron solidi es and cools in the mold, metallurgical changes occur that are similar in many respects to those discussed in the heat treatment of steel. In fact, the mold cooling conditions should be considered a heat treatment—the only one that many gray irons ever receive. The size and the distribution of ake graphite are controlled by the relationship between composition and section size, and by several phases of foundry practice. The matrix behaves like steel does; therefore, the composition de nes the extent of hardening under given cooling conditions. Light sections that cool rapidly are inclined to freeze white, with the carbon in the form of iron carbide (cementite). Higher carbon and silicon inoculation, and nickel and copper additions, reduce the chilling power and produce graphitic and machinable structures in light sections. High carbon and silicon alone, however, are likely to produce large akes and weak structures in heavy sections, and so castings with heavy sections are usually made from controlled and inoculated nickel irons.

A pearlitic matrix is highly desirable for a high-strength, wear-resistant iron; and the usual alloying elements—nickel, molybdenum, and copper—are used to increase the ability of gray-iron matrix to harden to this structure either in the mold or by subsequent heat treatment.

Gray iron has a low modulus of elasticity, varying with the strength level, that ranges from one-third to two-thirds that of steel. For gearing this is an advantage, since it tends to reduce the compressive stress (Hertz stress) developed by a given tooth load. It also tends to reduce slightly the beam strength, because the teeth bend more and there is more sharing of load. Gray iron is so brittle, though, that it should not be considered in gears subject to serve shock.

The AGMA standard, the *Gear Materials Manual*, gives a wide range of data on steel, cast irons, and certain nonferrous

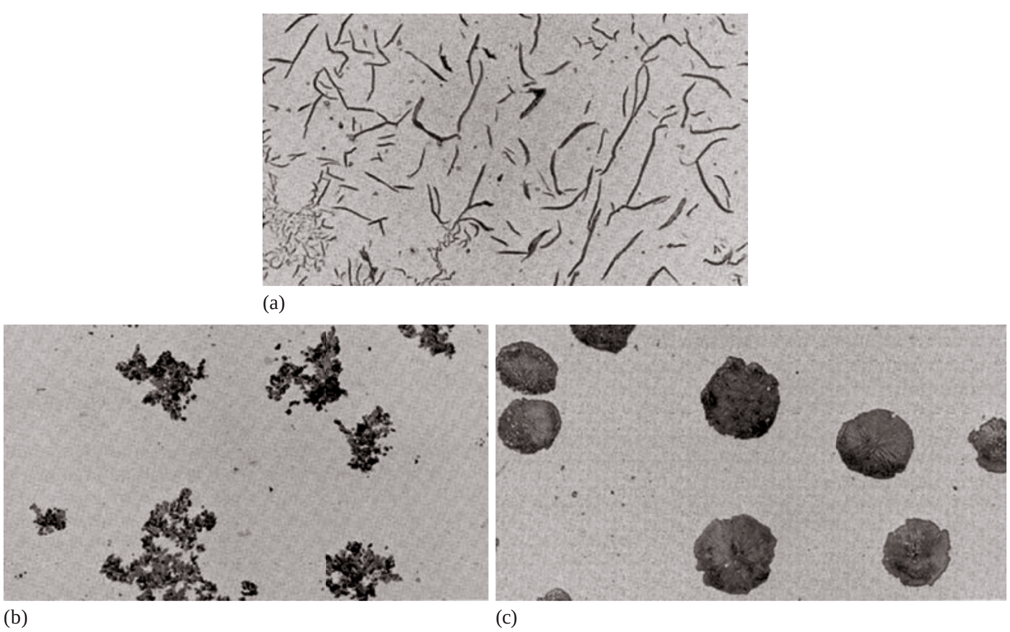


FIGURE 6.23 Comparison of structures (all unetched, ×250). Dark particles are graphite. (a) Gray iron; (b) malleable iron; and (c) ductile iron.

TABLE 6.15
Classes of Gray Cast Iron and Approximate Data for Hardness and Ultimate Tensile Strength

Class No.	Brinell Hardness, Min.	Tensile Strength, Min. (psi)
20	155	20,000
30	185	30,000
35	205	35,000
40	220	40,000
50	250	50,000
60	285	60,000

materials. This standard is in process of revision and, therefore, can be expected to soon have even more and better data than it now has. Gear designers should keep abreast of this work and use the many items of information about trade practices, test bars, normal acceptance limits, etc. In Tables 6.15 through 6.17 some general data have been given as a guide, but no attempt will be made to cover the whole range of data now available (or soon to become to available).

Table 6.15 shows the classes of gray cast iron and their data for minimum hardness and minimum tensile strength.

Alloy cast irons are needed as size increases and as the part complexity in section thickness increases. Table 6.16 shows some typical alloy cast iron compositions and the expected modulus of elasticity.

The nominal properties of malleable cast iron are given in Table 6.17. This kind of cast iron will take much more shock than gray cast iron. Note the elongation values.

6.3.2 Ductile Iron

Up until 1948 malleable iron was the only type of cast iron available that would provide a useful degree of toughness or ductility. In malleable iron, the free graphite forms from white iron during annealing in clusters of tiny akes. Malleable iron, compared with gray iron, has ductility and better yield and elastic properties. Malleable iron has not been used widely for gears, however, because of its poor wear resistance and limitation to parts of relative light section size.

Production of iron with true spheroidal graphite in the as-cast condition was announced in 1948. The material is variously referred to as *ductile iron*, *spheroidal-graphite iron*, or *nodular iron*. Figure 6.23, which shows ake graphite in gray

TABLE 6.16
Data on Alloy Cast Irons

	Composite Range			
Class No.	Nickel	Molybdenum	Chromium	Modulus of Elasticity, Min. (psi)
30	0.5–1.0	Optimal	0.2–0.4	14×10^6
40	1.0–2.0	Optimal	0.3–0.5	16×10^6
50	1.5–2.0	0.3–0.4	Optimal	18×10^6
60	2.0–2.5	0.4–0.5	0.20 maximum	20×10^6
70	2.5–3.0	0.5–0.6	0.20 maximum	22×10^6
80	3.0–3.5	0.6–0.7	0.20 maximum	24×10^6

TABLE 6.17
Mechanical Properties of Malleable Iron

Commercial Designation	Typical Brinell Hardness Range	Tensile Strength, Min. (psi)	Yield Strength, Min. (psi)	Elongation in 2 in., Min. (%)
40010	160–210	60,000	40,000	10.0
43010	160–210	60,000	43,000	10.0
45007	165–220	65,000	45,000	7.0
48005	180–230	70,000	48,000	5.0
50005	180–230	70,000	50,000	5.0
53007	195–240	75,000	53,000	4.0
60003	195–245	80,000	60,000	3.0
70003	205–265	85,000	70,000	3.0
80002	240–270	95,000	80,000	2.0
90001	265–305	105,000	90,000	1.0

TABLE 6.18
Typical Properties of the Different Grades of Ductile Iron

Commercial Designation	Recommended Heat Treatment	Brinell Hardness Range	Tensile Strength, Min. (psi)	Yield Strength, Min. (psi)	Elongation in 2 in., Min. (psi)
60-40-18	Annealed	140–180	60,000	40,000	18.0
65-45-12	As cast or annealed	150–200	65,000	45,000	12.0
80-55-06	Quenched and tempered	180–250	80,000	55,000	6.0
100-70-03	Quenched and tempered	230–285	100,000	70,000	3.0
120-90-02	Quenched and tempered	270–330	120,000	90,000	2.0

iron and spheroidal graphite in ductile iron, clearly demonstrates that the term *nodular iron* is not a completely accurate term to describe the new material.

The spheroidal form of graphite, furthermore, provides a material with a combination of properties different from what is possible with malleable iron. Ductile iron may be made by treating a low-phosphorus gray-iron base composition with a magnesium or equivalent additive. It has tensile strength ranging from 60,000 to 180,000 psi, yield strength from 45,000 to about 150,000, and elongation of up to 25%. The modulus of elasticity is about 24×10^6 psi. The standard grades listed are shown in Table 6.18.

Ductile iron can be made with a high carbon content and still have low carbide content. It is, therefore, not surprising that both its machinability and its wear resistance are excellent. The combination of (a) strength, (b) toughness, (c) wear and fatigue resistance, and (d) susceptibility to heat treatment offered by ductile iron has been of considerable interest to gear designers. This material has been used to advantage in applications formerly filled by carbon and alloy through-hardened steels, flame-hardened steel, gray cast iron, and gear bronze.

6.3.3 Sintered Iron

The art of making metal parts from powdered metals, or sponge metal, is not new. The ancient Egyptians forged tools from sponge iron. Platinum wire was made in the 19th century from sponge platinum powder.

The art of powder metallurgy is applied to the making of structural parts, in direct connection with other methods of manufacture. Improvements in the process have made it possible to make many parts by this method, which can compete successfully on a cost–strength basis with other methods of fabrication. Gears make from pressed and sintered iron powder have been made by the millions for such low-cost pieces of machinery as washing machines and food mixtures.

The usual base material for sintered gears is iron or brass metal powder, but when used alone these metals have inherently low strength. They are suitable only for light-duty gearing, for example, in timing devices, small appliances, or toys. Copper impregnating pressed-iron gears, though, can considerably increase their strength. The impregnation is accomplished by putting a small piece of copper on top of a sintered-iron gear and then heating the gear in an atmosphere-controlled furnace so that molten copper soaks through the porous sintered gear. The copper fills the voids and bonds the material into a strong structure.

Generally accepted trade standards for the composition of powder-metal products have not yet been established. Table 6.19 shows the composition and strength of sintered metals commonly used for gears.

Because of the limitations of die design, only spur gears have been made in large quantities by the powder-metal process. Helical gears can be made if the helix is not too large an angle. Blanks for any kind of gear can be made by sintering if the teeth are cut after sintering. There is not much incentive to do this, though, as the biggest cost saving in the sintering process comes from finishing the part by sintering and avoiding the cost of gear tooth cutting.

Sintered gears can carry moderately heavy loads at fairly fast speeds. They resist wear very well, and they are easy to lubricate.

TABLE 6.19
Typical Sintered Metals Used for Gears

	Tensile Strength (psi)		% Elongation	
Type	As Sintered	Copper-Impregnated	As Sintered	Copper-Impregnated
Alloy A (iron, 7.5% copper)	35,000	70,000	0.8	1.0
Alloy B (iron, 1.0% copper)	35,000	85,000	0.8	1.0
Alloy C (iron)	20,000	60,000	15	20
Alloy D (80% iron, 20% copper)	30,000	–	20	–
High-strength (1% carbon, 17% copper) balance iron	–	85,000	–	1.0

6.4 NONFERROUS GEAR METALS

The metals copper, zinc, tin, aluminum, and manganese are used in various combinations as gear materials. The most important, perhaps, is the alloy called *bronze*.

Bronzes have become a widely used family of materials for gears, largely because of their ability to withstand heavy sliding loads. Like cast iron, bronzes are easy to cast into complex shapes, while certain types are available in wrought forms. Because bronze casting as a rule will be more expensive than cast iron, the designer will select the material primarily for its ability to meet special service requirements. Most gear designers are of the opinion that bronze will withstand high-speed rubbing—such as in a worm gearset—better than cast iron does. When the best tin bronzes are compared with the best gray cast irons, the bronzes indeed seem to be superior as a "bearing" material, although individual tests have been recorded in which certain bronzes were found inferior to some cast irons. Even in low-speed applications, bronze may be the most desirable material. Although the tables of hobbing machines do not turn very fast, experience indicates that higher table speeds can be used without danger of scoring when the hobbers have bronze index wheels than when they have cast-iron wheels.

The value of bronze as a bearing material seems to depend on the fact that the microstructure of the material is made up of two phases, one hard and one soft. It is believed that, when a bronze gear or bearing wears into good contact, the hard constituent of the microstructure has become aligned to fit the rubbing surface and to carry the bearing load. The softer constituent has been soft enough to allow this aligning process to occur.

A number of special alloys are used for die-casting gears. In this type of work, a low–melting point material with good casting characteristics is essential. Zinc-base alloys such as the alloy SAE 903 are very popular. This alloy contains about 4% aluminum and about 0.04% magnesium, with the remainder zinc. It melts at about 700°F and has a tensile strength of around 40,000 psi. Aluminum-base die castings have about the same tensile strength, but they weigh only about 40% as much as the zinc castings. Their melting point is around 1100°F. A good composition for gear work is the aluminum alloy with 10% silicon and 0.5% magnesium.

6.4.1 Kinds of Bronze

Like the terms *steel* and *cast iron*, *bronze* is really the name of a family of materials that may vary over a wide range of composition and properties. The "bronze" family is an alloy of copper and tin, compared with "brass," which is an alloy of copper and zinc. In practice, however, certain bronzes contain both tin and zinc, and aluminum and manganese bronzes, for example, may contain very little or no tin. It might have been less confusing if these had been called brasses.

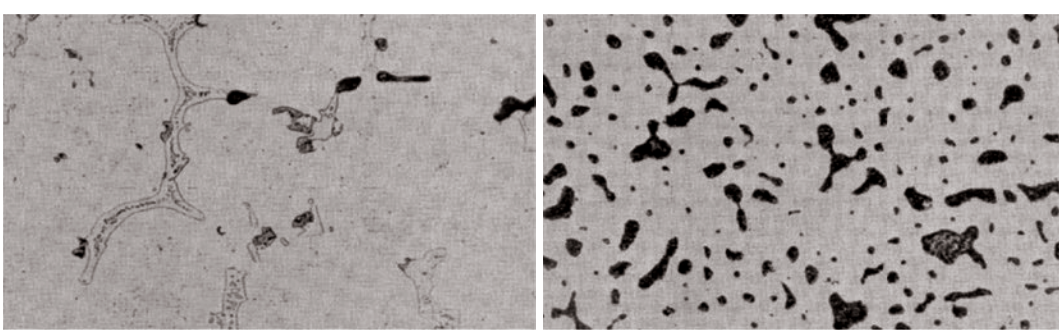
The basic bronze is an alloy of 90% copper and 10% tin that exhibits the desired two-phase structure. The best strength and bearing properties are developed if the hard and soft constituents are finely and intimately dispersed. This is usually accomplished by carefully chilling the casting. See Figure 6.24a for the "delta" constituent.

Zinc is added to copper–tin bronze to increase strength, but with some sacrifice in bearing properties. Nickel also increases strength and has a further beneficial effect on hardness and uniformity of the structure.

Lead added to bronze does not combine with the base metal but remains distributed throughout the casting to act as a solid lubricant, much as graphite does in cast iron. See Figure 6.24b for lead particles in bronze.

Lead is weak; therefore, it softens and weakens the bronze but improves machinability and allows quicker wearing in with a mating part. If bronze is not melted with the proper techniques, the tin may oxidize, and very hard crystals of tin may be formed. These may be as destructive to the bearing surface as emery added to the lubricant! Proper deoxidation will prevent this danger. The tin bronzes deoxidized with phosphor are usually the best for gear applications. The term *phosphor bronze* means literally a bronze deoxidized with phosphor.

Aluminum bronzes are more complex than the copper–tin grades and may not have such good bearing properties. These alloys have a high strength as cast, however, and the strength can be increased by heat treatment. Aluminum bronzes have



(a) (b)

FIGURE 6.24 Two typical tin-base worm-gear bronzes (both ×250). (a) Etched phosphor bronze; light islands are the delta constituents, the hard particles in the structure. (b) Unetched lead bronze.

been used in many gear applications where sliding velocity is not high.

Manganese bronzes have high zinc content along with smaller amounts of manganese, iron, and aluminum. Their strength far exceeds that of the tin bronzes. Super–manganese bronze has a tensile strength above 100,000 psi. With steel worms of 300 or more Brinell hardness, it makes about the strongest worm-gear combination that can be obtained for slow-speed applications, outside of using steel on steel.

Manganese bronzes are also available as forgings or as cold-drawn or extruded bars. Wrought manganese bronze is lower in alloy content and strength than super–manganese bronze, but has surprisingly good bearing properties. Some worm-gear applications with rubbing speeds of up to 50 m/s (10,000 fpm) have been handled with wrought manganese bronze. It is also suitable for many low-speed, heavy-load applications.

Silicon–alloy bronzes are available in both cast and wrought forms. Silicon increases strength only moderately in the cast grades, while the wrought forms can be further strengthened and hardened by cold drawing. There has been only limited use of silicon bronze for gears.

Table 6.20 shows some of the nominal compositions, tensile properties, and uses of the several kinds of bronze. It should be understood that there are a host of other bronzes which differ somewhat in composition from the typical kinds shown in the table. The strength and hardness values shown are *average* values. There is a considerable spread in properties for each composition, depending on the size of the part, the rate of cooling of the casting, and the quality of the bronze.

6.4.2 Standard Gear Bronzes

At present, three kinds of copper–tin bronzes are speci ed as standard materials by AGMA for use in worm-gear applications. The AGMA standard shows material for cast-bronze gear blanks. Besides copper–tin bronzes, aluminum bronzes and manganese bronzes are shown. The compositions are not given, but expected physical properties are.

Many organizations other than AGMA have set up standards for bronze. The situation is somewhat confusing, because there are no standards that are accepted by all bronze users. Frequently, contracts will specify that bronze parts are to be made of some kind of bronze that may not be appropriate for the kind of gearing that is required. In such cases, the gear designer should obtain approval to substitute an appropriate gear bronze. Some of the organizations which have set up bronze standards are the U.S. Navy Department, the Naval Aircraft Factory, the U.S. Air Corps, the Society of Automotive Engineers, and the American Society of Testing Materials.

TABLE 6.20
Types of Gear Bronzes and Typical Examples of Each Type

Type	Composition (%)	Strength (psi)	HB (500 kg)	Uses
Bronze (zinc-deoxidized)	Cu 88, Sn 10, Zn 2	ult. 46,000 y.s. 19,000 e.l. 12,000	65	Spur, bevel, worm-gears
Phosphor bronze (chill-cast)	Cu 88, Sn 10, Pb 0.25	ult. 50,000 y.s. 22,000	85	Medium-speed worm-gears
Nickel–phosphor bronze (chill-cast)	Cu 88, Sn 10.5, Ni 1.5, Pb 0.2	ult. 55,000 y.s. 28,000	90	Medium-speed worm-gears
Lead phosphor bronze (sand-cast)	Cu 87.5, Sn 11, Pb 1.5	ult. 50,000 y.s. 22,000	75	High-speed worm-gears
Aluminum bronze (sand-cast)	Cu 89, Al 10, Fe 1	ult. 65,000 y.s. 27,000 e.l. 20,000	120	Spur, bevel, low-speed worm gears
Aluminum bronze	Same as above but heat-treated	ult. 95,000 y.s. 60,000 e.l. 50,000	210	Heavy-duty low-speed gears
Super–manganese bronze (sand-cast)	Cu 64, Zn 23, Fe 2.75, Mn 3.75, Al 6.75	ult. 110,000 y.s. 70,000 e.l. 52,000	236	Heavy-duty low-speed gears
Manganese bronze (forged)	Cu 58, Zn 37.5, Al 1.5, Mn 2.5, Si 0.5	ult. 75,000 y.s. 40,000	135	Moderate-load gears; small high-speed gears
Silicon bronze (sand-cast)	Cu 95, Si 4, Mn 1	ult. 45,000 y.s. 20,000	80	Low-speed gears or moderate load

Note: e.l.: elastic limit; ult.: ultimate strength; y.s.: yield strength.

6.5 NONMETALLIC GEARS

In early times, gears were often made of wood. At present, few wooden gears are made (see Appendix A). A large quantity of "nonmetallic" gears is used, however. The modern use of nonmetallic gears goes back to the early 1900s, when John Miller conceived the idea of making a gear out of textile fibers held in a state of compression. He patented his idea in 1913. His original gears were made by taking layers of canvas cloth or nonwoven fabric and pressing them between steel plates. The steel plates were bolted together to hold the canvas in a state of compression. Later on, fibers were used instead of cloth, and the gear blank was impregnated with oil. The gear was sold under the trade name Fabroil.

For several years Fabroil, which was made of textile fibers, was the only type of nonmetallic gear in general use. Then another type of nonmetallic gear, made with phenolic resin, appeared on the market. Phenolic resins—such as Bakelite, invented by Dr. Bakeland—were adopted as a means of holding textile fibers in compression. In these gears, layers of cloth were bonded together with a phenol formaldehyde resin binder. After processing, the gear blank becomes a solid structure which did not require metal shrouds and steel studs to hold it together, as Fabroil did.

The cotton-phenolic type of gear belongs to a general class of material called *thermosetting laminates*. In addition to the laminates, several other plastic materials are used for gears. Bakelite and similar plastics are used to injection-mold complete toothed gears. These materials do not have the strength of the laminates, but they are inexpensive and easy to manufacture, and so suitable when large quantities of small, light-duty gears are needed.

Later, several remarkable new plastics have been appeared on the market. Nylon, for instance, had high strength for a plastic and possesses some good gear properties. *Delrin* and several other plastic gears are in common use. (See Table 6.21 for general families of nonmetallic gears.)

6.5.1 Thermosetting Laminates

A wide variety of sheet materials, such as paper, asbestos, cotton fabric or mat, wood veneer, nylon fabric, and glass fabric, may be used in making laminates. The binders may be phenolic resins, melamine resin, or silicones. The laminates used for gears are generally of cotton fabrics, although paper is used occasionally.

Laminates are made by coating the sheet material with the liquid binder. After drying, the sheets are cut, stacked between metal plates, and bounded to form a board under a pressure of 1000 to 2500 psi at a temperature between 270°F and 350°F. Such a board will show directional properties, because the components differ in strength from one direction to another.

The teeth of gears cut from these prelaminated boards will differ in strength, depending on the position of the teeth with respect to the grain of the board. When the highest strength is needed, it is possible to prepare special gear blanks with the fibers for the laminate running in all possible directions. Then all teeth will have equal strength.

The laminates are sold under a variety of trademarks. Compositions and properties vary considerably. Table 6.21 shows typical properties for several kinds of nonmetallic gears, including phenolic laminates.

Phenolic-laminated material has several characteristics which make it attractive for gearing. Being nonmetallic, it runs quietly even when meshed with a steel pinion. Nonmetallic gears show very little tendency to vibrate or respond to vibrations.

Nonmetallic gears weigh only about one-fifth to one-sixth as much as steel gears of the same size. Even though their strength is only from one-tenth to one-thirteenth that of steel, it is possible, in some applications, to have nonmetallic gears carry about the same load as a cast-iron gear or low-hardness steel gear of the same size. If both the steel gear and the nonmetallic gear are made to commercial accuracy limits, the effect of tooth errors on the steel gearset is much greater because the teeth deform less under load, and so small

TABLE 6.21
Physical Properties of Nonmetallic Gears

Property	Polycarbonate	Polyamide	Acetal	Phenolic Fabric, LE	
				Crosswise	Lengthwise
Tensile strength ($\times 10^3$ psi)	9–10.5	8.5–11	10	9.5	13.5
Flexural strength ($\times 10^3$ psi)	11–13	14.6	14	13.5	15
Elongation (%)	60–100	60–300	15–75	–	–
Impact strength (ft lb/in.)	12–16	0.9–2.0	1.4–2.3	1.00	1.25
Water absorption (%/24 h)	0.3	1.5	0.4	See Note 2	
Coefficient of thermal linear expansion (in./in./°F)	3.9	5.5	4.5	1.1	–
Heat resistance (continuous) (°F)	250–275	250	–	250	–
Representative trade names	Lexan	Nylon, Zytel	Derlin	Phenolite, Textolite	

Source: Plastic Division, General Electric Company, Pittsfield, Massachusetts.

Note: 1. The tabulated data are average values and should not be used for specifications; 2. The water absorption percentages for phenolic fabric are as follows: 1/8″, 1.3; 1/4″, 0.95; 1/2″, 0.70; and 1″, up to 0.55.

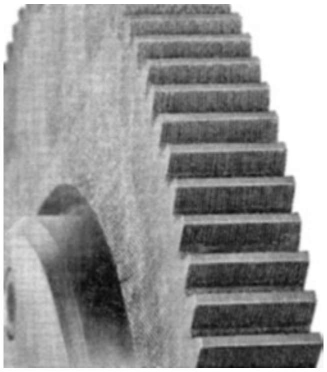


FIGURE 6.25 Phenolic laminated gear with a steel hub, after testing at a pitch-line velocity of 40 m/s (8000 fpm). (Courtesy of General Electric Co., Lynn, Massachusetts.)

errors create severe dynamic overloads in the steel gear teeth. The nonmetallic gear deforms about 30 times more than steel. This ensures good sharing of load between teeth and a low Hertz stress as a result of the wide contact band.

Laminated nonmetallic gears have been run together in some applications with only water used as a lubricant. A steel pinion and a laminated gear will often run with less lubrication than a pair of steel gears.

In designing nonmetallic gears, the relatively large tooth de ection creates a problem. The driven gear—because of de ection—has a tendency to gouge the driver at the *rst point of contact*. If the driver is a steel or cast-iron gear, this problem is not serious because the nonmetallic driven gear does not have enough hardness to gouge the hard metal. If the driven gear is steel, the gouging tendency can be controlled fairly well by making the driver with a very long addendum and the driven gear with little or no addendum.

For the highest load capacity, nonmetallic gears are mated with steel or cast-iron gears. The best wear resistance is obtained when the metal gear of the set has a hardness of 300 Brinell (or more).

In selecting a phenolic laminate, the pitch should be considered. When the teeth are 1.5 module (16 pitch) or coarser, a 15 oz/yd^2 tightly woven duck is about right. Teeth of 1 module (24 pitch) require a fabric base of about 6.5 oz duck. Finer-pitch gears use about 3 oz ne cambric.

Laminated nonmetallic gears have been used in a wide variety of applications. A few typical applications are air compressors, automotive timing gears, shoemaking machinery, electric clocks, and household appliances, bottling machinery, and calculating machinery. When mated with a good steel gear, sliding velocities up to about 15 m/s (3000 fpm) can be handled. Figure 6.25 shows a gear after testing at 40 m/s (8000 fpm).

6.5.2 Nylon Gears

One of the newer gear materials is nylon. This versatile plastic material has surprisingly high strength for a molded plastic. It has good bearing properties and considerable toughness.

Nylon may be injection-molded to form a complete toothed gear, or it may be molded in the form of rods or sheets from which gears may be machined. The best accuracy is obtained when the teeth are cut. The lowest cost, of course, is obtained when the complete part is molded.

A commonly used nylon for gears is the DuPont material FM-10001. Some of the properties of this material are shown in Table 6.22.

The values in Table 6.22 indicate that nylon has almost as much strength as the best laminates. Its weight is slightly less than that of the laminates. Nylon will deform under load about 75 times as much as steel!

One of the attractive features of nylon gears is their ability to operate with marginal lubrication. Small nylon gears have been able to run at high speed under light load with no lubricant at all. This is important in some applications, e.g., where a lubricant would soil yarn or lm handled by a machine. Nylon gears have worked well in may gear applications, such as in movie cameras, textile machines, food mixers, and timing devices. In some cases, it has been reported that nylon did not stand up well. It is proven by tests that nylon gears are capable of running at 40 m/s (8000 fpm) pitch-line velocity and tooth loads up to those carried by low-hardness steel gears. This indicates that nylon—when properly processed into a well-designed gear—is a surprisingly good gear material. See Figure 6.26.

TABLE 6.22
Some Properties of FM-10001 Nylon

Tensile Strength	
At −70°F	15,700
At 77°F	10,000
At 170°F	7600
Modulus of elasticity at 77°F	400,000
Rockwell hardness (R)	118
Speci c gravity	1.14
Mold shrinkage (in./in.)	0.015

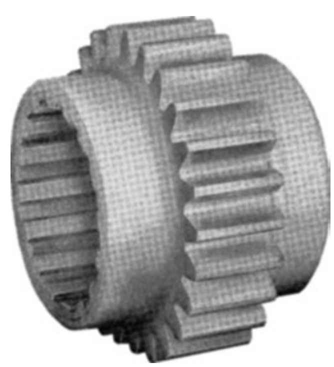


FIGURE 6.26 Solid nylon gear after testing at a pitch-line velocity of 40 m/s (8000 fpm) and overload rating. (Courtesy of General Electric Co., Lynn, Massachusetts.)

7 Direct Gear Design for Asymmetric Tooth Gears

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7.1 INTRODUCTION

The flanks of a gear tooth are functionally different for many gear drives. Tooth load on one flank is significantly higher and is applied for longer periods than that on the opposite one. An asymmetric tooth gshape reflects this functional difference (Figure 7.1). A design objective of asymmetric gear teeth is to improve the performance of primary drive profiles at the expense of the performance of opposite coast profiles. The coast flanks are unloaded or lightly loaded during a relatively short work period.

The first asymmetric gears had a buttress tooth shape with a low pressure angle at the drive tooth flanks, while supporting coast flanks had a high pressure angle. According to Woodbury (1958a): “In Leonardo da Vinci we find some drawings of tooth form—one very like a buttress tooth.” Willis (1841) had shown the asymmetric buttress gear teeth with the following explanation: “If a machine be of such a nature that the wheels are only required to turn in one direction, the strength of the teeth may be doubled by an alteration of form.” Leutwiler (1917) applied involute profiles for both drive and coast flanks of the buttress or, as he called them, “hook-tooth” gears. He suggested the 15° pressure angle for drive flanks and the 35° pressure angle for coast flanks.

Many gear researchers (Bolotovsky et al., 1984; DiFrancesco and Marini, 1997; Gang and Nakanishi, 2001; Brecher and Schafer, 2005; Karpat et al., 2005; Yang, 2007; Pedersen, 2010; Wang et al., 2011) have defined asymmetric gear geometry traditionally by preselected asymmetric generating gear rack parameters, which are typically modified from the standard symmetric rack parameters by increasing the pressure angle of one flank. The opposite flank and other rack tooth proportions remain unchanged.

It is well known that gear transmission load capacity and power density depend mainly on the tooth flank surface durability, which is defined by the contact stress level and scuffing resistance. From this point, the application of a higher pressure angle for drive tooth flanks and a lower pressure angle for coast tooth flanks is more promising than the buttress tooth shape. In addition, this tooth form provides lower stiffness and better gear mesh impact dampening.

Professor E. B. Vulgakov (Vulgakov and Rivkin, 1976; Vulgakov and Vasina, 1978) had applied his theory of generalized parameters to asymmetric gears, defining their geometry without using rack generation parameters.

Publications (Kapelevich, 1984, 1987, 2000; Vulgakov and Kapelevich, 1986) suggested an asymmetric tooth formed with two involutes of two different base circles. Figure 7.2 shows that the overlaid symmetric tooth with identical drive pressure angle and tooth thicknesses at the reference and tip diameters has an involute active involute flank much shorter than that of the drive flank of an asymmetric tooth.

The Direct Gear Design approach (Kapelevich, 2013), based on Professor E. B. Vulgakov’s theory of generalized parameters, allows for maximizing the performance of asymmetric tooth gears. Asymmetric tooth profiles make it possible simultaneously to increase the contact ratio and operating pressure angle beyond those limits achievable with conventional symmetric gears. The main advantage of asymmetric gears is contact stress reduction on the drive flanks that results in higher power transmission density (load capacity per gear size). Another important advantage is the possibility of designing the coast tooth flanks independently from the drive tooth flanks, i.e., managing tooth stiffness while keeping a desirable pressure angle and contact ratio of drive flanks. This allows for an increase in tooth tip deflection, thus damping tooth mesh impact and resulting in a reduction of gear noise and vibration.

7.2 GEOMETRY OF ASYMMETRIC TOOTH GEARS

Two involute flanks of the asymmetric tooth (see Figure 7.3) are unwound from two different base diameters d_{bd} and d_{bc}. The subscript d is used for the drive flank and the subscript c is used for the coast flank of an asymmetric tooth. A diameter d_x at the drive flank point x can be defined from the following expression:

$$d_x = \frac{d_{bd}}{\cos\alpha_{xd}} = \frac{d_{bc}}{\cos\alpha_{xc}}. \tag{7.1}$$

Then, the tooth asymmetry factor K is

$$K = \frac{d_{bc}}{d_{bd}} = \frac{\cos\alpha_{xc}}{\cos\alpha_{xd}}. \tag{7.2}$$

For many applications, the drive flank profile angle α_{xd} is greater than the coast flank profile angle α_{xc}. This means that $d_{bd} < d_{bc}$ and the asymmetry factor $K > 1.0$.

At the coast flank base circle d_{bc}, the coast flank profile angle $\alpha_{xc} = 0°$ and the drive flank profile angle α_{xd} is calculated from Equation 7.2:

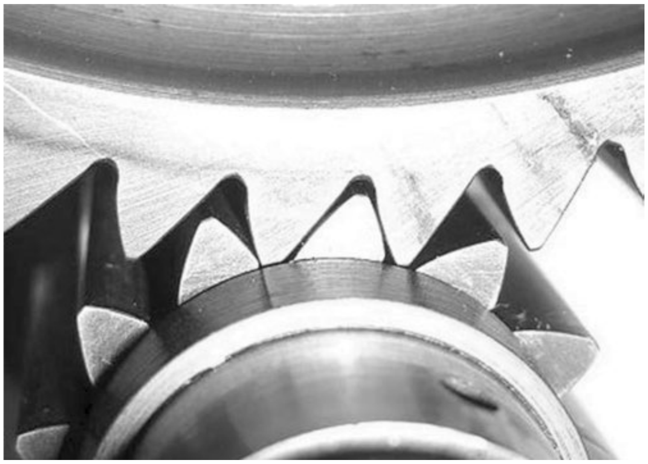


FIGURE 7.1 Gears with asymmetric teeth.

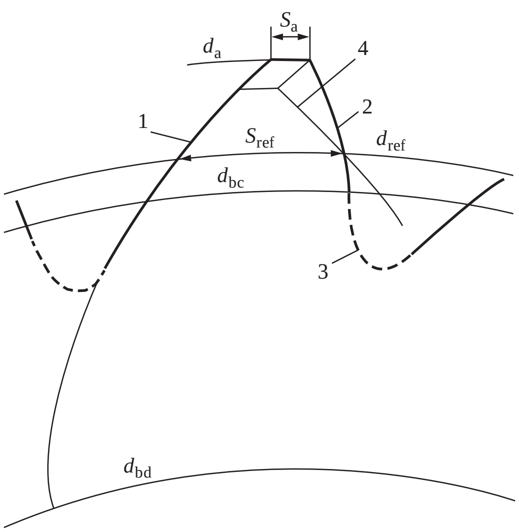


FIGURE 7.2 Asymmetric tooth constructed with two involutes. 1: drive ank from base diameter d_{bd}; 2: coast ank from base diameter d_{bc}; 3: root llet; 4: symmetric tooth pro le with the same drive ank and tooth thicknesses S_{ref} and S_a at the reference diameter d_{ref} and the tooth tip diameter d_a.

$$\alpha_{xd} = \cos^{-1}\left(\frac{1}{K}\right). \tag{7.3}$$

The base tooth thickness of the asymmetric tooth can be de ned only at the coast ank base circle d_{bc}:

For external tooth,

$$S_b = \frac{d_{bc}}{2}\left[\operatorname{inv}\nu_d + \operatorname{inv}\nu_c - \operatorname{inv}\cos^{-1}\left(\frac{1}{K}\right)\right]. \tag{7.4}$$

For internal tooth,

$$S_b = \frac{d_{bc}}{2}\left[\frac{2\pi}{z} - \operatorname{inv}\nu_d - \operatorname{inv}\nu_c + \operatorname{inv}\cos^{-1}\left(\frac{1}{K}\right)\right]. \tag{7.5}$$

The tooth thickness at the diameter d_x is

For external tooth,

$$S_x = \frac{d_x}{2}(\operatorname{inv}\nu_d + \operatorname{inv}\nu_c - \operatorname{inv}\alpha_{xd} - \operatorname{inv}\alpha_{xc}) \tag{7.6}$$

or

$$S_x = \frac{d_{bd}}{2\cos\alpha_{xd}}(\operatorname{inv}\nu_d + \operatorname{inv}\nu_c - \operatorname{inv}\alpha_{xd} - \operatorname{inv}\alpha_{xc}). \tag{7.7}$$

For internal tooth,

$$S_x = \frac{d_x}{2}\left(\frac{2\pi}{z} - \operatorname{inv}\nu_d - \operatorname{inv}\nu_c + \operatorname{inv}\alpha_{xd} + \operatorname{inv}\alpha_{xc}\right) \tag{7.8}$$

or

$$S_x = \frac{d_{bd}}{2\cos\alpha_{xd}}\left(\frac{2\pi}{z} - \operatorname{inv}\nu_d - \operatorname{inv}\nu_c + \operatorname{inv}\alpha_{xd} + \operatorname{inv}\alpha_{xc}\right). \tag{7.9}$$

An asymmetric tooth pro le also includes the tip land and the root llet between teeth (see Figures 7.4 and 7.5). The tooth tip diameter d_a can be de ned from Equation 7.1:

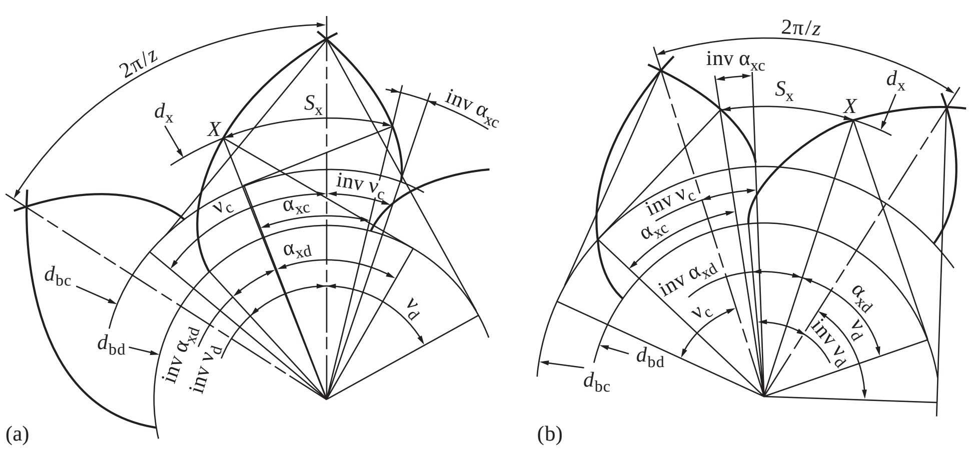


FIGURE 7.3 Involute anks of (a) external and (b) internal asymmetric gear teeth. z: number of teeth.

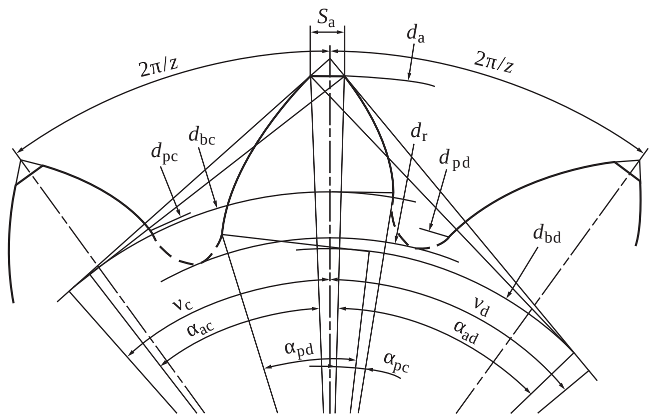


FIGURE 7.4 External asymmetric gear tooth.

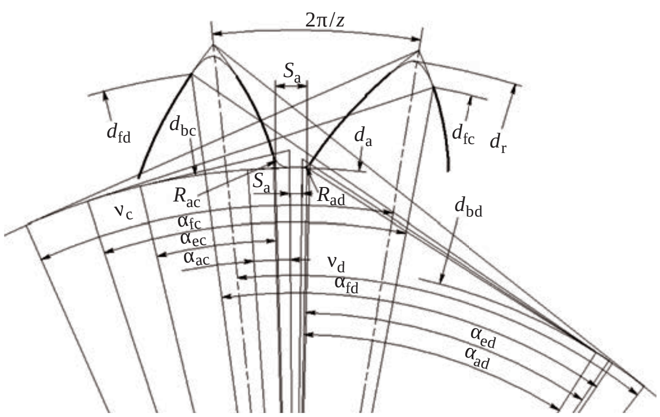


FIGURE 7.5 Internal asymmetric gear tooth.

$$d_a = \frac{d_{bd}}{\cos\alpha_{ad}} = \frac{d_{bc}}{\cos\alpha_{ac}}, \tag{7.10}$$

where α_{ad} and α_{ac} are the drive and coast profile angles at the diameter d_a.

The tooth tip land S_a of the external gear tooth is defined considering tooth tip radii equal to zero from Equation 7.6 or 7.8:

For external tooth (Figure 7.4),

$$S_a = \frac{d_a}{2}(\text{inv}\,\nu_d + \text{inv}\,\nu_c - \text{inv}\,\alpha_{ad} - \text{inv}\,\alpha_{ac}) \tag{7.11}$$

or

$$S_a = \frac{d_{bd}}{2\cos\alpha_{ad}}(\text{inv}\,\nu_d + \text{inv}\,\nu_c - \text{inv}\,\alpha_{ad} - \text{inv}\,\alpha_{ac}). \tag{7.12}$$

For internal tooth (Figure 7.5),

$$S_a = \frac{d_a}{2}\left(\frac{2\pi}{z} - \text{inv}\,\nu_d - \text{inv}\,\nu_c + \text{inv}\,\alpha_{ad} + \text{inv}\,\alpha_{ac}\right) \tag{7.13}$$

or

$$S_a = \frac{d_{bd}}{2\cos\alpha_{ad}}\left(\frac{2\pi}{z} - \text{inv}\,\nu_d - \text{inv}\,\nu_c + \text{inv}\,\alpha_{ad} + \text{inv}\,\alpha_{ac}\right) \tag{7.14}$$

The diameters d_{pd} and d_{pc} at the points of contact near the root fillet and related profile angles α_{pd} and α_{pc} are defined considering a mesh with the mating gear. The root diameter d_r is defined as a result of the root fillet profile optimization.

7.3 GEAR MESH CHARACTERISTICS

Asymmetric external and internal gear meshes are shown in Figure 7.6.

In all figures and equations describing gears with asymmetric teeth, the indices d and c are for parameters related to the drive and coast tooth flanks accordingly.

The pinion and gear tooth thicknesses S_{w1} and S_{w2} at the operating pitch diameters $d_{w1,2}$ are defined by Equations 7.7 and 7.9 as

$$S_{w1} = \frac{d_{bd1}}{2\cos\alpha_{wd}}(\text{inv}\,\nu_{d1} + \text{inv}\,\nu_{c1} - \text{inv}\,\alpha_{wd} - \text{inv}\,\alpha_{wc}), \tag{7.15}$$

For external gearing,

$$S_{w2} = \frac{d_{bd2}}{2\cos\alpha_{wd}}(\text{inv}\,\nu_{d2} + \text{inv}\,\nu_{c2} - \text{inv}\,\alpha_{wd} - \text{inv}\,\alpha_{wc}). \tag{7.16}$$

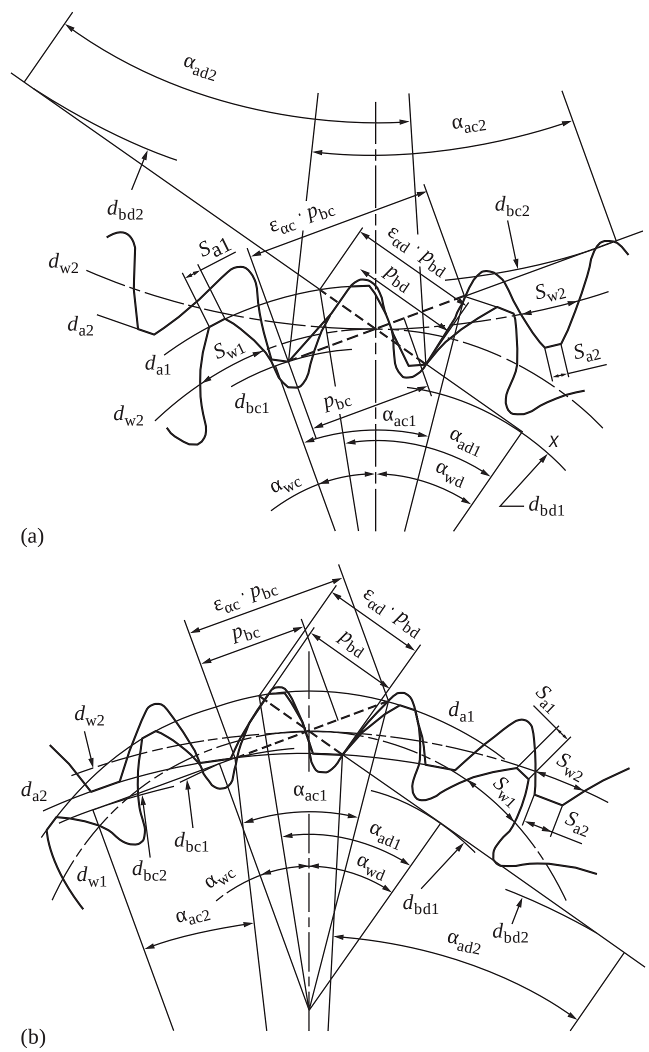


FIGURE 7.6 Asymmetric gear mesh. (a) External; (b) internal.

For internal gearing,

$$S_{w2} = \frac{d_{bd2}}{2\cos\alpha_{wd}}\left(\frac{2\pi}{z_2} - \mathrm{inv}\,\nu_{d2} - \mathrm{inv}\,\nu_{c2} + \mathrm{inv}\,\alpha_{wd} + \mathrm{inv}\,\alpha_{wc}\right). \tag{7.17}$$

The mating gear rack tooth thickness at the operating pitch line S_{w2} is

$$S_{w2} = \pi m - \frac{d_{bd1}}{2\cos\alpha_d}(\mathrm{inv}\nu_{d1} + \mathrm{inv}\nu_{c1} - \mathrm{inv}\alpha_d - \mathrm{inv}\alpha_c)\ \text{(metric)}, \tag{7.18}$$

$$S_{w2} = \frac{\pi}{DP} - \frac{d_{bd1}}{2\cos\alpha_d}(\mathrm{inv}\nu_{d1} + \mathrm{inv}\nu_{c1} - \mathrm{inv}\alpha_d - \mathrm{inv}\alpha_c)\ \text{(English)}. \tag{7.19}$$

The operating pressure angles for the drive anks α_{wd} and for the coast anks α_{wc} are de ned by substitution of S_{w1} and S_{w2} from Equations 7.15 and 7.16 or Equation 7.17 into the operating circular pitch equation:

$$p_w = S_{w1} + S_{w2}. \tag{7.20}$$

Then, for external gear,

$$\mathrm{inv}\,\alpha_{wd} + \mathrm{inv}\,\alpha_{wc} = \frac{1}{1+u}\left[\mathrm{inv}\,\nu_{d1} + \mathrm{inv}\,\nu_{c1} + u(\mathrm{inv}\,\nu_{d2} + \mathrm{inv}\,\nu_{c2}) - \frac{2\pi}{z_1}\right]. \tag{7.21}$$

For internal gear,

$$\mathrm{inv}\,\alpha_{wd} + \mathrm{inv}\,\alpha_{wc} = \frac{1}{u-1}[u(\mathrm{inv}\,\nu_{d2} + \mathrm{inv}\,\nu_{c2}) - \mathrm{inv}\,\nu_{d1} + \mathrm{inv}\,\nu_{c1}]. \tag{7.22}$$

The relation between pressure angles for the drive anks α_{wd} and for the coast anks α_{wc} is de ned from Equation 7.2 as

$$\cos\alpha_{wc} = K\cos\alpha_{wd}. \tag{7.23}$$

The pro le angles at the points of contact near the llet are

For external gear mesh, drive anks,

$$\alpha_{pd1} = \tan^{-1}[(1+u)\tan\alpha_{wd} - u\tan\alpha_{ad2}] \tag{7.24}$$

$$\alpha_{pd2} = \tan^{-1}\left(\frac{1+u}{u}\tan\alpha_{wd} - \frac{1}{u}\tan\alpha_{ad1}\right). \tag{7.25}$$

For external gear mesh, coast anks,

$$\alpha_{pc1} = \tan^{-1}[(1+u)\tan\alpha_{wc} - u\tan\alpha_{ac2}] \tag{7.26}$$

$$\alpha_{pc2} = \tan^{-1}\left(\frac{1+u}{u}\tan\alpha_{wc} - \frac{1}{u}\tan\alpha_{ac1}\right). \tag{7.27}$$

For internal gear mesh, drive anks,

$$\alpha_{pd1} = \tan^{-1}[u\tan\alpha_{ad2} - (u-1)\tan\alpha_{wd}] \tag{7.28}$$

$$\alpha_{pd2} = \tan^{-1}\left(\frac{u-1}{u}\tan\alpha_{wd} + \frac{1}{u}\tan\alpha_{ad1}\right). \tag{7.29}$$

For internal gear mesh, coast anks,

$$\alpha_{pc1} = \tan^{-1}[u\tan\alpha_{ac2} - (u-1)\tan\alpha_{wc}] \tag{7.30}$$

$$\alpha_{pc2} = \tan^{-1}\left(\frac{u-1}{u}\tan\alpha_{wc} + \frac{1}{u}\tan\alpha_{ac1}\right). \tag{7.31}$$

If asymmetry factor $K > 0$, interference occurs rst for the coast involute anks, which are unwound from the larger base circle. If the pro le angles α_{pc1} or α_{pc2} in the external mesh or angle α_{pc1} in the internal mesh are less than zero, then their involute anks close to the base diameters are interfering with the mating tooth tips. This leads to the involute ank pro le undercut.

The transverse contact ratios are as follows:

For external gear mesh, drive anks,

$$\varepsilon_{\alpha d} = \frac{z_1}{2\pi}[\tan\alpha_{ad1} + u\tan\alpha_{ad2} - (1+u)\tan\alpha_{wd}]. \tag{7.32}$$

For external gear mesh, coast anks,

$$\varepsilon_{\alpha c} = \frac{z_1}{2\pi}[\tan\alpha_{ac1} + u\tan\alpha_{ac2} - (1+u)\tan\alpha_{wc}]. \tag{7.33}$$

For internal gear mesh, drive anks,

$$\varepsilon_{\alpha d} = \frac{z_1}{2\pi}[\tan\alpha_{ad1} - u\tan\alpha_{ad2} + (u-1)\tan\alpha_{wd}]. \tag{7.34}$$

For internal gear mesh, coast anks,

$$\varepsilon_{\alpha c} = \frac{z_1}{2\pi}[\tan\alpha_{ac1} - u\tan\alpha_{ac2} + (u-1)\tan\alpha_{wc}]. \tag{7.35}$$

7.4 ASYMMETRIC TOOTH GEARING LIMITS

Figure 7.7 presents ranges of the drive pressure angles for a different number of teeth and asymmetry factors K.

The pressure angle limit for the external spur gears with $z_1 + z_2 \to \infty$ is (Kapelevich, 1984)

$$\alpha_{wlim} = \tan^{-1} K. \tag{7.36}$$

A chart of the pressure angle limit α_{wlim} as a function of the asymmetry factor K is shown in Figure 7.8.

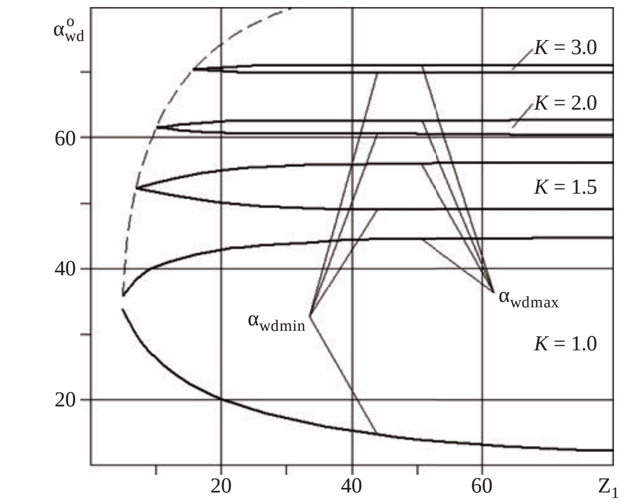


FIGURE 7.7 Minimum and maximum pressure angles for external spur gears with gear ratio $u = 1$ and various asymmetry factors K.

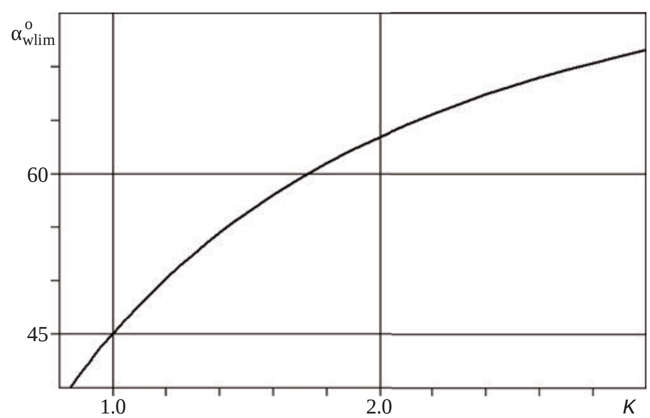


FIGURE 7.8 Pressure angle limits for external spur gears.

Direct Gear Design allows to greatly expand the transverse contact ratio range. Its maximum value depends on type of gearing (external or internal), tooth pro le (symmetric or asymmetric), number of teeth, and relative tooth tip thicknesses.

Maximum transverse contact ratios for spur external reversible asymmetric gears with the relative tooth tip thicknesses $m_{a1,2} = 0$ are shown in Table 7.1.

The Table 7.1 data indicate that the maximum contact ratio of the reversible asymmetric gears is just slightly greater than it is for the symmetric gears, and asymmetry of such gears is very low.

The practical maximum pressure angle and the transverse contact ratio are limited by the minimum tooth tip thickness. For a case of hardened teeth, it should be suf cient to avoid the hardening through the tooth tip. For gears made out of soft metals and plastics, it should be suf cient to exclude tooth tip bending. Minimum relative tooth tip thickness typically is $m_{a1,2} = 0.25 \div 0.30$. The practical minimal contact ratio for conventional spur gears is about $\varepsilon_{a\ min} = 1.10 \div 1.15$. For the high–contact ratio (HCR) gears, it is about $\varepsilon_{a\ min} = 2.05 \div 2.10$. These minimal contact ratio values are chosen to avoid its reduction below 1.0 for conventional spur gears and below 2.0 for the HCR spur gears, because of manufacturing and assembly tolerances, and tooth tip chamfers or radii. These conditions also identify the practical maximum pressure angle. The practical minimal pressure angle for symmetric gears is de ned by the beginning of the tooth involute undercut, when the involute pro le angles at the lowest contact points $\alpha_{pd1,2} = 0°$, and where the transverse contact ratio reaches its maximum value $\varepsilon_{a\ max}$.

Practical maximum drive ank pressure angles for conventional and HCR asymmetric gears are shown in Tables 7.2 and 7.3 (Kapelevich, 2013).

The maximum drive pressure angle values in Tables 7.2 and 7.3 assume some possible small undercut of the coast ank near the root, especially for gears with a low number of teeth ($15 \div 30$). However, this does not reduce the coast ank contact below $\varepsilon_{ac} = 1.0$.

7.5 TOOTH GEOMETRY OPTIMIZATION

7.5.1 Asymmetry Factor *K* Selection for Reversible Asymmetric Tooth Gears

There are many gear drives where a gear pair transmits the load in both load directions, but with signi cantly different magnitude and duration (Figure 7.9). In this case, the asymmetry factor K for a gear pair is de ned by equalizing potential accumulated tooth surface damage de ned by operating contact stress and number of the tooth ank load cycles. In

TABLE 7.1
Maximum Drive Contact Ratios for Reversible Asymmetric Gears

Number of Teeth, $Z_{1,2}$	10	15	20	30	40	50
Drive contact ratio, ε_{ad}	1.53	1.931	2.288	2.924	3.49	4.015
Coast contact ratio, ε_{ac}	1.0	1.0	1.0	1.0	1.0	1.0
Drive ank pressure angle, α_{wd}	25.67	22.02	19.77	17.02	15.34	14.157
Coast ank pressure angle, α_{wc}	20.58	15.79	13.19	10.47	8.73	7.692
Drive ank tooth tip angle, $\alpha_{ad1,2}$	43.87	38.97	35.71	31.48	28.75	26.77
Coast ank tooth tip angle, $\alpha_{ac1,2}$	41.51	36.19	32.85	28.71	26.03	24.147
Drive ank lowest involute angle, $\alpha_{pd1,2}$	0.0	0.0	0.0	0.0	0.0	0.0
Asymmetry factor, K	1.039	1.038	1.035	1.028	1.025	1.022

TABLE 7.2
Practical α_{wdmax} for Conventional Asymmetric Gears ($m_{a1,2}$ = 0.3, ε_{ad} = 1.1, α_{wc} = 15°)

	z_1						
z_2	**15**	**20**	**30**	**40**	**50**	**70**	**100**
15	43.5						
20	44.5	45.5					
30	45.9	46.4	47.3				
40	47	47.3	47.7	48.2			
50	47.6	47.8	48	48.3	48.9		
70	48	48.2	48.6	48.7	49	49.5	
100	48.7	48.9	49.2	49.5	49.1	49.6	50

TABLE 7.3
Practical α_{wdmax} for HRC Asymmetric Gears ($m_{a1,2}$ = 0.3, ε_{ad} = 2.1, α_{wc} = 15°)

	z_1						
z_2	**20**	**25**	**30**	**40**	**50**	**70**	**100**
20	19.3						
25	20.5	21.5					
30	21.5	22.4	23				
40	23	23.6	24.1	25			
50	24.1	24.6	25	25.6	26.1		
70	25.5	25.8	26.1	26.6	26.9	27.5	
100	26.7	27	27.2	27.5	27.7	28.1	28.5

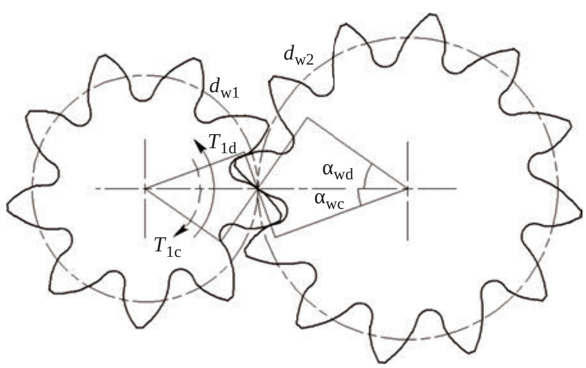


FIGURE 7.9 Asymmetric gear pair. T_{1d} and T_{1c}: pinion torques applied to the drive and coast tooth anks, respectively.

other words, the contact stress safety factor S_H should be the same for the drive and coast tooth anks. This condition can be presented as

$$S_H = \frac{\sigma_{HPd}}{\sigma_{Hd}} = \frac{\sigma_{HPc}}{\sigma_{Hc}}, \tag{7.37}$$

where

σ_{Hd}, σ_{Hc}—operating contact stresses for the drive and coast tooth anks

σ_{HPd}, σ_{HPc}—permissible contact stresses for the drive and coast tooth anks that depend on the number of load cycles

Then, from Equation 7.37,

$$\frac{\sigma_{Hd}}{\sigma_{Hc}} = \frac{\sigma_{HPd}}{\sigma_{HPc}}. \tag{7.38}$$

The contact stress at the pitch point (Standard ISO 6336, 2006) is

$$\sigma_H = Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_{w1} b_w} \times \frac{u \pm 1}{u}}, \tag{7.39}$$

where

$Z_H = \sqrt{2\cos\beta_b \cos\alpha_{wt} / \cos^2\alpha_t \sin\alpha_{wt}}$ is the zone factor that, for the directly designed spur gears, is

$$Z_H = \frac{2}{\sqrt{\sin(2\alpha_w)}}, \tag{7.40}$$

where

Z_E—elasticity factor that takes into account gear material properties (modulus of elasticity and Poisson's ratio)

Z_ε—contact ratio factor; its conservative value for spur gears is $Z_\varepsilon = 1.0$

Z_β—helix factor; for spur gears $Z_\beta = 1.0$

F_t—nominal tangent load, that, at the pitch diameter d_{w1}, is $F_t = 2T_1/d_{w1}$

T_1—pinion torque

b_w—contact face width

(The + sign is for external gearing; the − sign is for internal gearing.)

Then, for the directly designed spur gears, the contact stress at the pitch point can be presented as

$$\sigma_{\mathrm{H}} = Z_{\mathrm{E}} \frac{2}{d_{\mathrm{w1}}} \sqrt{\frac{2T_1}{b_{\mathrm{w}} \sin(2\alpha_{\mathrm{w}})} \times \frac{u \pm 1}{u}}. \tag{7.41}$$

Some parameters of this equation, Z_{E}, d_{w1}, b_{w}, and u, do not depend on the load transmission direction and Equation 7.38, since the pitch point contact can be presented as

$$\frac{\sin(2\alpha_{\mathrm{wc}})}{\sin(2\alpha_{\mathrm{wd}})} = A, \tag{7.42}$$

where the parameter A is

$$A = \frac{T_{\mathrm{1c}}}{T_{\mathrm{1d}}} \left(\frac{\sigma_{\mathrm{HPd}}}{\sigma_{\mathrm{HPc}}} \right)^2. \tag{7.43}$$

According to Standard ISO 6336 (2006), “The permissible stress at limited service life or the safety factor in the limited life stress range is determined using life factor Z_{NT}.” This allows for replacing the permissible contact stresses in Equation 7.43 for the life factors

$$A = \frac{T_{\mathrm{1c}}}{T_{\mathrm{1d}}} \left(\frac{Z_{\mathrm{NTd}}}{Z_{\mathrm{NTc}}} \right)^2. \tag{7.44}$$

When the parameter A is defined and the drive pressure angle is selected, the coast pressure angle is calculated by using Equation 7.43 and the asymmetry coefficient K from the common solution of Equations 7.42 and 7.23:

$$K = \frac{\sqrt{1 + \sqrt{1 - A^2 \sin^2(2\alpha_{\mathrm{wd}})}}}{\sqrt{2} \cos \alpha_{\mathrm{wd}}}. \tag{7.45}$$

If the gear tooth is equally loaded in both main and reversed load application directions, then both the coefficient A and the asymmetry factor K are equal to 1.0 and the gear teeth are symmetric.

Example 7.1

The drive pinion torque T_{1d} is two times greater than the coast pinion torque T_{1c}. The drive tooth flank has 10^9 load cycles and the coast tooth flank has 10^6 load cycles during the life of the gear drive. From the S-N curve (Standard ISO 6336, 2006) for steel gears, an approximate ratio of the life factors $Z_{\mathrm{NTd}}/Z_{\mathrm{NTc}} = 0.85$. Then, the coefficient $A = 0.85^2/2 = 0.36$. Assuming the drive pressure angle is $\alpha_{\mathrm{wd}} = 36°$, the coast pressure angle from Equation 7.42 is $\alpha_{\mathrm{wd}} = 10°$ and the asymmetry factor from Equation 7.45 is $K = 1.22$.

7.5.2 Root Fillet Optimization

In Direct Gear Design, the tooth fillet is designed after the involute flank parameters are completely defined. One goal is to achieve a minimum of stress concentration on the tooth fillet profile. In other words, the maximum bending stress should be evenly distributed along the large portion of the root fillet. The initial root fillet profile is a trajectory of the mating gear tooth tip in the tight (zero backlash) mesh. This allows for the exclusion of interference with the mating gear tooth.

The fillet optimization method was developed by Dr. Y. V. Shekhtman (Kapelevich and Shekhtman, 2003, 2009). It utilizes the following calculation processes:

- Definition of a set of mathematical functions used to describe the optimized fillet profile: Such a set may contain the trigonometric, polynomial, hyperbolic, exponential, and other functions and their combinations. Parameters of these functions are defined during the optimization process.
- Finite element analysis (FEA) with the triangle linear elements is used to calculate stress. This kind of finite element makes possible the achievement of satisfactory optimization results within a reasonable time.
- A random search method (Rastrigin, 1969) is used to define the next step in the multiparametric iteration process of the fillet profile optimization.

This fillet optimization method establishes the approximate fillet center (Figure 7.10). It is defined as the center of the best-fitted circular arc and is connected to the finite element nodes located on the initial fillet profile. The first and last finite element nodes of the initial fillet profile

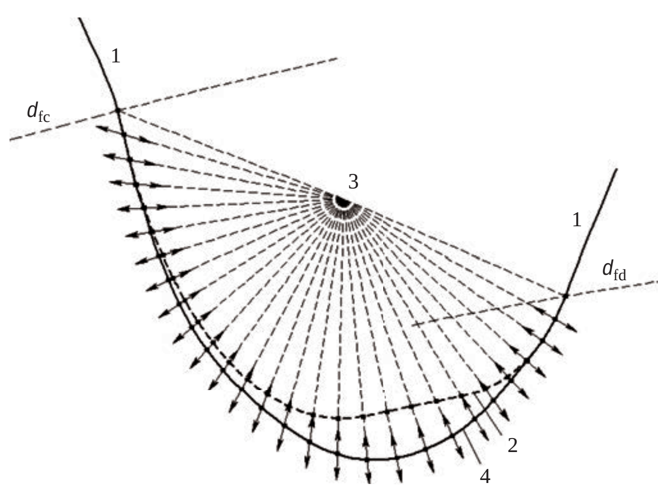


FIGURE 7.10 Tooth fillet profile optimization. 1: involute tooth flanks; 2: initial fillet profile; 3: fillet center; 4: optimized fillet profile; d_{fd} and d_{fc}: drive and coast flank form circle diameters, respectively.

located on the form diameter circle cannot be moved during the optimization process. The rest of the initial llet nite element nodes are moved along the straight lines that pass through the llet center. The bending stresses are calculated for every llet pro le con guration iteration. Variable parameters of the llet pro le functions that describe the llet pro le for the next iteration are de ned depending on the stress calculation results of the previous iteration. If it provided stress reduction, the optimization process moves the llet nodes in the same direction. If stress was increased, the nodes are moved in opposite directions. After the speci ed number of iterations, the optimization process is considered done. When more nite element nodes are placed on the llet pro le, stress calculation results are more accurate, but this requires more iterations, and the llet pro le optimization takes more time. During the optimization process, the llet nodes cannot be moved inside the initial llet pro le because this may cause interference with the mating gear tooth tip. This is one of the main constraints of llet optimization.

Figure 7.11 presents the gear tooth stress distribution comparison before and after root llet optimization.

Figure 7.12 presents a comparison of different tooth root llet pro le options. The involute anks, face widths, and tooth load and its application point are the same for all llet pro le options. Results of the FEA stress calculation, along with other root llet parameters, are shown in Table 7.4. Calculation results for llet pro le option 1 generated by the standard 20° pressure angle rack pro le are considered to be the 100% benchmark values. Parameters of other llet pro le options are de ned relative to option 1.

The root llet pro le comparison results presented in Table 7.4 indicate considerable root stress concentration reduction provided by the llet optimization. At the maximum tensile stress, the point of the optimized llet has signi cantly larger llet radius R_f, as well as the smaller distance H and root clearance C. It has the lowest maximum bending stress, which is evenly distributed along the large portion of the llet

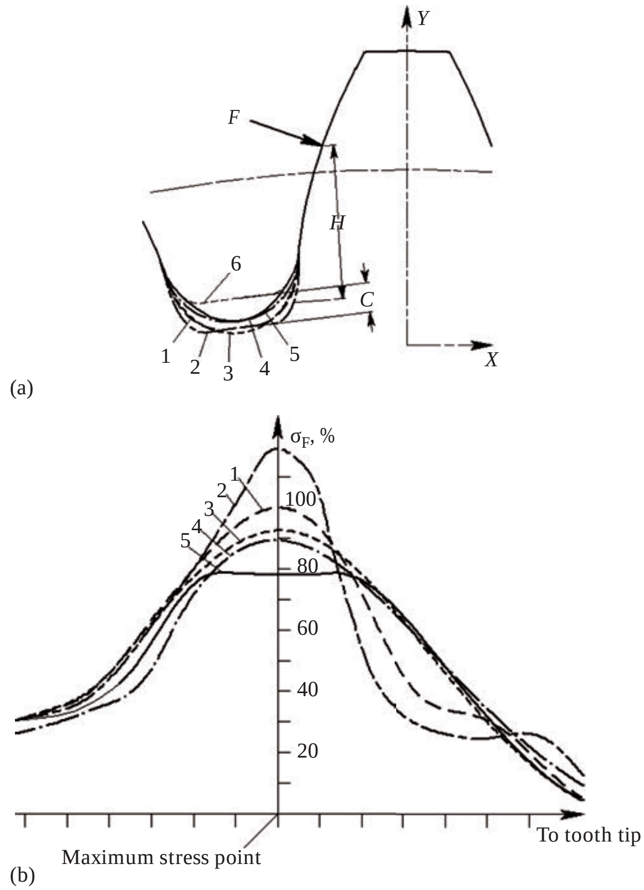


FIGURE 7.12 Root llet comparison. (a) Gear tooth with different llet pro les; (b) stress chart along the llet. 1: llet pro le generated by the standard coarse pitch rack with tip radius 0.3 m; 2: llet pro le generated by the standard ne diametral pitch rack with the tip radius equal to zero; 3: llet pro le generated by the full tip radius rack; 4: circular llet pro le; 5: optimized llet pro le; 6: trajectory of the mating gear tooth tip in tight mesh; F: applied load; H: radial distance between load application and maximum stress points; C: radial clearance; σ_F: tensile stress.

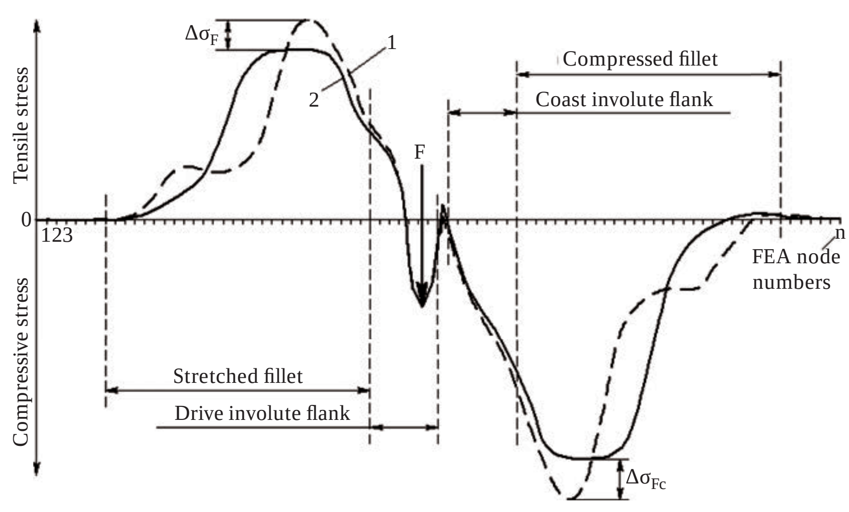


FIGURE 7.11 Tooth pro le stress distribution charts before (1) and after (2) root llet optimization. $\Delta\sigma_F$: tensile stress reduction; $\Delta\sigma_{Fc}$: compressive stress reduction.

TABLE 7.4
Fillet Profile Comparison (Figure 7.12)

	Rack with Tip Radius R = 0.3 m	Rack with Tip Radius R = 0	Rack with Full Tip Radius	Circular Fillet Profile	Optimized Fillet
Fillet pro le no.	1	2	3	4	5
R_f (%)	100	58	118	121	273
H (%)	100	103	100	88	82
C (%)	100	100	118	79	76
$\alpha_{F\ max}$	100	119	90	88	78

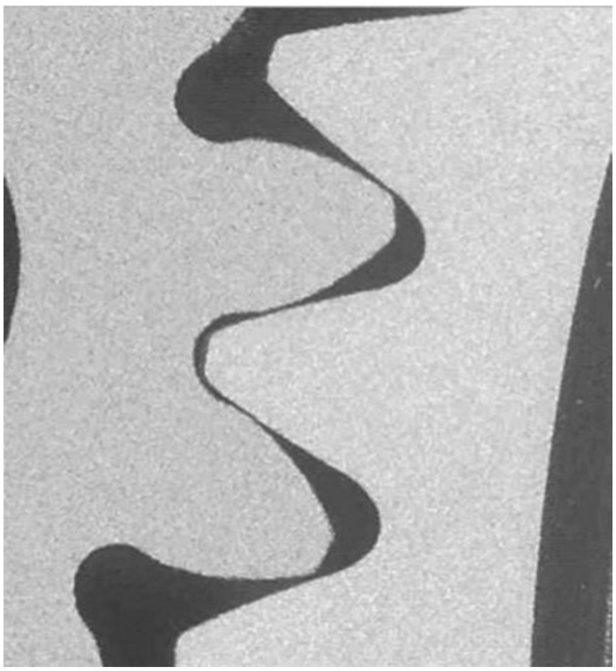


FIGURE 7.13 Gears with optimized root llets.

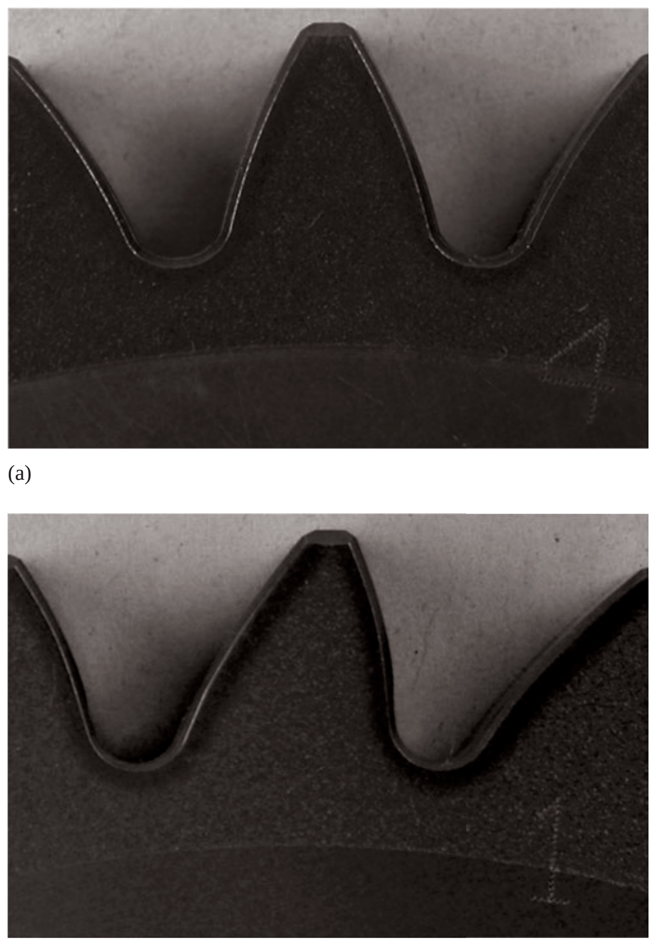
pro le. Other llet pro les have signi cantly greater and more sharply concentrated maximum stress.

Analysis of the llet optimization results had indicated that the optimized llet pro le practically does not depend on the force value and its application point on the involute ank, except in the case when the application point is located very close to the form diameter. In this case, compression under the applied force may affect the optimized llet pro le. Such load application should not be considered for llet optimization, because it induces minimal tensile stress in the root llet in comparison to other load application points along the tooth ank.

Gears with optimized root llets are shown in Figure 7.13.

7.6 ANALYTICAL AND EXPERIMENTAL COMPARISON OF SYMMETRIC AND ASYMMETRIC TOOTH GEARS

Directly designed gears with an asymmetric involute gear tooth form were analyzed to determine their bending and contact stresses relative to those of symmetric involute gear tooth form, which is representative of the helicopter main drive gears developed by the rotorcraft division of the Boeing Company (Brown et al., 2011). Asymmetric and baseline (symmetric) toothed gear test specimens were designed, fabricated, and tested to experimentally determine their single-tooth bending fatigue (STBF) strength and scuf ng resistance. The gear test specimens are presented in Figure 7.14.



(a)

(b)

FIGURE 7.14 Test specimen gear tooth pro les. (a) Baseline gear teeth; (b) asymmetric gear teeth. (Courtesy of Boeing Co., Philadelphia, Pennsylvania.)

The gear parameters and FEA-calculated bending stresses for the STBF test gears are presented in Table 7.5.

Similarly, the scuffing test gears are within the design experience range of typical main transmission helicopter power gears. The gear parameters for the scuffing test gear specimens are presented in Table 7.6.

Fatigue results of the STBF tests of the asymmetric tooth and the baseline specimens are presented in Figure 7.15.

Figure 7.16 shows the scuffing results for baseline and asymmetric gears. The 35° pressure angle asymmetric gears showed an improvement of approximately 25% in mean scuffing load (torque) compared to the baseline symmetric tooth specimens. The mean 3-sigma levels are also shown, based on a population of eight baseline data points and six asymmetric data points.

Test results demonstrated higher bending fatigue strength for both asymmetric tooth forms compared to baseline designs. Scuffing resistance was significantly increased for the asymmetric tooth form compared to a traditional symmetric involute tooth design.

TABLE 7.5
STBF Test Gear Specimen Parameters

	Symmetric Gears with Circular Fillets (Baseline)	Asymmetric Gears with Circular Fillets
Number of teeth of both mating gears	32	32
Diametral pitch (in.$^{-1}$)	5.333	5.333
Pressure angle	25°	35°/15°[a]
Pitch diameter (in.)	6.000	6.000
Base diameter (in.)	5.4378	4.9149/5.7956[a]
Outside diameter (in.)	6.3975	6.3864
Root diameter (in.)	5.571	5.558
Form diameter (in.)	5.6939	5.6581/5.8110[a]
Circular tooth thickness (in.)	0.2895	0.2895
Face width (in.)	0.375	0.375
Torque (in. lb)	5000	5000
Load application radius (in.)	3.06	3.06
Calculated maximum bending stress (psi)	57,887	54,703 (−5.5%)

[a] Drive/coast flank parameter.

TABLE 7.6
Scuffing Test Gear Specimen Parameters

	Symmetric Gears with Circular Fillets (Baseline)	Asymmetric Gears with Circular Fillets
Number of teeth of both mating gears	30	30
Diametral pitch (in.$^{-1}$)	5.000	5.000
Pressure angle	25°	35°/18°
Pitch diameter (in.)	6.000	6.000
Base diameter (in.)	5.4378	4.9149/5.7063[a]
Outside diameter (in.)	6.400 max	6.403 max
Root diameter (in.)	5.459 max	5.510
Form diameter (in.)	5.6864	5.6415/5.7607[a]
Circular tooth thickness (in.)	0.3096	0.3096
Face width (in.)	0.50	0.50
Drive contact ratio	1.417	1.25
Torque (in. lb)	6000	6000
Calculated maximum contact stress (psi)	193,180	174,100 (−9.9%)

[a] Drive/coast flank parameter.

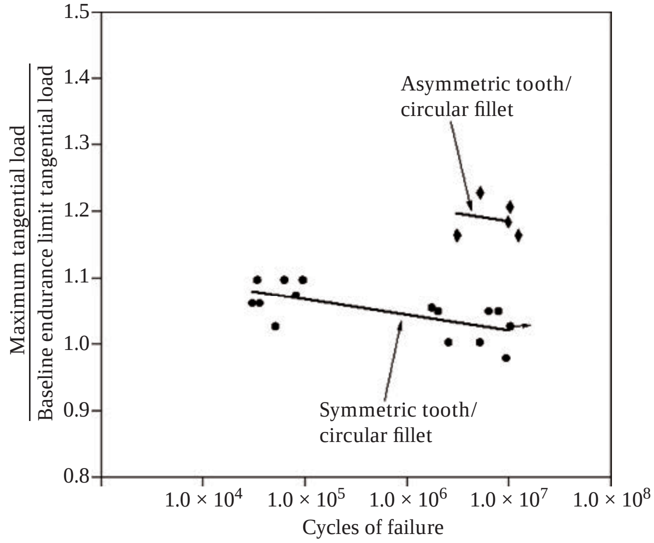


FIGURE 7.15 STBF data for asymmetric gears and optimized root llet gears along with baseline symmetric tooth/circular llet test data.

7.7 IMPLEMENTATION OF ASYMMETRIC TOOTH GEARS

The rst application of gears with asymmetric teeth in the aerospace industry was for the TV7-117S turboprop engine gearbox (Novikov et al., 2008). The engine and gearbox were developed by the Klimov Corporation (St. Petersburg, Russia) for a commuter airplane Ilyushin Il-114. The main characteristics of its gearbox are presented in Table 7.7.

The TV7-117S gearbox arrangement is shown in Figure 7.17. The rst planetary-differential stage has three planet gears. The second star-type coaxial stage has ve planet (idler) gears and a stationary planet carrier. The rst-stage sun gear is connected to the engine turbine shaft via spline. Its ring gear is connected to the second-stage sun gear and its planet carrier is connected to the second-stage ring gear and the output propeller shaft. This arrangement makes it possible to transmit about 33% of the engine power through the rst-stage carrier directly to the propeller shaft, bypassing the second stage. This allows for reducing the size and weight of the second stage, because it transmits only 67% of the engine power from the rst-stage ring gear to the second-stage sun

TABLE 7.7
TV7-117S Turboprop Engine Data

Data Parameters	Values
Input turbine RPM	17,500
Output prop RPM	1200
Total gear ratio	14.6:1
Overall dimensions (mm)	
Diameter	520
Length	645
Gearbox weight (N)	1050
Maximum output power (hp)	2800
Extreme output power (hp)	3500

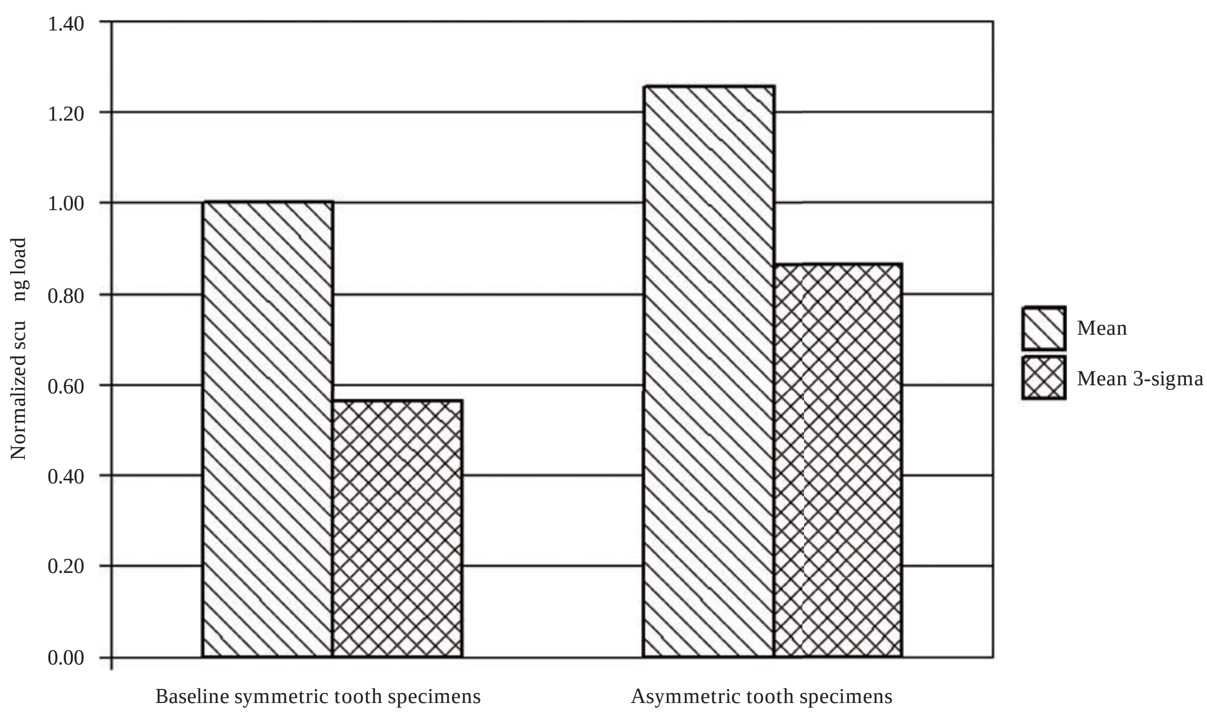


FIGURE 7.16 Results of baseline symmetric and asymmetric gear scuf ng tests.

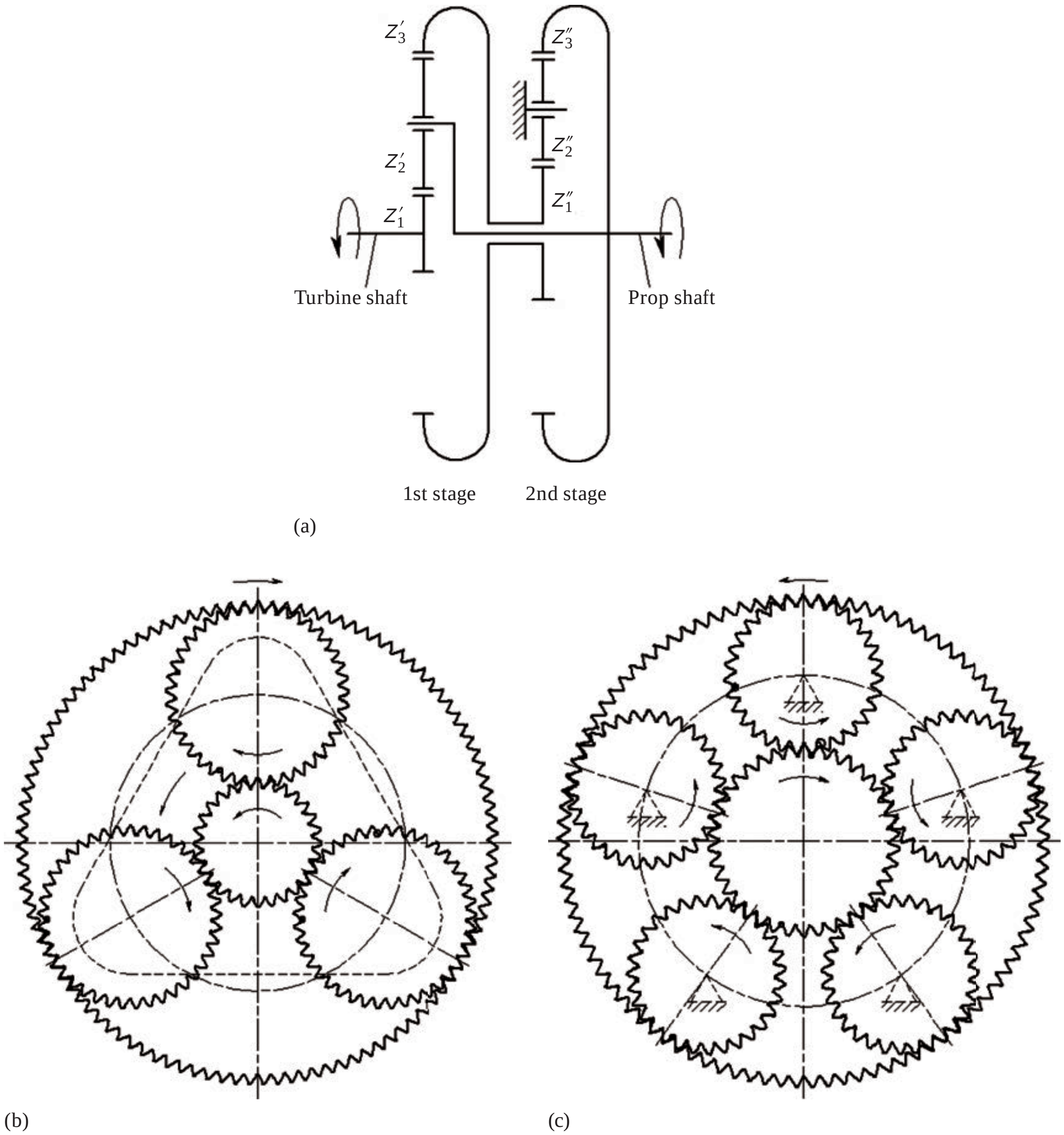


FIGURE 7.17 (a) Gearbox arrangement; (b) rst and (c) second stages with rotation directions (view from input shaft.)

gear, and then through the planets to the second-stage ring attached to the propeller shaft.

All gears have an asymmetric tooth pro le. Gear geometry parameters, operating torques and stresses are presented in Table 7.8.

All gears were made out of the forged blanks of the steel 20KH3MVF (EI-415). Its chemical composition includes the following: Fe, base material; C, 0.15%–0.20%; S, <0.025%; P: <0.030%; Si: 0.17%–0.37%; Mn: 0.25%–0.50%; Cr: 2.8%–3.3%; Mo: 0.35%–0.55%; W: 0.30%–0.50%; Co: 0.60%–0.85%; and Ni: <0.5%.

Application of the asymmetric teeth helped to provide extremely low weight to output torque ratio, signi cantly reduced the noise and vibration level, and cut down on the duration and expense of operational development (Novikov et al., 2008).

Photos of the gear assemblies of the TV7-117S gearbox are shown in Figure 7.18.

TABLE 7.8
Gear Geometry Data

	First Stage			Second Stage		
Gear	**Sun**	**Planet**	**Ring**	**Sun**	**Planet**	**Ring**
Number of gears	1	3	1	1	5	1
Nominal number of teeth	28	41	107	38	31	97
Module (mm)	3.000	3.000	3.000	3.362	3.362	3.362
Nominal drive pressure angle (°)	33	33, 25	25	33	33, 25	25
Nominal coast pressure angle (°)	25	25, 33	33	25	25, 33	33
Nominal pitch diameter (mm)	84.000	123.000	321.000	127.756	104.222	326.114
Drive base diameter (mm)	70.448	103.156, 111.476	290.925	107.145	87.408, 94.457	295.560
Coast base diameter (mm)	76.130	111.476, 103.156	269.213	115.786	94.457, 87.408	273.502
Tooth tip diameter (mm)	90.02/90.16	128.44/128.60	323.88/324.11	134.07/134.23	110.93/111.07	329.67/329.90
Root diameter (mm)	76.55/77.05	114.55/115.05	337.50/337.70	118.56/119.06	95.45/95.95	345.00/345.20
Tooth thickness at nominal pitch diameter (mm)	4.773/4.814	4.325/4.365	−0.667/−0.621	4.972/5.018	5.253/5.299	−1.104/−1.059
Face width (mm)	34.75/35.00	31.75/32.00	25.48/26.00	37.75/38.00	34.75/35.00	27.48/28.00
Center distance (mm)		103.50 ± 0.01			116.00 ± 0.01	
			Tolerance Analysis Results			
Operating drive pressure angle (°)	32.98/33.02		29.87/29.93	32.99/33.03		29.88/29.93
Operating coast pressure angle (°)	24.97/25.03		36.64/36.68	24.98/25.04		36.65/36.68
Operating dive contact ratio	1.18/1.26		1.33/1.36	1.20/1.28		1.35/1.38
Operating coast contact ratio	1.33/1.42		1.18/1.21	1.36/1.44		1.21/1.24
Operating normal backlash (mm)	0.196/0.322		0.197/.406	0.189/0.320		0.206/0.414
			Stress Analysis Results			
Maximum power (hp)				2800		
RPM	17,500	−11,132	–3063	–3063	3755	1200
Torque per mesh (N m)	374	548	1430	858	700	2191
Bending stress (MPa)						
Tension	240	257 280	298		318 345	371
Compression	−384	−403 −306	−355		−500 –385	–430
Contact stress (MPa)	960		604	1043		309

FIGURE 7.18 First- and second-stage assemblies.

8 Finite-Element Analysis of Gears

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8.1 MOTIVATION

Gears in modern applications operate ever closer to the limits of their performance. It has become imperative to have very precise, quantitative understanding of the state of load, stress, and impending failure in gears. Engineers responsible for making trade-offs between durability and noise, or between safety and weight need good tools for making informed decisions. Computational tools such as the boundary-element method (BEM) and the finite-element method (FEM) have become the tool of choice in such situations. BEM breaks up the surface of a gear into numerous discrete triangular or rectangular boundary elements for mathematical analysis. The FEM breaks up the gear volume into discrete tretrahedral and hexahedral finite elements.

The FEM is particularly advantageous when accounting for the effects of the following:

- Geometry and microgeometry: stress concentration due to fillet geometry, surface relief
- Material nonuniformity
- Load sharing between teeth and load distribution over the tooth face
- Thin rims
- Centrifugal loads, thermal expansion, press fits, and interference
- Dynamic loads, friction
- Manufacturing tolerance-related deviations

8.2 HOW IT WORKS: FINITE ELEMENT BASICS

The main idea behind the FEM is to break up a gear's volume into a large number of elemental volumes called finite elements (Figures 8.1 and 8.2). The response of the material within each element is written in terms of the displacement of the material at specific points called *nodes*. The potential and kinetic energies of the gear are expressed as functions of the nodal displacements. Finally, a suitable energy method such as the Lagrange equation is used to help create the equations that can be solved for the nodal displacements. The number of these equations can get quite large, and much technological effort has gone into efficiently solving these equations. Once nodal displacements are determined, most dependent values such as stresses and deformations anywhere within the gear can be obtained by simple *postprocessing* calculations.

Figure 8.3 shows a hexahedral finite element with its nodes marked by circles. A tetrahedral element is shown in Figure 8.4.

The deformation of the model is described by a displacement field $\bar{u}(P)$ which represents the three-dimensional displacement vector at any point of interest P:

$$\bar{u}(P) = (u_x,\ u_y,\ u_z)^T. \tag{8.1}$$

The FEM depends on the concept of shape functions. A shape function $N_i(P)$ is a weighting function for a node number i, and it describes the influence that that node has at the point P. These shape functions are zero everywhere except inside the volume of the finite elements that are connected to the node i. For linear Lagrangian finite elements, the shape functions are first-order polynomial functions of three element local coordinates. For parabolic elements, they are second-order polynomial functions. The functions are carefully chosen to meet accuracy, continuity, and compatibility requirements. Each node is assigned a three-dimensional nodal displacement vector $\bar{u}_i$:

$$\bar{u}_i = (u_{ix},\ u_{iy},\ u_{iz})^T. \tag{8.2}$$

The displacement field $\bar{u}(P)$ is assumed to be the weighted average of all the nodal displacement vectors $\bar{u}_i$ with the nodal shape functions $N_i(P)$ at the point of interest P as the weighting function:

$$\bar{u}(P) \approx \sum_i N_i(P)\bar{u}_i. \tag{8.3}$$

In compact matrix form, this weighting relationship can be written as

$$\bar{u}(P) \approx \mathbf{N}\cdot\mathbf{u}, \tag{8.4}$$

where $\mathbf{u}$ is a column vector containing all the individual nodal displacement vector components:

$$\mathbf{u} = (u_{1x}, u_{1y}, u_{1z}, u_{2x}, \ldots, u_{ix}, u_{iy}, u_{iz}, \ldots)^T, \tag{8.5}$$

and N is a large matrix containing the shape functions of the individual nodes:

$$\mathbf{N}(P) = \begin{bmatrix} N_i(P) & 0 & 0 \\ 0 & N_i(P) & 0 \\ 0 & 0 & N_i(P) \end{bmatrix}. \tag{8.6}$$

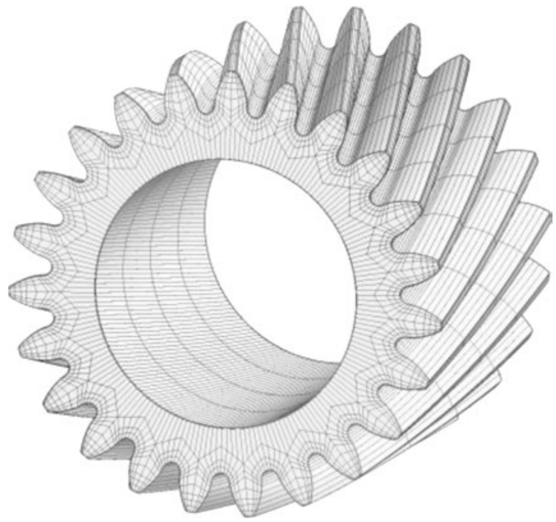


FIGURE 8.1 A nite-element model of a helical gear.

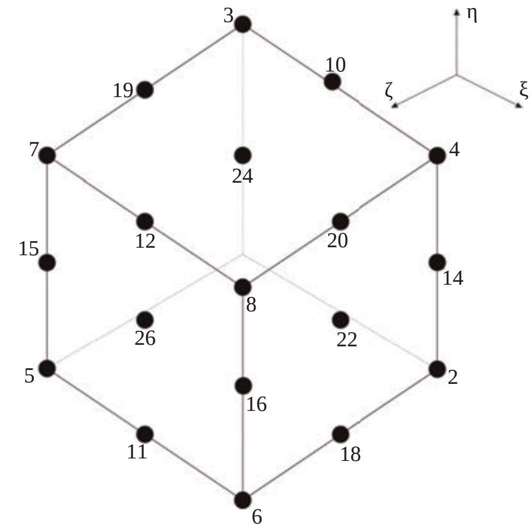


FIGURE 8.3 A parabolic hexahedral nite element.

Once the relationship between the deformation of the model and the nodal displacements is speci ed, the next item of interest is the strain $\bar{\varepsilon}(P)$ and the stress $\bar{\sigma}(P)$ at *P*:

$$\bar{\varepsilon}(P) = (\varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz}\Upsilon_{yz}\Upsilon_{xz}\Upsilon_{xy})^T, \tag{8.7}$$

$$\bar{\sigma}(P) = (\sigma_{xx}\sigma_{yy}\sigma_{zz}\tau_{yz}\tau_{xz}\tau_{xy})^T. \tag{8.8}$$

For small deformations, the strain $\bar{\varepsilon}(P)$ is simply related to various spatial derivatives of the displacement eld $\bar{u}(P)$:

$$\bar{\varepsilon}(P) = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \Upsilon_{yz} \\ \Upsilon_{xz} \\ \Upsilon_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \cdot \bar{u}(P). \tag{8.9}$$

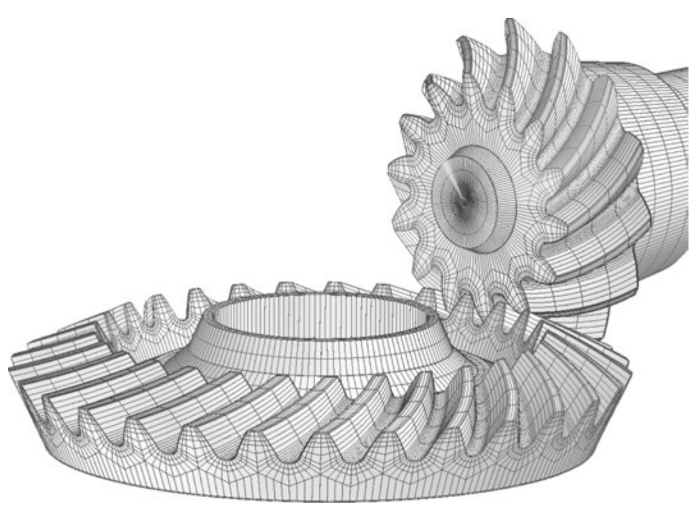


FIGURE 8.2 A nite-element model of a spiral bevel gear set.

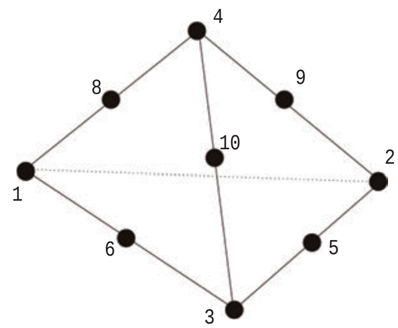


FIGURE 8.4 A parabolic tetrahedral nite element.

If we replace $\bar{u}(P)$ by the nite-element approximation in terms of the nodal displacements (Equation 8.4), the strains take the following matrix form:

$$\bar{\varepsilon}(P) \approx \mathbf{B}(P) \cdot u. \tag{8.10}$$

The matrix $\mathbf{B}(P)$ contains the spatial derivatives of the nodal shape functions at P:

$$\mathbf{B}(P) = \begin{bmatrix} & & & \frac{\partial N_i}{\partial x} & 0 & 0 & & & \\ & & & 0 & \frac{\partial N_i}{\partial y} & 0 & & & \\ & & & 0 & 0 & \frac{\partial N_i}{\partial z} & & & \\ \cdots & \cdots & \cdots & 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} & \cdots & \cdots & \cdots \\ & & & \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} & & & \\ & & & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 & & & \end{bmatrix}. \tag{8.11}$$

The stress $\bar{\sigma}(P)$ linearly depends on the strain $\bar{\varepsilon}(P)$ through Hooke's law for a linear elastic material undergoing small deformations:

$$\bar{\sigma}(P) = \mathbf{D} \cdot \bar{\varepsilon}(P), \tag{8.12}$$

$$\bar{\sigma}(P) = \mathbf{D} \cdot \mathbf{B} \cdot u. \tag{8.13}$$

This relationship is not valid if the material is in its plastic deformation regime, or the material is undergoing large displacements. Fortunately, most gear applications fall well within the bounds of these restrictions. The matrix $\mathbf{D}$ for an isotropic material is

$$\mathbf{D} = \begin{bmatrix} E_{11} & E_{12} & E_{12} & & & \\ E_{12} & E_{11} & E_{12} & & 0 & \\ E_{12} & E_{12} & E_{11} & & & \\ & & & G & 0 & 0 \\ & 0 & & 0 & G & 0 \\ & & & 0 & 0 & G \end{bmatrix}. \tag{8.14}$$

The constants E_{11} and E_{12} are

$$E_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \tag{8.15}$$

and

$$E_{12} = \frac{E\nu}{(1+\nu)(1-2\nu)}. \tag{8.16}$$

E is Young's modulus, v is Poisson's ratio, and G is the shear modulus:

$$G = \frac{E}{2(1+\nu)}. \tag{8.17}$$

In order to form the equations of motion for the nite-element model, we rst need expressions for the potential and kinetic energies of the model. Then we can use the Lagrange equation to get the equations of motion. The strain energy carried per unit volume of deformed material at any P inside the nite element is simply

$$\frac{1}{2}\bar{\sigma}^T\bar{\varepsilon}. \tag{8.18}$$

If this strain energy density is integrated over the entire volume V of the model, we obtain an expression for the total potential energy of the model in its deformed state:

$$U = \frac{1}{2}\int \bar{\sigma}^T\bar{\varepsilon}\, dV. \tag{8.19}$$

In matrix form,

$$U = \frac{1}{2}u^T\left(\int \mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B}\, dV\right)u, \tag{8.20}$$

or

$$U = \frac{1}{2}\mathbf{u}^T \cdot \mathbf{K} \cdot \mathbf{u}. \tag{8.21}$$

The matrix $\mathbf{K}$,

$$\mathbf{K} = \int \mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B}\, dV, \tag{8.22}$$

is called the *stiffness matrix* of the model.

The kinetic energy per unit volume at point P is

$$\frac{1}{2}\rho\bar{u}^T\bar{u}\ . \tag{8.23}$$

The overdot represents the time derivative of nodal displacements, or the nodal velocity. Again, we integrate over the entire volume V of the model to get an expression for the total kinetic energy:

$$T = \frac{1}{2}\int \rho\bar{u}^T\bar{u}\, dV. \tag{8.24}$$

In matrix form,

$$T = \frac{1}{2}u^T\left(\int \rho\mathbf{N}^T \cdot \mathbf{N}\, dV\right)u \tag{8.25}$$

or

$$T = \frac{1}{2}\mathbf{u}^T \cdot \mathbf{M} \cdot \mathbf{u}. \tag{8.26}$$

The matrix

$$\mathbf{M} = \int \rho\mathbf{N}^T \cdot \mathbf{N}\, dV \tag{8.27}$$

is called the *mass matrix* of the finite-element model. Now that we have expressions for the potential and kinetic energies, we may use Lagrange equation to obtain an equation of motion for the finite-element model. Lagrange equation consists of derivative of the potential and kinetic energies with respect to the nodal displacements and the velocities:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \mathbf{u}}\right) - \frac{\partial T}{\partial \mathbf{u}} + \frac{\partial U}{\partial \mathbf{u}} = \mathbf{f}. \tag{8.28}$$

Here, **f** is a column vector containing externally applied forces at the finite-element nodes:

$$f = (f_{1x}, f_{1y}, f_{1z}, f_{2x}, \ldots, f_{ix}, f_{iy}, f_{iz}, \ldots)^T. \tag{8.29}$$

On substituting the expressions from Equations 8.21 and 8.26 into the Lagrange equation (Equation 8.28) to give the equation of motion

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{K} \cdot \mathbf{u} = \mathbf{f}. \tag{8.30}$$

Frequently, an additional viscous damping term $\mathbf{C}u$ is added:

$$\mathbf{M} \cdot \mathbf{u} + \mathbf{C}\mathbf{u} + \mathbf{K} \cdot \mathbf{u} = \mathbf{f}. \tag{8.31}$$

C is called the damping matrix. A common method for estimating the damping matrix **C** is to use a combination of the mass and stiffness matrices:

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}. \tag{8.32}$$

The constants α and β are called *Rayleigh* coefficients and are usually empirically adjusted until the vibration energy dissipation of the model resembles that of a real gear. Frequently, we are interested in only the static response of a gear. In that case, the mass and damping terms may be dropped to obtain the static stiffness equation:

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{f}. \tag{8.33}$$

8.3 SOLVING THE SYSTEM OF EQUATIONS

If the forces acting on a gear are known, then the column vector **f** is known and this system of linear equations can be solved for the nodal displacement column vector **u**. Once the nodal displacements are available, stresses and strains are easy to obtain by substituting for **u** in Equations 8.10 and 8.12 to get $\bar{\varepsilon}(P)$ and then $\bar{\sigma}(P)$ at any point of interest P.

As the number of teeth in a gear increases, the size of the matrix **K** in the static system of equations in Equation 8.33 can be quite large, of the order of hundreds of thousands, or even millions of unknowns. For such large systems, simple direct solution methods like Gaussian elimination get unwieldy, and the time a computer needs to solve it can get extremely large. To circumvent this problem, modern finite-element solvers use iterative solvers which are well suited for use with parallel computers.

Frequently, the force vector **f** on the right hand side of Equation 8.33 is not known beforehand because it depends on how the tooth contact forces are distributed among gear teeth, and along the face width of the gears. In such cases, a special contact solver needs to be used. These contact solvers solve the system of equations in Equation 8.33 repeatedly, moving the forces around until a valid force distribution is found. The contact solver places an even larger burden on the linear equation solvers. In this case, even parallel computers are not sufficiently fast. Substructuring techniques may be used to make the computations manageable.

8.4 SUBSTRUCTURING

The term *substructuring* or *superelements* refer to a technique of breaking down a large stiffness equation system into several smaller ones. The substructuring methods often yield large savings in computer time, especially when the finite-element model has symmetry that can be exploited. It is possible to build a stiffness relationship like Equation 8.33 for a single tooth shown in Figure 8.5.

There are a few nodes of the tooth model that interact with elements of adjacent teeth. We designate these nodes as *master* nodes. Nodes in this model that do not connect with elements of any other tooth are designated as *slave* nodes. If we rearrange the rows and columns in the system of Equation 8.33 so that all the slave nodes appear first followed by all the master nodes:

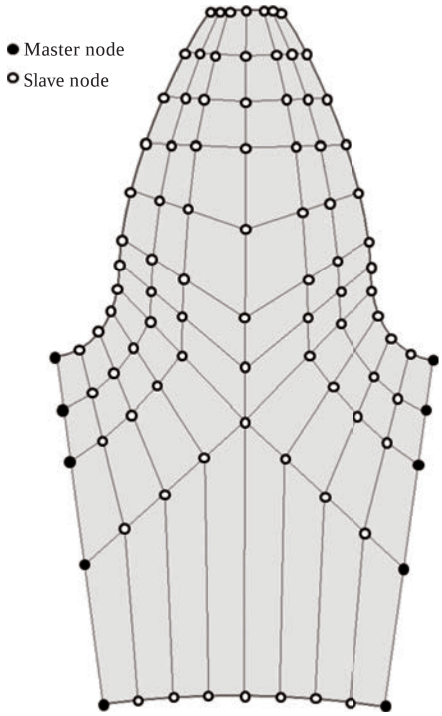


FIGURE 8.5 Nodes of nite-element model of a gear tooth separated into master nodes (dark circles) and slave nodes (light circles).

$$\mathbf{u} = \begin{Bmatrix} \mathbf{u}_s \\ \mathbf{u}_m \end{Bmatrix}, \tag{8.34}$$

$$\mathbf{f} = \begin{Bmatrix} \mathbf{f}_s \\ \mathbf{f}_m \end{Bmatrix}, \tag{8.35}$$

and

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sm} \\ \mathbf{K}_{ms} & \mathbf{K}_{mm} \end{bmatrix}, \tag{8.36}$$

then we get a partitioned system of equations:

$$\begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sm} \\ \mathbf{K}_{ms} & \mathbf{K}_{mm} \end{bmatrix} \cdot \begin{Bmatrix} \mathbf{u}_s \\ \mathbf{u}_m \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_s \\ \mathbf{f}_m \end{Bmatrix}. \tag{8.37}$$

If we use the top partition to isolate the upper slave displacement vector $\mathbf{u}_s$,

$$\mathbf{u}_s = \mathbf{K}_{ss}^{-1} \cdot (\mathbf{f}_s - \mathbf{K}_{sm} \cdot \mathbf{u}_m), \tag{8.38}$$

and substitute for $\mathbf{u}_s$ in the lower master partition, we get

$$\mathbf{K}_{ms} \cdot \mathbf{K}_{ss}^{-1} \cdot (\mathbf{f}_s - \mathbf{K}_{sm} \cdot \mathbf{u}_m) + \mathbf{K}_{mm} \cdot \mathbf{u}_m = \mathbf{f}_m \tag{8.39}$$

or

$$(\mathbf{K}_{mm} - \mathbf{K}_{ms} \cdot \mathbf{K}_{ss}^{-1} \mathbf{K}_{sm}) \cdot \mathbf{u}_m = \mathbf{f}_m - \mathbf{K}_{ms} \cdot \mathbf{K}_{ss}^{-1} \mathbf{f}_s, \tag{8.40}$$

We can de ne two new matrices $\mathbf{K}_{mm}^{*}$ and $\mathbf{f}_m^{*}$ as

$$\mathbf{K}_{mm}^{*} = \left(\mathbf{K}_{mm} - \mathbf{K}_{ms} \cdot \mathbf{K}_{ss}^{-1} \cdot \mathbf{K}_{sm}\right), \tag{8.41}$$

$$\mathbf{f}_m^{*} = \mathbf{f}_m - \mathbf{K}_{ms} \cdot \mathbf{K}_{ss}^{-1} \cdot \mathbf{f}_s. \tag{8.42}$$

Then Equation 8.40 turns into a form that is remarkably similar to the original system of equations in Equation 8.33:

$$\mathbf{K}_{mm}^{*} \cdot \mathbf{u}_m = \mathbf{f}_m^{*}. \tag{8.43}$$

This equation is called a condensed system of equations, and the process of obtaining it from Equation 8.33 is called *static condensation*. The matrix $\mathbf{K}_{mm}^{*}$ is called the condensed stiffness matrix, and the column vector $\mathbf{f}_m^{*}$ is called the condensed force vector. Solving Equation 8.44 gives us the solution $\mathbf{u}_m$ for the master nodes. The solution $\mathbf{u}_s$ for the slave nodes can then be obtained by using Equation 8.38.

In most situations, the number of master nodes is much smaller than the number of slave nodes. The condensed system of equations in Equation 8.43 is much smaller and faster to solve than the original system of equations in Equation 8.33. It is possible to build out the stiffness equation for entire gears like those in Figures 8.1 and 8.2 by rst condensing the matrices for individual tooth matrices into a single matrix for the entire gear. The resulting gear system is much smaller and more ef cient than a system of equations for the same gear without the use of static condensation.

8.5 DEFORMATION MODEL

In gears, the contact forces act on a very narrow strip. One drawback of the FEM is that the nite elements do not perform well in the vicinity of concentrated loads. The stress and the deformations can contain very large errors when sampled at a point one or two element lengths away from narrow contact zones as shown in Figure 8.6. Inaccuracies in local deformation can cause the contact force predictions from a contact solver to become erratic. For good results, at least 10 elements need to span the width of the contact zone. But using such a high level of re nement would imply either that the entire tooth model be extremely ne, or that a new mesh be created each time the position of contact changes.

This is a fundamental shortcoming of the FEM. The work-around is to use the nite-element model to calculate deformations at a safe distance away from the contact forces

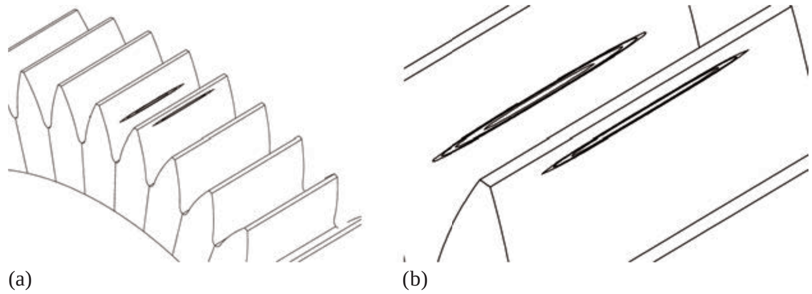


FIGURE 8.6 Contour lines of contact pressure inside an instantaneous contact zone on a crowned spur gear. The width of the contact zone is much narrower than the height of a tooth. A close-up view of (a) is (b).

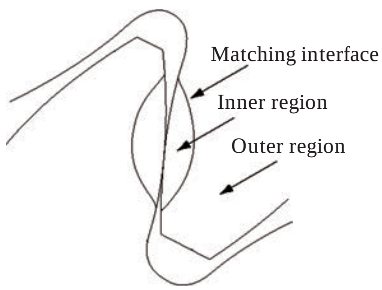


FIGURE 8.7 Finite-element results are extracted in the outer region at a safe distance away from the contact forces, and combined with a more suitable method in the inner region around the contact.

(the outer region), and use a more appropriate method to calculate the deformation within the intermediate volume, (the inner region), as shown in Figure 8.7. The Hertz deformation formula is frequently used for this purpose. Others use the Boussinesq solution, which describe the deformation of an elastic half space under point load.

8.6 GEOMETRY

The forces acting in gears are extremely sensitive to the geometry of the teeth. Geometric corrections called surface modifications are often introduced by gear designers in order to adjust the force distribution over the tooth surface, and to allow for a smoother entry and exit of the teeth. These surface modifications are very small in magnitude compared to the size of the finite elements. However, they have noticeable impact on the performance of the gears. In order for the finite-element model to be useful to the gear designer, it has to be sensitive to these surface modifications, and accurate in its prediction of their effects. Much care must be exercised when building the gear model in selecting the order of the elements. They must be of sufficient geometric fidelity.

8.7 CONTACT SOLVER

If the forces acting on a gear finite-element model are known, then the deflections and the stress can be calculated simply by solving the linear system of Equation 8.33 or 8.43. But this is very seldom the case. The total torque flowing through the gear set is known, but it is not sufficient to set up the nodal forces on the finite-element model. This is where a contact solver is useful. The contact solver takes information about the torque, uses the stiffness equations Equation 8.33 or 8.43, and simultaneously calculates the nodal forces and deformations.

There are several different mathematical implementations of contact solvers. Penalty methods, Lagrange multiplier methods, and linear programming methods are the most commonly used methods. The best choice for a contact solver will depend on its stability, accuracy, and computational efficiency. The third category of solvers, based on the linear programming, has been found to be most useful for gear modeling.

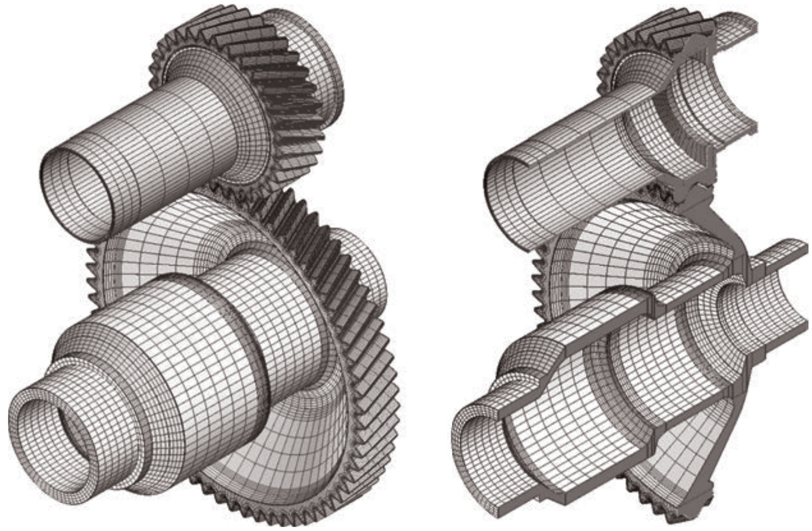
8.8 POSTPROCESSING

An example of a very lightweight gear design that needs to be processed is shown in Figure 8.8. It has a finite-element mesh that is refined near the gear fillets, and is progressively coarsened away from the teeth.

After the model is processed by the contact solver, it can be postprocessed. This postprocessing step converts the finite-element results into metrics that can be used by the gear designer. The instantaneous contact force distribution calculated by the contact solver is shown in Figure 8.9.

The pressure distribution can tell the designer how successful the surface modifications were. A close examination of Figure 8.9 would tell the designer that the abrupt start of tip relief had the unintended consequence of increasing the curvature and the pressure at the start of relief, even though it successfully relieved pressure near the tip. A plot of the exaggerated deformation (Figure 8.9) can be examined for further clues to explain the pressure distribution. The stress distribution (Figure 8.10) is useful to determine whether there are any locations on the gear that might yield due to high stress. Only the tensile stress (maximum principal normal stress) (Figure 8.11) is displayed in the figure, but the finite-element model can provide the designer with many other stress metrics, including compressive (minimum principal normal stress), maximum shear stress and von Mises stress. Only the instantaneous stress and the contact pressure distribution snapshots have been shown, but if the model is run over several closely spaced time instances, then consecutive snapshots can be used to capture a contact pattern (Figure 8.12).

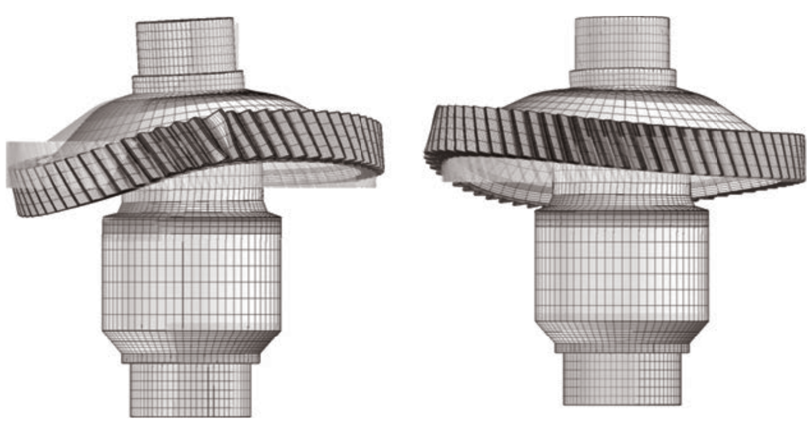
The consecutive time snapshots can also be used to examine the stress history experienced by each individual point in



(a) (b)

FIGURE 8.8 Finite-element model of a lightweight gear design. (a) Full model and (b) cross section of model.

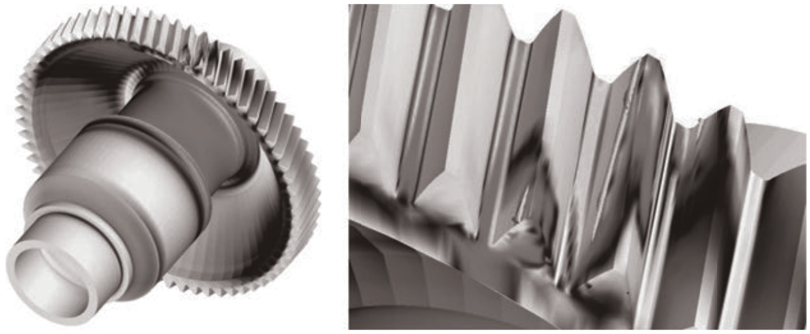
FIGURE 8.9 The distribution of contact pressure acting between the gear and the pinion, after the finite-element equations were processed by the contact solver.



(a) (b)

FIGURE 8.10 The deformed shape of the gear and the pinion. The deformation has been exaggerated by a factor of 400. (a) Top view and (b) side view.

the gear fillets. From the stress history, it is possible to calculate the alternating stress and mean stress. This calculation can be carried out at a large number of points spread over the entire fillet of the gear tooth. If all the calculations are then plotted on a modified Goodman diagram (Figure 8.13), then valuable insights can be obtained about the amount of fatigue damage accumulated due to the loading. If the finite-element model is run for each load level spanning its duty cycle, then Miner's rule can be used to predict the total damage and whether or not the gear can survive the duty cycle.



(a) (b)

FIGURE 8.11 The distribution of tensile (maximum principal normal) stress on the gear. The close-up view of (a) is (b).

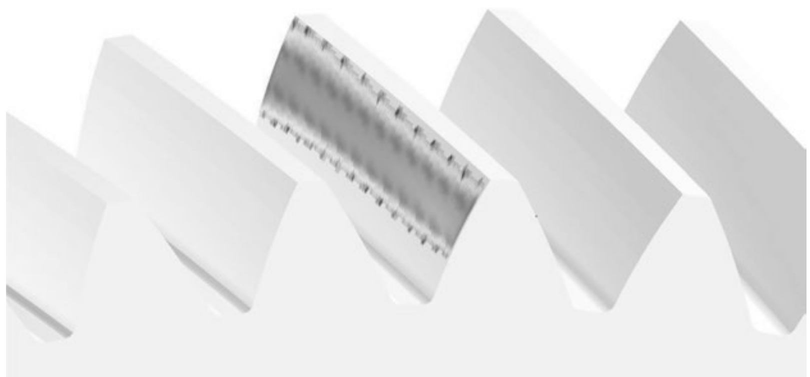


FIGURE 8.12 A contact pattern obtained by stitching together pressure distribution data from several closely spaced snapshot.

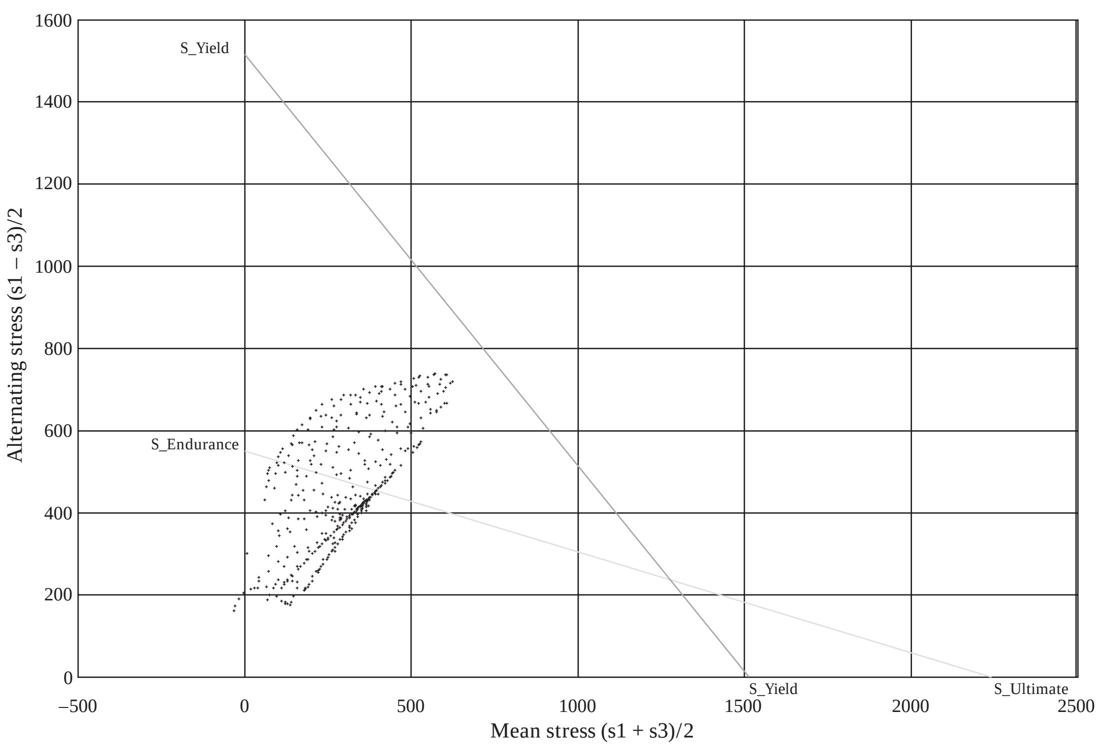


FIGURE 8.13 A modi ed Goodman diagram containing samples of alternating and mean stress at each point in the tensile llet of a gear tooth nite element model.

9 Load Rating of Gears

9.1 CONSIDERATIONS

In the present state of the art, gear capacity ratings must be considered only as an estimation. This is true because the gear metallurgy and the application conditions, such as load histograms, gear dynamics, and tooth alignment, can differ from estimations and influence both the tooth strength and the surface resistance of gears. Roughness and irregularities of the tooth fillet may reduce the tooth strength, whereas tooth surface finish and lubrication conditions influence the surface resistance in various ways.

The present rating procedures cover most influences by means of proper "factors" so that they can serve as a guideline for designers and enable them to achieve reliable designs. The rating systems are only a means of applying the designer's competence, not a substitute for it.

For instance, detailed calculations of the geometry factors for pitting resistance are given. They may be complicated, but with a computer they can be calculated in a few seconds at most. They should not be considered more important because they are detailed. A proper choice of the application factors is often much more important for the final result and for the success of the gear pair than the method used.

The choices of design application factors must be based on experience or searched in specific books for gear design, such as this book.

Here it is emphasized that the choice must be consistent. For example, common spur gears are usually inconsistent with high speed, and helical gears are inconsistent with low tooth accuracy. These types of inconsistencies can lower the reliability of gear ratings.

In this chapter,

RH—resistance to Hertzian pressure, that is, in general, pitting resistance. But the pressure limitation usually averts or limits other surface failures too, such as progressive wear or cold scoring. An approximate estimation for hot scoring for industrial gears is given.

RF—resistance of the field of the tooth root, or so-called *tooth strength*.

RHA and RFA—ratings based fundamentally on AGMA's methods reported in the standards that substantially coincide except for a pair of items that are clarified.

RHI and RFI mean that the original ISO method is taken as a basis. The DIN method has the same general conception as ISO's; some peculiarities are reported when important. The ISO method is discussed in Nieman and Winter (1983), Henriot (1981), and Castellani and Zanotti (1980).

All the above-mentioned methods relate to involute gear teeth. Simplifications or adaptations of any syntheses are specified.

9.2 MAIN NOMENCLATURE

AGMA symbols are used as far as possible for ISO factors, when the meaning is the same. Symbols that occur in a single item and ISO symbols are specified when necessary. The units are included in the formulas.

A, B—generic coefficients or exponents
A_H, A_F—adaptation factors for RH, RF
C—center distance
C_H, C_f, C_S, C_L, C_R—RH factors for hardness, finish, size, life, and reliability, respectively
C_P—pressure ratings: elastic coefficient
C_{SF}—service factor for RH
d_b—base diameter
d_o—tooth outside diameter, that is, gear outside diameter for external or gear internal diameter for internal gears
d_P, d_G—operating pitch diameters of pinion and gear, respectively
E—Young's elasticity modulus
F—net face width, that is, overlap face width
f—surface finish, that is, roughness, arithmetic average
f_d—ratio of the faced width over the pinion pitch diameter, $f_d = F/d_P$
G_H—unified geometry factor for RH
I, J—AGMA's geometry factors for RH and RF
J_n—unified geometry factor for RF, related to normal module or diametral pitch
K—synthetic surface loading factor for RH
K_{sh}, K_a, K_v, K_m—overload derating factors, each one ≥ 1, for power sharing, application, gear dynamics (velocity), and load distribution (misalignment), respectively
K_f, K_s, K_L, K_R—RF factors for fillet notch, size, life, and reliability, respectively
K_{SF}—service factor for RF
K_y—coefficient of the resistance to yielding for RF ($K_y < 1$)
m, m_n—transverse and normal modules, nominal
m_G—gear ratio ($m_G = N_G/N_P$)
m_P, m_F—transverse (profile) and face contact ratios
m_{pE}, m_{pA}—operating addendum contact ratios of pinion and gear (Equations 9.46 and 9.47)
m_e, m_N—contact line coefficient, load-sharing ratio
max (x, y), min (x, y)—maximum and minimum of two numbers
N—number of tooth loading cycles or tooth number
N_G, N_P—tooth numbers of gear and pinion ($N_G \geq N_P$)
n—revolutions per minute

P—power
p_{bt}—transverse base pitch
p_d, p_{nd}—transverse and normal diametral pitches, nominal
Q_H, Q_F—loading factors for RH and RF, respectively
S_2—S_2 = 1 for external; S_2 = −1 for internal gear or gear pair
s_c—contact pressure, that is, Hertzian pressure
s_t—stress at tooth fillet (notch effect included)
T—torque
U_L—unit load for RF
V—overall overload derating factor ($V \geq 1$)
v_t—tangential velocity
W_t—operating tangential load
X_B—curvature coefficient at the *lower point of single contact* (*LPSC*) of pinion
x_G, x_P—addendum modification coefficients of gear and pinion with reference to m_n or P_{nd}
v_{40}—kinematic viscosity in centistokes at 40°C
ϕ_{ns}—normal standard pressure angle
ϕ_s—transverse standard pressure angle
ϕ_t—transverse operating pressure angle
ψ_b—base helix angle
ψ_s—standard helix angle
ψ—operating helix angle

Subscripts

G, P—gear or pinion
H, F—RH or RF
lim—relating to a conventional fatigue limit
m—relating to the middle of the net face width for bevel gears
v—for bevel gears, relating to virtual gears with parallel axes
V—relating to the vertex of the life curve for the conventional fatigue limit
W—relating to the maximum load or pressure or stress of a life curve
y—relating to yield

For definitions of RH, RHA, RHI, RF, RFA, and RFI, see Section 9.1.

9.3 COPLANAR GEARS (INVOLUTE PARALLEL GEARS AND BEVEL GEARS)

The ratings are basically presented for involute gears with parallel axes. As proposed by Radzevich (2012a), gears with parallel axes, that is, spur gears, helical gears, and so forth, are referred to as *parallel-axis gears* (or P_a *gears*, for simplicity); gears with intersected axes, that is, straight bevel gears, spiral bevel gears, and so forth, are referred to as *intersected-axis gears* (or I_a *gears*, for simplicity); and gears with crossing axes, that is, hypoid gears, Spiroid gears, worm gears, and so forth, are referred to as *crossed-axis gears* (or C_a *gears*, for simplicity).

Adaptations are given for bevel gears.

9.3.1 Power, Torque, and Tangential Load

The pinion torque T_P can be given directly; otherwise, it is deduced from the power:

$$T_P\ (\mathrm{N\,m}) = \frac{9549.3P\ (\mathrm{kW})}{n_P\ (\mathrm{rpm})}, \tag{9.1}$$

$$T_P\ (\mathrm{lb\ in.}) = \frac{63.025P\ (\mathrm{hp})}{n_P\ (\mathrm{rpm})}. \tag{9.2}$$

The ratings are based on the operating tangential load W_t. For gears with parallel axes, the operating pitch diameters are

$$d_P = \frac{2C}{m_G + S_2}, \tag{9.3}$$

$$d_G = m_G d_P. \tag{9.4}$$

Hence,

$$W_t\ (\mathrm{N}) = 2000\left[\frac{T_P\ (\mathrm{N\,m})}{d_p\ (\mathrm{mm})}\right], \tag{9.5}$$

$$W_t\ (\mathrm{lb}) = 2\left[\frac{T_P\ (\mathrm{lb\ in.})}{d_p\ (\mathrm{in.})}\right]. \tag{9.6}$$

9.3.1.1 Loading Levels

It is easy to calculate the tangential load, but the definition of the power or torque to be introduced in formulas is sometimes the most difficult task for gear designer. The loading conditions are often variable. If load histograms of similar applications are available, or if they can be foreseen even summarily, it is useful to consider various loading levels where not only the load but also the rotation velocity can change. In this case a so-called *equivalent load* sometimes is calculated, but this is risky. In fact, if a nondamaging load is applied for a higher number cycles, this leads to underestimating the equivalent load. Therefore, it is better to assess separate damages and culminate them by an approximate approach according to *Miner's rule*.

If, on the contrary, a single main loading condition is identified, then the simple calculation of the service factor is often preferred. This includes the consideration of possible short higher loads and rules out further estimations of life and reliability.

9.3.2 RH and RF, Resistance to Yielding

Both the flank and the root of a gear tooth can yield or break if an excessive load induces plastic deformations.

9.3.3 RH and RF, Life Curves

Both surface and root resistance are fatigue resistances and can be approached in similar ways provided that no yielding occurs.

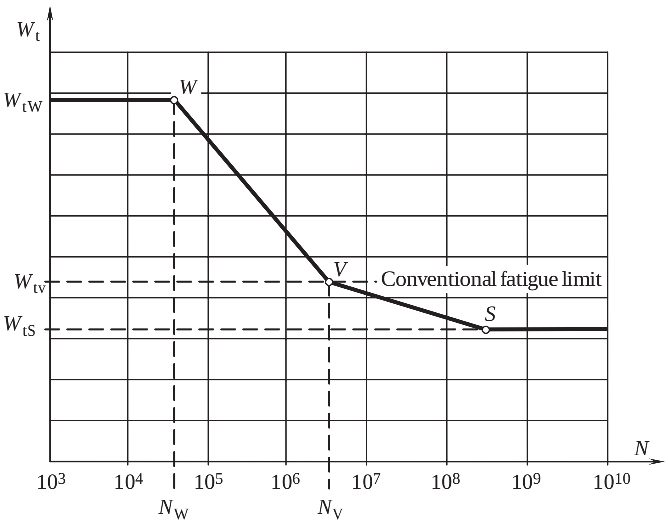


FIGURE 9.1 Schematized life curve (N, W_t.)

In Figure 9.1, a generic life curve is schematized. The initial maximum load W_{tW} (that is, W_{tWH} or W_{tWF}) should mean yielding, although separate estimates are preferred for practical reasons.

A two-slope curve W_V, V_S is used for life predictions. The second sloped stretch has been introduced by AGMA and is not de ned by ISO, but the concept can be used for RHI and RFI life ratings. AGMA does not de ne the vertex S but there are some who consider that no failure occurs below a certain load W_{tS}. This is not con rmed experimentally, but it can be adopted as a criterion in the life range speci ed for a gear pair.

In the case of slow gears in boundary lubrication regime, the progressive wear can become a more important item than pitting for tooth ank damage, but the load limitation implied by the reference to life curves also helps in reducing wear.

9.3.4 RH and RF, Conventional Fatigue Limits

Conventional fatigue limits of both Hertz pressure and root bending stress are obtained either from eld experiments or from laboratory tests. These lead to a conventional fatigue limits $W_{t\,lim\,H}$ or $W_{t\,lim\,F}$ of the load by excluding reliability and life factors from the ratings. They usually correspond to vertex V except for RFA, where a small difference follows the data of the life factor according to AGMA or ANSI/AGMA standards.

The gear capacity rating stops here if a single loading level is considered and then service factors that exclude life calculations can be assessed. Otherwise, the numerical de nition of the life curves provides damage, life, and reliability ratings.

9.3.5 RH—Synthetic Surface Loading Factor *K*

A factor K, whose square root is proportional to the Hertzian pressure, is de ned as follows:

$$K\ (\text{N/mm}^2) = \frac{W\ (\text{N}) \times [1 + S_2 (N_P / N_G)]}{d_P\ (\text{mm}) \times F\ (\text{mm})}, \qquad (9.7)$$

$$K\ (\text{lb/in.}^2) = \frac{W\ (\text{lb}) \times \left[1 + S_2 (N_P / N_G)\right]}{d_P\ (\text{in.}) \times F\ (\text{in.})}. \qquad (9.8)$$

9.3.6 RF—Unit Load U_L

A unit load U_L can be used for tooth bending strength:

$$U_L\ (\text{N/mm}^2) = \frac{W_t\ (\text{N})}{F\ (\text{mm}) \times m_n\ (\text{mm})}, \qquad (9.9)$$

$$U_L\ (\text{lb/in.}^2) = \frac{W_t\ (\text{N})}{F\ (\text{in.}) \times m_n\ (\text{in.})}. \qquad (9.10)$$

ISO allows the F value to be increased for a gear (pinion or gear) that has a face width greater than the net one. The maximum permitted increase is equal to 1 module at each gear side.

9.3.7 Adaptation for Bevel Gears (W_t, K, U_L)

The ratings are related to the middle of the net face width. This contributes a small supplementary margin for reliability, as the result of the distributed load applies to a point somewhat more external (for perfectly aligned teeth). The corresponding mean pitch diameters are shown in Figure 9.2:

$$d_{Pm} = d_P - F \sin \gamma_P, \qquad (9.11)$$

$$d_{Gm} = d_G - F \sin \gamma_G, \qquad (9.12)$$

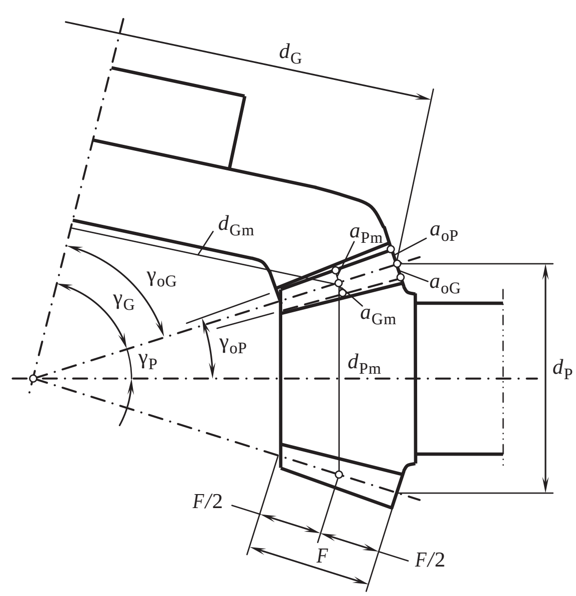


FIGURE 9.2 Geometrical data of a bevel gear pair for load capacity ratings.

where γ_P and γ_G are the pitch angles of pinion and gear. The main pitch diameter of pinion d_{Pm} must be introduced in Equations 9.5 and 9.6 instead of d_P for calculating W_t.

A virtual gear pair with parallel axes must be considered for capacity ratings (pressure angle and helix angle are maintained). Virtual tooth numbers N_{Pv} and N_{Gv}, and virtual pitch diameter d_{Pmv} are introduced in the K (Equations 9.7 and 9.8) for RH instead of N_P, N_G, and d_P:

$$N_{Pv} = \frac{N_P}{\cos\gamma_P}, \tag{9.13}$$

$$N_{Gv} = \frac{N_G}{\cos\gamma_G}, \tag{9.14}$$

$$d_{Pmv} = \frac{d_{Pm}}{\cos\gamma_P}. \tag{9.15}$$

Note that $N_{Pv}/N_{Gv} = (N_P/N_G)^2$ if $\gamma_P + \gamma_G = 90°$.

A mean normal module m_{nm} or diametral pitch P_{ndm} are introduced in the U_L (Equations 9.9 and 9.10) for RF instead of m_n and P_{nd}:

$$m_{nm} = \frac{m\cos\psi_s d_{Pm}}{d_p}, \tag{9.16}$$

$$P_{ndm} = \frac{P_d d_P}{d_{Pm}\cos\psi_s}, \tag{9.17}$$

where $m = d_p/N_P$ and $P_d = N_P/d_p$ are the usual nominal module or diametral pitch with reference to Figure 9.2 for the nominal pinion pitch diameter d_P, as operating and standard diameters coincide for bevel gears.

Example 9.1: Bevel Gear Pair

N_P = 13; N_G = 38; m = 5.08 or P_d = 5; ψ_S = 35°; F = 30.48 mm = 1.2 in.; T_P = 320 N m = 2832 lb in.; γ_P = 18.8861°; γ_G = 71.1139°; d_P = 66.04 mm = 2.6 in.; d_{Pm} = 56.174 mm = 2.2116 in.; W_t = 11.393 N = 2561 lb; d_{Pmv} = 59.37 mm = 2.337 in.; K = 7.03 N/mm^2 = 1020 lb/in.^{2}; m_{nm} = 3.5396 or P_{ndm} = 7.1759; U_L = 105.6 N/mm^2 = 15.320 lb/in.^{2}

A simpler approach to the K calculation for 90° bevel gear pairs is given by Dudley (1970), which calculates W_t at the middle point of the face width as in the foregoing, but introduces outer d_P diameter and real tooth numbers in the K formula. It is most useful especially for immediate estimation in connection with special tables for bevel gears and gives results that are not too different, especially for higher gear ratios.

The K rating based on the virtual tooth numbers is useful especially for programmed computations. The real d_{Pmv} diameter must be used for calculation of W_t! The same program can be used for bevel gears as well as cylindrical gears for unification reasons, and the same K tables can be consulted. The special tooth alignment conditions of bevel gears must be taken into account when assessing the load distribution factor K_m.

Specific methods for bevel gears are given in ANSI and DIN standards and are being elaborated by ISO.

9.3.8 Use of the Synthetic Factors K and U_L

The K and U_L factors serve for the first step in a detailed capacity analysis, as well as for simplified estimates or for the first tentative assumptions of the size of gear pairs in new designs.

9.4 COPLANAR GEARS: SIMPLIFIED ESTIMATES AND DESIGN CRITERIA

9.4.1 Direct Assumption of the Synthetic Factors

Table 9.1 gives values of K as used in industrial gears with case or induction surface hardening. The table can serve as an estimation. The K values as used in industry often fail to satisfy detailed capacity analyses. Wider tables are given in specific books for gear design.

Gears without surface hardening have far lower K values and are somewhat more uncertain as they are more sensitive to both tooth flank roughness and load histogram. A summary estimation of acceptable K values can be made in advance if specific field experience is available. Otherwise, detailed ratings of similar cases can give useful indicators.

The unit load U_L for RF varies in ampler ranges. For instance, U_L values in the range of 40 to 170 N/mm^2 (6000 to 25,000 lb/in.^{2}) can be found for case-hardened gears. Note that the highest U_L values of this range often do not satisfy the AGMA estimations. It is advisable not to exceed U_L values of 120 N/mm^2 (17,500 lb/in.^{2}) even for the best nitrided teeth and for induction-hardened teeth with the tooth root correctly hardened. Lesser values are opportune for larger nitrided gears and far lesser values for induction hardening not extended to the tooth root. Industrial gear without surface hardening usually have lower U_L because their general size is greater to ensure RH. Thus, they do not create problems for RF.

The K and U_L values work in two ways as far as simplified estimations are concerned:

1. They enable a rough assessment of the gear capacity for given gear data. (In this case, they can also be the first step for a detailed analysis.)
2. They are not the basis for new gear designs.

9.4.2 Design Procedure

The usual procedure consists of adopting a provisional K value, establishing the general gear size, and choosing module or diametral pitch such that an acceptable U_L is obtained.

TABLE 9.1
Guideline Surface Factors *K* as Used in Industry for Case- and Induction-Hardening Gears

Application		Materials[a] P	Materials[a] G	Finish P	Finish G	K,[b] N/mm² (lb/in.²)	Speed	Service
Ordinary industrial speed reducers		C	C	sh. or gr.	sh. or gr.	5.6–10 (810–1450)	Low–mean	Nom.
		C	I	gr.	cut.	3.6–6.3 (520–910)		
		I	I	cut.	cut.	2.8–5 (400–720)		
Planetary speed reducers	Sun/planets	C	C	sh. or gr.	sh. or gr.	4.5–8 (650–1150)	Low–mean	Nom.
	Planets/ring	C	I	sh. or gr.	cut.	2.8–5 (400–720)		
Parallel gear pairs for high power		C	C	gr.	gr.		Low–mean	Nom.
Machine tools for metal cutting or grinding		C	C	gr.	gr.		Low–mean	Mean
Various types of machines		C	C	sh. or gr.	sh. or gr.	4.5–8 (650–1150)		Mean
						3.2–5.6 (460–810)		Heavy
		C	C		cut.	4.5–7.1 (650–1030)	Low–mean	Mean
					cut.	2.8–5 (400–720)		Heavy
		C	I	gr.	cut.	2–3.6 (290–520)		Mean
								Heavy

Note: P: pinion; G: gear; C: case-hardened alloy steel, HRC = 58–62; I: induction-hardened alloy steel, HRC = 55. Tooth finishing: cut.: hobbing or cutting; sh.: shaving; gr.: grinding. Service: Nom.: nominal, $C_{SF} = 1$; mean: $C_{SF} = 1.25–1.6$; heavy: $C_{SF} = 1.8–2.25$.

[a] Other material and manufacturing combinations may be adopted: For instance, specialist gear manufacturers may grind induction-hardened gear, which requires a great deal of specialized experience.

[b] Maximum K values in the table are usually adopted for good-precision helical gears, good tooth alignment, and mean speed. Misalignment problems often imply lower values to be adopted for bevel gears. Higher K values are sometimes allowed if a short life, that is, a small overall number of cycles, is required. Design, manufacturing, or operating anomalies oblige adoption of lower K values. Lower K values are often used for high-power gears for reasons not related to pitting resistance, e.g., bearing life—or gear life efficiency, scoring resistance, and, indirectly, tooth breakage resistance. In fact, lower modulus (higher diametral pitches) may be advisable, which will signify an increase in general size.

Pinion torque T_P and desired gear ratio m_G are given. Thus, two cases must be distinguished:

1. The center distance C.
2. All geometrid parameters are free.

In the first case the pitch diameter of the pinion is calculated from Equations 9.3 and 9.4 and the necessary face width is determined from Equations 9.7 and 9.8.

In the second case the ratio of the face width divided by the pinion pitch diameter can be adopted according to the kind of gear and application:

$$f_d = \frac{F}{d_P}. \tag{9.18}$$

Thus, the necessary pitch diameter is determined:

$$d_P\ (\text{mm}) = \sqrt{\frac{2000T\ (\text{N m}) \times \left[1 + (N_P/N_G)S_2\right]}{f_d K\ (\text{N/mm}^2)}}, \tag{9.19}$$

$$d_P\ (\text{in.}) = \sqrt{\frac{2T_P\ (\text{lb in.}) \times \left[1 + (N_P/N_G)S_2\right]}{f_d K\ (\text{lb/in.}^2)}}. \tag{9.20}$$

Both cases are shown in Figure 9.3.

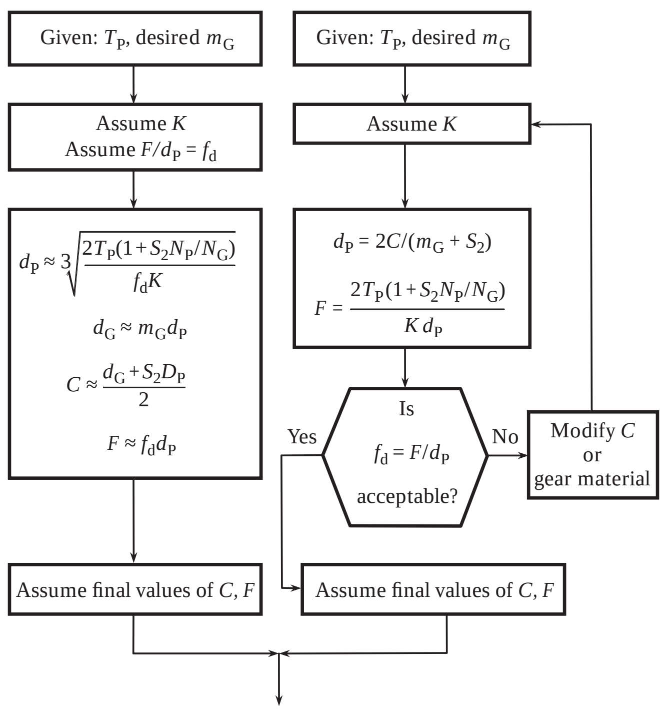


FIGURE 9.3 Design flowchart for the tentative choice of overall size of a gear pair. Metric units: T_p (N mm); K (N/mm²); F, d_P, d_G, C (mm). English units: T_p (lb in.); K (lb/in.²); F, d_P, d_G, C (in.).

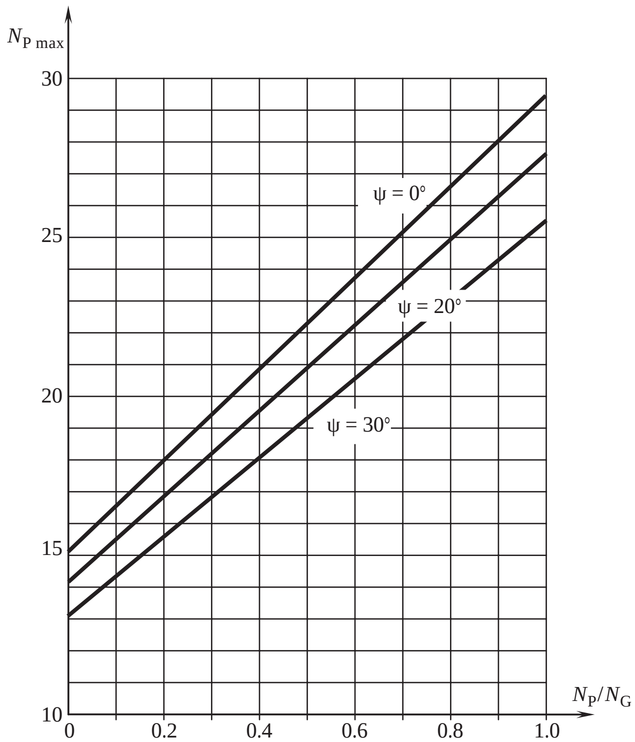


FIGURE 9.4 Tentative choice of tooth numbers for case-hardened gears: $N_{P\,max}$, maximum pinion tooth number for RF if RH is fully exploited. External gears, unidirectional loading.

For bevel gears obvious adaptations are necessary.

Finally, a standard module or diametral pitch is chosen on the basis of Equations 9.9 and 9.10 and the tooth numbers are established. This may necessitate some slight retouching of the previous calculations according to the actual m_G ratio.

An alternative way for choosing the tooth numbers is given in Figure 9.4 for case-hardened gears subjected to unidirectional load. The diagram gives the maximum tooth number of the pinion that is compatible with RF if RH is fully exploited, that is, for higher K values.

9.5 COPLANAR GEARS: DETAILED ANALYSIS, CONVENTIONAL FATIGUE LIMITS, AND SERVICE FACTORS

The conventional fatigue limits are calculated for both RH and RF. The analysis can stop here with the determination of the equivalent service factors in the case of one loading level, or it can go on by rating RH and RF fatigue damages and gear life, as well as reliability factors.

9.5.1 RH and RF, Overload Derating Factors

The actual tooth load generally increases with regard to W_t and reaches a maximum value in a given instant and in a given tooth point for various reasons that are taken into account by means of speci c overload factors. They all derate the load capacity of the gear pair and each one of them is assumed ≥1:

1. The power sharing may not be uniform in the case of multiple drivings such as that of planet gears, or of a wheel driven by two pinions. Power sharing factor: K_{sh}. (*Note*: This factor is not directly included in the methods of AGMA and ISO.)
2. The operating conditions of driving motor or of the driven machine can cause systematic overloads. Application factor: K_a.
3. The dynamic behavior of the gear pair itself can cause overloads. Dynamic factor: K_v (velocity factor).
4. Tooth misalignment of pro le and tooth spacing errors cause local overloads. Load distribution factor: K_m.

The applied factor and the dynamic factor should be assumed equal to 1 and substituted by a complete dynamic analysis of the system (motor/gears/driven machine) for a rational approach, which would lead to a greater W_t value. But separate factors K_a and K_v are usually maintained for practical reasons.

An overall overload factor is de ned as

$$V = K_{sh}K_aK_vK_m. \tag{9.21}$$

9.5.2 Power Sharing Factor K_{sh}

Assume that the tangential load W_t is calculated by considering a uniform power sharing. Thus, the K_{sh} factor accounts for the actual differences by referring to the most loaded tooth meshing.

The best power sharing for common planetary speed reducers is obtained by the adoption of three planets that make the structure nearly isostatic. Then, a K_{sh} = 1.1 to 1.2 can be assumed. Grater values should be adopted for two, or for four or more planets. Increase K_{sh} by 10% to 40% for drive types with two planets on each axis if no special manufacturing equipment is used to enable perfect tooth phasing.

For ordinary gearing, K_{sh} = 1.1 to 1.2 for double driving by two electric motors according to the motor type. For example, K_{sh} = 1.1 to 1.2 for usual motors; K_{sh} = 1.05 to 1.1 for high-sliding motors (only used for lower powers); and K_{sh} = 1 for motors with electronic control of torque balance as used for double driving of big mills. (Note that, in the latter case, possible different amount of failures of the two pinions usually depends on other causes.)

Double driving with single input shaft and single output shaft: K_{sh} = 1 is allowed if there is some phasing device or if the manufacturing equipment enables perfect tooth phasing. Otherwise, K_{sh} = 1.1 to 1.4 and larger.

9.5.3 Application Factor K_a

The application factor K_a is usually assumed to be in the range 1 to 2, sometimes larger. Consult speci c design books or load capacity standards.

9.5.4 Dynamic Factor K_v

9.5.4.1 Tangential Velocity

$$v_t\ (\text{m/s}) = \frac{\pi d_P\ (\text{m/s}) \times n_P\ (\text{rpm})}{60{,}000} \tag{9.22}$$

$$v_t\ (\text{ft/min}) = \frac{\pi d_P\ (\text{in.}) \times n_P\ (\text{rpm})}{12} \tag{9.23}$$

9.5.4.2 RHA and RFA

For Q_v = 6 to 11,

$$B = (12 - Q_v)^{0.667}, \tag{9.24}$$

$$K_v = \left[1 + \frac{\sqrt{v_t\ (\text{m/s})}}{7.56 - B}\right]^{B/4}, \tag{9.25}$$

$$K_v = \left[1 + \frac{\sqrt{v_t\ (\text{ft/min})}}{106 - 14B}\right]^{B/4}. \tag{9.26}$$

The factor Q_v is generally meant as the AGMA gear quality number, but lesser values are suggested if any cause of vibration or tooth resonance is suspected.

9.5.5 RHI and RFI, K_a

A simplified method is given here, based on the so-called method C of the DIN standards rather than on the original ISO method, as it gives a wider validity range.

The dynamic factor can be made to depend on a factor A:

$$A = \frac{v_t\ (\text{m/s}) \times N_P/100}{\sqrt{1 + (N_P/N_G)^2}}, \tag{9.27}$$

$$A = \frac{v_t\ (\text{ft/min}) \times N_P/19{,}700}{\sqrt{1 + (N_P/N_G)^2}}, \tag{9.28}$$

$$K_v = 1 + A\left[\frac{55B_1\,(0.65)^{Q_v-7}}{K_{sh}K_aW_t\ (\text{N})/F(\text{mm})} + B_2\right], \tag{9.29}$$

$$K_v = 1 + A\left[\frac{314B_1\,(0.65)^{Q_v-7}}{K_{sh}K_aW_t\ (\text{lb})/F(\text{in.})} + B_2\right], \tag{9.30}$$

where

Q_v = the AGMA quantity number
$B_1 = 1$, $B_2 = 0.02$ for spur gears
$B_1 = 0.89$, $B_2 = 0.009$ for helical gears with $m_F \geq 1$

For helical gears with $m_F < 1$, the factor K_v has to be obtained by interpolation:

$$(K_v) = K_{v\,hel} + (1 - m_F)(K_{v\,spur} - K_{v\,hel}). \tag{9.31}$$

The method refers properly to steel gear pairs with $N_P < 50$ and is applicable with a worse approximation for metal gears with a lower Young's modulus. General application is allowed for $A < 3$. The calculation can be accepted for higher A values insofar as the gear pair does not enter a resonance condition. Simplified analyses show resonance risk in the range A = 10 to 14 for steel gears (see the following section), but this can be altered by any peculiarity of the dynamic system. The AGMA rating with a lower Q_v number may be preferred in any doubtful case.

Note: The K_v equations are estimations. Even if the computer gives 16 digits, the first decimal digit itself is uncertain and in fact its ISO or DIN value often differs from the AGMA one. Equations 9.29 and 9.30 often gives lower (more optimistic) results than AGMA does, especially for heavy loaded helical gears. Experience says it is acceptable, in its validity field, if the tooth profile errors are not the contrary kind of tip or root reliefs. Otherwise, Equations 9.25 and 9.26 is preferred with a cautious choice of Q_v especially if the A parameters are rather high as a rough indication of vibration risk.

9.5.6 Dynamic Problems of Gears: Resonance and Vibration Conditions

The teeth act as a spring connecting pinion and gear masses: therefore, the gear pair itself has a natural vibration frequency and may incur a resonance condition according to tooth contact frequency, that is, according to speed and tooth number. A main peak load occurs at resonance speed and lower peaks at submultipliers and multipliers of resonance speed. In Figure 9.5, n_P is pinion speed in revolutions/min; n_{rp} is the pinion resonance speed.

The subject has been thoroughly investigated, both theoretically and experimentally, by Kubo (1980), who gives a synthetic description of dynamic problems.

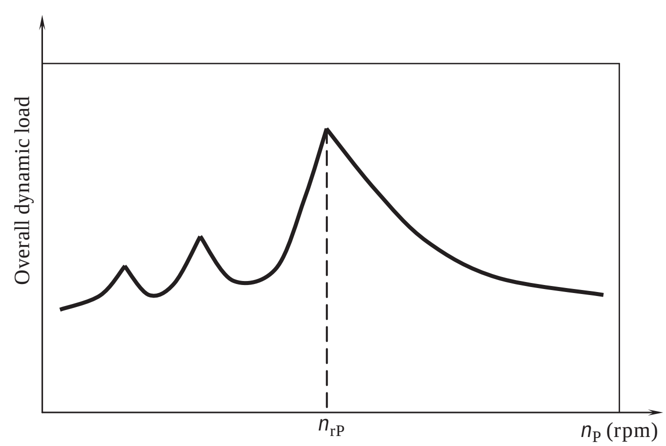


FIGURE 9.5 Pinion resonance speed.

A procedure for the calculation of n_{rp} is given by ISO and DIN but does not consider masses external to the net face of the gear pair. The following equations indicate a very simple way of generalizing the calculation. The polar moments of inertia of pinion and gear are J_P and J_G (in kg m^2 ≡ N m s^2 [lb in. s]); but any mass rigidly connected with pinion or gear must be included in the calculation of J_P and J_G. Then equivalent masses of pinion and gear are de ned as though on tooth contact line (in kg m^2 ≡ N m s^2 [lb in. s]):

$$m_{cP} = \frac{4J_P}{d_{bP}^2}, \tag{9.32}$$

$$m_{cG} = \frac{4J_G}{d_{bG}^2}, \tag{9.33}$$

where the base diameters d_{bP} and d_{bG} are in meters (inches), and an equivalent single mass is determined:

$$m_c = \frac{1}{1/m_{cP} + 1/m_{cG}}. \tag{9.34}$$

Finally,

$$n_{rP} = \frac{30}{\pi N_P}\sqrt{\frac{10^6 G\ (\mathrm{N/\mu m\ mm}) \times F\ (\mathrm{mm})}{m_c\ (\mathrm{kg})}}, \tag{9.35}$$

$$n_{rP} = \frac{30}{\pi N_P}\sqrt{\frac{G\ (\mathrm{lb/in^2}) \times F\ (\mathrm{in.})}{m_c\ (\mathrm{lb\,s^2/in.})}}, \tag{9.36}$$

where G is the overall tooth stiffness.

For steel gears, ISO and DIN give either a detailed method for its calculation or a mean value for G (that is, G = 20 N/µm mm   3 × 10^6 lb/in.^{2}). AGMA indicates G = (1.5 to 2.0) × 10^6 lb/in.^{2} 10 to 14 N/µm mm, but limits its validity to load distribution analysis, where de ections other than those of the gears are in uential. German researchers show a certain stiffness diminution due to both gear and external causes. The proper value for Equations 9.35 and 9.36 may be situated between 14 and 20 N/µm mm, or between 2 × 10^6 and 3 × 10^6 lb/in.^{2}, and varies, of course, with tooth proportions and the cooperation of various tooth pairs, depending on tooth spacing errors, etc.

Example 9.2

Consider a gear pair with C = 16 in. = 406.4 mm, N_P/N_G = 27/43, total face width F_P = 5.9 in. = 149.86 mm, F_G = 5.5 in. = 139.7 mm, P_{nd} = 2.5 in.$^{-1}$ (that is, m_n = 10.16 mm), ϕ_{ns} = 20°, ψ_s = 29°; steel gears with a specific mass density of 0.0007345 lb s^2/in^4 = 7850 kg/m^3; and d_{bP} = 11.400 in. = 289.57 mm, d_{bG} = 18.156 in. = 461.17 mm.

CALCULATION IN ENGLISH UNITS

Real masses m_P = 0.5186, m_G = 1.226; polar moments of inertia J_P = 9.875, J_G = 59.22, equivalent masses m_{cP} = 0.3040, m_{cG} = 0.7186; equivalent single mass m_c = 0.2136; n_{rP} = 2840 rpm.

CALCULATION IN METRIC UNITS

m_P = 9081, m_{FP} = 214.7; polar moments of inertia J_P = 1.116, J_G = 6.691, equivalent masses m_{cP} = 125.8; equivalent single mass m_c = 37.4; n_{rP} = 2840 rpm.

Note that the resonant tangential velocity is v_t = 9100 ft/min = 46 m/s, and the factor A in Equations 9.27 and 9.28 is equal to 10.6, that is, it is in the range of 10 to 14, as mentioned previously.

Provided that the assessment of n_{rP} = 2840 rpm is correct and that there is no causes of dynamic overloads external to the gear pair, the standard K_v factors may be considered as valid in a eld of n_P = 0 to 0.25 n_{rP}, that is, for pinion speeds less than about 700 rpm. For greater speeds, it is advisable to adopt the AGMA method with a cautious choice of Q_v. For instance, diminish Q_v by 1 or 2 units in the range 0.25 $n_{rP} < n_P <$ 0.7 n_{rP}, and by 2 to 3 units for $n_P >$ 0.7 n_{rP}. Of course, the actual situation may be far better, as the speed may be in the interval between two load peaks, but the assessment of resonant or vibration conditions presents a number of uncertainties.

The main problems are as follows:

1. The real overall tooth stiffness.
2. How masses in uence actual resonance speed when they are not adjacent and rigidly connected with pinion or gear. For instance, the polar moment of inertia of the gear of the previous stage, adjacent to a pinion, certainly must be added to that of the pinion, but the effect of an external coupling is uncertain.
3. External excitations can cause resonance. As mentioned previously, a rational approach should substitute the dynamic analysis of the entire system for the product K_aK_v. Long and elastic external transmission shafts often exclude effects of masses that are far from gears, but in particular cases can themselves originate vibration conditions. Figure 9.6 refers to a starting process, that is, to a transient regime: the vibrations are much stronger in Figure 9.6b, and the only difference consists of a more elastic nal transmission. The gure was computed, but the investigation was promoted by some anomalous failures of industrial gears.

If there is any doubt regarding the third problem—or any explicable failure has occurred—it is advisable to interpose elastic couplings or hydraulic couplings.

Special design features must be adopted when calculations show that the speed approaches resonance. Gears can work in a resonance condition (it should be avoided if

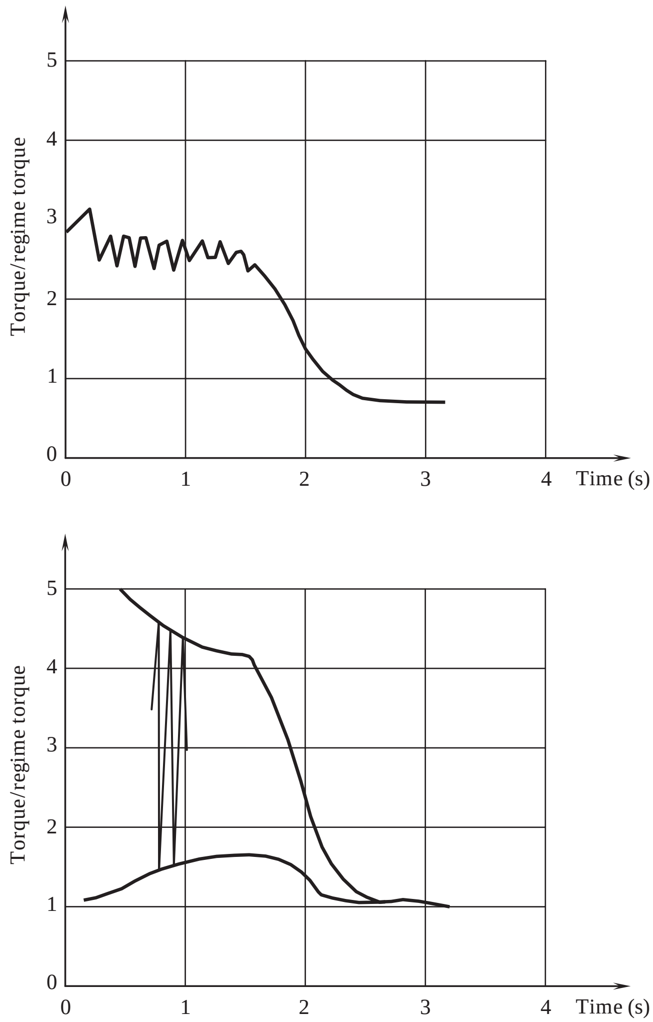


FIGURE 9.6 Gear vibration in transient condition depending on the stiffness of the external output shaft.

possible); a number of marine gears do, as their speed is variable and can incur resonance. But they must be built with very good precision and must be helical gears with higher overlap ratios.

Note that when the calculation indicates a resonance risk, it may be easier to avoid it by diminishing rather than increasing the resonance speed. This has also the advantage that uncertainties of load peaks corresponding to resonance submultipliers are avoided. This can be accomplished by increasing either pinion mass or pinion tooth number or both, with the aim of operating above the resonance range. Higher tooth numbers, that is, lower modules, also reduce the risk of scoring. On the other hand, this often makes RF more restrictive than RH and requires careful verifications.

Gears that are to operate above the resonance speed should be precision gears. Unground tool-finished marine gears operated in the past above the resonance speed. In this case, medium precision may be insufficient as the gear pair may incur a multiple a multiple of the resonance speed. Then it will be subjected to a load peak, even though it may be lower than the one corresponding to the main resonance condition.

For gear speeds either in the resonance range or above it, high-precision gears with AGMA number not less than 11, and AGMA K_v factor rating with a Q_v factor not greater than 10, are advisable as a summary general criterion. More optimistic estimates, as considered by AGMA for very accurate gearing, may be allowed if the system is simple enough and the calculation indicates that the speed is far enough from both the main resonance speed and its multiples.

Spur gears should not operate in the resonance range, although they do in particular cases. Then specific tests are available.

The procedure for the calculation of n_{rP} cannot be considered valid for planetary gears, although the general concepts are the same.

9.5.7 Load Distribution Factor, K_m

Consult AGMA and ISO documents for the original methods. For AGMA, $K_m = C_m = C_{mf}C_{mt}$, respectively, for face and transverse load distribution factors, but usually $C_{mt} = 1$. ISO defines the products $K_{H\beta}K_{H\alpha}$ for tooth surface and $K_{F\beta}K_{F\alpha}$ for tooth fillet where $K_{F\beta}$ is somewhat less than $K_{H\beta}$.

Both standards give simplified and analytical methods. (Note that ANSI/AGMA standards change the “analytical” rating for helical gears with regard to AGMA standard, and obtains larger, more severe values of K_m—in fact, this is the only important difference between the ratings of the two standards.)

Warning: The “empirical” or simplified methods of ISO and AGMA standards largely differ from the analytical ones, especially when the shaft of the wheel deforms elastically, as often happens in multistage gear units. However, the analytical methods require preliminary computation of the tooth misalignment.

A preliminary indication only is given in the following section. Assume that K_m varies from 1.1 to 1.8; greater values would imply bad general design or unacceptable manufacturing errors. Table 9.2 helps in choosing K_m.

Effects of helix corrections are known from many sources. Special attention must be given to bevel gears.

Carter deformations often are an important item for tooth alignment. They should be investigated by finite-element method (FEM), at least for large, important gear units or for mass production. This is not an easy task because of the complex form of the carters, usually obtained by the assembling of different parts. Experimental checks are useful when possible.

Note on overload factors: The choice of the single overload factors or the direct assumption of their synthetic product V is within the competence of the gear designer, even if the indications and the formulas of the standards are helpful. Their assessment is generally more important for the final results than the geometry calculations that follow, even if the latter are also necessary.

TABLE 9.2
Guideline for Assumption of the Load Distribution Factor K_m

	Stiffness of the Shafts of Pinion and Wheel[a]				
Good	$A = 0$				
Average	$A = 1$				
Bad	$A = 2$				
	Gear Helix Precision and Shaft Parallelism in the Housing[b]				
Good	$B = 0$				
Average	$B = 1$				
Bad	$B = 2$				
	Choice of K_m[c]				
$A + B$	0	1	2	3	4
K_m	1.1	1.25	1.4	1.6	1.8

Note: This table is valid for gears designed and manufactured following normal criteria and for normal housing stiffness. A low housing stiffness may produce opposite effects:

1. It may compensate for tooth misalignment for one stage gear pairs; or
2. For multistage gear trains, it may help some gear pairs, especially those under most load, and worsen the misalignment of others.

[a] In normal multistage gear trains, deflection of the second pinion shaft (which carries the first wheel) causes misalignment of the first gear pair, and so on.

[b] Helix precision: what is of most interest in the difference in pinion and wheel helices. Shaft eccentricity in journals is due to elastic deformations of the bearing and to clearances: internal bearing clearance and possible clearances between bearing and housing and between bearing and shaft. Remember that overhung gear mounting causes shaft eccentricity and gear misalignment even in the case of preloaded bearings, because of elastic deformations in the bearing, especially when an overhung pinion is the gear on a shaft under greatest load. This is typical on most bevel pinions.

[c] Lower K_m by 10%–20% (minimum $K_m = 1.1$) for appropriate helix correction or longitudinal tooth flank correction or crowning. (*Warning*: Inappropriate corrections or crowning may cause worse misalignment!) Increase K_m by 10%–30% for bad tooth spacing or profile precision, especially in helical gears. Increase K_m by 10%–20% because of torsional pinion deflection for the first gear pair of multistage speed reducers if the gear ratio is high and the pinion is sited at the side of the power input.

AGMA and ISO overload factors should be interchangeable, according to their definitions, and in fact it may be necessary to alternate them to make direct assumptions, for example, when the validity field of an equation is limited or when the standard indications are doubtful for a given application.

9.6 RH—CONVENTIONAL FATIGUE LIMIT OF FACTOR K

A unified formula of K_{lim} for both RHA and RHI is as follows:

$$K_{lim} = G_H \left(\frac{s_{c\,lim}}{C_P} \right)^2 \frac{A_H}{V}, \tag{9.37}$$

where all parameters are in consistent units: K_{lim} and $s_{c\,lim}$ in N/mm², and C_P in (N/mm²)$^{0.5}$, or K_{lim} and $s_{c\,lim}$ in lb/in.², and c in (lb/in.²)$^{0.5}$. The overall overload factor V has been defined (Equation 9.21). The other factors are given in the following.

9.6.1 RH—Preliminary Geometric Calculations

The pressure and helix angles are

$$\psi_s = \tan^{-1}\left(\frac{\tan\phi_{ns}}{\cos\psi_s} \right), \tag{9.38}$$

$$\psi_b = \sin^{-1}\left(\frac{\sin\psi_s}{\cos\phi_{ns}} \right), \tag{9.39}$$

$$B = \frac{2C\,(\text{mm}) \times \cos\psi_s}{m_n\,(\text{mm}) \times (N_G + S_2 N_P)} = \frac{2C\,(\text{in.}) \times P_{nd}\,(\text{in.}^{-1}) \times \cos\psi_s}{N_G + S_2 N_P}, \tag{9.40}$$

$$\phi_t = \cos^{-1}\left(\frac{\cos\phi_s}{B} \right), \tag{9.41}$$

$$\psi = \tan^{-1}(B \tan\psi_s). \tag{9.42}$$

The base diameters are

$$d_{bP} = d_P \cos\phi_t, \tag{9.43}$$

$$d_{bG} = d_G \cos\phi_t. \tag{9.44}$$

The profile (transverse) contact ratio is

$$m_p = m_{pA} + m_{pE}, \tag{9.45}$$

where m_{pA} and m_{pE} are the operating addendum contact ratios relating to the stretches AC and CE in Figures 9.7 and 9.8:

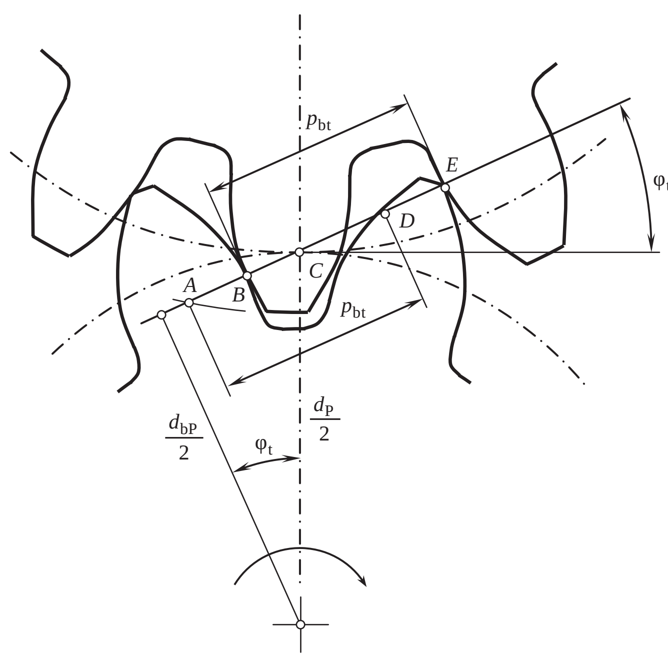


FIGURE 9.7 Involute meshing of an external gear pair in point B, LPSC of pinion or higher point of single contact of gear.

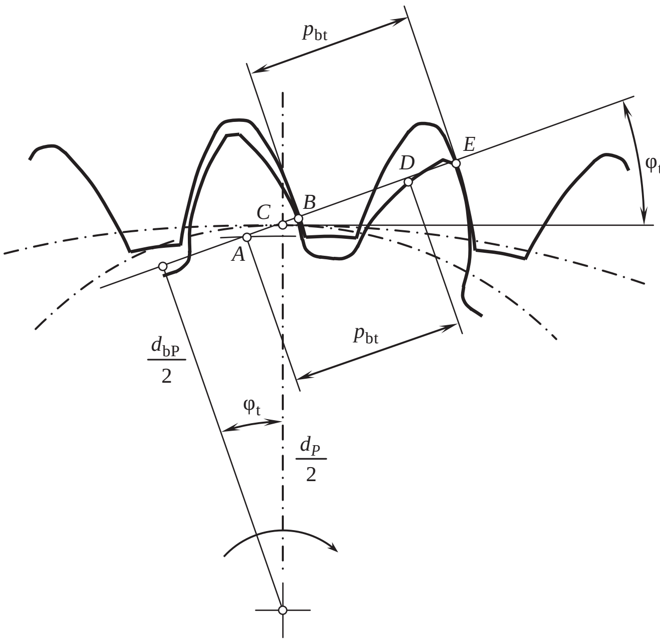


FIGURE 9.8 Involute meshing of an internal gear pair in point *B*, LPSC of pinion or HPSC of gear.

$$m_{pE} = \left[\sqrt{\left(\frac{d_{oP}}{d_{bP}}\right)^2 - 1} - \tan\phi_t\right]\frac{N_P}{2\pi}, \tag{9.46}$$

$$m_{pA} = \left(\sqrt{\left(\frac{d_{oG}}{d_{bG}}\right)^2 - 1} - \tan\phi_t\right)\frac{S_2 N_G}{2\pi}. \tag{9.47}$$

Outside and base diameters d_o and d_b can be given in any consistent units. If the contact does not extend as far as the tooth tip because of semitopping or other reasons, then the diameters at the contact limit must be introduced instead of d_{oP} and d_{oG}. Design criteria of gear teeth with semitopping can become an important item for the surface load capacity, as they can greatly affect the contact ratio. This involves a variation of the overall contact length of helical teeth.

The curvature coef cient X_B is related to the LPSC of the pinion involutes, *B*, in Figures 9.7 and 9.8:

$$M_E = \frac{2\pi(1 - m_{pE})}{\tan\phi_t}, \tag{9.48}$$

$$X_B = \left(1 - \frac{M_E}{N_P}\right)\left(1 + S_2\frac{M_E}{N_G}\right). \tag{9.49}$$

The X_B formula is valid for usual gears with $1 < m_p < 2$.

The LPSC *B* depicts a real step in tooth meshing for spur gears, whereas it works as a reference point for helical gears and enables the trend of the pro le curvature to be followed. In usual design cases with a lower gear ratio and a larger path *CE* with regard to *AC* (Figure 9.9), the relative curvature of

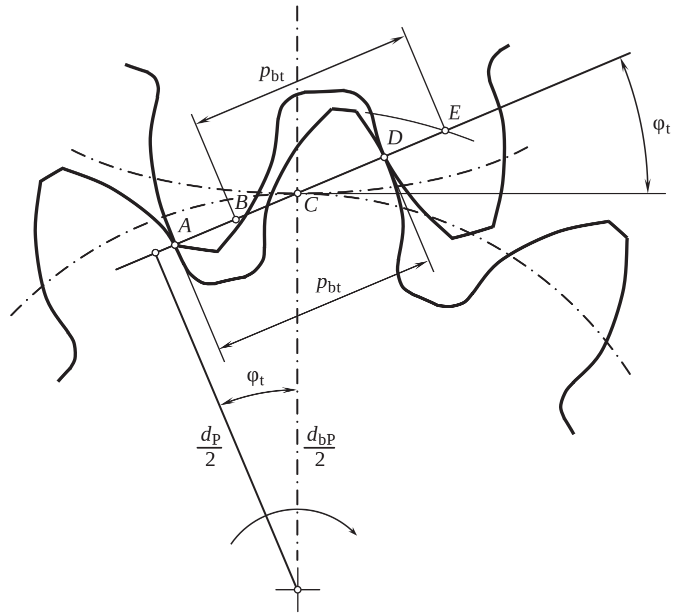


FIGURE 9.9 Involute meshing of an external gear pair in point *D*, pinion HPSC.

the involutes may be worse in the *higher point of single contact* (*HPSD*) *D*. Then a coef cient X_d in every formula of the *geometry factor* G_H is

$$M_A = \frac{2\pi(1 - m_{PA})}{\tan\phi_t}, \tag{9.50}$$

$$X_D = \left(1 - S_2\frac{M_A}{N_G}\right)\left(1 + \frac{M_A}{N_P}\right). \tag{9.51}$$

The *face contact ratio* (*overlap ratio*) is

$$m_F = \frac{F\ (\text{mm}) \times \sin\psi_s}{\pi m_n\ (\text{mm})} = \frac{F\ (\text{in.}) \times P_{nd}\ (\text{in.}^{-1}) \times \sin\psi_s}{\pi}. \tag{9.52}$$

A single face width must be considered here for double-helical gear pairs.

The following contact line coef cient m_1 and load-sharing ratio m_N are for RHA only (as well as for RFA). For spur gears,

$$m_1 = 1, \tag{9.53}$$

$$m_N = 1. \tag{9.54}$$

For helical gears, m_{pi} and m_{Fi} are the integer parts of m_p and m_F, respectively, and m_{pd} and m_{Fd} the decimal parts:

$$m_1 = \frac{m_p m_{Fi} + m_{pi} m_{Fd} + \max\left(m_{Pd} + m_{Fd} - 1.0\right)}{m_p m_F}, \tag{9.55}$$

$$m_N = \frac{\cos\psi_b}{m_1 m_P}. \tag{9.56}$$

9.6.2 Adaption for Bevel Gears

For pressure and helix angles Equations 9.38 and 9.39 are valid. Standard and operating angles coincide for bevel gears: $\phi_t = \phi_s$ and $\psi = \psi_s$.

The addenda in the middle point of the face width (Figure 9.10) are

$$a_{Pm} = a_{oP} - (F/2)\tan(\gamma_{oP} - \gamma_P), \tag{9.57}$$

$$a_{Gm} = a_{oG} - (F/2)\tan(\gamma_{oG} - \gamma_G). \tag{9.58}$$

The virtual outside diameters are

$$d_{oPv} = d_{Pmv} + 2a_{Pm}, \tag{9.59}$$

$$d_{oGv} = d_{Gmv} + 2a_{Gm}. \tag{9.60}$$

The virtual base diameters are

$$d_{bPv} = d_{Pmv}\cos\phi_t, \tag{9.61}$$

$$d_{bGv} = d_{Gmv}\cos\phi_t. \tag{9.62}$$

Then Equations 9.46 and 9.47 can be applied by introducing all virtual data, virtual tooth number included, while in Equation 9.52 the mean normal module m_{nm} or the mean normal diametral pitch P_{ndm} must be introduced. No variation affects Equations 9.45, 9.48 through 9.51, and 9.53 through 9.56.

9.6.3 RH—Unified Geometry Factor G_H

9.6.3.1 RHA

For spur gears

$$G_H = X_B \frac{\sin(2\phi_t)}{4}. \tag{9.63}$$

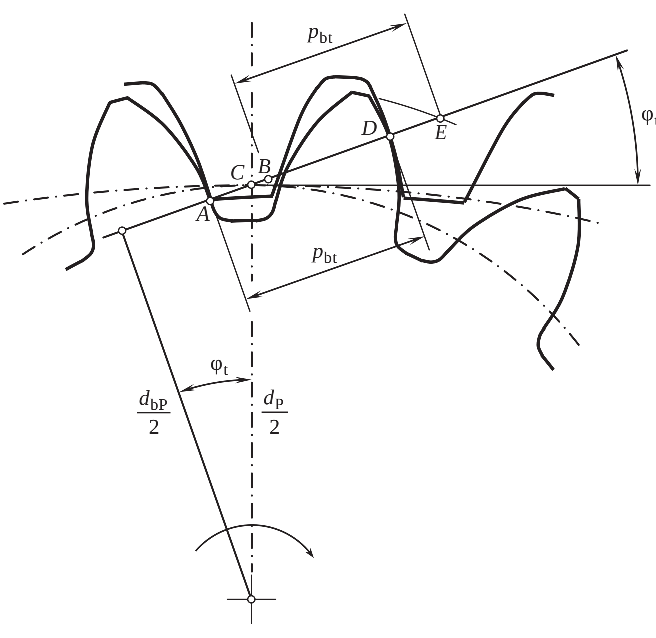


FIGURE 9.10 Involute meshing of an internal gear pair in point *D*, pinion HPSC.

For helical gears

$$G_H = \frac{X_B}{0.9}\left[\frac{\sin(2\phi_t)}{4m_N}\right]. \tag{9.64}$$

Note: AGMA's original geometry factor *I* is, as a concept,

$$I = \frac{G_H}{1 + S_2(N_P/N_G)}. \tag{9.65}$$

Thus, Equations 9.63 and 9.65 give exactly the same *I* factor as the AGMA standards for spur gears.

For helical gears, Equation 9.64 is simplified with regard to the standards but maintains the trend depending on the profile curvature, whereas the results are somewhat more conservative in most common cases, for example, for small tooth numbers of the pinion. No modification is adopted for the so-called LCR gears, as the m_N ratio accounts for their load sharing. (*LCR* is an AGMA term that means "low contact ratio"—which is disputable, as it refers to overlap or face contact ratio and not to transverse contact ratio.)

9.6.3.2 RHI—Spur and Helical Gears

$$G_H = \frac{X_B}{0.9}\left[\frac{\sin(2\phi_t)}{4\cos\psi_b\cos\psi_s Z_\varepsilon^2}\right], \tag{9.66}$$

where Z_ε is a factor of the contact ratios according to ISO:

$$Z_\varepsilon^2 = \frac{4 - m_p}{3}[1 - \min(m_F, 1)] + \frac{\min(m_F, 1)}{m_p}. \tag{9.67}$$

Note: Equation 9.66 leads to the same final result as the original ISO factors, expected for the effect of the correction coefficient $X_B/0.9$ that is introduced as a simple criterion for taking into account the trend of the variation of the relative profile curvature. The DIN method of 1987 is equivalent to introducing the factor X_B only for spur gears and only when it is less than 1, without dividing it by 0.9. It is possible that the final draft of the ISO method coincides with DIN's. See the following comparisons and discussion.

In practice the X_B coefficient can be maintained as calculated from Equations 9.49 and 9.51, also when $m_p > 2$ as an estimation, for both RHA and RHI. Specific tests must be performed if one wants to fully exploit the performances of such an unusual kind of gears. Note that involute gear teeth with higher contact ratios can be obtained by adopting higher tooth numbers and longer teeth. Low tooth numbers are possible for the pinion if the tooth profiles differ from the involute, but this does not regard this chapter.

Tables 9.3 and 9.4 indicate G_H values for typical meshing cases of external and internal gear pairs with nominal addendum = m_n or $1/P_{nd}$. The addendum modification coefficients x_P and x_G are obtained by dividing the modifications of the nominal addendum by m_n or multiplying by P_{nd} before

TABLE 9.3
Geometry Factors for RHA and RHI, External Gear Pairs

ψ_s	N_P/N_G	x_P	x_G	ϕ_t	m_{pA}	m_{pE}	m_p	G_{HA}	G_{HI}
0°	13/82	0.4	0	21.240	0.592	0.893	1.485	0.149	0.198
	13/82	0.4	0.4	22.338	0.634	0.847	1.481	0.148	0.196
	13/41	0.4	0	22.085	0.587	0.858	1.444	0.152	0.199
	13/41	0.4	0.4	23.807	0.650	0.784	1.434	0.152	0.197
	13/21	0.4	0	23.132	0.573	0.813	1.386	0.161	0.205
	13/21	0.4	0.4	25.529	0.657	0.709	1.366	0.162	0.205
	26/82	0.0	0	20	0.914	0.810	1.725	0.146	0.214
	26/82	0.4	0	21.099	0.629	0.986	1.615	0.167	0.233
	26/82	0.4	0.4	22.085	0.701	0.904	1.605	0.167	0.233
	26/41	0.0	0	20	0.859	0.810	1.670	0.152	0.217
	26/41	0.4	0	21.714	0.636	0.935	1.570	0.169	0.232
	26/41	0.4	0.4	23.173	0.736	0.811	1.547	0.173	0.234
	26/26	0.0	0	20	0.810	0.810	1.621	0.158	0.222
	26/26	0.4	0.4	23.929	0.746	0.746	1.493	0.182	0.242
15°	13/82	0.4	0	21.810	0.561	0.861	1.423	0.228	0.249
	13/82	0.4	0.4	22.851	0.602	0.817	1.419	0.226	0.246
	13/41	0.4	0	22.610	0.559	0.828	1.387	0.228	0.248
	13/41	0.4	0.4	24.254	0.620	0.757	1.377	0.225	0.245
	13/21	0.4	0	23.608	0.550	0.785	1.334	0.223	0.254
	13/21	0.4	0.4	25.912	0.631	0.684	1.315	0.230	0.251
	26/82	0.0	0	20.647	0.866	0.774	1.641	0.263	0.287
	26/82	0.4	0	21.677	0.597	0.946	1.542	0.281	0.307
	26/82	0.4	0.4	26.610	0.666	0.867	1.533	0.280	0.305
	26/41	0.0	0	20.647	0.818	0.774	1.592	0.267	0.291
	26/41	0.4	0	22.258	0.606	0.897	1.503	0.280	0.305
	26/41	0.4	0.4	23.646	0.703	0.779	1.482	0.280	0.305
	26/26	0.0	0	20.647	0.774	0.774	1.548	0.272	0.297
	26/26	0.4	0.4	24.370	0.717	0.716	1.432	0.286	0.312
30°	13/82	0.4	0	23.745	0.475	0.766	1.241	0.212	0.257
	13/82	0.4	0.4	24.616	0.510	0.728	1.238	0.209	0.254
	13/41	0.4	0	24.413	0.480	0.737	1.217	0.215	0.261
	13/41	0.4	0.4	25.818	0.534	0.675	1.209	0.211	0.257
	13/21	0.4	0	25.261	0.480	0.700	1.180	0.225	0.273
	13/21	0.4	0.4	27/274	0.554	0.610	1.163	0.220	0.268
	26/82	0.0	0	22.796	0.731	0.669	1.400	0.257	0.312
	26/82	0.4	0	23.635	0.505	0.827	1.332	0.272	0.331
	26/82	0.4	0.4	24.413	0.566	0.760	1.325	0.271	0.329
	26/41	0.0	0	22.796	0.699	0.699	1.368	0.265	0.322
	26/41	0.4	0	24.118	0.520	0.785	1.305	0.276	0.336
	26/41	0.4	0.4	25.294	0.607	0.682	1.290	0.275	0.334
	26/26	0.0	0	22.796	0.669	0.669	1.338	0.275	0.334
	26/26	0.4	0.4	25.919	0.627	0.627	1.254	0.284	0.346

Note: General data: $\phi_{ns} = 20°$. Nominal addendum $m_n = 1/P_{nd}$. x = the addendum modi cation ÷ m_n or addendum modi cation × P_{nd} (x_P and x_G for pinion and gear, respectively), that is, $d_o = Nm_n/\cos\psi_s + 2m_n(1 + x)$. Helical gear pairs: $m_1 = 0.95$ for G_{HA} and $m_F \geq 1$ for G_{HI}.

any further correction of the tooth outside diameters that the gear designer may adopt for any reason. The coef cients are positive if they involve an increase in the diametral size. (Note that the Germans adopt the opposite convention for internal gears.)

TABLE 9.4
Geometry Factors for RHA and RHI, Internal Spur Gear Pairs

N_P/N_G	x_P	x_G	ϕ_t	m_{pA}	m_{pE}	m_p	G_{HA}	G_{HI}
13/82	0.4	0	17.959	0.343	1.027	1.369	0.153	0.194
13/82	0.4	0.4	20.000	0.412	0.944	1.356	0.147	0.185
13/41	0.4	0.4	20.000	0.431	0.944	1.376	0.145	0.185
26/82	0.0	0	20.000	0.863	0.810	1.673	0.135	0.193
26/82	0.4	0	17.406	0.204	1.285	1.490	0.186	0.247
26/82	0.4	0.4	20.000	0.412	1.077	1.489	0.172	0.228

Note: General data. $\phi_{ns} = 20°$. $\psi_s = 0°$. Nominal addendum $m_n = 1/P_{nd}$. x = the addendum modi cation ÷ m_n or addendum modi cation × P_{nd} (x_P and x_G for pinion and gear, respectively). Addendum modi cations are assumed to be > 0, for both pinion and internal gear, when they involve an increase in the diametral size. Therefore, for the pinions, $d_o = Nm_n + 2m_n(1 + x_P)$. For the internal gears, the addenda are shortened by $0.2m_n = 0.2/P_{nd}$, thereby avoiding false contacts and widening the range of usable cutters, that is, $d_o = Nm_n - 2m_n(1 - x_G - 0.2)$.

9.6.4 RH—Comments and Comparisons on the Unified Geometry Factor G_H

The trend of the ISO and DIN geometry factors, depending on the helix angle according to Equation 9.66, is based on German researchers reported by Niemann et al. (1962). On the other hand, more important differences between the various methods depend on the choice of the point along the contact line where the relative pro le curvature is rated.

Figures 9.11 and 9.12 show the geometry factor as depending on the addendum modi cation coef cients (see above). When $x_G = x_P$, the center distance is unmodi ed; otherwise, it is modi ed. No addendum shortening has been considered as it is unnecessary in both cases.

AGMA-V and ISO-V refer to the variants that are adopted in this chapter, Equations 9.64 and 9.66. They have been presented by Castellani (1981, 1982) and other examples were investigated. Equation 9.63 for spur gears agrees with all the AGMA standards mentioned in the following.

Both the variants for RHA and RHI show a very similar trend of AGMA standards with regard to the in uence of x_P and x_G coef cients, but G_H is far higher according to the AGMA standards for helical pinions with a low tooth numbers (Figure 9.12) than requested for surface-hardened gears if surface and root resistances have to be balanced. Of course, the success of gears such as these that have been designed according to AGMA standards depends on other items, for example, on a conservative choice of the allowable Hertzian pressure. But a more cautious choice of G_H, as given by Equation 9.64, becomes opportune especially if higher values of the Hertzian pressure are allowed according to ANSI/AGMA standards.

On the whole, a far greater gap between the G_H factors or helical and spur gears can be observed for AGMA ratings with regard to ISO's. The DIN method reduces G_H as opposed to ISO for spur gears, when $X_B < 1$. The eld experience often

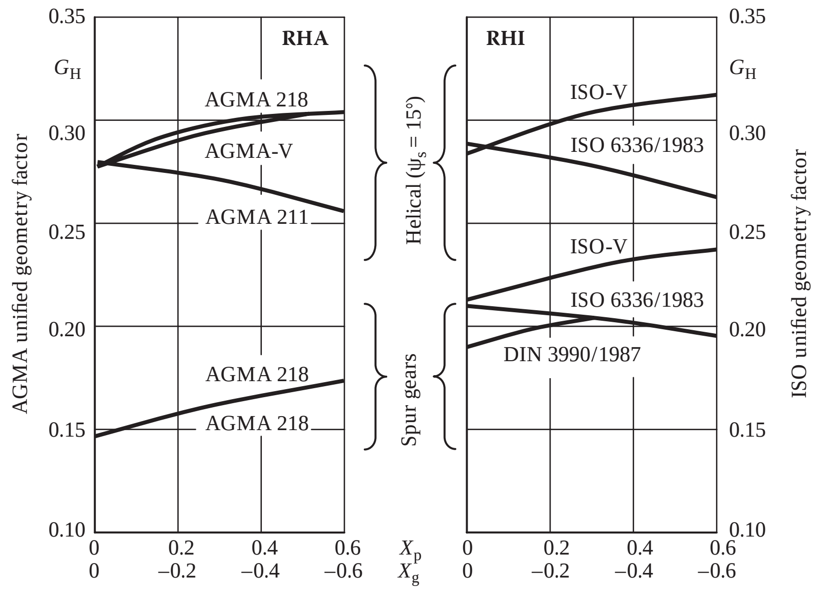


FIGURE 9.11 Uni ed geometry factor for external teeth with unmodi ed center distance. ϕ_{ns} = 20°, N_P/N_G = 26/82, m_F = 1 for the helical gears.

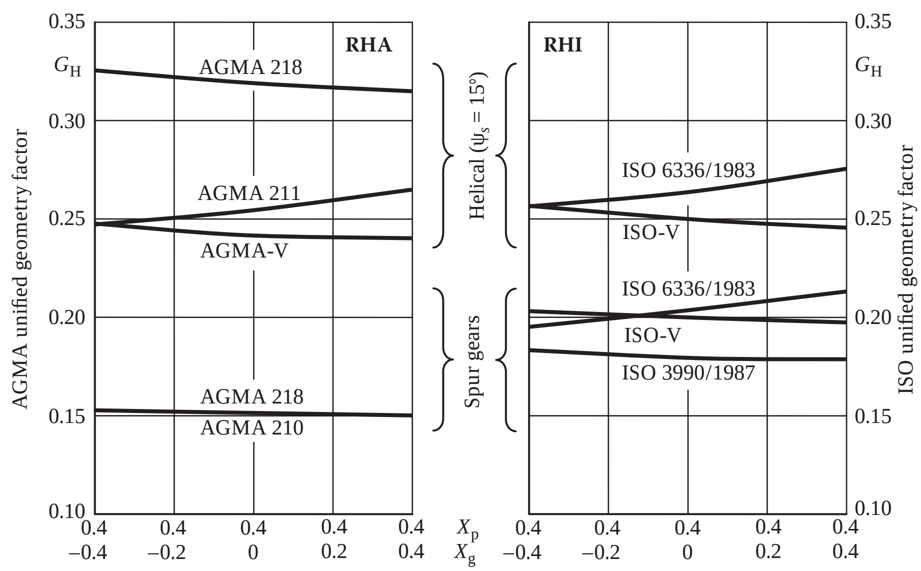


FIGURE 9.12 Uni ed geometry factor for external teeth with unmodi ed center distance and a low tooth number of the pinion. ϕ_{ns} = 20°, N_P/N_G = 13/82, m_F = 1 for the helical gears.

disagrees with such a gap. Cases are known where helical gears, substituting for spur ones, did not avoid pitting. A lot of common industrial planetary units built with case-hardened spur gears, which have been working successfully for many years, would soon have failed if they behaved according to AGMA or even to DIN.

9.6.5 RH—Elastic Coefficient C_P and Conventional Fatigue Limit $s_{c\,lim}$ of the Hertzian Pressure

Table 9.5 gives the elastic coef cient C_P (ISO symbol: Z_E) for gear pairs made from the same category of materials. Otherwise, C_P must be calculated.

For metal materials with Poisson’s ratio μ = 0.3:

$$C_P\ [(\mathrm{N/mm^2})^{0.5}] = \sqrt{0.175 E_m\ (\mathrm{N/mm^2})}\ , \qquad (9.68)$$

$$C_P\ [(\mathrm{lb/in.^2})^{0.5}] = \sqrt{0.175 E_m\ (\mathrm{lb/in.^2})}\ , \qquad (9.69)$$

where E_m is the mean Young’s modulus of elasticity of pinion (E_P) and gear (E_G):

$$E_m = \frac{2E_P E_G}{E_P + E_G}\ . \qquad (9.70)$$

TABLE 9.5
Elastic Coefficient C_P for RH

Pairing of Materials	E_m, N/mm² (lb/in.²)	E_m, N/mm² (lb/in.²)
Steel/steel	207,000 (30,000,000)	190 (2290)
Nodular or malleable iron/nodular or malleable iron	172,000 (25,000,000)	173 (2090)
Cast iron/cast iron	138,000 (20,000,000)	155 (1870)
Steel/nodular or malleable iron	188,000 (27,000,000)	180 (2170)
Steel/cast iron	165,000 (24,000,000)	170 (2050)
Nodular or malleable iron/cast iron	153,000 (22,000,000)	163 (1960)

Table 9.6 gives values near to the maximum indicated by the standards for $s_{\text{c lim}}$ (AGMA symbol: s_{ac}; ISO symbol: $\sigma_{\text{H lim}}$). *Warning*: The notes to the table are essential!

9.6.6 RH—Adaptation Factor A_H

The values are meant for gears similar to those of previous eld experience or of laboratory tests. Adaptation factors aim to adjust the calculation to the manufacturing and operating peculiarities of the examined gear pair. In this item AGMA and ISO present great differences.

9.6.6.1 RHA

$$A_H = \frac{C_H^2}{C_s C_f C_T^2}\,. \tag{9.71}$$

TABLE 9.6
RH and RF Conventional Fatigue Limits

Material	Flank and Root Hardness	$s_{\text{c lim}}$, N/mm² (lb/in.²) RHA	$s_{\text{c lim}}$, N/mm² (lb/in.²) RHI	$s_{\text{c lim}}$, N/mm² (lb/in.²) RFA	$s_{\text{c lim}}$, N/mm² (lb/in.²) RFI
Cast iron	175 HB	470 (68,000)	360 (52,000)	55 (8000)	150 (22,000)
	200 HB	530 (77,000)	400 (58,000)	80 (12,000)	165 (24,000)
Nodular iron	180 HB	560 (81,000)	500 (73,000)	190 (27,000)	370 (54,000)
	240 HB	680 (98,000)	580 (84,000)	230 (33,000)	410 (59,000)
Malleable iron	180 HB	510 (74,000)	480 (69,000)	80 (12,000)	370 (54,000)
	240 HB	620 (89,000)	550 (80,000)	130 (19,000)	410 (59,000)
Through-hardened and tempered steel	220 HB	700 (102,000)	690 (100,000)	240 (35,000)	550 (80,000)
	260 HB	795 (115,000)	745 (108,000)	265 (38,000)	580 (84,000)
	300 HB	890 (129,000)	800 (115,000)	290 (42,000)	610 (88,000)
Induction-hardened steel	~55 HRC	1250 (180,000)	1300 (190,000)	340 (50,000)	740 (105,000)
Carburized case-hardened steel	58–62 HRC	1500 (215,000)	1550 (225,000)	430 (63,000)	930 (135,000)
Case-nitrided steel, 2%–3% Cr	700–750 HV	1300 (190,000)	1380 (200,000)	400 (58,000)	840 (120,000)

Note: For cast steels ISO suggests lowering $s_{\text{c lim}}$ by 10% and $s_{\text{t lim}}$ by 20%. Data for surface-hardened steels require proper hardened depth. The tabulated data are not applicable to large-sized gears, induction- or case-hardened, or to medium-sized to large gears, gas nitride. Core hardness is a determining factor. $s_{\text{t lim}}$ values for induction hardening refer to root hardening. $s_{\text{t lim}}$ must be reduced by 30% for alternating load. Intermediate values should be adopted for bidirectional loads according to the number of cycles in each direction: maximum reduction of 30% for one load application. Tabulated data take no account of shock sensitivity. All mentioned steels are alloy steels. All materials are considered tested and in correct metallurgical condition. ANSI/AGMA standards allow as much as $s_{\text{c lim}}$ = 1910 N/mm² (275,000 lb/in.²) and $s_{\text{t lim}}$ = 5200 N/mm² (75,000 lb/in.²) for carburized case-hardened steels with good metallurgical quality and certi ed cleanliness (AGMA grade 3).

The factors at the denominator, that is, for size, flank finish, and temperature, are usually assumed equal to unity. They should be >1 if anomalous conditions arise. The hardness factor C_H equals 1 except in two cases for the gear.

Case 1: For non-surface-hardened gears, if the Brinell hardnesses H_{BP} and H_{BG} of pinion and gear are in the range

$$1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7, \tag{9.72}$$

then

$$C_H = 1 + \frac{\left[1.08(H_{BP}/H_{BG}) - 1\right](m_G - 1)}{120}. \tag{9.73}$$

Case 2: If the pinion teeth are surface hardened with minimum $HRC_P = 48$ Rockwell hardness, and the gear Brinell hardness is in the range $180 \le H_{BG} \le 400$,

$$C_H = 1 + \frac{450 - H_{BG}}{1333 e^{0.0125 f_P}}, \tag{9.74}$$

where $e = 2.718$ (base of the natural logarithms), and f_P is the flank of the pinion (roughness, arithmetic average; μin.).

Note: The factor C_H implies work hardening of the gear, but field experience shows that in case 2 the pinion teeth have a file effect rather than a work-hardening effect on the gear teeth if f_P is higher, with disastrous wear. The pinion of such gear pairs should not be finished with $f_P > 40$ μin. or 1 μm.

9.6.6.2 RHI

$$A_H = (Z_X, Z_W, Z_L, Z_v, Z_R). \tag{9.75}$$

The size factor Z_X is usually assumed equal to unity.

The hardness factor $Z_W > 1$ only if the pinion has surface-hardened teeth with a flank finish $f_P > 1$ μm or 40 μin. (roughness, arithmetic average) and the gear is in the range $130 \le H_{BG} \le 400$. Then

$$Z_W = 1.2 - \frac{H_{BG} - 130}{1700}. \tag{9.76}$$

The factors Z_L, Z_v, and Z_R for lubrication, velocity, and roughness are the most typical ISO factors. Simplified equations are given for them, and some modifications for Z_R are suggested.

9.6.6.3 RHI Lubrication Factor

$$Z_L = \left(\frac{v_{40}}{167}\right)^A, \tag{9.77}$$

where v_{40} is the nominal kinematic velocity of the lubricant at 40°C (104°F) in centistokes (1 cSt = 10^{-6} m^2/s). Assume $A = 0.05$ for surface-hardened teeth, and $A = 0.1$ for non-surface-hardened teeth (maximum $Z_L = 1.25$).

9.6.6.4 RHI Velocity Factor

$$Z_v = \left[\frac{v_t\ (\text{m/s})}{10}\right]^B, \tag{9.78}$$

$$Z_v = \left[\frac{v_t\ (\text{ft/min})}{1970}\right]^B, \tag{9.79}$$

where $B = 0.025$ for surface-hardened teeth (minimum $Z_v = 0.93$), and $B = 0.05$ for non-surface-hardened teeth (minimum $Z_v = 0.85$).

9.6.6.5 RHI Roughness Factor

Tooth flank roughness is the mean value of some teeth of pinion f_P and of f_G. Then a reduced value f_r is calculated, which relates to the size of test gears:

$$f_r\ (\mu\text{in.}) = \frac{f_P\ (\mu\text{m}) + f_G\ (\mu\text{m})}{2} \sqrt[3]{\frac{200}{d_P\ (\text{mm}) + d_G\ (\text{mm})}}, \tag{9.80}$$

$$f_r\ (\mu\text{in.}) = \frac{f_P\ (\mu\text{in.}) + f_G\ (\mu\text{in.})}{2} \sqrt[3]{\frac{7.87}{d_P\ (\text{in.}) + d_G\ (\text{in.})}}. \tag{9.81}$$

Consider f_r as arithmetic average as above. Then

$$Z_R = \left[\frac{0.5}{f_r\ (\mu\text{m})}\right]^C, \tag{9.82}$$

$$Z_R = \left[\frac{20}{f_r\ (\mu\text{in.})}\right]^C, \tag{9.83}$$

where $C = 0.08$ for surface-hardened teeth, and $C = 0.15$ for non-surface-hardened teeth. This agrees with the original ISO data with good approximation, but greater C exponents are advisable when tool-finished gears are subjected to a continuous and constant load, except in the case of case-hardened gears, where $C = 0.08$ seems sufficiently derating.

Note: The ISO surface factors have an experimental basis, but surface fatigue tests always give a lot of scattering and ISO explicitly states that such factors are only an estimation. However, they account for the roughness disturbances with Z_R as well as for the lubrication regime, even if only summarily, by means of Z_L and Z_v. The factor Z_L is somehow disputable as it refers to a nominal viscosity; Z_v apparently indicates a right trend as speedy gears in EHD conditions are favored, whereas slow gears in boundary lubrication regime are routinely derated, as they will meet progressive wear even if not always pitting.

There is actually a surface condition effect, which is sometimes more important than it is thought to be, which risks misleading one's appreciation of the real nature of some field experience. For instance, a lot of big gears probably work in

much more severe application conditions than those considered by K_a and do not have pitting just because of a more effective work hardening than that allowed by C_H. It would be risky both to rely on K_a not directly checked and to assume optimistic C_H especially if not proven for different kinds of materials.

Sometimes the gear designer should take into account other favorable or unfavorable circumstances not considered by the standards. This task can accomplish by varying directly the A_H factor. Shot-peening increases the pitting resistance. The type of lubrication apparatus and the lubrication and cooling conditions are also in uential parameters, because the lubricant action and viscosity in uence EHD conditions. Synthetic oils are generally favorable to pitting resistance according to their type and the gear material. Oil additives, even if conceived for other purposes, have sometimes a favorable in uence, and any operating condition affecting tooth temperature can be indirectly important.

9.6.7 RH—Hertzian Pressure

The value of the Hertzian pressure s_c (ISO symbol: σ_H) does not directly enter the ratings based on the K factor and is given merely for information. AGMA's and ISO's equations differ for RHA and RHI:

RHA

$$s_c = C_p \sqrt{\frac{VKC_sC_f}{G_H}} \tag{9.84}$$

RHI

$$s_c = C_p \sqrt{\frac{VK}{G_H}} \tag{9.85}$$

Both formulas are ctitious for various reasons, but this does not affect the reliability of the general calculations.

9.6.8 RH—Service Factor C_{SF} (Only for One Loading Level)

Two cases are distinguished for the procedure.

Case 1: Gear sizes and load are given; K and K_{lim} have been calculated. Then such data are equivalent to a service factor

$$C_{SF} = \frac{K_a K_{lim}}{K}, \tag{9.86}$$

which substitutes and summarizes application as well as life and reliability problems.

Case 2: K_{lim} has been rate for a new design with provisional data and a given C_{SF} is required. Then the gear design must be newly sized by assuming

$$K = \frac{K_a K_{lim}}{C_{SF}}. \tag{9.87}$$

The service factor is often mistaken for the application factor. This does not supply any margin for reliability, unless a cautious $s_{c\,lim}$ value has been chosen, but an explicit margin C_{SF}/K_a is preferable for the sake of clearness. The margin must be increased for a long gear life and may be reduced for a short one, even to the point where C_{SF} itself may be <1. In such cases the direct estimation of tooth damage and gear life is preferable.

The C_{SF} procedure normally excludes life analysis, and vice versa.

9.6.9 Power Capacity Tables

Tables 9.7 through 9.12 give a general survey of performance of industrial case-hardened gears.

As the present rating standards require detailed geometrical data, Tables 9.7 and 9.8 give them for gear pairs with parallel axes, which can be built by metric standardized tools. Of course, the performances are not too different for gears built with

TABLE 9.7
Spur Gears: Data and General Assumptions for Table 9.8

		Addendum Modification Coefficients			Center Distance C (mm)			
					50	100	200	300
Gear Ratio m_G	Tooth Nos. N_P/N_G	x_P	x_G	Operating Pressure Angle ψ_t	Normal Module m_n (mm)			
1.0	25/25	0	0	20°	2.00	4.0	8	–
1.6	22/35	0.1394	−0.0673	20.3899°	1.75	3.5	7	–
2.5	19/47	0.2849	0.0607	21.5190°	1.50	3.0	6	12
4.0	16/64	0.2981	−0.2981	20°	1.25	2.5	5	10
6.3	14/86	0.3628	−0.3628	20°	1.00	2.0	4	8

Note: External teeth: normal standard pressure angle ϕ_{ns} = 20°. Outside diameters $d_o = Nm_n + 2m_n(1 + x)$. Carburized case-hardened alloy steel, 58–62 HRC; ground teeth. Service factor C_{SF} = 1.5, including application, reliability, and life factors. Power-sharing factor K_{sh} = 1. Dynamic factors K_v rated according to RHA or RHI methods for AGMA quality number = 11. Load distribution factor K_m = 1.25.RHA: $s_{c\,lim}$ = 1500 N/mm²; A_H = 1. RHI: $s_{c\,lim}$ = 1500 N/mm²; A_H rated for f = 0.8 μm (32 μin.), v_{40} = 200 cSt, and pitch velocity (other in uential factor = 1).

TABLE 9.8
Surface Capacity of Case-Hardened Spur Gears

		Center Distance *C*/Net Face Width (mm/mm)							
		50/17.5		100/35		200/70		400/140	
		Power (kW) (Multiply by 1.341 for Power in hp)							
Gear Ratio m_G	Pinion Speed n_P (rpm)	RHA	RHI	RHA	RHI	RHA	RHI	RHA	RHI
1.0	100	0.6	0.7	4.7	5.8	37	50	–	–
	250	1.5	1.8	12.0	15.0	91	128	–	–
	500	2.9	3.6	23.0	31.0	178	257	–	–
	710	4.1	5.2	32.0	44.0	250	(364)	–	–
	1000	5.7	7.4	45.0	62.0	–	–	–	–
	1420	8.0	11.0	63.0	88.0	–	–	–	–
	1700	9.5	13.0	74.0	104.0	–	–	–	–
1.6	100	0.4	0.5	3.4	4.4	27	35	–	–
	250	1.1	1.3	8.3	11.0	66	91	–	–
	500	2.1	2.6	16.0	22.0	129	185	–	–
	710	2.9	3.7	23.0	32.0	182	264	–	–
	1000	4.1	5.3	32.0	45.0	253	370	–	–
	1420	5.8	7.6	45.0	64.0	354	(520)	–	–
	1700	6.9	9.1	54.0	76.0	–	–	–	–
2.5	100	0.28	0.32	2.2	2.7	18	22	139	191
	250	0.7	0.8	5.5	6.8	43	58	342	(492)
	500	1.4	1.6	11.0	14.0	85	118	–	–
	710	1.9	2.4	15.0	20.0	120	170	–	–
	1000	2.7	3.4	21.0	29.0	167	240	–	–
	1420	3.8	4.8	30.0	41.0	235	340	–	–
	1700	4.6	5.8	36.0	49.0	279	405	–	–
4.0	100	0.15	0.17	1.2	1.4	9.2	12	73	99
	250	0.36	0.42	2.9	3.5	23	30	179	256
	500	0.7	0.8	5.7	7.2	45	62	352	(523)
	710	1.0	1.2	8.0	10.0	63	89	(494)	(746)
	1000	1.4	1.7	11.0	15.0	88	126	–	–
	1420	2.0	2.5	16.0	21.0	123	180	–	–
	1700	2.4	3.0	19.0	26.0	147	215	–	–
6.3	100	0.08	0.09	0.6	0.7	4.8	6.0	38	50
	250	0.19	0.22	1.5	1.8	12	15	95	131
	500	0.38	0.44	3.0	3.7	24	32	186	268
	710	0.50	0.60	4.2	5.3	33	45	262	385
	1000	0.70	0.90	5.9	7.6	47	65	365	(546)
	1420	1.10	1.30	8.3	11.0	66	93	(512)	(778)
	1700	1.30	1.50	9.9	13.0	78	111	–	–

Note: For gear and rating data, see Table 9.7. See the text for due reservations about table validity.

TABLE 9.9
Helical Gears: Data and General Assumptions for Table 9.10

Gear Ratio m_G	Tooth Nos. N_P/N_G	Helix Angle ψ_s	Addendum Modification Coefficients x_P	x_G	Transverse Operating Pressure Angle ψ_t	Center Distance C (mm) 50	100	200	300
						Normal Module m_n (mm)			
1.0	23/23	21°	0.1903	0.1903	23.3437°	2	4	8	–
1.6	20/32	22°	0.3036	0.2566	23.9929°	1.75	3.5	7	–
2.5	18/45	18°	0.2413	−0.0245	21.8755°	1.5	3	6	12
4.0	15/60	19°	0.3218	0.0270	22.2820°	1.25	2.5	5	10
6.3	13/82	17°	0.3736	−0.0366	21.8078°	1	2	4	8

Note: External teeth: normal standard pressure angle ϕ_{ns} = 20°. Outside diameters $d_o = Nm_n/\cos\psi_s + 2m_n(1 + x)$. Carburized case-hardened alloy steel, 58–62 HRC; ground teeth. Service factor C_{SF} = 1.5, including application, reliability, and life factors. Power-sharing factor K_{sh} = 1. Dynamic factors K_v rated according to RHA or RHI methods for AGMA quality number = 11. Load distribution factor K_m = 1.25. RHA: contact line coef cient m_1 rated according to the actual tooth data; $s_{c\,lim}$ = 1500 N/mm^2; A_H = 1. RHI: $s_{c\,lim}$ = 1500 N/mm^2; A_H rated for f = 0.8 μm (32 μin.), v_{40} = 200 cSt, and pitch velocity (other in uential factor = 1).

standardized diametral pitches and general sizes and tooth numbers near to those of the tables. The addendum modi cations are so chosen that the so-called *Almen* factors are balanced; that is, the product of theoretical Hertzian pressure multiplied by sliding velocity and access or recess length is the same at the extreme points of the contact path. This criterion is suitable for both speed-reducing and speed-increasing gear pairs.

The addenda suggested by Gleason are adopted for bevel gears in Tables 9.11 and 9.12.

Tables 9.8, 9.10, 9.11, and 9.12 refer to the surface, that is, to pitting and generic wear. The tooth numbers are chosen such that the tooth strength generally is no problem in the case of RFI rating and unidirectional loading. RFI rating and/or bidirectional loading may give more restrictive results.

Higher power may require oil cooling especially in continuously transmitted load; otherwise, the real power capacity will be reduced, apart from other drawbacks. Anomalous dynamic conditions, for example, big masses rigidly connected to pinions or external causes of vibration excitation, may invalidate the data (see Section 9.5).

The values in parentheses in the tables are doubtful for scoring resistance; see Section 9.9. Otherwise, scoring should not be a problem, provided that good EP oils are used. If the scoring capacity of the lubricant is lower, problems may arise for the highest gear sizes and speeds indicated in the tables.

Greater powers can be transmitted at higher speeds, but lower modulus, that is, greater tooth numbers are advisable, and this can make RF more restrictive than RH. Each case must be solved by itself according to the risk of operating in resonance ranges (see Section 9.5) and according to the loading conditions, the prescribed rating method, and the desired reliability against tooth breakage; no general tabulated indications can be given.

Service and load distribution factors, as well as quality numbers, tooth roughness, and lubricant viscosity, as indicated in Tables 9.7, 9.9, 9.11, and 9.12, must be considered only as reference values.

9.7 RF—CONVENTIONAL FATIGUE LIMIT OF FACTOR U_L

A uni ed formula for both RFA and RFI can be given as follows:

$$U_{L\,lim} = \frac{J_n A_F s_{c\,lim}}{K_s V}, \tag{9.88}$$

where $U_{L\,lim}$ and $s_{c\,lim}$ are in consistent units, either newtons/square millimeter or pounds/square inch. The overall overload factor V has been de ned in the foregoing. See the following discussion for the other factors.

9.7.1 RF—Geometry Factor J_n

The geometry factor accounts for tooth form and stress concentration at llet. The HPSC, D for the pinions (Figures 9.9 and 9.10) and B for the gears (Figures 9.7 and 9.8), is taken as the rating point for spur gears in RFA and for both spur and helical gears in RFI, the so-called method B, according to the procedure of Castellani and Castelli (1980) that is followed entirely here. Tip meshing is considered in RFA normal method for helical gears instead. Both methods introduce adjusting factors.

The detailed computation of the geometry factor requires programming. The equations can be deducted from the original standards or from the cited references that de ne the actual llet form as generated by pinion cutter as well as by rack cutter or by hobs. Then, a formally uni ed geometry factor can be deduced from the original factors by means of the following equations:

For RFA

$$J_n = \frac{J}{\cos\psi_s}, \tag{9.89}$$

TABLE 9.10
Surface Capacity of Case-Hardened Helical Gears

		Center Distance *C*/Net Face Width (mm/mm)							
		50/17.5		100/35		200/70		400/140	
		Power (kW) (Multiply by 1.341 for Power in hp)							
Gear Ratio m_G	Pinion Speed n_P (rpm)	RHA	RHI	RHA	RHI	RHA	RHI	RHA	RHI
1.0	100	1.1	1.0	8.5	8.1	67	69	–	–
	250	2.6	2.5	21	21	165	180	–	–
	500	5.2	5.1	41	43	(323)	(367)	–	–
	710	4.9	5.1	58	62	–	–	–	–
	1000	6.9	7.2	81	89	–	–	–	–
	1420	9.7	10.0	113	126	–	–	–	–
	1700	12.0	13.0	135	152	–	–	–	–
	100	0.44	0.41	5.60	5.70	45	48	–	–
	250	1.1	1.00	14.0	15.0	110	124	–	–
	500	2.2	2.10	27.0	30.0	216	255	–	–
1.6	100	4.9	5.1	39	43	303	366	–	–
	250	6.9	7.2	54	62	(422)	(519)	–	–
	500	9.7	10.0	76	88	–	–	–	–
	710	12.0	13.0	90	106	–	–	–	–
	1000	0.44	0.41	3.5	3.40	28	28	219	242
	1420	1.10	1.00	8.6	8.60	68	74	(536)	(6.29)
	1700	2.20	2.10	17	18.0	134	151	–	–
2.5	100	3.0	3.0	24	26	188	218	–	–
	250	4.3	4.3	34	37	263	310	–	–
	500	6.0	6.2	47	52	368	443	–	–
	710	7.2	7.4	56	63	438	(531)	–	–
	1000	0.21	0.21	1.7	1.8	14	15	108	124
	1420	0.53	0.53	4.2	4.5	34	38	265	323
	1700	1.10	1.10	8.4	9.1	66	78	(520)	(664)
4.0	100	1.5	1.5	12	13	93	112	–	–
	250	2.1	2.2	17	19	130	160	–	–
	500	3.0	3.2	23	27	183	230	–	–
	710	3.5	3.8	28	33	217	276	–	–
	1000	0.10	0.10	0.8	0.8	6.6	7.0	52	58
	1420	0.26	0.26	2.1	2.1	16.0	18.0	129	153
	1700	0.51	0.51	4.1	4.3	32.0	37.0	254	315
6.3	710	0.7	0.7	5.8	6.2	46	53	358	453
	1000	1.0	1.0	8.1	8.9	64	76	(499)	(646)
	1420	1.4	1.5	11.0	13.0	89	109	–	–
	1700	1.7	1.8	13.0	15.0	107	131	–	–

Note: For gear and rating data, see Table 9.9. See the text for due reservations about table validity.

where J is the geometry factor as de ned in AGMA standards.

For RFI

$$J_n = \frac{\cos\phi_t}{Y_F Y_S Y_\beta \cos\phi_s}. \tag{9.90}$$

The RFI factors for tooth form (Y_F), stress correction ($Y_S = K_f$), and helix (Y_β), are clari ed in the cited references. The calculation of the helix factor Y_β is reported here because it shows how lower face contact ratio m_F affects J_n according to ISO for helical gears:

$$Y_\beta = 1 - \frac{\min(m_F, 1) \times \min(\psi_s, 30)}{120}. \tag{9.91}$$

This must be remembered when consulting the following J_n tables that refer to $m_F \geq 1$ for RFI.

TABLE 9.11
Surface Capacity of Case-Hardened Straight Bevel Gears

			Gear Pitch Diameter d_G, mm (in. [Approx.])							
			50 (2)		100 (4)		200 (8)		400 (16)	
	Tooth Nos.		Power (kW) (Multiply by 1.341 for Power in hp)							
Gear Ratio m_G	N_P/N_G	Pinion Speed n_P (rpm)	RHA	RHI	RHA	RHI	RHA	RHI	RHA	RHI
1.0	25/25	100	0.32	0.34	2.5	2.8	19	24	–	–
		500	1.5	1.7	11	14	86	111	–	–
		1000	2.9	3.4	22	27	160	197	–	–
		1420	3.9	4.7	30	36	–	–	–	–
		1700	4.6	5.6	35	42	–	–	–	–
1.6	18/29	100	0.12	0.13	0.9	1.0	7.3	8.5	–	–
		500	0.6	0.6	4.4	5.3	33	44	–	–
		1000	1.1	1.3	8.3	11	63	85	–	–
		1420	1.5	1.8	12	15	86	116	–	–
		1700	1.8	2.2	14	18	101	135	–	–
2.5	15/37	100	0.05	0.05	0.39	0.43	3.1	3.5	24	29
		500	0.25	0.25	1.9	2.2	14	18	108	151
		1000	0.50	0.50	3.6	4.4	27	36	203	(293)
		1420	0.60	0.70	5.0	6.2	37	51	–	–
		1700	0.80	0.90	5.8	7.5	44	61	–	–
4.0	14/56	100	–	–	0.15	0.16	1.2	1.3	9.2	11
		500	0.09	0.10	0.7	0.8	5.5	6.8	42	58
		1000	0.18	0.19	1.4	1.6	11	14	80	115
		1420	0.25	0.28	1.9	2.4	15	20	109	161
		1700	0.30	0.34	2.3	2.8	17	24	128	190
6.3	13/82	100	–	–	0.06	0.06	0.47	0.53	3.7	4.4
		500	–	–	0.28	0.32	2.2	2.7	17	23
		1000	0.07	0.08	0.5	0.6	4.3	5.5	32	46
		1420	0.10	0.11	0.8	0.9	5.9	7.8	45	65
		1700	0.12	0.13	0.9	1.1	7.0	9.4	53	78

Note: Shaft angle Σ; pressure angle ϕ_{ns} = 20°. Net face width = 0.3 cone distance. Tooth proportions recommended by Gleason Works, Rochester, New York. Carburized and case-hardened alloy steel, 58–62 HRC; tool finishing. Service factor C_{SF} = 1.5, including application, reliability, and life factors. K_{sh} = 1; K_v for AGMA quality number = 8; K_m = 1.4. RHA: $s_{c\,lim}$ = 1500 N/mm²; A_H = 1. RHI: $s_{c\,lim}$ = 1500 N/mm²; A_H rated for f = 2 μm (80 μin.); v_{40} = 200 cSt, and actual pitch velocity. See the text for due reservations about table validity.

The operating angular parameters, instead of the standard ones, have been introduced by AGMA for both form and stress correction factors. However, the differences are small, especially for small center-distance modifications. The tables of the cited references can be used for guidance, for both RFA and RFI.

Warning: AGMA standards give an alternative procedure for the so-called LCR helical gears with $m_F \leq 1$, but they often give more optimistic results. Instead, the normal procedure is followed in this chapter, as based on the load-sharing ratio m_N (see Equation 9.56) the same as for RH.

Tables 9.13 and 9.14 are supplied here. The stress correction factors K_f are added, not only for comparison purposes, but also because they must be considered again for excluding the stress concentration effect in the yielding calculations for RFA (which follow).

Note that the DIN method coincides with ISO's for spur gears, whereas it leads to more optimistic results for helical gears with regard to the RFI procedure considered here: With reference to Table 9.13, J_n increases by about 4% for ψ_s = 15°, and by as much as 17% to 21% for ψ_s = 30°.

In Table 9.13, a European standard hob or rack cutter has been considered (though tip edge rounding is considered rather small for cautionary reasons). On the other hand, the results are not too different for standard AGMA tools with smaller addendum and smaller tip rounding, respectively, or with greater addendum and full tip rounding. In fact, the bending arm of the tooth load and the notch effect at the fillet are in some measure self-compensatory. Full rounding gives somewhat better J_n values; remember that the tooth root surface must be good to ensure a real improvement of tooth strength. The table can obviously serve only

TABLE 9.12
Surface Capacity of Case-Hardened Spiral Bevel Gears

			Gear Pitch Diameter d_G, mm (in. [Approx.])							
			50 (2)		100 (4)		200 (8)		400 (16)	
	Tooth Nos.		Power (kW) (Multiply by 1.341 for Power in hp)							
Gear Ratio m_G	N_P/N_G	Pinion Speed n_P (rpm)	RHA	RHI	RHA	RHI	RHA	RHI	RHA	RHI
1.0	21/21	100	0.58	0.60	4.6	5.0	36	42	–	–
		500	2.8	3.1	21.0	26.0	164	217	–	–
		1000	5.4	6.3	41.0	52.0	31	423	–	–
		1420	7.4	8.9	57.0	74.0	(425)	(583)	–	–
		1700	8.8	11.0	67.0	87.0	–	–	–	–
1.6	16/25	100	0.21	0.21	1.7	1.8	1.3	15	–	–
		500	1.0	1.1	7.9	9.2	61.0	78	–	–
		1000	2.0	2.2	15.0	19.0	116	158	–	–
		1420	2.7	3.2	21.0	27.0	160	224	–	–
		1700	3.3	3.9	25.0	32.0	188	267	–	–
2.5	14/35	100	0.07	0.07	0.58	0.64	4.6	5.3	36	45
		500	0.36	0.38	2.8	3.3	22	28	166	236
		1000	0.7	0.8	5.4	6.7	41	57	315	(477)
		1420	1.0	1.1	7.5	9.6	57	81	–	–
		1700	1.2	1.4	8.9	12.0	68	98	–	–
4.0	13/52	100	–	–	0.23	0.24	1.8	2.0	14	16
		500	0.14	0.14	1.10	1.20	8.5	10	66	87
		1000	0.27	0.29	2.10	2.50	16	21	126	178
		1420	0.38	0.42	3.00	3.60	23	30	174	255
		1700	0.45	0.50	3.50	4.30	27	36	205	305
6.3	12/75	100	–	–	0.09	0.09	0.7	0.8	5.5	6.5
		500	–	–	0.43	0.47	3.3	4.0	26	34
		1000	0.11	0.11	0.80	0.90	6.5	8.2	50	70
		1420	0.15	0.16	1.20	1.40	9.1	12.0	70	100
		1700	0.18	0.19	1.40	1.70	11.0	14.0	82	120

Note: Shaft angle Σ; pressure angle $\phi_{ns} = 20°$; spiral angle $\psi_s = 35°$. Net face width = 0.3 cone distance. Tooth proportions recommended by Gleason Works, Rochester, New York. Carburized and case-hardened alloy steel, 58–62 HRC; lapped teeth. Service factor $C_{SF} = 1.5$, including application, reliability, and life factors. $K_{sh} = 1$; K_v for AGMA quality number = 9; $K_m = 1.4$. RHA: contact line coefficient m_1 rated according to the actual tooth data; $s_{c\,lim} = 1500$ N/mm²; $A_H = 1$. RHI: $s_{c\,lim} = 1500$ N/mm²; A_H rated for $f = 2$ μm (80 μin.); $v_{40} = 200$ cSt, and actual pitch velocity. See the text for due reservations about table validity.

as a general guide. A programmed computation is necessary if specific manufacturing conditions must be taken into account.

As regards internal teeth, some comparison with FEM analysis performed by Castellani and Castelli (1980) shows that the standard methods hardly agree with real fillet stresses. Thus, Table 9.14 serves more as pure information than for practical purposes. A 45° angle of the tangent to the fillet (instead of 30°) has been considered for RFI according to provisional suggestions by Castellani and Castelli (1980). (The original ISO and DIN methods simplify the procedure by considering rack instead of internal gears, but they fail to give adequate estimations of the fillet radius.)

In the cases of Gleason bevel gears, if a computation is performed, the tooth thickness correction must be considered as independent of the addendum modification. When using tables, note that a slight increase in J_n for the pinion and decrease for the gear should be considered if the well-known Gleason geometrical coefficient K has been introduced for the thickness correction and $K > 0$.

9.7.2 RF—Adaptation Factor A_F

A factor A_F corrects the geometry factor by taking into account any peculiarity of geometry and material condition at fillet: $A_F > 1$ for favorable, and $A_F < 1$ for unfavorable circumstances as a whole.

There is no similar overall factor in the standard methods, whereas there are some particular factors that may be thought of as a part of A_F.

Slim ring gears, both external and internal—the case is obviously more frequent internal ones—have as stress

TABLE 9.13
Geometry Factors for RFA and RFI, External Gear Pairs

ψ_s	N_P/N_G	x_P	x_G	J_{nAP}	K_{nAP}	J_{nAG}	K_{nAG}	J_{nIP}	K_{nIP}	J_{nIG}	K_{nIG}
0°	13/82	0.4	0.0	0.378	1.938	0.358	1.908	0.285	2.213	0.277	2.216
	13/82	0.4	0.4	0.380	1.906	0.388	1.935	0.282	2.208	0.261	2.547
	13/41	0.4	0.0	0.370	1.872	0.377	1.807	0.276	2.166	0.277	1.988
	13/41	0.4	0.4	0.372	1.817	0.388	1.857	0.271	2.155	0.266	2.380
	13/21	0.4	0.0	0.360	1.782	0.296	1.649	0.264	2.103	0.253	1.768
	13/21	0.4	0.4	0.362	1.698	0.377	1.733	0.256	2.082	0.261	2.187
	26/82	0.0	0.0	0.364	2.015	0.412	2.132	0.323	2.028	0.317	2.429
	26/82	0.4	0.0	0.434	2.120	0.386	2.008	0.311	2.527	0.297	2.322
	26/82	0.4	0.4	0.434	2.081	0.420	2.048	0.307	2.513	0.278	2.699
	26/41	0.0	0.0	0.353	1.972	0.379	2.035	0.313	1.991	0.319	2.144
	26/41	0.4	0.0	0.422	2.054	0.361	1.910	0.301	2.466	0.298	2.071
	26/41	0.4	0.4	0.419	1.988	0.417	1.980	0.294	2.435	0.285	2.512
	26/26	0.0	0.0	0.344	1.934	0.344	1.934	0.305	1.960	0.305	1.960
	26/26	0.4	0.4	0.406	1.909	0.406	1.909	0.283	2.368	0.283	2.368
15°	13/82	0.4	0.0	0.516	1.487	0.504	1.643	0.319	2.193	0.308	2.216
	13/82	0.4	0.4	0.525	1.458	0.542	1.642	0.316	2.188	0.290	2.522
	13/41	0.4	0.0	0.511	1.465	0.477	1.556	0.310	2.150	0.309	1.994
	13/41	0.4	0.4	0.524	1.419	0.541	1.563	0.304	2.139	0.296	2.361
	13/21	0.4	0.0	0.502	1.437	0.427	1.425	0.297	2.091	0.287	1.780
	13/21	0.4	0.4	0.518	1.374	0.530	1.451	0.288	2.070	0.291	1.172
	26/82	0.0	0.0	0.515	1.543	0.568	1.680	0.361	2.031	0.350	2.415
	26/82	0.4	0.0	0.573	1.602	0.545	1.647	0.343	2.488	0.329	2.318
	26/82	0.4	0.4	0.581	1.573	0.583	1.650	0.339	2.475	0.308	2.662
	26/41	0.0	0.0	0.499	1.543	0.527	1.613	0.351	1.996	0.354	2.143
	26/41	0.4	0.0	0.565	1.584	0.513	1.566	0.334	2.433	0.332	2.074
	26/41	0.4	0.4	0.573	1.542	0.576	1.582	0.326	2.405	0.315	2.483
	26/26	0.0	0.0	0.486	1.543	0.486	1.543	0.341	1.966	0.341	1.966
	26/26	0.4	0.4	0.562	1.520	0.562	1.520	0.314	2.343	0.314	2.343
30°	13/82	0.4	0.0	0.522	1.558	0.504	1.672	0.339	2.106	0.326	2.197
	13/82	0.4	0.4	0.531	1.536	0.533	1.671	0.336	2.102	0.308	2.430
	13/41	0.4	0.0	0.519	1.541	0.485	1.606	0.331	2.077	0.331	1.998
	13/41	0.4	0.4	0.531	1.505	0.535	1.611	0.326	2.068	0.313	2.289
	13/21	0.4	0.0	0.512	1.519	0.450	1.504	0.320	2.035	0.317	1.800
	13/21	0.4	0.4	0.526	1.469	0.530	1.525	0.311	2.018	0.310	2.116
	26/82	0.0	0.0	0.520	1.596	0.557	1.699	0.381	2.011	0.360	2.350
	26/82	0.4	0.0	0.564	1.643	0.539	1.675	0.355	2.354	0.343	2.279
	26/82	0.4	0.4	0.571	1.621	0.568	1.677	0.351	2.346	0.323	2.535
	26/41	0.0	0.0	0.508	1.596	0.527	1.650	0.372	1.985	0.369	2.117
	26/41	0.4	0.0	0.559	1.630	0.517	1.614	0.347	2.320	0.351	2.064
	26/41	0.4	0.4	0.565	1.597	0.565	1.626	0.340	2.300	0.330	2.381
	26/26	0.0	0.0	0.497	1.596	0.497	1.596	0.364	1.962	0.364	1.962
	26/26	0.4	0.4	0.556	1.580	0.556	1.580	0.330	2.256	0.330	2.256

Note: General data, ϕ_t, and contact ratios same as those for Table 9.3. Cutting data for $m_1 = P_{nd} = 1$:

- Hob or rack cutter without protuberance, addendum = 1.25, radius of tip edge rounding = 0.2
- Reduction of normal base thickness of pinion and gear = 0.02 for tooth backlash

concentration increase at the tooth root. The ANSI/AGMA standards introduce a derating "rim thickness factor" K_B and ("for informational purposes only") suggests $K_B > 1$ when the ratio of the rim thickness over the whole tooth depth is less than 1.2. This means that the adaptation factor A_F should be divided by K_B. On the other hand, the real root stress in such gears depends on many parameters, and direct analyses should be made if their tooth strength is central to the overall performance of a gear pair.

In the original AGMA method there is a factor $K_T \geq 1$ that derates strength for higher temperature. It may be included in A_F inversely; that is, it may lower A_F. There certainly is a

TABLE 9.14
Geometry Factors for RFA and RFI, Internal Spur Gears

N_P/N_G	x_P	x_G	Z_C	ρ_{aC}	x_C	J_{nAG}	K_{fAG}	J_{nIG}	K_{fIG}
13/82	0.4	0.0	25	0.0	0.4	0.380	2.278	0.377	2.191
	–	–	–	–	0.0	0.339	2.533	0.276	2.894
	–	–	–	–	−0.4	0.280	3.060	0.195	4.000
	–	–	–	0.2	0.4	0.423	2.064	0.414	2.013
	–	–	–	–	0.0	0.406	2.115	0.334	2.411
	–	–	–	–	−0.4	0.394	2.177	0.276	2.854
	–	–	40	0.0	0.0	0.301	2.854	0.197	4.000
	–	–	40	0.2	0.0	0.398	2.155	0.298	2.667
13/82	0.4	0.4	25	0.0	0.4	0.420	2.305	0.362	2.461
	–	–	–	–	0.0	0.375	2.577	0.256	3.392
	–	–	–	–	−0.4	0.306	3.154	0.213	4.000
	–	–	–	0.2	0.4	0.469	2.076	0.407	2.204
	–	–	–	–	0.0	0.455	2.124	0.330	2.653
	–	–	–	–	−0.4	0.446	2.170	0.278	3.101
	–	–	40	0.0	0.0	0.319	3.035	0.214	4.000
	–	–	40	0.2	0.0	0.447	2.162	0.289	2.998
13/41	0.4	0.4	15	0.0	0.0	0.443	2.660	0.331	3.141
	–	–	15	0.2	0.0	0.520	2.266	0.389	2.666
26/82	0.0	0.0	25	0.0	0.0	0.445	2.662	0.309	3.330
	–	–	25	0.2	0.0	0.528	2.290	0.386	2.719
	–	–	40	0.0	0.0	0.397	2.934	0.253	4.000
	–	–	40	0.2	0.0	0.515	2.313	0.339	3.045
26/82	0.4	0.0	25	0.0	0.0	0.365	2.654	0.290	3.039
	–	–	25	0.2	0.0	0.440	2.219	0.355	2.513
	–	–	40	0.0	0.0	0.323	2.982	0.217	4.000
	–	–	40	0.2	0.0	0.431	2.251	0.315	2.792
	0.4	0.4	25	0.0	0.0	0.417	2.688	0.269	3.644
	–	–	25	0.2	0.0	0.508	2.237	0.353	2.814
	–	–	40	0.0	0.0	0.354	3.144	0.242	4.000
	–	–	40	0.2	0.0	0.498	2.264	0.307	3.203

Note: General data, notes, ϕ_t, and contact ratios same as those for Table 9.4. Cutting data for $m_n = 1$ (mm) and $P_{nd} = 1$ (in.$^{-1}$):

- Shaper-cutter with Z_c tooth number, nominal addendum = 1.25, radius of tip edge rounding = ρ_{aC}, addendum modification = x_C relating to a given sharpening condition of the cutter
- Reduction of gear base thickness = 0.02 for tooth backlash

Angle of fillet tangent = 45° for RFI

The occasional value $K_{fIG} = 4$ is arbitrary (ISO's rating is out of range).

derating temperature influence, but it is a problem to make a reliable assessment of it.

As for ISO, there are two factors that can be thought of as included in A_F: the notch sensitivity factor $Y_{\delta_{rel}T}$ and the roundness factor $Y_{R_{rel}T}$. They are defined as *relative* factors, as they introduce modifications of the stress correction with regard to "test" gears.

The relative variation of the notch sensitivity has no great influence on steel gear strength according to ISO. However, a reduced notch sensitivity can be favorable for bad fillets, unfavorable for good ones (for example, for fillets better than those considered in Table 9.14), that is, the computation of K_{fI} gives too optimistic values in such a case.

A higher fillet roughness is unfavorable and tool scores are worse. It may be necessary to diminish A_F by 5% to 10%.

A factor $A_F = 0.95$ to 0.90 can be suggested for teeth cut without tool protuberance and shaved, if no other peculiarity occurs.

More severe restrictions are necessary for grinding steps that can be originated by many grinding conditions as investigated by Castellani and Zanotti (1980). The factor $A_F = 0.95$ to 0.75 in common cases. But A_F should be as low as 0.3 in some extreme cases as investigated by Winter and Wirth (1977)!

It must be stressed that even teeth cut with tool protuberance may present fillet anomalies.

The effect of short-peening is doubtful for planet gears and in any case of bidirectional loading. Otherwise, it is generally

favorable and suggests $A_F > 1$, especially if applied on low-roughness fillets free from any irregularities. Specific tests are advisable for important cases or for mass production gears.

Note: We may correct the previous indications for RFA ratings since the original AGMA values of fatigue limit stress are rather low (see the following). They have been deduced from field experience, contrary to ISO indications that originate from laboratory tests. Therefore, it may be supposed that AGMA's fatigue limits already take into account the most common cases of bad fillet conditions and $A_F = 1$ may be adopted in such cases.

On the contrary, an accurate A_F assessment is necessary for RFI ratings.

9.7.3 RF—Size Factor K_s

AGMA does not give any numerical indication for $K_s > 1$.

ISO prescribes $K_s = 1$ for any case if m_n (mm) ≤ 5, and $K_s = 1$ for m_n (mm) > 5. (Note that the original ISO factor is $Y_X = 1/K_s$.)

For RFI

$$m_n\ (\text{mm}) > 5 \quad K_s = \frac{1}{1 - A(B-5)}, \tag{9.92}$$

where

For steel or nodular iron gears, non-surface-hardened

$$A = 0.006 \quad B = m_n \text{ for } m_n \le 30 \quad B = 30 \text{ for } m_n > 30.$$

For surface-hardened steel:

$$A = 0.010 \quad B = m_n \text{ for } m_n \le 30 \quad B = 30 \text{ for } m_n > 30.$$

For cast iron

$$A = 0.015 \quad B = m_n \text{ for } m_n \le 25 \quad B = 25 \text{ for } m_n > 25.$$

Note: The meaning of the size factor K_s for ISO is as follows: It is known that equal materials with similar metallurgical structure and equal hardness have different fatigue resistance according to their size. Then K_s must affect the fatigue limit—as it does (see Equation 9.88)—but not the stress value (Equation 9.94). AGMA apparently considers it as a generic departing factor that takes into account any undetermined cause of stress increasing for big gear sizes: In fact, K_s is included in the calculation of s_t according to AGMA (see the following discussion, Equation 9.93).

9.7.4 RF—Conventional Fatigue Limit of the Fillet Stress $s_{t\,lim}$

Table 9.6 reports values near the maximum indicated by the standards for $s_{t\,lim}$ (AGMA symbol: s_{at}; ISO and DIN symbol: σ_{FE}). (Note that ISO intended a different parameter by the symbol $\sigma_{F\,lim}$, which is a nominal stress and amounts to one-half of σ_{FE} stress correction factor for test gears equals 2.)

The difference between AGMA and ISO indications is striking and is in part compensated for by the difference between K_f and J_n factors. For instance, ISO indications for case-hardened gears lead to greater $U_{L\,lim}$ so that Figure 9.4 generally gains in significance. However, the ISO RF assessments have been industrially tested, as they are a refinement of the original Niemann method that has been applied for more than 20 years. What is necessary in RFI ratings is a careful choice of the adaptation factors (see the foregoing), as well as a proper assumption of all factors influencing the final results.

The indication given Table 9.6 about reducing $s_{t\,lim}$ for bidirectional load, as applied for a number of cycles in every direction, is determined from Japanese research undertaken by Aida and Oda (1969).

9.7.5 RF—Tooth Root Stress at Fillet s_t

The tooth root stress does not directly enter the ratings based on the U_L factor, and is given only for information. Its value includes the stress correction factor K_f: Thus, it would mean the actual root stress, provided that no plastic phenomena occur (which is generally not true, at least for steel gears):

$$\text{RFA: } s_t = \frac{VU_L K_s}{J_n}, \tag{9.93}$$

$$\text{RFI: } s_t = \frac{VU_L}{J_n}. \tag{9.94}$$

The difference between the two equations depends on the nature of the K_s factor (see the foregoing discussion).

9.7.6 RF—Service Factor K_{SF} (Only for One Loading Level)

Once the detailed gear data have been established, the loading data are equivalent to the following service factor for RF:

$$K_{SF} = \frac{K_a U_{L\,lim}}{U_L}. \tag{9.95}$$

If this way is adopted, the service factor summarizes application and reliability problems as well as those of gear life and excludes detailed ratings of the latter.

9.8 COPLANAR GEARS: DETAILED LIFE CURVES AND YIELDING

In the following paragraphs equations and data are given for defining the life curves in a unified way for RH and RF. The simple approach may be disputed, especially for RH for the calculation of the cumulative damage according to Miner's rule, but it is usually accepted for industrial gears as best estimated values and is preferable to no checking at all. The adopted procedure facilitates the assumption of life curves different from

the indications of standards, if the designer prefers to follow the indications of specific gear design books or those of an available field experience. In fact, many factors can affect the real slope of the life curve and the real life for a given load.

More appropriate approaches are advisable for high-performance gears in special fields, and require specific experimentation in conditions similar to the applications as regards sizes, materials, lubrication, loading, and velocity.

The procedure is based on the definition of a loading factor Q_H or Q_F that coincides with the life factor of the standards K_L for RF, or with the squared C_L factor for RH, but works in an opposite way. The value of the life factor depends on the cycle number N and leads to determining a reliability factor C_R or K_R. Such a procedure cannot deal with the cases of more than one loading level. Instead, the Q_H or Q_F factor is calculated as depending directly on the given load, so that the cumulative damage and gear life are rated, and the reliability factor can be determined from the damage. Yielding ratings are facilitated.

Loading factors Q_H *and* Q_F: For each loading level the following ratio is called *loading factor*:

$$\text{RH: } Q_H = \frac{K}{K_{\text{lim}}}, \tag{9.96}$$

$$\text{RF: } Q_F = \frac{U_L}{U_{L\,\text{lim}}}. \tag{9.97}$$

9.8.1 Definition of the Life Curves and Gear Life Ratings for One Loading Level

As the loading factor is proportional to the tangential load W_t, the life curve in Figure 9.1 can be redrawn with Q_H or Q_F as ordinates; see Figure 9.13.

Q_W is the maximum initial value, Q_V corresponds to vertex V, and Q_S is a safety value below which some assume no failure will occur.

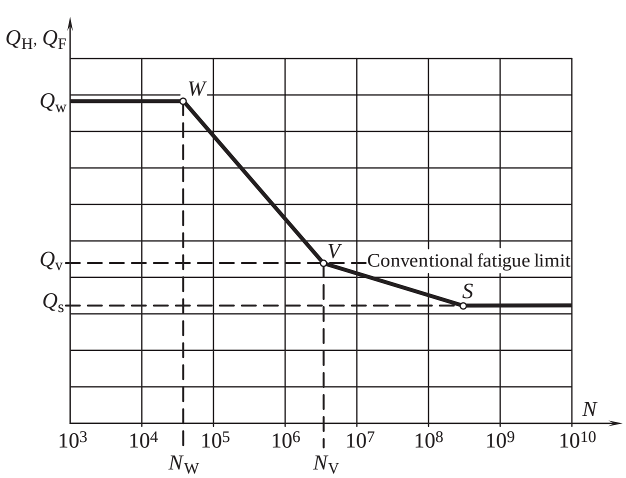


FIGURE 9.13 Unified definition of the characteristic points of the life curves for gear capacity ratings, RH curves (N, Q_H) or RF curves (N, Q_F).

The choice of Q_S is up to the gear designer. It is advisable not to assume Q_{HS} greater than 0.8 or Q_{FS} greater than 0.7, with a possible exception: higher Q_{FS} values might be allowed for RFA, especially if a proper adaptation factor A_F is adopted, because a safety margin is implied in the fatigue limits given by AGMA standards.

Lower values are necessary for an increased reliability, for example, when a gear failure may be dangerous for the operators (see Section 9.8.4), for example, for hoists and marine applications.

The vertex value Q_V equals unity with the exception of RFA, where $Q_{FV} = 1.04$.

The initial maximum value Q_W can be chosen by the designer, or it equals the maximum standard value of the life factor (squared for RH), C_{LW^2} for RH and K_{LW} for RF, reported in Table 9.15. In the same table, the standard cycle numbers for points W and V are given, that is, N_W and N_V.

AGMA does not consider any correction of such maximum. Thus,

$$\text{RHA: } Q_{HW} = C_{LW^2}. \tag{9.98}$$

$$\text{RFA: } Q_{FW} = K_{LW}. \tag{9.99}$$

Note that the concept of Q_W is "yielding" (for direct yielding assessments see the following). ISO suggests some corrections of the life curves that can be extended by assuming approximately that the adaptation factors, as well as size factor for RF, influence fatigue, but not yielding limits. Thus, such factors must be excluded, as they have been introduced in the calculation of the fatigue limit:

$$\text{RHI: } Q_{HW} = \frac{C_{LW^2}}{A_H}, \tag{9.100}$$

$$\text{RFI: } Q_{FW} = \frac{K_{LW} K_s}{A_F}. \tag{9.101}$$

Thus, the estimations are adjusted with regard to the gears that have been tested in the laboratory. Note that the corrections do not mean that the yielding assessment is modified: on the contrary, they mean that the yielding limit is considered as independent of the cited factors that induce the fatigue limit. The ratio Q_W is adjusted for this purpose. (A single further adjustment will be added for RFI for the direct assessment of the margin against yielding; in the case of reverse of bidirectional load, see the following.)

The sloped stretches W_V and V_S of the life curve follow the equations

$$\text{RH: } N_{Hf} = \frac{N_{HV}}{(Q_H)^A}, \tag{9.102}$$

$$\text{RF: } N_{Ff} = \frac{N_{FV}}{(Q_F/Q_{FV})^A}, \tag{9.103}$$

TABLE 9.15
Coordinates of Points Defining Life Curve

Initial point W, Figure 9.13:			
Number of cycles: N_W			
Maximum loading factor: Q_{HW} for RH, Q_{FW} for RF			
RHA	$N_W = 10^4$		All steels
	$Q_{HW} = C_{LW^2}$	$C_{LW^2} = 2.17$	All steels
RHI	$N_W = 10^5$		All materials
	$Q_{HW} = \dfrac{C_{LW^2}}{A_H}$	$C_{LW^2} = 1.69$	For cast iron and for gas-nitrided steels
		$C_{LW^2} = 2.56$	For all other materials
RFA	$N_W = 10^3$		All steels
	$Q_{FW} = K_{LW}$	$K_{LW} = 2.7$	For case-hardened steels
		$K_{LW} = 2.4\left(\dfrac{H_B - 250}{140}\right)$	For through-hardened steels in the range of $210 \le H_B \le 320$
RFI	$N_W = 10^3$		For cast iron and case-hardened steels
	$N_W = 10^4$		For all other materials
	$Q_{FW} = \dfrac{K_{LW} K_S}{A_F}$	$K_{LW} = 1.6$	For cast iron and gas-nitrided steels
		$K_{LW} = 2.5$	For all other materials
Vertex V, Figure 9.13:			
Number of cycles: N_V			
Conventional fatigue limit: Q_{HV} for RH, Q_{FV} for RF			
RHA	$N_V = 10^7$	$Q_{HV} = 1$	All steels
RHI	$N_V = 2 \times 10^6$	$Q_{HV} = 1$	For cast iron and gas-nitrided steels
	$N_V = 5 \times 10^7$	$Q_{HV} = 1$	For all other materials
RFA	$N_V = 3 \times 10^6$	$Q_{HV} = 1.040$	All steels
RFI	$N_V = 3 \times 10^6$	$Q_{HV} = 1$	All materials

Note: The table refers to the following materials: through-hardened and tempered steel; carburized and case-hardened steel; gas-nitrided steel; induction-hardened steel, with root hardening for RF. The RHI and RFI indications include cast iron, malleable iron (pearlitic), and nodular (ductile) iron.

where

$A = A_1$ for the first stretch
$A = A_2$ for the second stretch
N_f = the gear life (cycle number) until failure (pitting or tooth breakage)

The curve stretch WV is fully defined by the previous choice of the coordinates of points W and V. For $N_W \le N \le N_V$

$$A = A_1 = \frac{\log(N_V/N_W)}{\log(Q_W/Q_V)}. \tag{9.104}$$

The exponent A_2 for the stretch VS, that is, for $N > N_V$ and $Q_H > Q_{HS}$ or $> Q_{FS}$, must be chosen by the designer.

AGMA standards only give ranges that correspond:

- For RHA, to $A_{2\,\min} = A_1$, $A_{2\max} = 22$
- For RFA, to $A_{2\,\min} = 31, 56$

ISO does not give any indication. Of course, A_2 cannot be less than A_1, whose value is rather different from AGMA's in its turn.

Lower A_2 leads to lower curve and more pessimistic results. There are a lot of influencing parameters, but for RH the velocity is probably the most important. A low velocity involves boundary lubrication and suggests a lower curve, that is, lower exponent A_2, possibly $A_2 = A_1$; whereas a high velocity allows higher A_2 values if it makes possible EHD lubrication of accurate gears with low surface roughness.

For RF the choice is above all a matter of reliability from various points of view: a good and checked reliability of materials and manufacturing allows higher A_2, while a desired better reliability of gear resistance suggests lower A_2 values.

Different life curves can be established by adopting different coordinates of points W and V. Specific tests or indications of specific books for gear design can help. Any type of ordinates can be found there: pinion torque, tangential load, Hertzian pressure, root stress, and so forth. It is easy to translate them into Q_H and Q_F factors: a point that serves as the conventional fatigue limit should be chosen and a loading factor $Q_V = 1$ attributed. Remember that the Hertzian pressure must be squared when calculating the ratio Q_H. Finally, the exponent A_2 is chosen so that the desired curve is fully schematized.

Now, the cycle number per minute n_L may or may not be equal the gear revolutions per minute according to possible power sharing and uni- or bidirectional loading.

If N_f is the cycle number until failure for a given loading level, as rated by Equation 9.102 or 9.103, the gear life until failure, in hours, is

$$L_f = \frac{N_f}{60 n_L}. \tag{9.105}$$

Note: n_L can be different for RH and RF. For example, consider a common planet pinion. For RF, it completes a load cycle for each revolution of the planet pinion, relative to the sun pinion and ring gear, and includes the load applications on both tooth sides. (Of course, its strength is derated because of the reverse bending load: This affects the preliminary assumption of $s_{t\,lim}$.) For RH, the surface K factor relating to the planet-ring mesh is usually nondamaging for the planet gear. One cycle per revolution must be considered if the load transmitted by the sun gear is unidirectional; but one-half cycle per revolution, as an average, if the load is bidirectional and its application is borne equally by the two tooth sides.

Note: Uni- or bidirectional load is not the same as uni- or bidirectional motion. For instance, the speed reducer for a hoist has bidirectional motion but unidirectional loading, whereas the gears that control the traveling of a bridge crane have both bidirectional motion and load.

9.8.2 Yielding

As in the foregoing discussion, the Q_W value of the loading factor should mean *yielding*.

9.8.2.1 RH

AGMA does not say anything on this subject, whereas ISO explicitly attributes the meaning of surface yielding to the beginning of the life curve. No safety margin is de ned in this regard. Field experience usually does not show any failures for occasional loads at this level, for well-rated gears. In all probability, possible initial plastic deformations of the tooth surface turn into work hardening. Thus, a yielding loading factor Q_{Hy} is simply given by Equation 9.106.

9.8.2.2 RHI

$$Q_{Hy} = Q_{HW}. \tag{9.106}$$

Loading factors Q_H higher than Q_{HW} should not be allowed. Plastic destruction of the tooth surface is not uncommon if excessive loads are applied even for a short time.

9.8.2.3 RF

AGMA gives an independent rating based on a yielding stress s_{ty} (original symbol: s_{ay}), whereas ISO attributes the meaning of yielding to the initial maximum of the life curve. However, a safety margin is necessary for tooth root yielding and it is obtained by means of a factor de ned by AGMA: K_y that must be <1. The de nition of the loading factor facilitates the RF yielding ratings as follows: It must be $Q_F \le Q_{Fy}$, where Q_{Fy} is a yielding loading factor that includes K_y.

9.8.2.4 RFA

$$Q_{Fy} = \frac{K_y s_{ty} K_f}{A_F s_{t\,lim}}. \tag{9.107}$$

The following equation corresponds approximately to AGMA's indications for through-hardened gears and tempered steels in the range of $180 \le H_B \le 410$:

$$s_{ty} = A(H_B - 68), \tag{9.108}$$

where A is 3.3 for metric units (N/mm^2), and A is 480 for English units (lb/in.^{2}).

AGMA standards do not give any indications for surface-hardened steels. Here, the hardening depth is a determining factor. Furthermore, the yielding stress must not be thought of in general terms, as the AGMA geometry factor for bending strength refers to the tooth side where the load is applied. This has to do with normal kinds of fatigue failure, but the stress is higher at the opposite side because of compressive stress.

The following data can be assumed as a guideline, provided that the proper hardening depth be adopted and that a suitable K_y coef cient be introduced (see the following discussion):

Carburized case-hardened steels

$$s_{ty} = 1200 \text{ N/mm}^2 \text{ (175,000 lb/in.}^2\text{)}$$

Tooth root induction hardened

$$s_{ty} = 950 \text{ N/mm}^2 \text{ (140,000 lb/in.}^2\text{)}$$

Gas-nitrided steels

$$s_{ty} = 700 \text{ N/mm}^2 \text{ (100,000 lb/in.}^2\text{)}$$

(Same general speci cations as in Table 9.6.)

9.8.2.5 RFI

$$Q_{Fy} = \frac{K_y Q_{FV} s_{t\,lim\,T}}{s_{t\,lim}}, \tag{9.109}$$

where $s_{t\,lim\,T}$ is the tabulated or test $s_{t\,lim}$, while the adopted $s_{t\,lim}$ can be lower because of reverse or bidirectional load or for any reason.

Note that the introduction of the K_y factor can make $Q_{Fy} < Q_{FW}$. In such cases Q_{FW} serves solely for determining the life curve, but the maximum load level must be kept lower than Q_{Fy}.

AGMA suggests $K_y = 0.75$ for general use, and $K_y = 0.50$ for conservative purposes. Lower values may be advisable in the ratings of cast iron and bronze if the indications of the AGMA standards are followed for s_{ty}. Cautious values may also be used for nitride and for induction-hardened gears. The introduction of K_f in Equation 9.107 serves again to exclude the

notch effect for yielding, but this can be more easily accepted for ductile materials.

AGMA does not exclude K_s for yielding, whereas ISO does. This depends on the different concepts of the factor, but can be considered as an additional margin against yielding of big gears in the AGMA rating, provided that a K_s greater than 1 has been assumed.

9.8.3 Tooth Damage and Cumulative Gear Life

The assumption of the safety loading level Q_S is equivalent to assuming that no damage D_g occurs if $Q_H < Q_{HS}$ or $Q_F < Q_{FS}$: $D_g = 0$; that is, an unlimited life is supposed for RH or RF, respectively, although AGMA and other do not accept this view.

If, on the contrary, the loading level is $\geq Q_S$, the damage of the tooth flank or fillet is

$$D_g = \frac{L}{L_f}, \tag{9.110}$$

where L is the desired life in hours for the loading level considered, and L_f is the gear life until failure, as rated by Equation 9.105.

If x different damaging loads for RH or for RF are applied to the gear, a cumulative damage is rated:

$$D_{gc} = D_{g_1} + D_{g_2} + \cdots + D_{gx}. \tag{9.111}$$

The allowable cumulative gear life under damaging loads is, inversely,

$$L_{fc} = \frac{L_c}{D_g}, \tag{9.112}$$

where L_c is the desired cumulative life for the considered damaging levels. If there are further, nondamaging loading levels, the total allowable gear life increases proportionally to the desired gear life.

9.8.4 Reliability

The reliability factors, C_R for RH and K_R for RF, can be easily calculated in accordance with the definitions of the standards if all cycle numbers of the damaging loads are included either in the first stretch WV of the curve (range: $N_W \leq N \leq N_V$), or in the second stretch VS (range: $N > N_V$, $Q_H > Q_{HS}$ or $Q_F > Q_{FS}$). In this case,

$$\text{RH: } C_R = \left(\frac{1}{D_{gc}}\right)^{1/2A}, \tag{9.113}$$

$$\text{RF: } K_R = \left(\frac{1}{D_{gc}}\right)^{1/A}. \tag{9.114}$$

(ISO symbols: S_H for C_R and S_F for K_R.)

If the cycle numbers correspond to different stretches of the curves, iterative calculations would be necessary. The assessment of reliability factors is not necessary in itself: It is given here just because the standards define them, and indeed Equations 9.113 and 9.114 afford the calculations even in cases where the original methods do not; that is, when more than one loading level is considered. However, the damage assessment is sufficient in itself.

A damage limitation is necessary to get reliability: $D_{gc} < 0.3$ is usually suitable for RH: lower values for RF, especially if a gear failure is dangerous for the operators or costly because it stops a plant far more important than the gears themselves. In such cases, a severe limitation of the RH damage is necessary, too, as extended wear may induce tooth breakage.

Q_{HS} and Q_{FS} must be lowered accordingly when reliability must be increased.

9.9 COPLANAR GEARS: PREVENTION OF TOOTH WEAR AND SCORING

Lubrication regimes and the associated problems of tooth wear and cold and hot scoring are amply treated the book by Townsend (1992). The reader is referred to it for a thorough examination of them and for a detailed calculation of the hot scoring risk. They involve not only ratings in themselves, but designer competence and suitable tooth profile definition. This chapter is limited to some first estimates.

9.9.1 Progressive Tooth Wear

Fast gears in EHD lubrication regime were examined after 40 years of operation and had practically no flank wear. However, slow gears in boundary lubrication always wear. For instance, some very slow spur gears presented a large amount of metal removal in the single contact zone after 10 years—but they did operate for 10 years. In fact, wear was important if compared with cycle number, but the total cycle number was not great, due to the small number of revolutions per minute. The disturbance due to profile alteration was acceptable because of the slowness of motion.

In practice, the same limitation of the tooth pressure that is considered as a pitting prevention serves as a wear prevention, especially if an adaptation factor for tooth velocity was introduced, such as ISO's Z_v, that lowers the conventional fatigue limit for slow gears. If the desired cycle numbers involve the second sloped stretch of the life curve, then a low exponent of the life equation (Equation 9.102 or 9.103) is necessary for slow gears. And a proper lubricant with sufficient density and viscosity is necessary. Lubricant additives are useful if they are appropriate: for example, additives that favor run-in and/or reduce the friction coefficient may threaten wear in the long term, besides their possible action in pitting prevention.

9.9.2 Scoring and Scuffing

Gears can score even at low speed (cold scoring) if inappropriate lubricants are used and if the gear design does not prevent contacts in high-sliding zones. Cold scoring is distinguished

from progressive wear, not only because of a different tooth flank appearance, but because it can occur after a short operating time. "Scoring" usually means hot scoring, that is, tooth surface destruction of fast loaded gears at high contact temperature. Many gear features can influence scoring. For instance, edge contact of gear teeth may be an important item for scoring risk, and a proper tip relief can prevent it. Another important item is the tooth surface finish. A detailed rating method is based on the concept of flash temperature. Important indications on suitable profile features are also given there. Other detailed methods are given by the standards.

Scoring prevention does not concern common industrial gear applications, since EP lubricants have become usual, but only for fast, heavily loaded gears.

This chapter gives only a simplified equation, approximately determined from Niemann's original indications for good EP oils. For external gears with parallel shafts and $\phi_{ns} = 20°$, $v_t < 30$ m/s, $m_n < 10$ mm, $m_{PA} \le \cos \psi_s$, and $m_{PE} \le \cos \psi_s$,

$$W_{\mathrm{t\,scor}} = \left(\frac{d_P}{1 + N_P/N_G} \right)^3 \frac{0.48F \cos \psi_s}{V v_t \left(0.007 m_n^2 + \sqrt{m_n} \right)}, \quad (9.115)$$

where d_P, F, and m_n are in millimeters, v_t is in meters/second, and the scoring tangential load is determined in newtons. In the case of English units, it is better to translate then into metric units at first and apply the formula just as it is.

The equation emphasizes the risk of scoring for higher loads. It usually gives a safety margin, as the best EP oil may allow loads even greater by two or three times to be applied without scoring. There are many circumstances in the risk of scoring, and for any doubtful case it is advisable that proper tooth design and more detailed ratings be performed. For example, the basic gear temperature is an important item, and higher power losses mean higher gear temperature, if all other parameters are equal.

Gear pairs with higher contact rations require special investigations, preferably experimental ones.

9.10 CROSSED-HELICAL GEARS

Crossed-helical gears theoretically have point contact so that they can carry a very limited load for RH, while they present no problems for RF. The Hertz theory can be applied to the specific case by means of tabulated factors. However, such kinds of gears are often employed as small accessories for mass production, for example, of combustion engines. Thus, it is both desired and possible to perform preliminary tests.

A summary review of performance of crossed-helical gears, manufactured with typical materials, is given in Table 9.16. A variant can have case-hardened pinions with ground teeth and bath-nitrided gear with shaved teeth.

9.11 HYPOID GEARS

In theory, the load capacity rating of hypoid gears might be approached in a way not too different from what is adopted for bevel gears, if one takes into account the real pinion size and corrects the curvature radii of the profiles for RH. In practice, a lot of conflicting observations can be made. Longitudinal sliding may increase the risk of scoring. However, the profile design of such gears can be very accurate at present. The tooth surface can be made less sensitive to axial displacements and misalignment due to mounting and to elastic deflections. They may then carry greater loads compared to bevel gears as designed by more common criteria. Finally, their load capacity must be fully exploited as the reduction of their overall size is important in their usual application fields, for example, for vehicle transmissions. All such observations lead to one conclusion: it is better to size hypoid gears according to previous field experience and test them for their specific application than to calculate their

TABLE 9.16
Normal Capacity of Crossed-Helical Gears (Service Factor = 1)

Pinion Pitch Diameter d_P, mm (in.)	Gear Ratio m_G	Gear Pitch Diameter d_P (mm)	Center Distance C (mm)	Pinion Speed n_P (rpm) 500	1000	2000	3000	4000
				Power (kW) (Multiply by 1.341 for hp)				
25 (~1)	1	25	25	0.008	0.015	0.025	0.033	0.041
	3	75	50	0.028	0.052	0.091	0.120	0.150
	5	125	75	0.036	0.068	0.120	0.160	0.190
50 (~2)	1	50	50	0.063	0.120	0.200	0.270	0.330
	3	150	100	0.220	0.420	0.730	0.970	1.180
	5	250	150	0.290	0.540	0.940	1.260	1.530
75 (~3)	1	75	75	0.230	0.420	0.730	0.970	1.180
	3	225	150	0.810	1.510	2.640	3.510	4.260
	5	375	225	1.050	1.960	3.430	4.560	5.540

Note: The table is deduced from the first edition of the *Gear Handbook*, but the helix angle is unified to 45° to enable grinding of the pinion teeth. The given power serves only as a rough indication. Higher performance can be achieved by special materials in tested operating conditions. Crossed-helical gears should be avoided as far as possible in common industrial applications. Gear data: $\phi_n = 20°$, $\psi = 45°$. Pinion in alloy steel, carburized and case-hardened, 58–62 HRC, ground teeth. Gear is phosphorus bronze. Short run-in.

load capacity. An approximate idea of the performance of hypoid gears can be determined from the same Table 9.12 that deals with spiral bevel gears, corresponding to equal gear diameter.

9.12 WORM GEARING

Double-enveloping worm gears are not considered here: In fact, either they are used as precision gearing where the transmitted power is very low and so they are oversized and a capacity rating is not worthwhile—or they require speci c tests, as their operating behavior depends on a lot of speci c manufacturing and application conditions.

9.12.1 Cylindrical Worm Gearing

The existing standard methods for rating the capacity of worm gears are based on the Hertz theory for RH. They include some empirical factors or modi cations that do not take into due consideration the surface nish of both worm threads and wheel teeth (which may be only partly improved by a run-in period) and the actual tooth-bearing area. The hob that cuts the wheel teeth usually has larger diameter than the worm, and its parameters affect the contact conditions of the gear pair in an unforeseeable way, unless a speci c software is used for investigating them. The lubrication conditions and the lubricant types greatly in uence the real performance of the gear pair. Furthermore, the high sliding produces a gear amount of heat that must be dissipated through the housing. Therefore, the housing design and cooling are more important for the real performances of worm gears than for common coplanar gears, especially when a continuous load is transmitted.

Thus, the ratings for worm gears are less reliable than those for coplanar gears (even if the latter are not completely reliable either).

Table 9.17 gives approximate values of the allowable input power for speed-reducing worm gears manufactured

TABLE 9.17
Nominal Capacity of Cylindrical Worm Gear Pairs for Standard Speed Reducers (Service Factor C_{SF} = 1)

		Center Distance, mm (in.)									
		75	(3)	100	(4)	150	(6)	200	(8)	300	(12)
Ratio	Worm Speed (rpm)	Input Power, kW (hp)									
10	500	1.7	(2.3)	3.7	(4.9)	9.5	(13)	17	(23)	35	(47)
	710	2.1	(2.8)	4.6	(6.1)	12.0	(16)	22	(29)	45	(60)
	1000	2.6	(3.5)	5.7	(7.6)	15.0	(20)	27	(37)	57	(77)
	1420	3.2	(4.3)	7.1	(9.6)	19.0	(25)	35	(47)	73	(98)
	1700	3.6	(4.8)	8.0	(11.0)	21.0	(29)	39	(53)	83	(111)
20	500	1.0	(1.4)	2.3	(3.1)	5.9	(8.0)	11	(15)	23	(30)
	710	1.3	(1.7)	2.9	(3.9)	7.5	(10.0)	14	(19)	29	(39)
	1000	1.6	(2.1)	3.6	(4.8)	9.4	(13.0)	17	(23)	37	(49)
	1420	2.0	(2.7)	4.5	(6.0)	12.0	(16.0)	22	(30)	47	(63)
	1700	2.2	(3.0)	5.1	(6.8)	13.0	(18.0)	25	(33)	53	(71)
30	500	0.8	(1.1)	1.7	(2.3)	4.4	(5.9)	8.1	(11)	17	(21)
	710	1.1	(1.4)	2.2	(2.9)	5.6	(7.5)	10.0	(14)	22	(29)
	1000	1.3	(1.7)	2.7	(3.6)	7.0	(9.4)	13.0	(17)	28	(37)
	1420	1.6	(2.2)	3.4	(4.5)	8.8	(12.0)	16.0	(22)	36	(48)
	1700	1.8	(2.4)	3.8	(5.1)	9.9	(13.0)	19.0	(25)	40	(54)
40	500	0.7	(1.0)	1.4	(1.9)	3.6	(4.8)	6.6	(8.9)	14	(19)
	710	0.9	(1.2)	1.8	(2.4)	4.5	(6.1)	8.4	(11.0)	18	(24)
	1000	1.1	(1.5)	2.2	(3.0)	5.7	(7.6)	11.0	(14.0)	23	(31)
	1420	1.4	(1.8)	2.8	(3.7)	7.2	(9.6)	13.0	(18.0)	29	(39)
	1700	1.5	(2.0)	3.1	(4.1)	8.1	(11.0)	15.0	(20.0)	33	(44)
50	500	0.6	(0.8)	1.2	(1.5)	3.0	(4.1)	5.6	(7.5)	12	(16)
	710	0.7	(1.0)	1.4	(1.9)	3.8	(5.1)	7.1	(9.6)	15	(20)
	1000	0.9	(1.2)	1.8	(2.4)	4.8	(6.4)	9.0	(12.0)	19	(26)
	1420	1.1	(1.5)	2.3	(3.0)	6.0	(8.1)	11.0	(15.0)	25	(33)
	1700	1.3	(1.7)	2.5	(3.4)	6.8	(9.1)	13.0	(17.0)	28	(38)
60	500	0.5	(0.6)	1.1	(1.3)	2.5	(3.4)	4.7	(6.4)	10	(13)
	710	0.6	(0.8)	1.2	(1.6)	3.2	(4.3)	6.0	(8.1)	13	(17)
	1000	0.7	(0.9)	1.5	(2.0)	4.0	(5.4)	7.6	(10.0)	16	(22)
	1420	0.9	(1.2)	1.9	(2.5)	5.0	(6.8)	9.6	(13.0)	21	(28)
	1700	1.0	(1.3)	2.1	(2.8)	5.7	(7.6)	11.0	(15.0)	23	(31)

by specialized firms. The values must be multiplied by the efficiency for obtaining the output power. The table refers to service factors equal to 1 for gear durability.

Proper housing design and fan cooling ensure good heat dissipation for such speed reducers. Nevertheless, the power values must be thought of as mechanical rather than as thermal limits. The service factors usually account for the case of continuous and prolonged loading, as well as for application conditions and required life as usual.

The table relates to the following:

- Worms: alloy-steel, case-hardened—ground threads with involute profile
- Wheels: centrifugally cast special bronze—hobbing with good accuracy and surface finish
- Suitable gear diameters for optimizing efficiency and durability
- Suitable diameter difference between the hob (generating the wheel) and the worm, and suitable bearing zone between thread and wheel tooth

Either superfinishing techniques are used for both worm and wheel, or some run-in period is required.

There is a great difference between the allowable power transmission of such specialized speed reducers and that of worm gears applied, as a part of any machine, internally to a nonspecific housing. The same worm gears of said speed reducers are often employed in such case but must be essentially derated.

The problem of designing and manufacturing a worm gear pair for a generic mechanical application is that only the best conditions of materials, manufacturing, lubrication, and housing cooling give sufficient reliability to load capacity assessments. Each one of such items not only derates performance, but makes it more uncertain, if its condition is not the best.

In industrial use the differences in types of thread profile, given equal surface finish and accuracy, are less important. Nevertheless, worms with so-called straight profiles may have a capacity reduction depending on the diameter of the grinding wheel, as the ground profile usually is not really straight and differs from the profile of the hob that generates the worm wheel. Such a combination may in some cases obtain a favorable relief effect, but often reduces the tooth bearing area. A prolonged run-in can usually help, unless the profile combination is such that it produces a bad operating conjugation. Some tests show an advantage for special worms with concave thread flanks.

The load capacity essentially decreases for worms with nonground threads: in this case the usual assessments are absolutely unreliable.

9.13 STATE OF THE STANDARDS FOR LOAD RATING OF GEARS

For coplanar and bevel gears, detailed information on the state of AGMA, ISO, and DIN standards is given by French gear engineers. For worm gearing, the conception of a standard method is proposed by Octrue (1985, 1988).

10 Gear Manufacturing Methods

The many methods of making gear teeth must be considered by the gear designer. The size and the geometric shape of the gear or the pinion must be within the capacity range of some machine tool. If the lowest competitive cost is to be obtained, then the gear designer must make the gear of a size, shape, and material that will permit the most economical method of manufacture.

The purpose of this chapter is to review all the commonly used methods of making gears. Illustrative data concerning the sizes and the kinds of machine tools now available on the market will be given. Design limitations for each method of manufacture will be discussed so that the gear designer will have at least a general idea of what can be done by each method. Some data will be given on how fast gears can be made by each method. This is a controversial subject. The reader should use the data given with caution. They will not t all situations.

A subject like gear manufacturing methods is broad enough to require several books.* In this chapter, it will be possible to tell only a small part of the story. Further information may be found in the references at the end of the book.

Gear manufacturing terms used in this chapter are de ned in the glossary in Table 10.1.

Figure 10.1 shows a broad outline of the methods used to make gear teeth. Methods which are geometrically similar are grouped together. This gure shows that the gear designer will have to evaluate a large number of methods to choose the most suitable method for each job.

10.1 GEAR TOOTH CUTTING

A wide variety of machines is used to cut gear teeth. As shown in Figure 10.1, there are more or less four distinct ways to cut material from a gear blank so as to leave a toothed wheel after cutting. The cutting tool may be threaded and gashed. If so, it is a hob, and the method of cutting is called *hobbing.* When the cutting tool is shaped like a pinion or a section of a rack, it will be used in a cutting method called *shaping.* In the *milling* process, the cutting tool is a toothed disk with a gear tooth contour ground into the sides of the teeth. The fourth general method uses a tool, or series of tools, that wraps around the gear and cuts all teeth at the same time. Methods of this type are *broaching, punching,* and *shear cutting.*

10.1.1 Gear Hobbing

Spur, helical, crossed-helical (spiral), and worm gears can be produced by hobbing. All gears but worm gears are cut by feeding the hob across the face width of the gear. In the case of worm gears, the hob is fed either tangentially past the blank or radially into the blank. Figure 10.2 shows how a hob forms teeth on different kinds of gears.

A wide variety of sizes and kinds of hobbing machines are used. Machines have been built to hob gears all the way from less than 2 mm to over 10 m ($\frac{3}{32}$ to 400 in.) in diameter. Large double-helical gears are frequently hobbed with double-stanchion machines. These hobbers use two cutting heads 180° apart to cut both helices at once. Figures 10.3 through 10.5 show several examples of hobbing.

In designing gears to be hobbed, a number of things must be considered. A hob needs clearance to run out at the end of the cut. If the gear teeth come too close to a shoulder or other obstruction, it may be impossible to cut the part by hobbing. If the gear is double helical, a gap must be left between the helices for hob runout. The method of calculating the width of this gap is given in Section 11.2. If it is not possible to use the narrowest possible gap, the values shown in Table 10.2 may be used.

Some large hobbers do not have centers to mount the work. In such cases, the gear is normally clamped on its rim. This makes it necessary to provide rim surfaces that are true with the journals of the part. Most of the smaller hobbing machines mount the work on centers. If the part does not have shaft extensions with centers, it is necessary to provide tooling so that the part can be mounted on a cutting arbor with centers.

The hobbing process is quite advantageous in cutting gears with very wide face widths or gears that have a toothed section which is integrated with a long shaft. A very high degree of tooth spacing accuracy can be obtained with hobbing. High-speed marine and industrial gears with pitch-line speeds in the range of 15 to 100 m/s (3000 to 20,000 fpm) and diameters up to 5 m (200 in.) are very often cut by hobbing. A few large mill gears up to 10 m (400 in.) in diameter are hobbed. (These are not high-speed gears.)

Gears can be nished by setting the hob to full depth and making only one cut. Where highest accuracy is desired, it is customary to make a *roughing* and *nishing* cut. The roughing cut removes almost all the stock. The nishing cut may remove from 0.25 to 1.00 mm (0.010 to 0.040 in.) of tooth thickness, depending on the size of the tooth.

The time required to make a hobbing cut can be calculated from the following formula:

$$\text{Hobbing time (min)} = \frac{\text{no. of gear teeth} \times (\text{face} + \text{gap})}{\text{no. of hob threads} \times \text{feed} \times \text{hob (rpm)}}. \tag{10.1}$$

The reason for adding the gap to the face width is the fact that a hob has to travel a certain distance in going into the cut and coming out of the cut. This extra travel happens to equal

* The interested reader may wish to go for details of gear manufacturing methods to the monograph of Radzevich (2010).

TABLE 10.1
Glossary of Gear Manufacturing Nomenclature

Term	Definition
Broaching	A machining operation which rapidly forms a desired contour in a working surface by moving a cutter, called a *broach*, entirely past the workpiece. The broach has a long series of cutting teeth that gradually increase in height. The broach can be made in many different shapes to produce a variety of contours. The last few teeth of the broach are designed to finish the cut rather than to remove considerably more metal. Broaches are often used to cut internal gear teeth, racks, and gear segments on small gears, and are usually designed to cut all teeth at the same time.
Burnishing	A finishing operation which polishes a surface by rubbing.
Casting	A process of pouring molten metal into a mold so that the metal hardens into desired shape. Casting is often used to make gear blanks that will have cut teeth. Small gears are frequently cast complete with teeth by the die-casting process, which uses a precision mold of tool steel and low–melting point alloys for the gears.
Drawing	A metal-forming process used to make gear teeth on a small-diameter rod by pulling the rod through a small gear-shaped hole.
Extruding	A process that uses extreme pressure to push solid metal through a die of the desired shape.
Generating	Any gear-cutting method in which the cutter rolls in mesh with the gear being cut. A generating method allows a straight-sided cutter to cut a curved involute profile in the gear blank.
Grinding	A process that shapes the surface by passes with a rotating abrasive wheel. Grinding is not a practical way to remove large amounts of metal, so grinding is used to make very fine-pitch teeth, or to remove heat-treat distortion from large gears that have been cut and then fully hardened. Many different kinds of grinding operations are used in gear manufacture.
Hobbing	A precise gear tooth-cutting operation that uses a threaded and gashed cutting tool called a *hob* to remove the metal between the teeth. The rotating hob has a series of rack teeth arranged in a spiral around the outside of a cylinder (see Figure 11.9), so it cuts several gear teeth at one time.
Lapping	A polishing operation which uses an abrasive paste to finish the surfaces of gear teeth. Generally, a toothed, cast-iron lap is rolled with the gear being finished.
Milling	A machining operation which removes the metal between two gear teeth by passing a rotating cutting wheel across the gear blanks.
Molding	A process like casting which involves filling a specially shaped container (the mold) with liquid plastic or metal, so that the material has the shape of the mold after it cools. Injection molding machines use high pressure to force the hot plastic into steel gear molds.
Punching	A fast, inexpensive method of producing small gears from thin sheets of metal. The metal is sheared by a punching die which stamps through the sheet stock into a mating hole.
Rolling	A process which rapidly shapes fine gear teeth or worm threads by high pressure rolling with a toothed die.
Shaping	A gear-cutting method in which the cutting tool is shaped like a pinion. The shaper cuts while traversing across the face width and rolling with the gear blank at the same time.
Shaving	A finishing operation that uses a serrated gear-shaped or rack-shaped cutter to shave off small amounts of metal as the gear and cutter are meshed at an angle to one another. The crossed axes create a sliding motion which enables the shaving cutter to cut.
Shear cutting	A rapid gear-cutting process which cuts all gear teeth at the same time using a highly specialized machine. This method of cutting gear teeth is no longer used very frequently.
Sintering	A process for making small gears by pressing powder metal into a precision mold under great pressure and then baking the resulting gear-shaped briquette in an oven. Sintering is cost effective only for quantity production because the molds and tools are very expensive.
Stamping	Another word for punching.

the amount of space that is necessary for the gap between helices. Equation 10.1 works with either millimeters or inches for face and feed.

In the past, it was quite customary to use single-thread hobs for finish and single- or double-thread hobs for roughing. Single-thread hobs give the best surface finish and tooth accuracy. In many cases, though, multiple-thread hobs are used for both roughing and finishing. When gears are shaved or lapped after hobbing, surface finish in hobbing is not quite so important. If the gear has enough teeth and they are not an even multiple of the number of the hob threads, hobs with as many as five or seven threads may be quite profitably used. For best results, there should be about 30 gear teeth for each hob thread. This would mean that a five-thread hob would not be used to cut fewer than 151 gear teeth. For commercial work of moderate accuracy, the number of gear teeth may be as low as 15 per hob thread.

Hobbing feeds and speeds depend on how well the gear material cuts, the accuracy desired, the size of the gear, and the strength of the hobbing machine. Table 10.3 shows representative values for three classes of work. The *conservative* values show what would be used when best accuracy was needed. The *high-speed* values are about the highest that can be obtained when machine and hob designs receive special attention, and accuracy is not the most important thing. The size of hob will depend on the pitch. Section 11.2 shows standard hob sizes for different pitches.

Table 10.3 is based on a low-hardness steel of 100% machinability. For different hardnesses, the cutting speed should be reduced approximately as indicated in Table 10.4.

The above table is based on hobbers of rugged design cutting parts well within the machine capacity. In many cases, it is necessary to reduce feeds and speeds to about 60% of the values given to reduce wear on the hob caused by machine vibration.

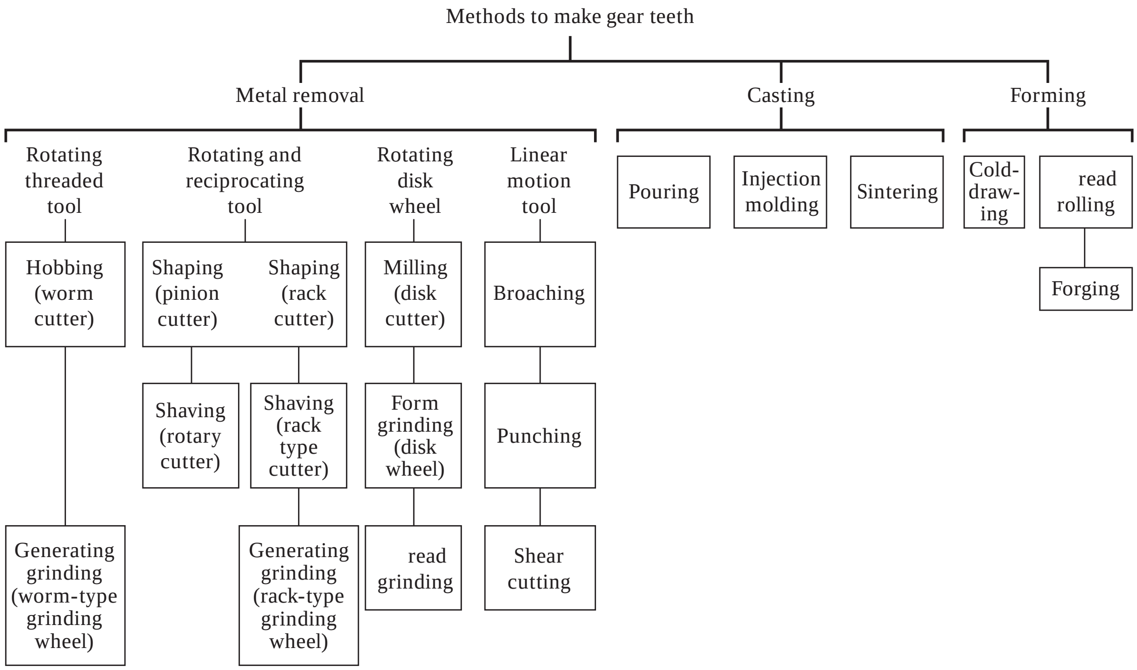


FIGURE 10.1 Outline of methods of making gear teeth.

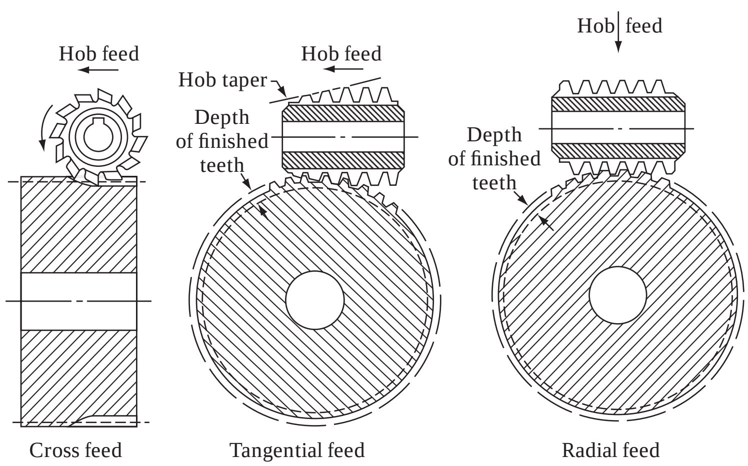


FIGURE 10.2 Comparison of different kinds of hobbing.

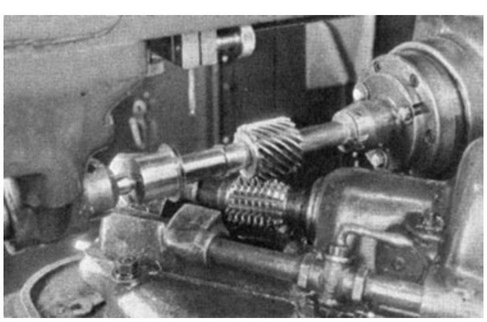


FIGURE 10.3 A tapered hob ready to nish cut a small helical gear. This picture shows the basics of gear hobbing.

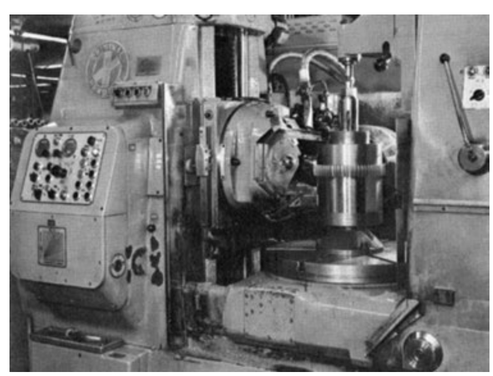


FIGURE 10.4 A large coupling hub with external spur teeth that has been precision hobbed on a medium-size hobbing machine. (Courtesy of Sier-Bath Gear Col, Inc., North Bergen, New Jersey.)

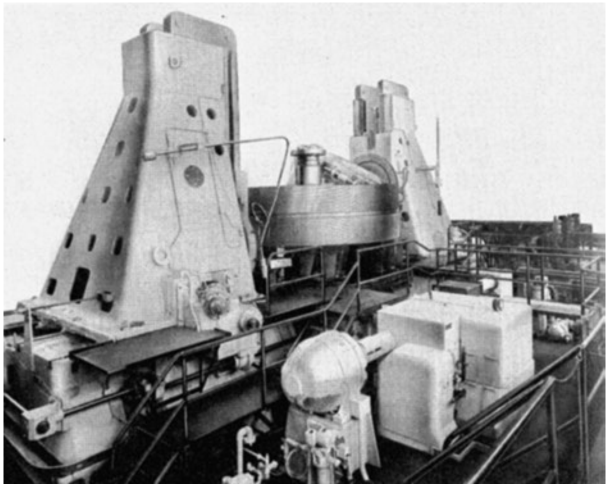


FIGURE 10.5 A large hobbing machine used to cut precision gears for large ships. (Courtesy of General Electric Co., Lynn, Massachusetts.)

In the 1960s and the 1970s and in later years, many advancements were made in machine tool design and cutting tool design. The art of gear cutting made notable improvements in both the accuracy and the production rates that could be achieved. Also, the diversity of equipment and methods became much greater than it was in earlier years. It is not possible in this book to cover all the latest things in gear hobbing.

To give the reader an appreciation of the state of the art in hobbing, Table 10.5 was prepared. This table shows the nominal production times for a small gearset and a large gearset. It also shows fast production time—what can be done using the most advanced hobbers, hobs, and hobbing techniques.

The data shown in Table 10.5 are a composite of survey data collected. This means that the data are somewhat average. It is possible to do even better under the most favorable conditions. (Of course, under poor conditions, Table 10.5 times for either job shop or fast production will not be achievable.)

TABLE 10.2
Nominal Gap Width

Tooth Size		Hob Diameter		Gap Width: 15° Helix		Gap Width: 30° Helix		Gap Width: 45° Helix	
Module, Normal	Normal Diametral Pitch	mm	in.	mm	in.	mm	in.	mm	in.
1.25	20	48	1.875	16	0.625	19	0.75	19	0.75
1.60	16	64	2.50	22	0.875	26	1.00	26	1.00
2.50	10	76	3.00	32	1.250	38	1.50	38	1.50
3.00	8	76	3.00	35	1.375	48	1.875	48	1.875
4.00	6	89	3.50	45	1.750	51	2.00	57	2.25
6.00	4	102	4.00	57	2.250	70	2.75	83	3.25
8.00	3	115	4.50	73	2.875	89	3.50	105	4.125

TABLE 10.3
Hobbing Feeds and Speeds

Hob Diameter		Feed per Revolution: Finishing, 1 Thread		Roughing, 1 Thread		Roughing, 2 Thread		Roughing, 3 Thread		Speed (rpm)		
mm	in.	mm	in.	mm	in.	mm	in.	mm	in.	Conservative	Normal	High
25	1	0.5	0.020	1.3	0.050	0.9	0.035	0.6	0.025	590	870	1150
47	1.875	0.7	0.030	1.5	0.060	1.1	0.045	0.7	0.030	270	370	490
75	3	1.1	0.045	2.2	0.085	1.7	0.065	1.0	0.040	145	210	270
100	4	1.4	0.055	2.5	0.100	1.9	0.075	1.3	0.050	105	150	200
125	5	1.5	0.060	2.5	0.100	1.9	0.075	1.3	0.050	80	115	150
150	6	1.7	0.065	2.8	0.110	2.0	0.080	1.4	0.055	65	95	125
200	8	1.8	0.070	3.0	0.120	2.3	0.090	1.5	0.060	50	70	95

Note: These values are based on AISI B1112 steel, 100% machinability rating.

TABLE 10.4
The Recommended Values for Reduction of Cutting Speed

Material	Hardness HV	Hardness HB	Percentage of Table 7.3 Speed
Steel	370	350	32
	320	300	40
	205	200	56
Cast iron	180	175	85

Table 10.5 brings out these important considerations:

- Fast production work with the best equipment may be three to five times faster than job shop work with more ordinary equipment.
- Finish cutting of the harder steels may take two or three times longer than pregrind cutting of lower-hardness steels.
- Skive hobbing of fully hardened steels is as fast as or faster than finish hobbing medium-hard steels.

10.1.2 Shaping Pinion Cutter

Spur, helical, and face gears and worms can be cut with a pinion-type cutter. Either internal or external gears can be cut. Parts from less than 1 mm to over 3 m (1⁄16 to over 120 in.) may be shaped. Relatively wide face widths may be cut, but in certain cases, shaping will not handle as much face width as the hobbing process. For example, a standard 1.25 m (50 in.) gear-shaping machine will have a face width capacity of 0.2 m (8 in.) regardless of helix angle, but a comparably sized hobbing machine would handle face width of 0.6 m (24 in.), depending on the helix angle.

Figures 10.6 and 10.7 show examples of shaper cutting.

Shaper-cutters need only a small amount of cutter-runout clearance at the end of the cut or stroke. Shaped teeth may be located close to the shoulders. A cluster gear can be readily shaper-cut where it may be impossible to hob because of insufficient hob-runout clearance. One numerically controlled (NC) shaper, pictured in Figure 10.8, could conceivably cut a cluster gear in one setup, depending upon gear data. Double-helical gears can be cut with very narrow gaps between helices. In fact, one design of shaper can actually produce a continuous double-helical tooth.

In designing gears to be shaped, it is necessary to machine a groove as deep as the gear tooth at the end of the face width for runout of the cutter. Normal values for the width of this groove are listed in Table 10.6.

It is difficult to mount parts between centers while they are being shaper-cut, since the bottom portion must be driven. At least one end must therefore be clamped to a fixture or gripped by a chuck. Gear parts which are integral with long shaft extensions may be supported at the upper end by a center or a steady rest, and a long shaft extension downward may be accommodated by a hole in the base of the machine bed and even by a recessed portion of the machine foundation.

Gear shaping is quite advantageous on parts with narrow face widths. In hobbing, it takes time for the hob to travel into and out of the cut. For helical gears, the hob travel must be proportionally increased. In shaping, there is a minimum of overtravel for spur gears, and this overtravel does not increase for helical gears. For instance, a 2.5-module (10-pitch) gear with 25 mm (1 in.) face width would have an extra travel in hobbing of about 32 mm (1¼ in.). In shaping, the extra travel would be only 5.56 mm (0.219 in.). In this example, the hob would cut across more than

TABLE 10.5
Some Examples of Production Time for Hobbing or Milling Gear Teeth

Examples of Gearsets	Number of Teeth	Module	Face Width (mm)	Material and Hardness	Production Time per Piece N	Production Time per Piece F
1	25	2	32	Alloy steel, pregrind cut at Vickers hardness	5 min	1.5 min
	102	2	30	of about 285	11 min	3 min
2	25	2	32	Alloy steel, preshave cut at Vickers	15 min	3 min
	102	2	30	hardness of about 375	30 min	6 min
3	25	15	307	Alloy steel, finish hob at Vickers hardness	6 h	2.5 h
	102	15	300	of about 325	22 h	9 h
4	25	15	307	Alloy steel, carburized and hardened to	4.5 h	1.7 h
	102	15	300	Vickers 700, finished by skiving hobbing	17 h	6.5 h

Note: 1. The above table is based on spur gears, or helical gears up to 15° helix angle, and 20° pressure angle.
2. The alloy steel would contain chromium, manganese, nickel, and molybdenum. Good examples are AISI 4320 and 4340.
3. Vickers 285 = approximately 270 HB; Vickers 375 = approximately 353 HB; Vickers 700 = approximately 58.5 HRC.
4. Production time cut: N, nominal job shop work. Milling or hobbing teeth, single-start conventional hobs or cutters. Work-holding fixtures and hobbing machines somewhat old and not too well suited to the exact work being done. F, a production hobbing facility doing fast repeat work. Multistart and coated hobs being used wherever possible. (A popular hard coating for extra performance is titanium nitride.) Work-holding fixtures of special design. Most modern, high-performance hobbing machines.

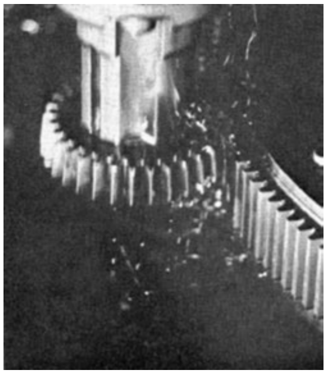


FIGURE 10.6 The basics of shaper cutting.

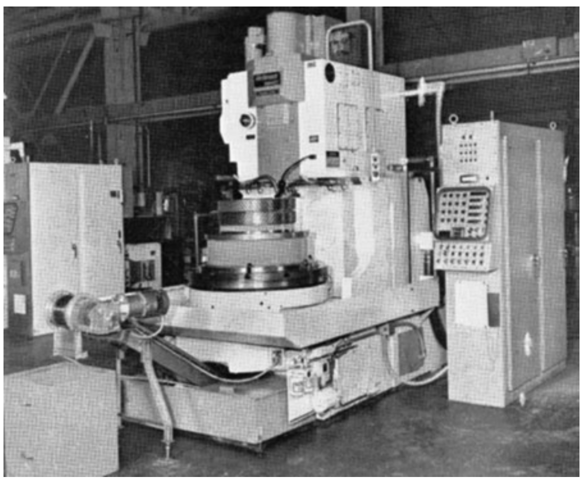


FIGURE 10.7 A shaping machine that has just cut a medium-sized double-helical gear. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

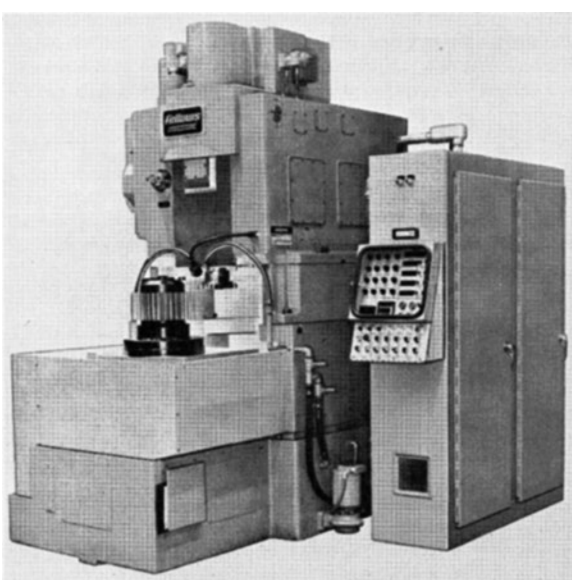


FIGURE 10.8 The *Hydrostroke* gear-shaping machine. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

TABLE 10.6
Normal Values of Width of the Runout Groove

Tooth Size		Width of the Runout Groove					
		Spur		15° Helix		23° Helix	
Module	Diametral Pitch	mm	in.	mm	in.	mm	in.
1.0	24	5.0	3/16	6.0	15/64	6.5	¼
1.8	14	5.0	3/16	6.5	¼	7.0	17/64
2.5	10	5.5	7/32	7.2	9/32	8.0	5/16
4.0	6	6.5	¼	7.2	9/32	8.0	5/16
6.0	4	7.2	9/32	9.0	11/32	9.5	3/8

twice the face width that a shaper-cutter would to do the same part. Furthermore, with the advent of the modern cutter spindle back-off type machines, it is not uncommon to see stroking rates of 1000 strokes per minute for face widths 25 mm (1 in.) or smaller. In fact, for narrow face widths of 6 mm (¼ in.), high-speed shapers capable of stroking rates of over 2000 strokes per minute are in use. Hydrostatically mounted guide and cutter spindle are necessary because of these high stroking rates.

The time in minutes required to shaper-cut a gear with a disk-type cutter may be estimated by the following formula:

$$\text{Shaping time} = \frac{\text{no. of gear teeth} \times \text{strokes per rev. of cutter} \times \text{no. of cuts}}{\text{no. of cutter teeth} \times \text{no. of strokes per min}}. \tag{10.2}$$

It should be noted here that a *stroke* is a cutting stroke and a return stroke. Thus, a stroke is really a double stroke.

In shaping the coarser pitches, it is necessary to take several roughing and nishing cuts. Table 10.7 shows some nominal sizes of cutters and the number of cuts taken when shaping steel teeth of about 200 HB.

TABLE 10.7
Nominal Sizes of Shaper Cutters and Number of Shaping Cuts

Tooth Size		Cutter Diameter		Number of Cuts	
Module	Diametral Pitch	mm	in.	Roughing	Finishing
1.25	20	50	2	1	1
		75	3	1	
2.5	10	75	3	1	1
		100	4	1	1
4	6	100	4	1 or 2	1
		115	4.5	1 or 2	1
6	4	100	4	2 or 3	1
		125	5	2 or 3	1
12	2	150	6	2 or 3	1
		175	7	2 or 3	1

Table 10.8 gives some representative values for the rotary feed rate per double stroke. These represent average values which might be used in cutting medium-precision gears.

The number of double strokes per revolution of the cutter can be calculated by the following:

$$\text{Strokes per revolution} = \frac{\text{cutter diameter} \times \pi}{\text{rotary feed rate}}. \quad (10.3)$$

The number of double strokes per minute is calculated from the following formula:

$$\text{Strokes per minute} = \frac{1000 \times \text{max. cutting speed (m/min)}}{\text{stroke length (mm)} \times \pi} \text{ (metric)}, \quad (10.4)$$

$$\text{Strokes per minute} = \frac{12 \times \text{max. cutting speed (ft/min)}}{\text{stroke length (in.)} \times \pi} \text{ (English)}. \quad (10.5)$$

The cutting speed depends on the face width of the gear, the hardness and machinability of the material, the degree of quality, the cutter material, and the desired cutter life. See Table 10.9.

Just as considerable advancements have been made in hobbing, there have been notable advancements in shaper-cutting gears in the 1960s and later. Perhaps the most notable are the following:

- NC machines
- Cutter spindle back-off (instead of worktable back-off)
- Cutter spindle hydraulically moved instead of by mechanical means
- New infeed methods
- Machine with column that moves and stationary worktable (conventional shapers work the opposite way)
- Better shaper cutter materials and special hard coatings

These things have led to signi cant improvements in the precision of shaped gears and considerable improvement in gear shop production of shaped gears.

Figure 10.9 shows the Hydrostroke machine, developed by Fellows Corporation. The details of how the cutter spindle

TABLE 10.8
Rotary Feed for Each Double Stroke of the Shaping Cutter

			Rotary Feed per Double Stroke							
	Brinell Hardness	Machin-	1.5 to 2.5 Module (10 to 17 DP)		2.5 to 4 Module (6 to 10 DP)		4 to 6 Module (4 to 6 DP)		6 to 9 Module (3 to 4 DP)	
Material	**(HB)**	**ability (%)**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**
Steel to be	135	100	0.5	0.020	0.5	0.020	0.6	0.024	0.6	0.024
case-hardened	185	80	0.5	0.020	0.5	0.020	0.6	0.024	0.6	0.024
	220	65	0.3	0.012	0.35	0.014	0.45	0.018	0.5	0.020
Through-hardened	172	72	0.4	0.016	0.4	0.016	0.5	0.020	0.6	0.024
alloy steel	217	55	0.3	0.012	0.35	0.014	0.4	0.016	0.6	0.024
	254	45	0.25	0.010	0.3	0.012	0.3	0.012	0.4	0.016
Plastics	–	130	0.20	0.008	0.3	0.012	0.3	0.012	0.35	0.014

Note: The above feed values are based on a roughing and a nishing cut for 6 module (4 diametral pitch) and smaller teeth. For large teeth, up to 10 module, two roughing cuts and one nishing cut are intended. For nishing cuts of high-precision quality, the rotary feed will need to be reduced to get the required nish and the required spacing accuracy. DP: diametral pitch.

TABLE 10.9
Typical Cutting Speeds for Shaper-Cutter

Gear Face Width		Machinability											
		20%		40%		80%		100%		120%		130%	
mm	**in.**	**m/min**	**fpm**	**m/min**	**fpm**	**m/min**	**fpm**	**m/min**	**fpm**	**m/min**	**fpm**	**m/min**	**fpm**
200	8	1.5	5	6.4	21	18.6	61	26.4	87	35.4	116	39.4	130
100	4	3.7	12	9.1	30	22.6	74	30.8	101	39.3	129	43.0	141
50	2	6.7	22	12.5	41	26.8	88	35.4	116	43.9	144	47.9	157
30	1.2	9.1	30	15.8	52	31.1	102	39.6	130	47.9	157	51.8	170
20	0.8	11.6	38	19.2	63	35.1	115	43.6	143	51.8	170	55.2	181
13	0.5	15.8	52	24.1	79	40.8	134	49.1	161	57.3	188	60.4	198

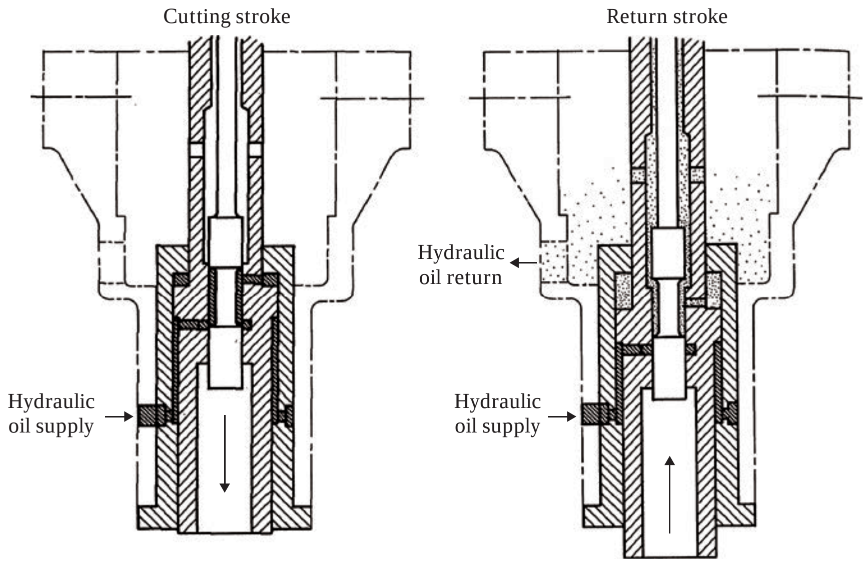


FIGURE 10.9 Schematic details showing the principle of the mechanism to reciprocate the cutter in a *Hydrostroke* gear shaper. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

is stroked back and forth by pressurized oil are shown in Figure 10.10.

The Hydrostroke machine is capable of doubling production rates under favorable conditions. It also has the potential to cut some gears so accurately that they do not need to be shaved (or ground) to get a relatively high precision. (The formulas just given for time required to shape gears do not apply to gears shaped in this machine.)

Figure 10.11 shows an example of a high-production machine with a movable column and a stationary worktable. Note the special automation tooling. Figure 10.8 shows in a schematic fashion how a high-production, moving column

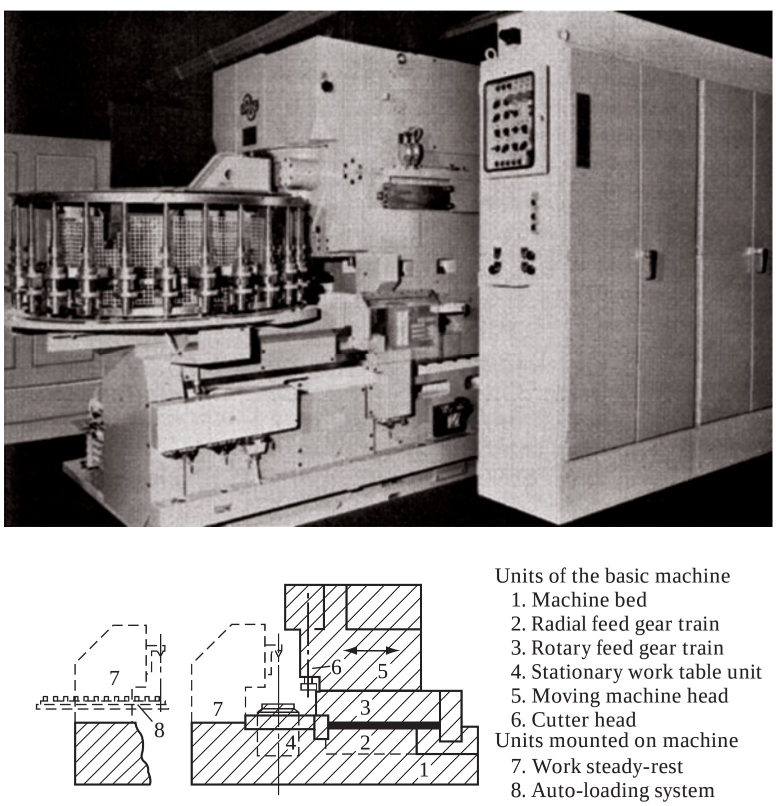


FIGURE 10.10 High-production, NC gear-shaping machine. (Courtesy of Lorenz, a subsidiary of Maag Gear Wheel Co., Ettlingen, German Federal Republic.)

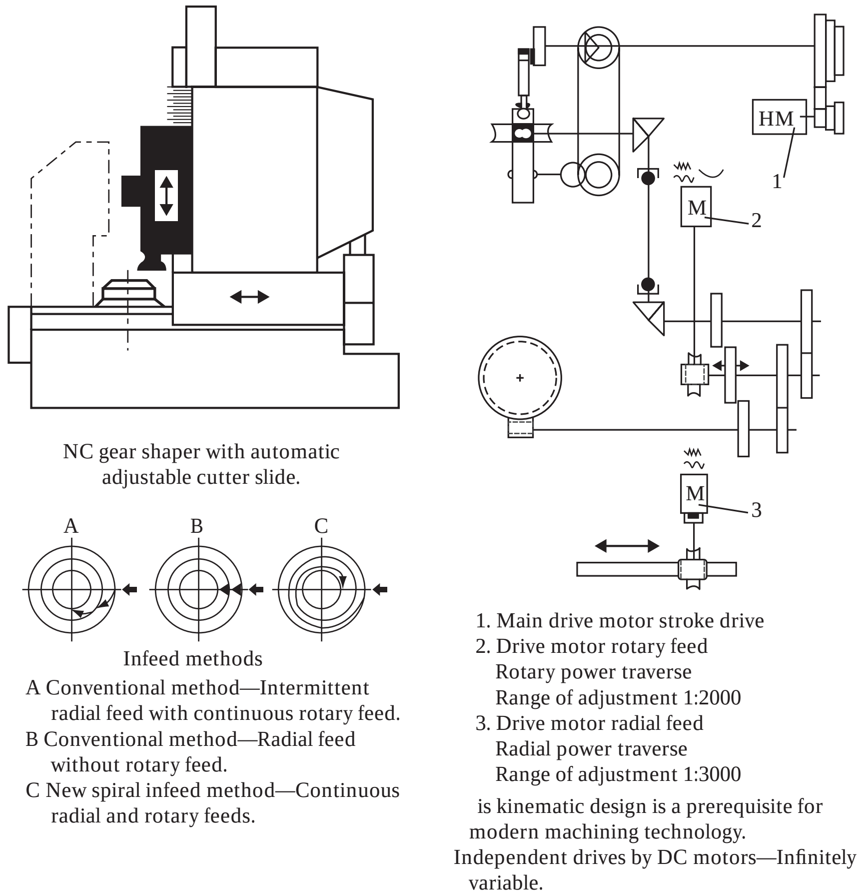


FIGURE 10.11 Schematic design of an NC gear-shaping machine. (Courtesy of Lorenz, a subsidiary of Maag Gear Wheel Co., Ettlingen, German Federal Republic.)

type of machine works. Basic motions are shown in the upper left-hand corner. The drive system is shown in the upper right-hand corner. Note that the drives use DC motors and are in nitely variable. The lower left-hand corner shows a spiral infeed method.

The machine just described has a cutter spindle back-off system and independent DC motor drives for stroking rates and for rotary and radial feed amounts. This machine provides a signi cant increase in productivity compared with the older-style table relief shapers that used feed cams and feed change gears with a single motor.

The special drives resulted in the developing of a new cutting method, sometimes referred to as *spiral infeed*. This cutting method tends to improve chip loading conditions and more evenly distributes tool wear around the cutter.

Table 10.10 shows estimates of production time needed to shape a variety of parts. The time shown tends to be much less than would be obtained by using equations in this chapter.

TABLE 10.10
Some Production Estimates of Shaping Time Using High-Production Equipment and Techniques

Part	No. of Teeth	Pitch Diameter mm	Pitch Diameter in.	Face Width mm	Face Width in.	Brinell Hardness	Cycle Time
Auto starter, spur pinion, AISI 4004	10	25.4	1.00	15.9	0.625	165	27.5 s
Auto transmission, spur gear, SAE 8620	21	67.3	2.65	17.1	0.673	160	1.6 min
Tractor transmission, helical gear, AISI 1045	27	119.6	4.71	44.5	1.750	140	4.7 min
Truck spur gear, SAE 5130	55	232.9	9.17	19.1	0.750	225	9.3 min
Truck, spur gear, malleable iron	47	223.8	8.81	15.9	0.625	255	1.67 min
Tractor, spur gear, AISI 4140	55	678.2	26.70	127.0	5.00	270	110 min
Mach., spur gear, AISI 1552	252	1066.8	42.00	49.8	1.960	300	72 min
Mach., internal gear, ductile iron	94	477.5	18.80	79.5	3.130	217	37.2 min
Mach., helical spline, 416 stainless	32	58.7	2.31	96.8	3.810	155	12 min

The equations, of course, are for job shop rather than high production conditions.

The values in Table 10.10 may, of course, be better by even more advanced shaping methods—and with somewhat unfavorable conditions, it may not be possible to cut a part as quickly as the table shows.

10.1.3 Shaping Rack Cutter

Spur and helical gears as well as racks may be shaped with a rack cutter. Although machines designed to use rack cutters are mostly used for external gears, attachments for cutting internal gears with a pinion-shaped cutter are available.

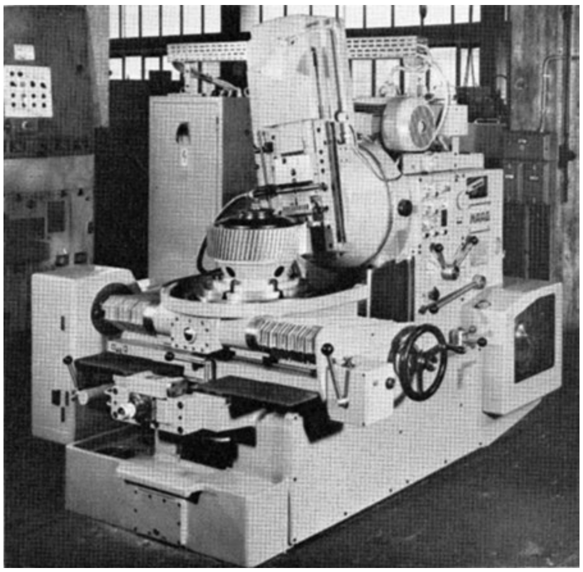


FIGURE 10.12 Rack shaping a helical gear. (Courtesy of Maag Gear Wheel Co., Zurich, Switzerland.)

The generating action is the same as rolling a gear along a mating rack. The rack tool, mounted in a clapper box, reciprocates while the gear rolls past its cutting tool. Cutting generally takes place during the downstroke, and the clapper box clears the tool from the work on the upstroke. See Figure 10.12.

Rack shapers cut only a few teeth in one generating cycle, then index to pick up the next teeth. The number of teeth per generation, the number of strokes per tooth (feed), and the stroking speed are all individually variable. The cutting ram is mounted in long guide-ways and is driven by a crank motion or, on heavy-duty versions, a multipitch screw.

The rack tool can be likened to one ute of a hob. It is not as expensive as a hob, and it does not require as much runout clearance as a hob. Also, there is no diameter/tooth size restriction, so a single-tooth rack tool can be made as large as the clapper box.

The most popular use for the rack shaper is for coarse pitches, high-hardness material, and narrow-gap double-helical gears. It is also ideal for cutting segments, since only the toothed portion must be rolled past the tool. The rack shaper can also machine two- and three-lobe rotors, as a result of the large tool size and the large number of strokes per tooth.

Rack shapers are commercially available in size that will handle gears all the way from a centimeter (0.4 in.) in diameter up to about 14 m (45 ft) in diameter. Face widths up to 1.55 m (61 in.) maximum may be cut. See Figure 10.13.

Rack shapers are generally built to mount the work in a xture instead of between centers. Gears with long shaft extensions are usually supported by a steady rest.

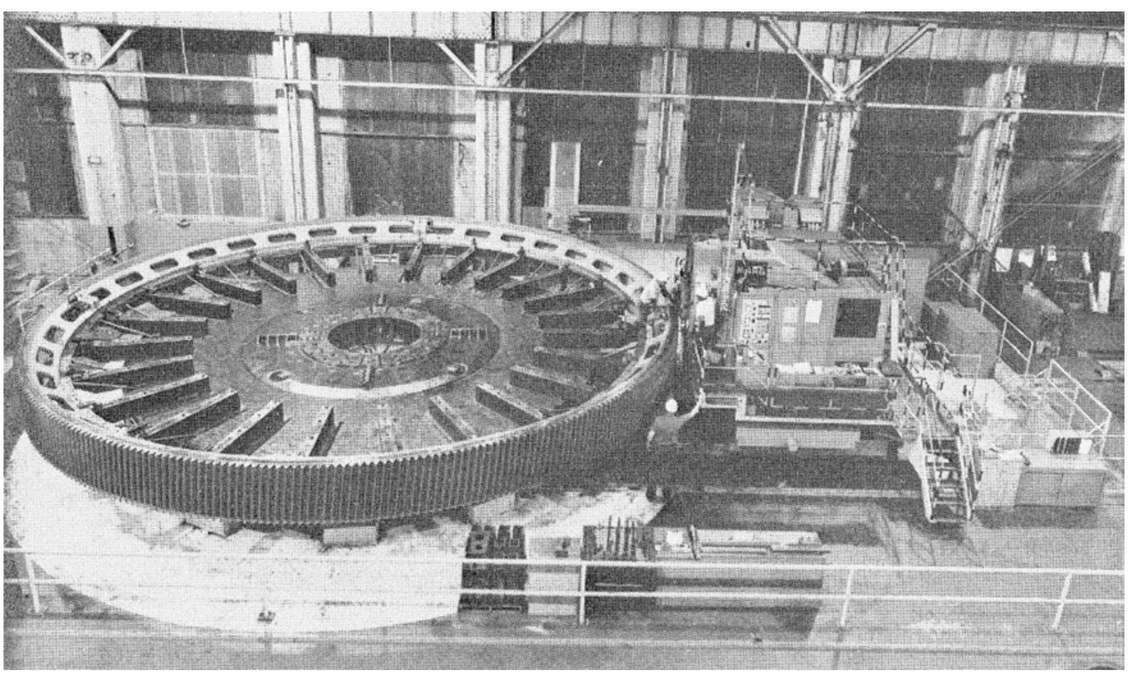


FIGURE 10.13 One of the largest gear-cutting machines in the world. A helical gear 11.9 m (469.9 in.) in diameter has just been rack shaped. (Courtesy of Fuller Company, Allentown, Pennsylvania.)

The time in minutes required to cut a gear on a rack shaper may be calculated by the following formula:

$$\text{Shaping time} = \text{no. of gear teeth} \times \left(\frac{\text{index time}}{\text{teeth per index}} + \frac{\text{strokes per tooth}}{\text{strokes per min}} \right) \times \text{no. of cuts.} \tag{10.6}$$

The indexing time varies from about 0.08 minute on small machines to around 1.0 minute on large machines. When n-ishing high-precision gears, the machines are indexed once per tooth so that the same tool nishes all the gear teeth, thus preventing tool pitch errors or mounting errors from being transferred to the gear. In roughing, more teeth may be cut per index, depending on the gear diameter and tooth size.

The number of strokes per tooth depends upon the number of teeth in the gear, the module (pitch), and the gear material. The fewer the strokes, the larger the generating marks on the given tooth. Gears of 100 to 200 teeth have almost straight line pro les, while smaller numbers of teeth have more pro le curvature. For this reason, it is necessary to use more strokes per tooth when nishing gears with small numbers of teeth. In ne pitches, the roughing cuts do not remove too much metal. This makes it possible to use fewer strokes per tooth in roughing than in nishing. In coarser pitches, however, a lot of metal has to be removed in roughing. This makes it necessary to use more strokes per tooth in roughing that in nishing. But with heavy-duty machines, coarser pitch gashing is usually done with step-type plunge cutters (Figure 10.14), which can remove a large volume of material with fewer strokes per tooth. Table 10.11 shows the average practice in strokes per tooth. More strokes may be needed in nishing if highest accuracy is desired, while fewer strokes may suf ce in commercial work.

The cutting speed, or the number of strokes per minute, is determined by both the size of the machine and the face width. In cutting a very narrow face width, the cutting speed in meters per minute (feet per minute) may be limited by the strokes-per-minute capability. In general, small machines are built to reciprocate much faster than large machines. Rack-type tools are limited to about the same cutting speed in meters per minute as other tools (see Table 10.12), but heavier cuts are possible than with hobs or disk-type shaping tools.

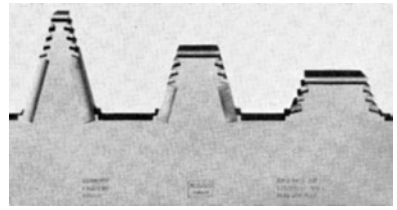


FIGURE 10.14 Special rack cutters for the fast, rough cutting of large gears. (Courtesy of Maag Gear Wheel Co., Zurich, Switzerland.)

Table 10.13 shows some typical values for number of cuts and strokes per minute for different kinds of work.

10.1.4 Cutting Bevel Gears

Straight bevel gears are produced by a generating machine which reciprocates a cutting tool in a motion somewhat like that of a shaper. The tools used do not resemble a pinion or a segment of a rack. The peculiar geometry of the gear makes the use of a special tool to cut each side of the bevel tooth desirable. These tools each have a single inclined cutting edge which generates the bevel gear tooth on one side or the other. The machines achieve a generating motion by rolling the work and the cutter head at a low rate, while the cutters rapidly reciprocate back and forth. See Figure 10.15.

The latest models of straight bevel gear generators have a design feature which permits the tooth to be cut with a slight amount of crown. A cam can be set so that the cutting tool will remove a little extra metal at each end of the tooth. This makes it possible to secure a localized tooth bearing in the center of the face width.

Bevel gears are overhung from the spindle of the generating machine during cutting. Those designing bevel gears should be careful to make the blank design suitable for mounting in

TABLE 10.11
Average Strokes per Tooth for Rack Shaping

	Strokes per Tooth											
	Roughing						Finishing					
No. of	2.5 m_t (10 P_t)		4 m_t (6 P_t)		12 m_t (2 P_t)		2.5 m_t (10 P_t)		4 m_t (6 P_t)		12 m_t (2 P_t)	
Teeth	Std.	HD	Std.	HD	Std.	HD	Std.	HD	Std.	HD	Std.	HD
15	60	–	103	–	325	175	15	–	22	–	40	39
20	55	–	90	–	295	160	14	–	16	–	30	32
30	50	–	85	50	260	140	11	–	13	12	22	23
50	42	–	75	44	230	130	7.5	–	9	9	16	17
80	38	–	65	38	200	110	7	–	7	7	13	13

Note: HD: heavy-duty machines; m_t: transverse module; P_t: transverse diametral pitch; Std.: standard machines.

TABLE 10.12
Cutting Speeds for Different Materials

		Cutting Speed					
		Conservative		Normal		High Speed	
Material	**Hardness (HB)**	**m/min**	**fpm**	**m/min**	**fpm**	**m/min**	**fpm**
Steel	350	7.6	25	11	35	15	50
	300	11	35	14	45	21	70
	200	17	55	26	85	37	120
Cast iron	250	14	45	18	60	27	90
	175	23	75	30	100	46	150
Bronze	90 (500 kg)	46	150	85	280	122	400
Laminated plastic	–	107	350	168	550	213	700

TABLE 10.13
Number of Cuts and Strokes per Minute for Typical Rack Shaping

				Number of Cuts		
Tooth Size		Pitch Diameter			Finishing	
Module	**Diametral Pitch**	**mm**	**in.**	**Roughing**	**Commercial**	**Precision**
1.6	16	100	4	1	1	1
2.5	10	100	4	1	1	1
2.5	10	200	8	1	1	1
4	6	200	8	2	1	2
4	6	1000	40	2	1	2
12	2	1000	40	3	1	2
12	2	2500	100	3	1	2
			Strokes per Minute			
			200	**80**	**55**	**32**
Face width, mm (in.)			25 (1)	100 (4)	200 (8)	400 (16)
Cutting speed, m/min (fpm)			13 (42)	27 (90)	27 (90)	27 (90)

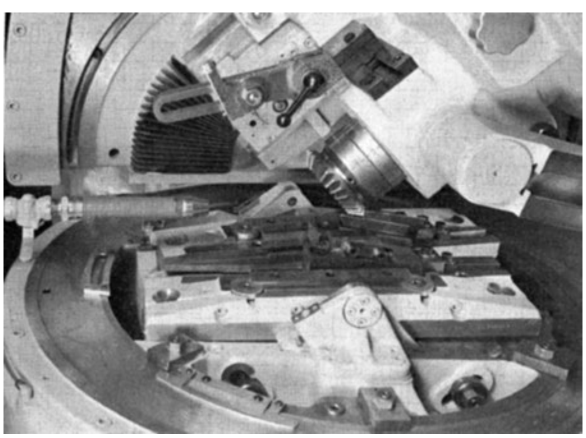


FIGURE 10.15 This view shows the generation of a Coni ex straight bevel gear. Note the two reciprocating tools which travel in a curved path to produce a slight crowning on the teeth. (Courtesy of Gleason Works, Rochester, New York.)

the bevel gear generator. Frequently, it is necessary to design the bevel gear as a ring which is bolted onto its shaft after cutting. This gets the shaft out of the way during cutting.

Bevel gears coarser than 1 module (25 pitch) are usually given a roughing and a nishing cut. Where production is high and parts are not over 350 mm (14 in.) in diameter, it is possible to use special high-speed roughing machines which do not have a generating motion. These machines rough faster than the generators, and they save the generators from the wear and tear of roughing. Teeth of 0.8 module (32 pitch) or ner are often cut in one cut. Special Duplete tools are used. These tools have two cutting edges in tandem—one edge roughs and one nishes.

The complications of bevel gear cutting make it hard to write any general formulas for estimating cutting time. In view of this situation, the best way to give some general information on cutting time is to give a table with cutting time shown for a range of sizes of bevel gears.

TABLE 10.14
Average Time to Cut Straight Bevel Gears

	Pitch-Cone	Tooth Size		Face Width		Time	
No. of Teeth	Angle (°)	Module	Pitch	mm	in.	Roughing	Finishing
12	8.5	1.1	24	9.5	0.375	1.1	1.1
20	22	1.1	24	6.4	0.250	1.7	1.4
30	45	1.1	24	3.2	0.129	2.6	2.1
50	68	1.1	24	6.4	0.250	4.3	3.5
80	81.5	1.1	24	9.5	0.375	3.4 DI	6.8
12	8.5	2.5	10	25	1.000	3.1	3.1
20	22	2.5	10	19	0.750	4.5	4.5
30	45	2.5	10	9.5	0.375	5.9	6.8
50	68	2.5	10	19	0.750	11.3	11.3
80	81.5	2.5	10	25	1.000	10.3 DI	20.7
12	8.5	6.4	4	64	2.500	14.4	10.0
20	22	6.4	4	51	2.000	20.0	14.0
30	45	6.4	4	38	1.500	21.0	16.1
50	68	6.4	4	51	2.000	50.0	35.0
80	81.5	6.4	4	64	2.500	96.0	66.6

Note: DI stands for double index.

Table 10.14 shows the average cutting time for steel gears of about 250 HB.

Spiral and Zerol bevel gears are cut with a generating machine that uses a series of cutting blades mounted on a circular toolholder. The toolholder is rotated to cause a cutting action while the work slowly rotates with the toolholder. The rotation of the work with respect to the toolholder causes a generating action to occur. After one tooth space is nished, the machine goes through an indexing motion to bring the cutter into the next tooth slot. See Figure 10.16.

The line of machines using face-mill cutters will handle gears from 5 mm (3⁄16 in.) to 2.5 m (100 in.) in diameter and teeth from 0.5 module (48 pitch) to 17 module (1.5 pitch). Large spiral gears up to about 2.5 m (100 in.) in diameter can also be cut on a planing type of generator which uses a single tool.

The average time required to cut spiral or Zerol bevel gears may be estimated from Table 10.15. If face widths are less than those shown in the table, there may be some reduction in cutting time, but it will not be in proportion to the reduction of face width. If very high accuracy is required, the time may be longer. No allowance is made for setup time which may be required to adjust the machine to produce the desired tooth contour. The time shown in Table 10.15 is the time required per piece after the machine has produced a few satisfactory pieces. Loading and unloading times are not included.

Figure 10.17 shows a hypoid gear being cut with a straddle type of cutting tool. Note that the face mill has *two* rows of cutting teeth. This method cuts gears faster, and it is believed that straddle-cut (or ground) gears have somewhat longer life.

Figure 10.18 shows a very large machine capable of cutting spiral bevel or hypoid gears up to 2.5 m (100 in.) in diameter. This machine makes large teeth for heavy industrial, marine, and of-the-road vehicle applications.

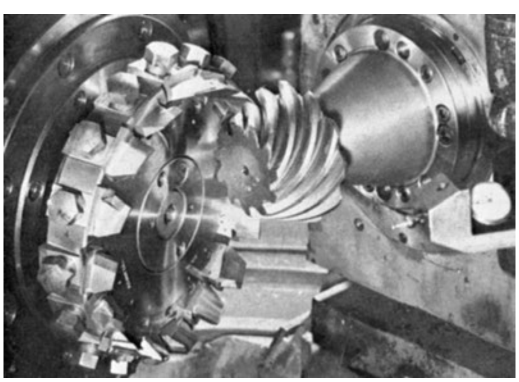


FIGURE 10.16 Spiral gears up to approximately 1.25 m (50 in.) in diameter are generated on machines employing circular face-mill cutters. (Courtesy of Gleason Works, Rochester, New York.)

10.1.5 Gear Milling

Worms, spur gears, and helical gears may be produced by the milling process. Bevel gear teeth are sometimes produced by this method, but the geometric limitations of trying to produce accurate bevel teeth by this process greatly restrict its use except for roughing cuts.

Conventional milling machines equipped with a dividing head may be used to mill gear teeth. A slot at a time is milled, and the machine is hand-indexed to the next slot. Several makes of special gear-milling machines, known in the trade as *gear-cutting machines*, are on the market. These are designed for the sole purpose of cutting gear teeth or clutches and the like with milling cutters. They are usually equipped with automatic indexing equipment.

TABLE 10.15
Average Time to Cut Spiral Bevel Gears

		Tooth Size		Face Width		Time (min)	
No. of Teeth	Pitch-Cone Angle (°)	Module	Pitch	mm	in.	Low Production	High Production
12	8.5	1.1	24	9.5	0.375	2.4	1.2
20	22	1.1	24	6.4	0.250	4.0	2.0
30	45	1.1	24	3.2	0.125	5.0	2.5
50	68	1.1	24	6.4	0.250	10.0	5.0
80	81.5	1.1	24	9.5	0.375	16.0	8.0
12	8.5	4.2	6	41	1.625	14.6	4.9
20	22	4.2	6	38	1.500	23.7	7.9
30	45	4.2	6	32	1.250	27.6	9.2
50	68	4.2	6	38	1.500	34.2	11.4
80	81.5	4.2	6	41	1.625	44.8	14.9
12	8.5	8.5	3	83	3.250	34.2	11.4
20	22	8.5	3	83	3.250	57.0	19.0
30	45	8.5	3	83	3.250	78.0	26.0
50	68	8.5	3	83	3.250	130.0	43.3
80	81.5	8.5	3	83	3.250	150.7	50.2

Note: The 24-pitch gears are assumed to be cut on a Gleason No. 423 Hypoid Generator, the 6-pitch are cut on a No. 641 G-PLETE Generator, and the 3-pitch are cut on a No. 645 G-PLETE Generator. For high production, the completing process is used to nish gear teeth in a single chucking from the solid blank. For lower production requirements, the ve-cut process can be used: rough and nish the gear member, rough the pinion member, then nish each side of the pinion teeth in a separate cutting operation.

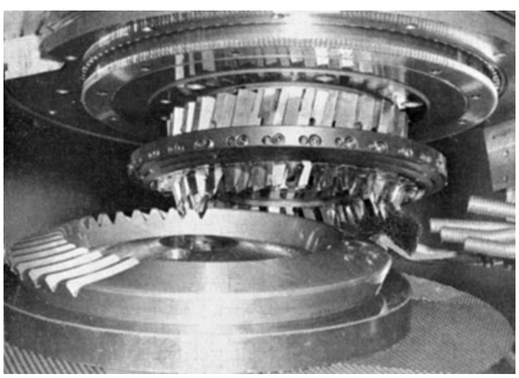


FIGURE 10.17 Straddle cutting a hypoid gear. (Courtesy of Gleason Works, Rochester, New York.)

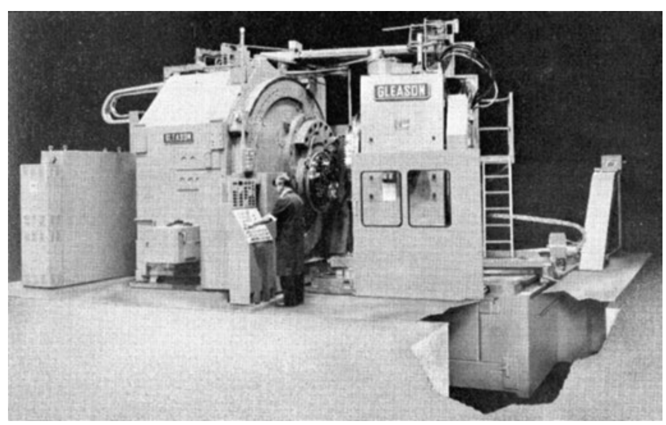


FIGURE 10.18 Large, heavy-duty machine for cutting spiral bevel and hypoid gears. (Courtesy of Gleason Works, Rochester, New York.)

The machines used to mill gears are a different type from those used to cut gear teeth. The worm-milling machine is essentially a thread-milling machine.

Gear-cutting machines cover the whole range of gear sizes up to more than 5 m (200 in.) in diameter. Pitches that are coarser than 34 module (¾ pitch) are often cut with an end mill. Helical rolling mill pinions in the circular pitch range of 100 to 200 mm (4 to 8 in.) are often produced by end milling. Figure 10.19 shows a gear being milled. Figure 10.20 shows an end mill used to mill some large gear teeth.

Gear-cutting machines and milling cutters are not so expensive as hobbing machines and hobs or gear shaper and shaper-cutters. On the other hand, gear-cutting machines do not produce accuracy comparable with that produced by hobbing and shaping. Some high-precision gears are produced by rough milling followed by hardening and nish grinding.

Parts to be milled must allow room for runout of the cutter at each end of the tooth. Some gear-cutting machines mount the work on centers, while others clamp the work on a xture. Wide face widths can be milled, and it is usually possible to have fairly long shaft extensions on the gear.

Gear teeth may be milled in one cut, or they may be given a rough cut almost to size and then nish milled. The time required to make a milling cut can be calculated from the following formula:

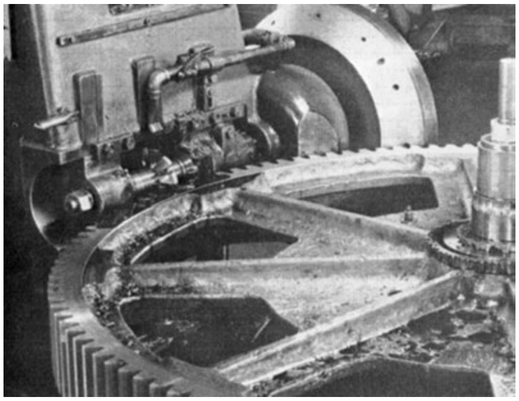


FIGURE 10.19 Milling a large gear.

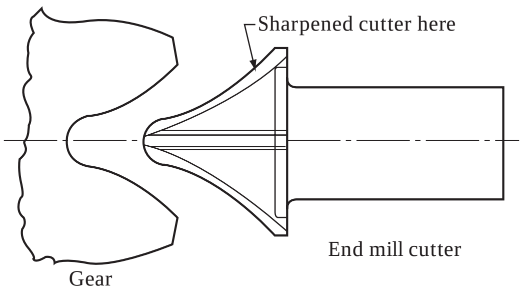


FIGURE 10.20 Coarse-pitch spur or helical gears can be cut on a hobbing machine with an end-mill type of cutter.

$$\text{Milling time (min)} = \text{no. of gear teeth} \times \left(\text{index time} + \frac{\text{face} + \text{overtravel}}{\text{feed per min}} \right) \quad (10.7)$$

The index time on automatic machines runs from 0.04 to 0.08 minute per tooth. Hand indexing takes much longer.

The minimum overtravel for milling a spur gear is

$$\text{Overlap} = 2\sqrt{\text{depth of cut} \times (\text{cutter diameter} - \text{depth of cut})}. \quad (10.8)$$

The feed per minute ranges from about 12 to 500 mm/min (½ to 20 in./min). The feed rate may be calculated from

$$\text{Feed per min} = \frac{\text{rpm of cutter} \times \text{no. of cutter teeth}}{\text{feed per tooth}}. \quad (10.9)$$

The revolutions per minute of milling cutters is based on cutting speeds that high-speed steel tools have been able to stand when cutting different materials. Table 10.12 gives some representative values of cutting speeds for different materials and different degrees of care in cutting. The conservative values represent the condition where long cutter life is desired, and best accuracy and nish are also sought. The high-speed values represent the condition where a rugged machine and a cutter are being worked to the limit.

In a few cases, cemented carbide cutters have been used to make gears instead of high-speed steel cutters. Much higher cutting speeds can be used with carbides, provided that the machine is rugged enough and powerful enough to drive the carbide cutter.

After the cutting speed is determined, the revolution per minute of the cutter is

$$\text{rpm of cutter} = \frac{1000 \times \text{cutting speed (m/min)}}{0.262 \times \text{outside dia. of cutter (mm)}} \quad \text{(metric)}, \quad (10.10)$$

$$\text{rpm of cutter} = \frac{12 \times \text{cutting speed (fpm)}}{0.262 \times \text{outside dia. of cutter (in.)}} \quad \text{(English)}. \quad (10.11)$$

The feed per tooth depends on the nish desired and on how well the material cuts. Some typical values are listed in Table 10.16.

There have been important developments in milling just as in hobbing and shaping. Special cutters can be used for very rapid stock removal. With very rugged machine, precision indexing, and precision cutters, very good accuracy can be achieved.

The milling process is particularly suitable for making racks, especially large racks. Figure 10.21 shows the milling

TABLE 10.16
Some Typical Values of Feed per Tooth

Feed per Tooth				
Steel Material		Cast Iron, Bronze, or Plastic		
mm	in.	mm	in.	Description
0.05	0.002	0.10	0.004	Conservative
0.08	0.003	0.15	0.006	Normal
0.13	0.005	0.25	0.010	Fast cutting

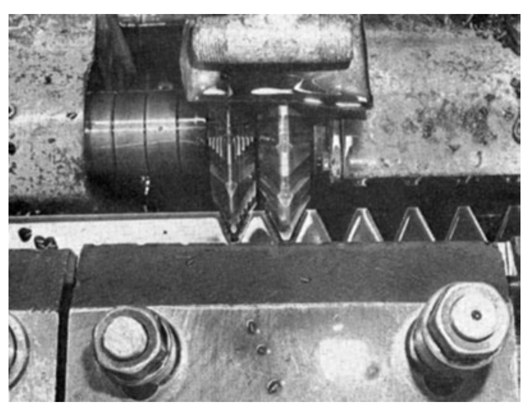


FIGURE 10.21 Milling large rack segments for a radar antenna application. (Courtesy of Sewall Gear Mfg. Col, St. Paul, Minnesota.)

of a large rack. Note the special roughing cutter that leads the precision nishing cutter.

10.1.6 Broaching Gears

Small internal gears can be cut in one pass of a broach provided that the gear designer does not put the teeth in a blind hole. Large internal gears can be made by using a surface type of broach to make several teeth at a pass. Indexing of the gear and repeated passing of the broach can make a complete gear. Gears as large as 1.5 m (60 in.) in diameter are made by this process.

Racks and gear segments are often made by broaching. The teeth formed by broaching may be either spur or helical.

Present broaching machines and broach-making facilities impose quite de nite limits to the size gear that can be made by a one-pass broach. The parts most commonly made range from about 6 to 75 mm (¼ to 3 in.) in diameter. Parts up to 200 mm (8 in.) in diameter have been made in production by broaching with a one-pass broach. However, when broaches get to this size, they become very costly. It is very dif cult to forge, heat treat, and grind a piece of high-speed steel that is 200 mm (8 in.) in diameter and several feet long and get a hardness of over 62 HRC and an accuracy within 0.005 mm (0.0002 in.). Figure 10.22 shows an example of a small broaching machine.

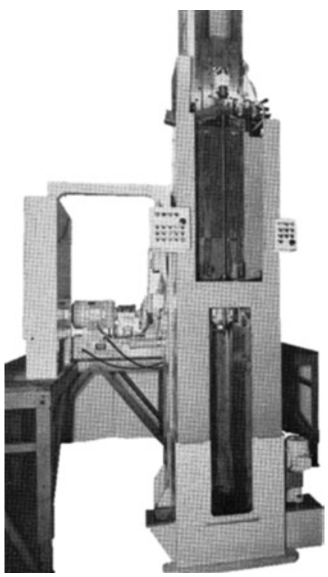


FIGURE 10.22 Broaching machine. (Courtesy of National Broach and Machine Co., a division of Lear Siegle, Detroit, Michigan.)

Broaching is a rapid operation. If the teeth are not too deep or the face width too wide, the gear can be completely cut in one pass. The time in minutes for a one-pass broaching cut is

$$\text{Broaching time} = \frac{\text{length of stroke}}{12 \times \text{cutting speed}} + \frac{\text{length of stroke}}{12 \times \text{return speed}} + \text{handling time.} \tag{10.12}$$

The rate at which a broach can cut will depend on the material being cut and the quality desired. Table 10.17 shows some typical broaching speeds.

On the return stroke, the broach is often moved as fast as the machine will operate. This may be on the order of 9 to 11 m/min (30 to 35 fpm). The handling time for loading and unloading the machine will run about 0.07 to 0.12 minute on a production setup.

The length of the broach stroke will be longer than the cutting portion of the broach, but usually not as long as the overall length. The cutting length of the broach will depend on tooth depth, face width, and how easily the gear cuts. Table 10.18 shows some typical broaching lengths.

A very important development in broaching was the *pot-broaching* technique. Conventional broaching pushes a long broach through the part. Pot broaching moves the part through the broach.

Figure 10.23 shows a tool for push-up pot broaching. The parts are pushed through this tool. The tool is made up of a number of small parts with cutting teeth, instead of a very large part with cutting teeth.

Figure 10.24 shows a diagram of a machine for pull-up pot broaching. When parts require a long cut, it is mechanically better to pull the parts through the pot broach. This avoids possible buckling of a long pushrod.

The pot-broaching method tends to make the production time much shorter, and the tool itself is much easier to make. Pot-broaching is a high-production method that has come into

TABLE 10.17
Typical Speed of Broach on the Cutting Stroke

		Broaching Speed					
		Conservative		Normal		High Speed	
Material	Hardness (HB)	m/min	fpm	m/min	fpm	m/min	fpm
Steel	350	1.2	4	3.0	10	4.9	16
	300	2.4	8	4.9	16	6.1	20
	200	4.9	16	6.7	22	9.1	30
Cast iron	250	4.3	14	6.1	20	7.3	24
	175	5.5	18	7.3	24	9.1	30
Bronze	90	5.5	18	7.3	24	9.1	30

TABLE 10.18
Typical Broach Lengths

		Broach Length							
		Free-Cutting Material				Tough Material			
		Cutting Portion		Overall		Cutting Portion		Overall	
Gear Whole Depth (in.)	Face Width (in.)	m	in.	m	in.	m	in.	m	in.
0.250	2	2.2	85	2.5	100	–	–	–	–
0.200	1	0.9	35	1.3	50	1.9	75	2.3	90
0.100	1	0.5	20	0.9	35	0.8	30	1.1	45
0.050	0.5	0.15	6	0.25	10	0.25	10	0.5	20

FIGURE 10.23 Push-up pot-type broach tool. Note parts finished up at top and blanks starting at the bottom. (Courtesy of National Broach and Machine Co., a division of Lear Siegler, Detroit, Michigan.)

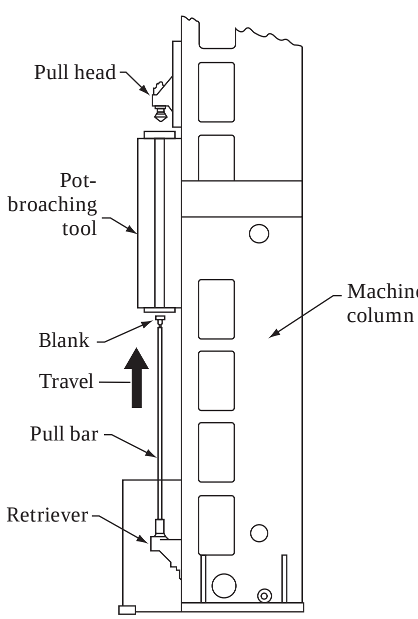


FIGURE 10.24 Sketch of broaching machine for pull-up pot broaching.

FIGURE 10.25 Some small automotive pinions pot broached at a rate of 195 parts per hour. RH pinion is 2.5 module (10 pitch), and LH pinion is 3.17 module (8 pitch).

very substantial use on high-volume jobs. Figure 10.25 shows a good example of pot-broach work.

10.1.7 Punching Gears

Punching is undoubtedly the fastest and cheapest method of making gear teeth. Unfortunately, punched gears do not have the same degree of accuracy as many of the cut gears, and punching is limited to narrow face widths.

The design of blanks for gears to be punched must suit the process. The punching machines can handle only thin sheet stock. The as-punched gear is a disklike wafer. The shaft can be attached to the punching by pressing a piece of rod through the hole in the punching. Punched gears are often mated with pinions that have much wider face widths than the gear. This makes positioning the punched gear precisely at the right distance from the end of the shaft unnecessary. In some cases, though, punched gears must be accurately positioned.

Both external and internal spur gears may be made by punching. Equipment currently in use will handle gears from 6 to 25 mm (¼ to 1 in.). The face width of the punching should not be greater than two-thirds of the whole depth of the tooth. Only fairly soft materials can be punched. Brass is popular material for punching. Sheet bronze, aluminum, and steel are also used.

One stroke of a punch can make a punched gear. The punching rate depends on the thickness and the hardness of

TABLE 10.19
Typical Punching Rates

Material	Hardness (HB)	Thickness mm	Thickness in.	Strokes/min
Brass	60	0.5	0.020	300
		2.0	0.080	250
Aluminum	–	0.5	0.020	300
		2.0	0.080	250
Steel	150	0.5	0.020	150
		2.0	0.080	100

the material as well as on the die life and gear quality desired. Table 10.19 gives some typical rates.

10.1.8 G-TRAC Generating

One of the surprising developments in gear cutting is the G-TRAC generator, made by Gleason Works. The machine uses rack-type cutting tools mounted on an endless chain. The gear being cut is rolling in the mesh with the passing rack teeth in the chain. Figure 10.26 shows the whole machine. Figure 10.27 shows a stack of parts being cut so as to have helical gear teeth.

The G-TRAC machine is intended for high production, low cost, and high accuracy. It is possible, though, to effectively use the machine for small production. In this case, a single row of cutting tools is used, and one tooth slot is formed at a time.

The simple rack tools can be made by gear shops that are using the G-TRAC. This feature should make it easy to change tooth module (pitch), tooth pressure angle, or tooth depth when a gear unit is under development. (Normally, gear-cutting tools have to be ordered from a tool-making company, and some month's time is lost when new, nonstandard gear tools are needed.)

The concept of G-TRAC machine opens several new possibilities in gear machine tools.

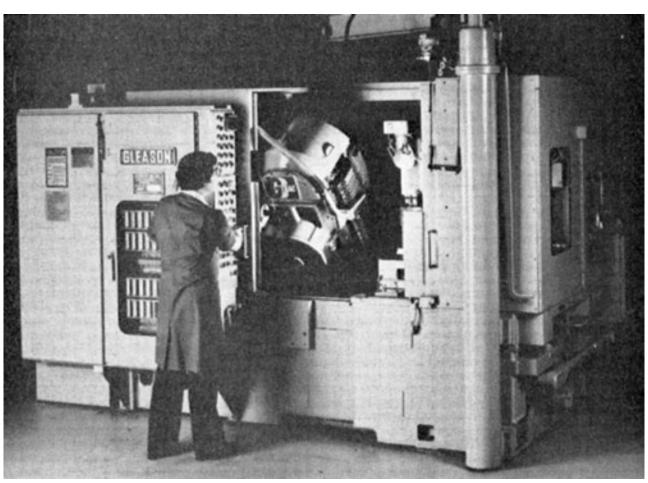


FIGURE 10.26 G-TRAC generator. (Courtesy of Gleason Works, Rochester, New York.)

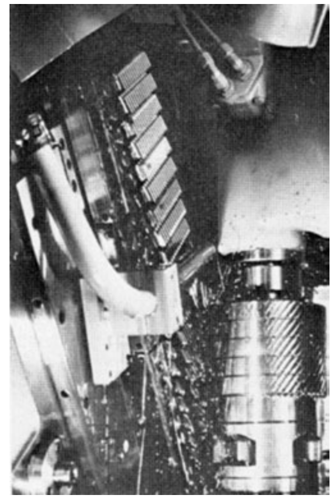


FIGURE 10.27 Generating helical gear teeth on a stack of gear blanks by the G-TRAC machine. (Courtesy of Gleason Works, Rochester, New York.)

10.2 GEAR GRINDING

In general, gear grinding is an operation that is performed after a gear has been cut and heat treated to a high hardness and has had journals or other mounting surfaces finish ground. Grinding is needed because it is very difficult to cut parts over 350 HB (38 HRC). Since fully hardened steels are needed in many applications and it is often difficult to keep the heat-treat distortion of a cut gear within acceptable limits, there is a large field of gear work in which the grinding process is needed.

In a few cases, medium-hard gears that could be finished by cutting are ground. This may be done to save the cost of expensive cutting tools like hobs, shapers, or shaving cutters; or it may be done to get a desired surface finish on accuracy on a gear that is difficult to manufacture.

In some of the fine pitches, gear teeth may be finish ground from the solid. For instance, the whole volume of stock removed in making an 0.8-module (32-pitch) tooth less than the amount of stock removed in finish grinding a good 4 module (6 pitch) tooth, even when the 0.8-module and 4-module teeth have equal face widths.

Figure 10.1 shows that most of the methods of cutting a gear tooth have a counterpart in a grinding method. Disk cutters are used to mill gear teeth, and disk-grinding wheels grind gear teeth. Threaded hobs cut gear teeth, and threaded grinding wheels grind gear teeth. Rack shaping is matched by generating grinders that make a disk wheel go through the motion of a tooth on a rack. There is, however, no cutting counterpart to the dished wheel, base circle–generating grinding method.

In the following sections, information is given on how to estimate grinding time. This is a controversial subject which is hard to handle. Things like the hardness of the grinding wheel, the accuracy required of the gear, and the toughness and hardness of the stock being ground all enter into the time

required to grind a gear. In general, medium-hard gears which are made by through-hardening steel can be ground faster than carburized gears at full hardness. Also, the less stock left for grinding, the faster the gear can be ground. The grinding times and the number of passes required shown in the various tables should be considered as only nominal values, subject to considerable revision either upward or downward in individual cases.

10.2.1 Form Grinding

Form grinders use a disk wheel to grind both sides of the space between two gear teeth. Their grinding action is very similar to the action of a machine for milling gear teeth. Form-grinding wheels have an involute form dressed into the side of the wheel, while a generating grinding wheel is straight-sided. Figure 10.28 shows a comparison of the two kinds of wheels for the same pinion.

The machines available to form-grind gears can handle external spur gears from about 10 mm (0.4 in.) to about 2 m (80 in.) in diameter. Internal gears from about 1 m (40 in.) major diameter to about 50 mm (2 in.) minor diameter can be ground. Some form grinders will grind spur as well as helical gears, external or internal gear teeth. Figure 10.29 shows a general-purpose form grinder.

Form-grinding machines usually mount the work on centers. The gear design must provide room for the wheel to run out at each end of the work. Pinions which require an undercut to allow them to mesh with a mating gear without interference cannot be ground with a single grinding wheel. However, pinions with undercut may be ground with two grinding wheels straddling one or more teeth.

10.2.1.1 Ceramic Form Grinding

It is a common practice to grind gears with ceramic wheels such as aluminum oxide or silicon carbide.

The time required to grind a gear has three parts. These are the following:

1. Time to rough grind
2. Time to nish grind
3. Time to dress the grinding wheel

In addition, there are some handling times, such as time to load the work, time to centralize the grinding stock, time to unload the work, and time to check the work for size.

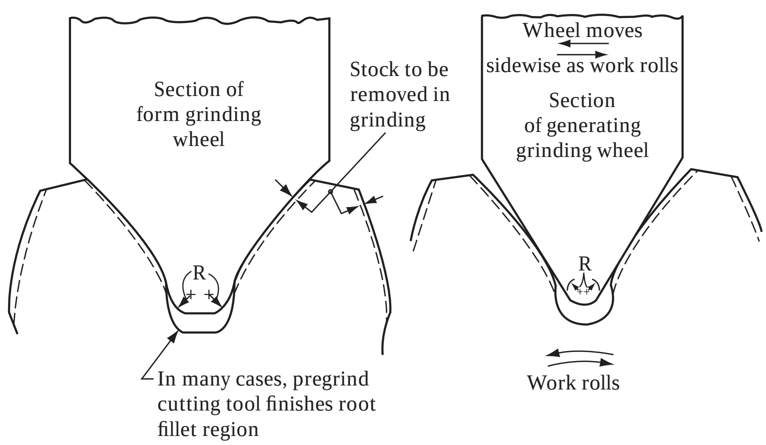


FIGURE 10.28 Comparison of form grinding and generating grinding.

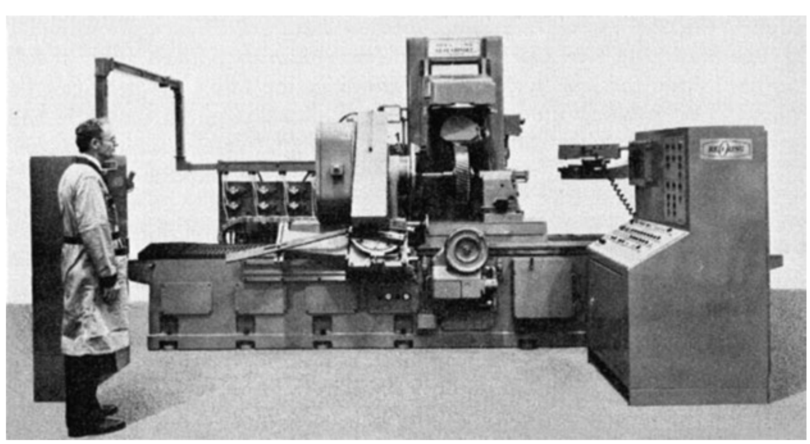


FIGURE 10.29 An automated general-purpose form-grinding machine capable of doing external or internal gears up to 600 mm (24 in.) pitch diameter and up to 45° helix angle. (Courtesy of National Broach and Machine Co., a division of Lear Siegler, Detroit, Michigan.)

TABLE 10.20
Typical Data for Dressing Form-Grinding Wheels

Size of Work			Wheel Diameter			
No. of Teeth	Module	Pitch	mm	in.	No. of Dressings per Gear	Minutes per Dressing
75	2.5	10	150	6	5	¾
20	2.5	10	150	6	2	¾
75	4	6	300	12	6	1
20	4	6	300	12	2	1
75	8	3	300	12	8	1¾
20	8	3	300	12	3	1¾

When ceramic grinding wheels are used, all but the smallest gears will require more than one wheel dressing per gear. The number of dressings required depends upon several factors, of which the principal ones are the number of gear teeth, the grinding wheel diameter, and the face width. The hardness of the gear and the accuracy required also enter into the picture.

Table 10.20 shows some typical data on the number of wheel dressings required and the time per dressing. It is assumed that the material is about 60 HRC and that moderate accuracy is required. Face width is assumed to be in these proportions (see Table 10.21).

The time required for either rough grinding or nish grinding by the form-grinding method may be estimated by Equation 10.13. In this equation, a cycle is the cutting action that occurs on a single tooth between successive feeds of the grinding wheel. The latest model of form grinder feeds down at each end of the stroke while roughing. Thus, a cycle is only a stroke across the tooth. The older-style machines feed down only once per revolution of the work. On each tooth, the grinding wheel makes a stroke across and back; in this case, a cycle includes both strokes:

$$\text{Grinding time (min)} = \frac{\text{no. of gear teeth} \times \text{no. of cuts}}{\text{cycles per minute}}. \quad (10.13)$$

TABLE 10.21
Tooth Size and Face Width Proportions

Tooth Size		Face Width	
Module	Pitch	mm	in.
2.5	10	40	1.5
4	6	50	2.0
8	3	100	4.0

The number of cuts will depend upon the amount of stock left for grinding and the amount removed per cut.

Enough stock must be removed to eliminate both the inaccuracy of the rough-cut gear teeth and the inaccuracy caused by heat-treat distortion. Large gears with thin webs tend to distort more than small gears of solid construction. Quenching dies can be used to considerably limit heat-treat distortion. With care and skill in heat treating, it is possible to hold heat-treat distortion to reasonably low limits.

Table 10.22 shows the general range between low amounts of heat-treat distortion and high amounts. The stock shown on tooth thickness is the sum of the amounts left on the two sides of the tooth. The relation between diameter over pins and tooth thickness is only approximate.

The worst condition that can occur before grinding is that the distortion is so bad that some tooth on the gear requires *all the stock to be ground off one side*. If the gear is worse than this, it will not clean up, and the gear might as well be scrapped instead of ground. Assuming that the gear distortion is within the stock allowed, the maximum number of cuts is

$$\text{No. of cuts} = \frac{2 \times \text{stock left for grinding}}{\text{stock normally removed per cut}}. \quad (10.14)$$

In Equation 10.14, the stock left for grinding is in terms of tooth thickness. The normal amount of stock removed is the amount that would be removed from the tooth thickness if the grinding wheel were cutting on both sides. The minimum number of grinding cuts is just one-half of that given by

TABLE 10.22
Amounts of Stock Normally Needed for Grinding

		Stock Left for Grinding							
		Low Heat-Treat Distortion				High Heat-Treat Distortion			
Tooth Size		Tooth Thickness		Diameter over Pins		Tooth Thickness		Diameter over Pins	
Module	Pitch	mm	in.	mm	in.	mm	in.	mm	in.
1.6	16	0.13	0.005	0.30	0.012	0.25	0.010	0.64	0.025
2.5	10	0.20	0.008	0.50	0.020	0.38	0.015	0.96	0.038
4.0	6	0.30	0.012	0.76	0.030	0.64	0.025	1.57	0.062
8.0	3	0.66	0.026	1.65	0.065	1.14	0.045	2.69	0.106
12.0	2	1.14	0.045	2.85	0.112	1.90	0.075	4.78	0.188

TABLE 10.23
Stock Removed per Form-Grinding Cut

Tooth Size		Rough Grinding									
		Tooth Flanks Only				Tooth Flanks and Root				Finish Grinding	
		45 HRC		60 HRC		45 HRC		60 HRC			
Module	Pitch	mm	in.	mm	in.	mm	in.	mm	in.	mm	in.
2.4	10	0.05	0.0020	0.04	0.0015	0.04	0.0015	0.025	0.0010	0.012	0.0005
4	6	0.05	0.0020	0.04	0.0015	0.04	0.0015	0.025	0.0010	0.012	0.0005
8	3	0.06	0.0025	0.05	0.0020	0.05	0.0020	0.040	0.0015	0.012	0.0005
12	2	0.08	0.0030	0.06	0.0025	0.06	0.0025	0.040	0.0015	0.020	0.0008

Equation 10.14. In an average case, the number of cuts might be expected to be somewhere about midway between the minimum and maximum number of cuts.

Roughing cuts usually take as much stock per cut as can be removed without burning the work. Finishing cuts take only a small amount of stock, and the last cut may just spark out, taking almost no stock. Table 10.23 shows the normal amounts of stock removed in form grinding per cut.

The rate at which a form-grinding machine strokes depends upon the face width being ground, the amount of overtravel, the kind of material being cut, and the range of speeds available on the machine. Some of the latest machines on the market can stroke up to 0.35 m/s (70 fpm). They completely rough out a tooth before indexing. This makes it possible to use a much smaller amount of overtravel than would be required if the wheel had to move clear of the work on each stroke to permit indexing.

Table 10.24 shows some typical cutting rates for form grinders. The table is based on a 300 mm (12 in.) grinding wheel for the multiple-cycle machine and 150 mm (6 in.) wheel for the single-cycle machine. The work is assumed to be 2.5 module (10 pitch). The coarser pitches take slightly longer to grind because of more overtravel. The table is based on 0.25 m/s (50 fpm) for roughing and 0.15 m/s (30 fpm) for finishing for the multiple-cycle machine. For the single-cycle type of machine, roughing speed is assumed to be 0.15 m/s (30 fpm) and finishing speed 0.10 m/s (20 fpm).

TABLE 10.24
Typical Cutting Rates for Form Grinding

Face Width of Gear		Multiple Cycles per Tooth Machine		Single Cycles per Tooth Machine	
mm	in.	Roughing (Cycles/min)	Finishing (Cycles/min)	Roughing (Cycles/min)	Finishing (Cycles/min)
20	¾	125	18	26	19
50	2	100	16	23	16
100	4	75	14	18	13
150	6	60	12	15	10
200	8	50	10	13	9

10.2.1.2 Borazon Form Grinding

Gear tooth grinding has long been done with ceramic wheels such as aluminum oxide or silicon carbide. These wheels have high abrasive performance.

Borazon is a kind of superabrasive material that is based on cubic boron nitride (CBN). This material began to be used to grind* gear teeth during about 1980. The popular name for CBN grinding is Borazon grinding. Borazon CBN is a trademark of General Electric, United States.

The CBN grinding wheel is being made as a highly precise metal wheel with a single layer of CBN particles galvanically bonded onto the surface. A precisely calibrated grit is used. A 65 μm (0.0025 in.) grit is used for gear finishing, and a 100 to 120 μm (0.005 in.) grit is used for gear roughing.

The CBN grinding wheel actually works somewhat like a milling cutter. Each exposed crystal of CBN tends to cut very tiny metal chips. The chips produced by CBN grinding look quite different from the particles produced by a vitrified, ceramic grinding wheel.

It is claimed that the CBN grinding wheel is about 3000 times more wear resistant than a typical aluminum oxide, ceramic wheel.

Borazon gear tooth grinding is done quite differently from conventional form grinding with ceramic wheels:

- No wheel dressing. (After a considerable amount of gear grinding, the CBN wheel is stripped and replated with a new layer of CBN particles.)
- Slow feed. (The wheel slowly progresses across the face width, removing all the stock in one pass.)
- Cool grinding. (With plenty of coolant and a free cutting characteristic, the CBN wheel tends to remove stock in a very predictable and controllable manner. Random gear tooth errors or variations in hardness do not seem to be serious obstacle to the very hard and strong CBN wheel. Local overheating is probably less likely to occur.)

Since the CBN wheel is not dressed, a different wheel is needed for each pitch, each number of teeth, and each style of profile modification (or pressure angle variation). For this

* Super abrasives had been used to sharpen hobs and cutters in the 1970s.

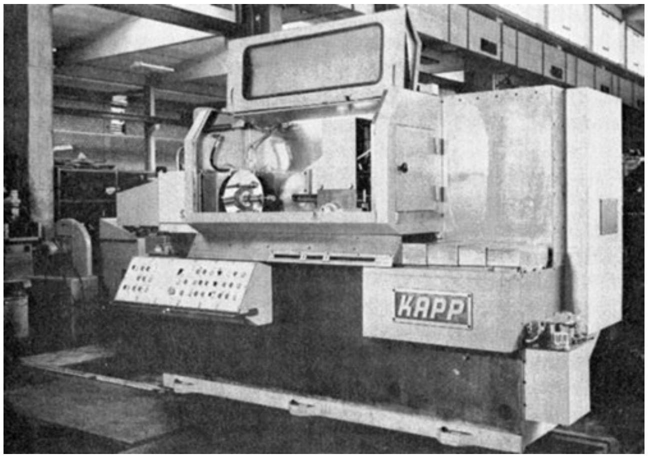


FIGURE 10.30 Form-grinding machine developed for Borazon grinding. (Courtesy of Kapp Co., Coburg, German Federal Republic.)

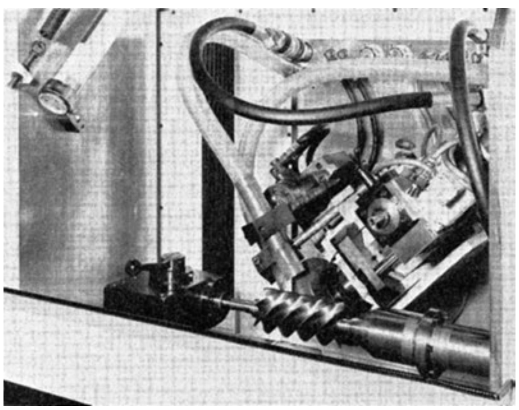


FIGURE 10.31 Form grinding a helical gear part. (Courtesy of Kapp Co., Coburg, German Federal Republic.)

FIGURE 10.32 Examples of gear parts ground by Borazon form grinding. (Courtesy of Kapp Co., Coburg, German Federal Republic.)

reason, the CBN process is less attractive for small-lot production than it is for volume production.

Figure 10.30 shows a form grinder used for Borazon grinding. Figure 10.31 shows a close-up of the grinding wheel and work. Some of the many kinds of parts that can be efficiently ground by the Borazon grinding method are shown in Figure 10.32.

10.2.2 Generating Grinding—Disk Wheel

There are two basic types of generating grinding with disk wheels.

One type uses a single wheel which is dressed to the shape of the basic rack tooth. The workpiece is rolled past the grinding area while the wheel reciprocates past the face width.

The other type uses two saucer-shaped wheels which are concave toward the tooth flank, so that only a narrow rim contacts the tooth while the workpiece is rolled past the grinding area. Since this latter type of grinder is now made only by Maag Gear Wheel Company, it will be referred to here as the Maag process, while the dressed single-wheel type will be referred to as the conical wheel process.

Conical wheel grinding machines can handle gears in the range of about 25 mm (1 in.) to 3.5 m (140 in.) in diameter, while the largest Maag machines can handle gears of up to 4.7 m (184 in.) in diameter.

Both types must have room for the grinding wheel to run out at each end of the cut. Smaller machines usually mount the work between centers. Large machines usually vertically mount the work with a fixture. Since the grinding pressure is small, supports are not usually needed.

The generating grinding machines can make both spur and helical external gears. Internal spur and helical gears can be made with one model, which can handle up to 0.85 m (33½ in.) base-circle diameter.

The action of a conical wheel grinding machine is similar to that of a rack shaper using a single point tool. The grinding wheel is dressed to the desired basic rack form and is reciprocated past the face width while the gear rolls past the grinding field on its pitch circle. Because the wheel must maintain a definite form, it must be of a relatively hard bond, and so it requires a coolant to minimize burning of the work. The Maag process is somewhat similar for their larger machines, except that two saucer-shaped wheels form the rack tooth. The wheel contact area is limited to its outer edge, where wheel wear can be sensed and continuously compensated for by automatic wheel adjustment. Relatively soft bonded wheels are used, so that no coolant is needed.

On the smaller Maag machines, the process is somewhat different. The two grinding wheels are still saucer-shaped and contact the work only at their outer edges, so that wear compensation can be used for soft wheels without coolant. But with the smaller machines, up to 1 m (40 in.) in diameter, the grinding wheels do not form a rack tooth, but are parallel, and always contact the workpiece involute at a point on a line tangent to the base circle. This is known as *zero-degree* grinding because the grinding wheels are set at a pressure angle of 0°. (See Figure 10.33.) The gear is rolled on the base-circle

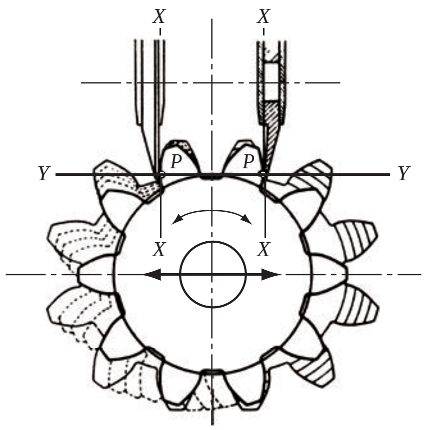


FIGURE 10.33 Zero-degree pressure angle grinding with saucer wheels. (Courtesy of Maag Gear Wheel Col, Zurich, Switzerland.)

tangent plane *Y–Y*. The grinding wheel contact planes are shown as *X–X*, and the contact points are shown as *P*.

Longitudinal corrections are made on all disk wheel–generating grinders by moving the grinding wheel to cut deeper or shallower as it passes across the face. Pro le modi cations on conical wheel machines are made by dressing the wheel. On Maag machines, where there is no wheel form to dress, pro le modi cations are made by moving the wheels to cut deeper or shallower along the pro le. Modi cation by means of wheel movement also allows topological modi cation, in which both pro le and helix may be continuously varied across the face width.

The time needed to make gears by generating grinding with disk wheels is made up of essentially the same elements as the time required to form grinding a gear. The time required to dress a grinding wheel (when necessary) may be estimated from Table 10.25. The time in minutes required to do either rough grinding or nishing grinding may be estimated using the following formula:

$$\text{Grinding time} = \text{no. of gear teeth} \times \text{no. of cuts} \times \left(\text{index time} + \frac{\text{strokes per tooth}}{\text{strokes per minute}}\right). \quad (10.15)$$

TABLE 10.25
Typical Frequency and Length of Time Required to Dress Conical Grinding Wheel

Size of Work		Wheel Diameter		Number of Dressing per Gear		Minutes per Dressing
mm	**in.**	**mm**	**in.**	**45 HRC**	**460 HRC**	
750	30	500	20	4	5	2½
450	18	300	12	3	4	1
300	12	300	12	3	3	1
150	6	300	12	2	3	¾
50	2	300	12	2	2	¾

Equation 10.14 can be used to get the number of cuts for generating grinding as well as for form grinding. In solving this equation, though, Table 10.26 should be used to get the amount of stock that is normally removed per cut. Because the amount of stock to be removed does not depend on the method of grinding, this item may be obtained from Table 10.22.

The time required for generating grinding machines to index varies from 0.04 to 0.30 min, depending on the make and the size of the machine. The strokes per tooth and the strokes per minute vary with machine designs, tooth numbers, and face widths.

When the workpiece is lightweight, the machine elements used to generate can also be relatively light. In these cases, the workpiece is rolled back and forth very rapidly in generation, while the stroking action is slow. The process lends itself to horizontal mounting of the work. Heavy workpieces need heavy generating elements that cannot be rapidly moved. These parts are vertically mounted and are slowly generated, while the wheel stroking motion is rapid. The result is that vertical machines use many fast passes along the face for each tooth, while horizontal machines use many fast generating motions, but only one pass along the face for each tooth for each cut. Table 10.27 shows some typical values for vertical machines, and Table 10.28 shows some typical values for horizontal machines.

Figure 10.34 shows a generating grinding machine of the type that uses a conical wheel. A close-up of the grinding wheel and a double-helical gear being ground are shown in Figure 10.35.

TABLE 10.26
Typical Amount of Stock Removed per Cut when Generating Grinding

		Roughing Grinding									
		Tooth Flanks Only				Flanks and Root				Finishing Grinding	
Tooth Size		45 HRC		60 HRC		45 HRC		60 HRC			
Module	**Pitch**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**
2.5	10	0.13	0.005	0.08	0.003	0.08	0.003	0.04	0.0015	0.013	0.0005
4	6	0.15	0.006	0.08	0.003	0.08	0.003	0.04	0.0015	0.013	0.0005
12	2	0.23	0.009	0.10	0.004	0.10	0.004	0.06	0.0025	0.020	0.0008

TABLE 10.27
Typical Stroking Rates for Vertical Grinders

Tooth Size		Strokes per Tooth				Strokes per Minute		
		Roughing Grinding		Finish Grinding		Face Width		
Module	Pitch	15 Tooth	75 Tooth	15 Tooth	75 Tooth	50 mm (2 in.)	100 mm (4 in.)	150 mm (6 in.)
2.5	10	30	14	40	20	200	135	90
4	6	35	17	55	28	200	135	90
8	3	40	20	65	35	150	100	65
12	2	45	25	70	50	60	45	30

TABLE 10.28
Typical Stroking Rates for Horizontal Grinders

Tooth Size		Generating Strokes per Minute		Stroking Rate (Seconds per Tooth, per Cut), Rough Grinding[a]					
				15 Tooth—Face Width			75 Tooth—Face Width		
Module	Pitch	15 Tooth	75 Tooth	50 mm (2 in.)	100 mm (4 in.)	150 mm (6 in.)	50 mm (2 in.)	100 mm (4 in.)	150 mm (6 in.)
2.5	10	120	240	3.5	7.0	10.5	1.8	3.6	5.4
4	6	130	148	2.8	5.6	8.4	2.3	4.6	6.9
8	3	130	75	2.8	5.6	8.4	4.6	9.2	13.8
12	2	110	40	3.5	7.0	10.5	12.0	24.0	36.0

[a] For nish grinding, multiply seconds per tooth by a factor of 3.

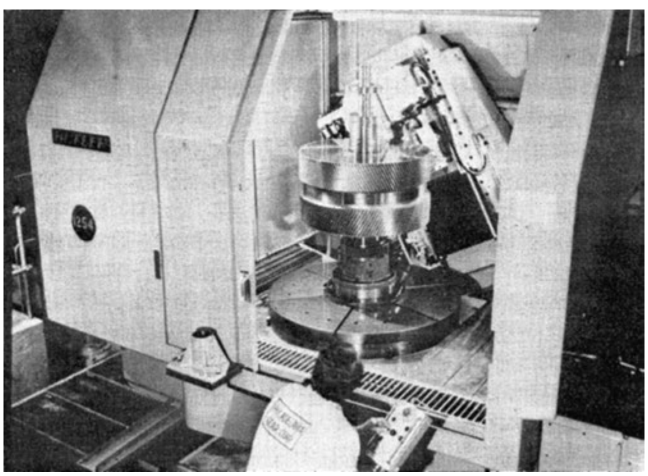


FIGURE 10.34 Generating grinding a double-helical gear with a conical wheel. (Courtesy of BHS-Ho er, Ettlingen, German Federal Republic.)

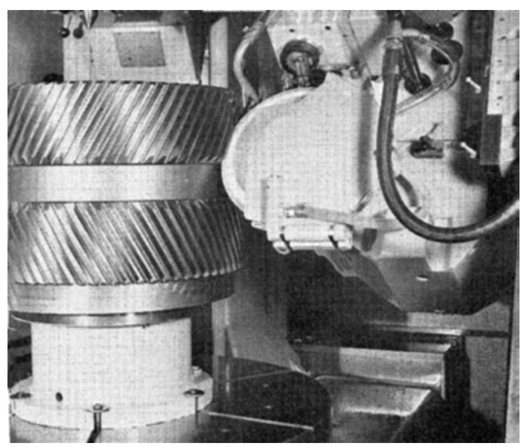


FIGURE 10.35 Close-up view of conical grinding wheel and work. (Courtesy of BHS-Ho er, Ettlingen, German Federal Republic.)

A grinding machine using double saucer wheels is shown in Figure 10.36. A close-up of the wheel and the work is shown in Figure 10.37.

10.2.3 Generating Grinding—Bevel Gears

Zerol, spiral, and hypoid gears can be nished by grinding. The machinery available to grind these gears uses a generating motion. The grinding wheel cuts somewhat like a face cutter. The wheel is shaped like a cup rather than like the disk wheels used to grind spur and helical gears. As the bevel gear is ground, the work and the grinding wheel roll through a generating motion. In principle, the grinding wheel acts like one tooth of a circular rack which is rolling through the mesh with the gear being ground.

Bevel gear grinding machines index after each tooth space has been ground with the generating motion. The motions of the machine are so smooth that the process appears to be almost as continuous as a hobbing or a shaping process.

No feed motion is needed to travel the grinding wheel across the face width of the work. Since the wheel is like

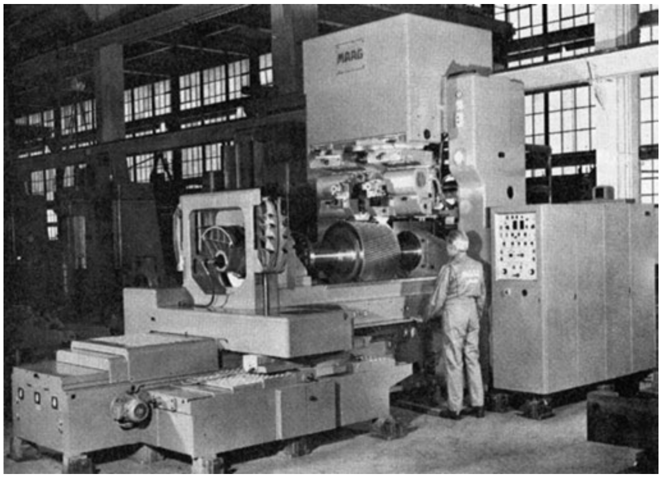


FIGURE 10.36 Generating grinder using two saucer-shaped wheels on a single-helical gear. (Courtesy of Maag Gear Wheel Co., Zurich, Switzerland.)

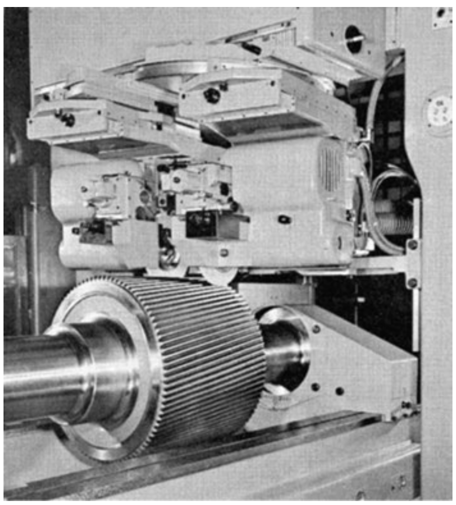


FIGURE 10.37 Close-up view of saucer-shaped grinding wheels and work.

a hoop laid across the face width, the whole face width is ground in the same operation. The fact tends to make bevel gear grinding fast. With spur or helical gears, a feed motion is needed (unless the face width is very narrow). This requires time for both the grinding of the tooth pro le and the advance of the wheel across the face width. In bevel gears, the face width is done just as soon as the tooth pro le is done.

At the present time, there are several sizes of bevel gear grinders for general-purpose work on the market. These are more or less universal machines intended to take a range of pitches, ratios, and spiral angles. Many special bevel gear grinders have been built for the high-production manufacture of a particular gearset.

The designer of bevel gears that require grinding should be careful to design something that is within the range of available machinery, unless the job is important enough to warrant the development of special machine tools.

The capacity ranges of bevel gear grinders are rather involved. The maximum gear diameter that a machine will do changes rather substantially as the gear ratio changes and as the spiral angle changes. In some cases, bevel gears up to 1 m (40 in.) can be ground on the larger bevel gear grinders. Most bevel gears which are ground seem to be in the size range of about 75 mm (3 in.) to 500 mm (20 in.) in diameter.

There are so many complications to bevel gear grinding that it is desirable for most designers to check with the shop that is to build their gears as soon as they have laid out a preliminary design.

The complications of bevel gear grinding make it hard to write any formula for grinding time. The best way to give information on grinding time appears to be to give a table of approximate times for different combinations. Table 10.29 shows approximate times, assuming that distortion is not serious and that moderate precision is desired.

The total time per piece shown in Table 10.29 includes the automatic wheel-dressing cycle. This is approximately 30 s per dress.

Figure 10.38 shows a typical spiral-bevel-gear grinding machine.

TABLE 10.29
Average Time to Grind Spiral Bevel Gears

No. of Teeth	Pitch-Cone Angle (°)	Tooth Size		Face Width		No. of Passes	No. of Dresses	Seconds per Tooth	Total Time (min)
		Module	Pitch	mm	in.				
12	8.5	4	6	41	1.625	4	1	2.7	2.7 per side
20	22	4	6	38	1.500	6	2	2.7	6.4 per side
30	45	4	6	32	1.250	7	2	2.7	10.5
50	68	4	6	38	1.500	8	2	2.7	19.0
80	81.5	4	6	41	1.625	9	3	2.7	33.9
12	8.5	8	3	82	3.250	4	1	4.5	4.1 per side
20	22	8	3	82	3.250	6	2	4.5	10.0 per side
30	45	8	3	82	3.250	7	2	4.5	16.7
50	68	8	3	82	3.250	8	2	4.5	31.0
80	81.5	8	3	82	3.250	9	3	4.5	55.0

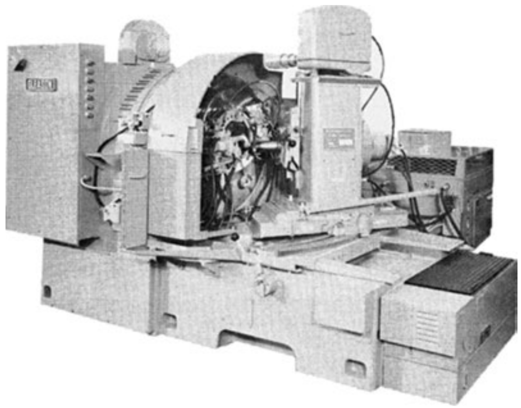


FIGURE 10.38 Grinding a spiral bevel gear. (Courtesy of Gleason Works, Rochester, New York.)

10.2.4 Generating Grinding—Threaded Wheel

Gears can be ground by a grinding machine that uses a threaded grinding wheel. The elements of this kind of machine are very similar to those of a hobbing machine. The process can grind either spur or helical gears, provided they are external.

The threaded wheel grinding machines are available up to 0.77 m (30 in.) outside diameter capacity. Either spur or helical, external gear teeth can be ground. Helix angles up to 45° are ground, but the maximum work diameter has to be considerably reduced. The sizes of teeth produced by this method range from about 0.2 module (120 pitch) to 8 module (3 pitch).

Grinding machines of around 350 mm (13¾ in.) maximum work capacity use threaded grinding wheels about 350 mm in size, while the larger machines around 0.77 m capacity use wheels about 400 mm (15¾ in.) in size.

The smaller machines (350 mm) are using grinding wheel speeds up to 1900 rpm.

Gears to be ground with a threaded wheel must allow room for the wheel to run out. (See Table 10.30.)

Several roughing cuts are used to bring the gear down to size. One or more nishing cuts—depending on the nish accuracy required—are used to nish the gear. In most cases, it is not necessary to stop to dress the grinding wheel while a gear is being ground. As Table 10.30 shows, several gears can usually be done before the wheel needs dressing. This table shows some typical numbers of gears ground per wheel dressing and amounts of overtravel.

It takes about 12 min to dress the grinding wheel. Diamond wheels are used to dress threads on the grinding wheel.

The time required to grind a gear by the threaded grinding wheel process may be calculated in much the same way as hobbing time is calculated. The time in minutes is

$$\text{Grinding time (min)} = \frac{\text{no. of gear teeth} \times (\text{face} + \text{overtravel}) \times \text{ no. of cuts}}{\text{no. of threads} \times \text{feed} \times \text{rpm of wheel}}. \tag{10.16}$$

The amount of stock removed with an average cut and the average rates of feed are shown in Table 10.31. The number of

TABLE 10.30
Number of Gears Ground per Wheel Dressing and Approximate Overtravel

Tooth Size		Face Width			Gears per Dressing		Overtravel	
Module	**Pitch**	**mm**	**in.**	**Helix Angle**	**25 Teeth**	**75 Teeth**	**mm**	**in.**
1.6	16	10	0.375	Spur	200	100	1	0.06250
		25	1.000	15°	100	50	6	0.25000
2.5	10	10	0.500	Spur	100	50	2	0.09375
		50	2.000	15°	40	20	6	0.25000
4.0	6	19	0.750	Spur	20	10	3	0.12500
		100	4.000	15°	5	2	9	0.37500

TABLE 10.31
Typical Values for Feed and Stock Removal When Grinding with a Threaded Wheel

		Feed per Revolution of Work				Stock Removed per Cut, Roughing or Finishing	
Tooth Size		Roughing		Finishing			
Module	**Pitch**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**
1.6	16	1.8	0.07	0.63	0.025	0.01	0.0004
2.5	10	1.8	0.07	0.63	0.025	0.01	0.0004
4	6	1.8	0.07	0.63	0.025	0.01	0.0004

cuts may be gured from the stock left to grind, except that extra cuts may remove almost no stock.

Wheel wear is quite uniform with the threaded grinding wheel. When one spot gets worn, the wheel is axially shifted to let a new part of the wheel generate the work. This is one of the reasons for why a lot of grinding can be done between the wheel dressings.

Gear teeth ner than 0.8 module (32 pitch) may be quite satisfactorily ground from the solid. Larger teeth are cut before grinding.

The threaded wheel grinder is used for high-precision gears. It is also being increasingly used for medium-precision gears which are hardened—and were formerly used without grinding after hardening. Closer quality control and more concern over gear noise make it harder to produce accep gears for lower-speed applications. Table 10.32 shows some comparisons of grinding time that may be expected. Note the substantial reduction in grinding time with lower accuracy. These less-accurate gears are adequate for vehicle gears and other certain industrial gears that do not run at high speeds.

Figure 10.39 shows a threaded wheel grinding machine being used to make high-speed helical pinions. Figure 10.40 shows a close-up of a threaded wheel and a high-speed helical gear.

10.2.5 Thread Grinding

Single-enveloping worms may be nished by grinding in a thread grinder. Ordinary worms are milled, hardened, and then ground. In ne pitches—5 mm (0.200 in.) linear pitch and less—it is possible to make the thread complete by grinding. Pitches as coarse as 40 mm (1.60 in.) linear pitch may be ground on available grinding machines. Some grinders limit the lead angle of the worm to 30°, while others will go up to as much as 50°. Worms up to 300 mm (12 in.) outside diameter may be ground with presently available equipment. See Figure 10.41 for an example of worm thread grinding.

Worm designs to be ground must allow room for a relatively large grinding wheel to run out. A commonly used size of wheel is 500 mm (20 in.).

When the worm-grinding wheel has a straight-sided pro le, it will produce a worm thread with a convex curvature. This curve is not an involute. The amount of this curvature may be calculated by the method shown in Section 11.4. To produce a straight-sided worm thread, a slight convex curvature in the grinding wheel pro le is required.

The time required to grind worm threads may be estimated by the following formula:

$$\text{Grinding time (min)} = \frac{\text{no. of threads}}{\text{threads per cut}} \times \left(\text{index time} + \frac{\text{thread length}}{\text{feed rate}}\right) \times \text{no. of cuts.} \tag{10.17}$$

The indexing time runs from about 0.1 to 0.5 min. The developed length of the thread is

$$\text{Length of thread} = \frac{3.14 \text{ worm dia.}}{\cos(\text{lead angle})} \times \frac{\text{face width}}{\text{lead of worm}}. \tag{10.18}$$

TABLE 10.32
Comparison of Estimate Grinding Times Using the Threaded Grinding Wheel Method for High-Precision and Medium-Precision Gears

Tooth Size			Face Width		Stock to Remove		Grinding Time
Module	Pitch	No. of Teeth	mm	in.	mm	in.	(min)
			High-Precision Gears (Turbine Drives)				
2.5	10	25	32	1.25	0.24	0.0095	6.3
2.5	10	102	32	1.25	0.24	0.0095	20.6
2.5	10	102	76	3.0	0.24	0.0095	49.3
5.0	5	25	76	3.0	0.30	0.0120	14.3
5.0	5	102	76	3.0	0.38	0.0150	65.4
5.0	5	102	152	6.0	0.46	0.0180	150.0
			Medium-Precision Gears (Vehicle Drives)				
2.5	10	25	32	1.25	0.24	0.0095	3.2
2.5	10	102	32	1.25	0.24	0.0095	8.8
2.5	10	102	76	3.0	0.24	0.0095	20.7
5.0	5	25	76	3.0	0.30	0.0120	9.1
5.0	5	102	76	3.0	0.30	0.0120	24.2
5.0	5	102	152	6.0	0.30	0.0120	48.1

Note: The high-precision gears are ground with a single-thread wheel. The medium-precision gears are ground with a double-threaded (two-start) wheel. In all cases, the gears are rough ground and then nish ground.

FIGURE 10.39 Grinding helical pinions on a threaded grinding wheel machine. (Courtesy of Sier-Bath Gear Co., North Bergen, New Jersey.)

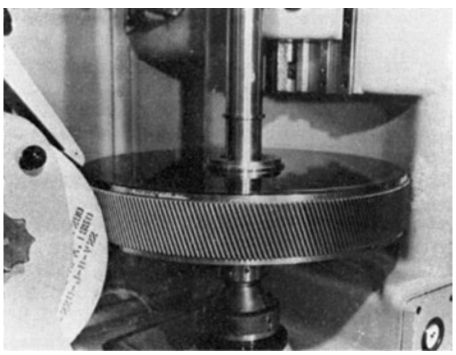


FIGURE 10.40 Threaded grinding wheel and helical gear. (Courtesy of Reishouer Corp., Elgin, Illinois, and Reishouer Ltd., Zurich, Switzerland.)

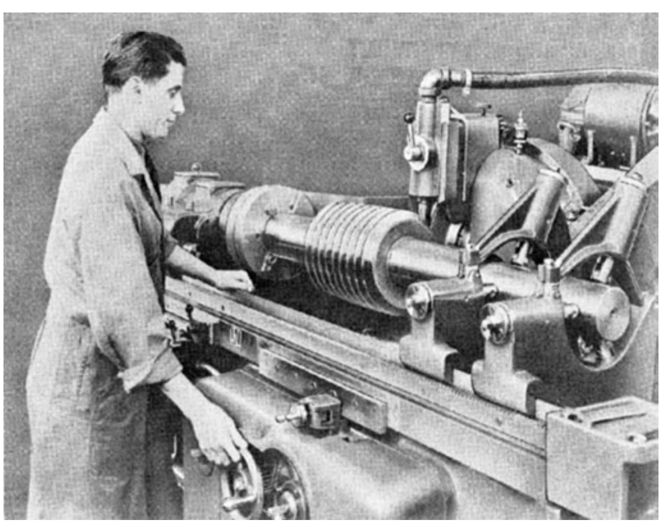


FIGURE 10.41 Grinding a large precision index worm. (Courtesy of Jones and Lamson, Springfield, Vermont.)

In fine-pitch worms, it is sometimes possible to use two or three ribs on the grinding wheel and grind all worm threads at once. This saves time.

The number of cuts will depend on both the pitch and the amount of stock left for grinding. Table 10.33 shows the number of cuts and the feed rates normally used for different pitches of worms. It is assumed that a nominal amount of stock is left for finishing. This would be about 0.25 mm (0.010 in.) on tooth thickness for 12 mm (0.500 in.) linear pitch, and 0.65 mm (0.025 in.) for 32 mm (1.250 in.) linear pitch.

10.3 GEAR SHAVING, ROLLING, AND HONING

There are three different ways of finishing involute gear teeth that involve a gear-like tool rolling with the work on a crossed axis:

- Shaving
- Rolling
- Honing

The shaving process finishes by cutting. The rolling process cold-forms the metal by very small amounts of cold flow on the gear teeth, produced by pressure from the hardened roll. The honing process abrasively removes very small amounts of metal from the gear tooth surfaces.

Gear shaving is strictly a finishing operation. Compared with grinding, shaving is generally a much faster process.

Grinding is not limited to hardnesses. It is usually quite practical to shave gears up to 350 HB (38 HRC). Gears up to 450 HB (47 HRC) have been cut and shaved with fair results. At the higher hardnesses, tool wear is very fast, and special techniques and lubricants are required. Also, the shaving cutter needs to be extra hard.

TABLE 10.33
Number of Cuts and Feed Rates Normally Used in Worm Grinding

Linear Pitch		No. of Cuts		Feed Rate per Min			
				250 mm (10 in.) Wheel		500 mm (20 in.) Wheel	
mm	in.	Roughing	Finishing	mm	in.	mm	in.
6	0.250	2	1	625	25	1000	40
12	0.500	3	1	500	20	875	35
19	0.750	4	1	375	15	750	30
25	1.000	5	2	300	12	625	25
31	1.250	7	2	250	10	500	20

Shaving is a corrective process. Most people in the trade express the view, "Shaving will not make a bad gear good, but it will make a good gear better!" Shaving readily improves surface nish and reduces gear runout. If the gear and the shaving cutter are properly designed, it is possible to considerably improve pro le accuracy. Tooth-to-tooth spacing is improved by shaving, but accumulated spacing error is not changed by a large amount unless the accumulated error comes mostly from eccentricity effects. On narrow-face-width gears, helix errors can be controlled by shaving, but a narrow shaving cutter has little or no control on wide-faced gears.

Since shaved gears are not heat-treated between cutting and shaving there is no heat-treat distortion on clean up, and only a small amount of stock is needed for shaving. Shaved gears are often fully hardened after shaving. When this is done, heat-treat distortion is held to a minimum or controlled in such a manner that uniform distortion results, and this distortion is allowed for in the shaving process.

There are two general methods of shaving, rotary shaving and rack shaving. The rotary method uses a pinion-like cutter with serrated teeth, while the rack method uses an actual rack with serrated teeth. Figure 10.42 shows rotary shaving, while Figure 10.43 below shows rack shaving.

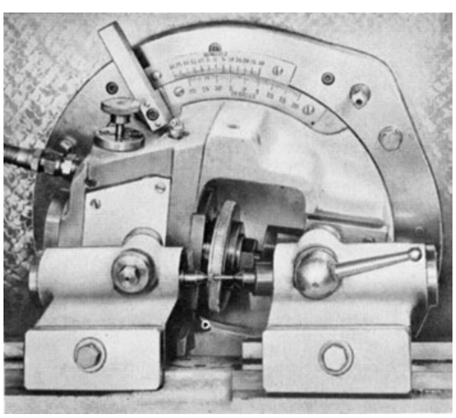


FIGURE 10.42 Shaving an instrument pinion. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

10.3.1 Rotary Shaving

The rotary shaving cutter has gear teeth ground with a pro le that will conjugate with that of the part to be shaved. The cutter teeth are serrated, with many small rectangular notches. As the shaving cutter rotates with the gear, these notches scrape off little shavings of material; hence the name *shaving.* There is no index gearing in the shaving process. Small parts are driven by the cutter that is shaving them, while large parts drive the cutter instead. The shaving cutter is essentially a precision-ground gear made of tool steel. The reason that shaving can produce very high accuracy is that there is an averaging action as the cutter rolls with the work. This tends to mask the effect of any slight indexing errors that may have been ground into the cutter.

The shaving cutter does not have the same helix angle in degrees as that of the part it is shaving. This makes the axis of the cutter sit at an angle with the axis of the work. The amount of this crossed-axis angle governs the shaving action. The greater the angle, the more the cutter cuts. The crossed-axis angle causes a cutter tooth to slide sideways as it rolls with the gear tooth. It is this motion that makes the shaving cutter scrape off metal. If shaving cutters were serrated in a radial direction and the crossed-axis angle were zero, a shaving cutter would not cut at all unless it was axially reciprocating. It so happens that one design of shaving machine does use a rapid reciprocating motion to shave internal gears. In general, shaving cutters do not have any rapid reciprocating motion.

Both external and internal gears may be shaved. The gears may be either spur or helical. Small external gears may be shaved by feeding the cutter either parallel to the gear axis or at some angle to the axis. If the cutter is fed at right angles to the gear axis, the cutter must be as wide as or slightly wider than the work. If the cutter is fed in a diagonal direction, there is a relation between the cutter minimum width and the gear face width. When the cutter is fed parallel to the gear axis, there is no geometric requirement on the cutter face width. With this direction of feed, satisfactory results have been achieved by shaving a gear as wide as 0.5 m (20 in.) face width with a 25 mm (1 in.) cutter.

Figure 10.44 shows a shaving machine for small gears that will shave with the feed at an angle to the axis. A vertical

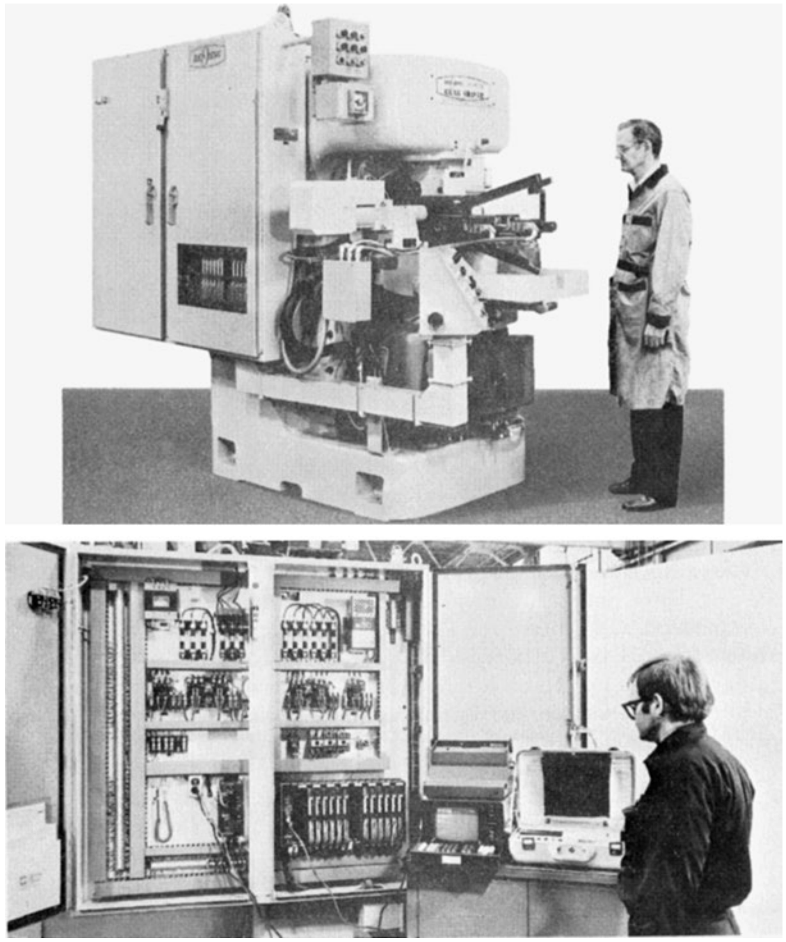


FIGURE 10.43 Gear-shaving machine equipped with programmable controller. (Courtesy of National Broach and Machine Co., a division of Lear Siegler, Inc., Detroit, Michigan.)

axis shaving machine is shown in Figure 10.45. This type of machine is often used for large gears.

Gears which are to be shaved should allow room for the cutter runout. Shaving-cutter runout is hard to figure, because the cutter sits at an angle with the work and contacts the work on an oblique line. Gears to be shaved are often cut (before shaving) with a cutting tool which has a slight protuberance in its tip. This is helpful because the shaving cutter is not intended to remove metal from the root fillet of the gear. The protuberance on the cutting tool produces a slight relief, or undercut. This undercut allows the tip of the shaving cutter to freely roll instead of hitting a shoulder where the shaving action stops at the bottom of the gear tooth.

Gears to be shaved must have tooth designs that permit sufficient teeth to be in contact with the shaving cutter. Since

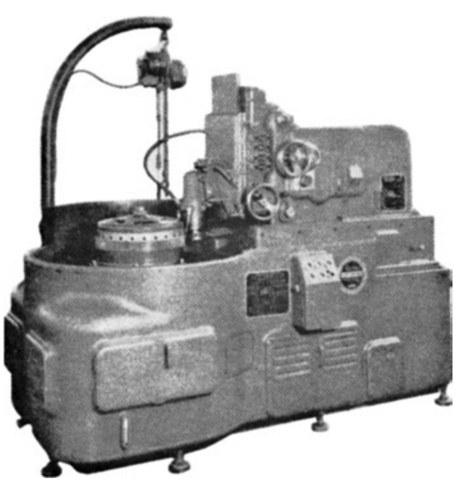


FIGURE 10.44 A rotary gear shaver for finishing medium-sized internal and external gears. (Courtesy of National Broach and Machine Co., a division of Lear Siegler, Detroit, Michigan.)

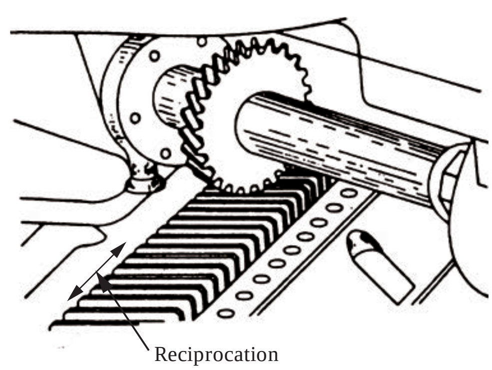


FIGURE 10.45 The rack shaving method.

the cutter either drives or is driven by work, the gear teeth must be capable of smoothly transmitting power. For instance, shaving standard spline teeth with a 30° pressure angle and a height which is stubbed to 50% of full proportions is usually impractical. These teeth do not have enough involute pro le to smoothly transmit power when they roll. In general, the pressure angle should be in the range of 14.5° to 25°, and the tooth height should be at least 75% of full depth to permit satisfactory rolling conditions. Clearance in the root llet should be at least $0.3m_n$ $(0.3/P_d)$ to permit a suitable design of a preshaving type of cutting tool.

Spur pinions with small numbers of teeth are somewhat more dif cult to shave. There are problems with obtaining good involute when shaving pinions with 10 teeth and fewer. It is also dif cult to design a suitable shaving cutter for internal gears with fewer than 40 teeth. When internal gears have 25 teeth, it is just about impossible to get a cutter inside that will shave on the crossed-axis principle.

The time required to shave gears may be estimated by one of two formulas. If the feed is parallel to the gear axis, Equation 10.19 should be used. If the feed is at an angle to the axis, Equation 10.20 is the one to use.

Feed parallel to axis:

$$\text{Shaving time (min)}$$
$$= \frac{0.262 \times \text{pitch dia.} \times (\text{face width} + \text{overtravel}) \times \text{no. of cuts}}{\text{shaving speed} \times \text{feed}}. \tag{10.19}$$

Feed at an angle to axis:

$$\text{Shaving time (s)} = \text{time per stroke} \times \text{no. of cuts}. \tag{10.20}$$

The overtravel in shaving may be estimated at about 1.5 times per cutter width. The face width and the pitch diameter in Equation 10.19 are those of the gear being shaved.

The number of cuts required will depend on the amount of stock left for shaving and the amount taken per cut. Sometimes extra cuts may be needed to secure helix angle correction. The amount of stock that may be removed in shaving is fairly limited. It is practical to put in only a certain amount of undercut in the preshave cutting. If too much stock is removed, the undercut allowance will be exceeded, and there will be trouble from cutter bottoming. Table 10.34 shows the approximate amount of stock that may be left for shaving.

The stock removed per cut and the rate of feed both depend somewhat on the size of the gear and considerably upon the degree of quality and surface nish desired. Table 10.35 gives some representative values that can be used for estimating purposes.

The rolling velocity of a shaving cutter is generally described as *shaving speed*. The shaving speed, of course, considerably varies depending on the hardness and the size of

TABLE 10.34
Amounts of Stock Left for Shaving

Tooth Size		Stock Left on Tooth Thickness			
		Minimum (High Accuracy)		Maximum (Medium Accuracy)	
Module	Pitch	mm	in.	mm	in.
1.6	16	0.025	0.0010	0.050	0.002
2.5	10	0.038	0.0015	0.075	0.003
4	6	0.050	0.0020	0.100	0.004
12	2	0.075	0.0030	0.150	0.006

TABLE 10.35
Shaving Stock Removal and Feed Rates

Kind of Work			Stock Removal per Cut		Cross Feed Rate per Revolution	
Diameter						
mm	in.	Accuracy	Roughing, mm (in.)	Finishing, mm (in.)	Roughing, mm (in.)	Finishing, mm (in.)
150	6	High precision	0.018 (0.0007)	0.008 (0.0003)	0.38 (0.015)	0.15 (0.006)
150	6	Medium precision	0.038 (0.0015)	0.018 (0.0007)	0.50 (0.020)	0.50 (0.020)
600	24	High precision	0.018 (0.0007)	0.008 (0.0003)	0.38 (0.015)	0.20 (0.008)
600	24	Medium precision	0.038 (0.0015)	0.018 (0.0007)	0.63 (0.025)	0.63 (0.025)
2400	96	High precision	0.018 (0.0007)	0.008 (0.0003)	0.38 (0.015)	0.25 (0.010)
2400	96	Medium precision	0.038 (0.0015)	0.013 (0.0005)	0.63 (0.025)	0.38 (0.015)

TABLE 10.36
Seconds per Cycle When Shaving Cutter Is Fed at an Angle to the Gear Axis

Gear Diameter		Feed 90° to Gear Axis		Feed 60° to Gear Axis		Feed 30° to Gear Axis	
mm	in.	Commercial	Precision	Commercial	Precision	Commercial	Precision
75	3	28	40	32	45	36	50
150	6	45	65	50	70	54	75
300	12	57	85	63	90	68	95
450	18	70	100	76	105	82	120
600	24	85	125	91	130	97	149

the gear parts being shaved. Some typical values of shaving speed are as follows:

Small steel parts, 250 BHN, 25 to 100 mm (1 to 4 in.) in diameter	120 to 150 m/min (400 to 500 fpm)
Large gears 250 BHN, 1 to 2.5 m (40 to 100 in.) in diameter	90 to 120 m/min (300 to 400 fpm)
Medium gears, 350 BHN, 100 mm to 1 m (4 to 40 in.) in diameter	75 to 90 m/min (250 to 300 fpm)

When gears are shaved with the cutter traveling at an angle to the axis, the cutter has to be wide enough to shave the whole gear face width in one stroke. This process will not handle as wide work as the parallel method. Typical machinery on the market can handle gears up to 0.6 m (24 in.) in diameter and 100 mm (4 in.) face width by this process. By comparison, when the feed is parallel to the axis, available machinery will handle gears up to 5 m (200 in.) in diameter and 1.5 m (60 in.) face width.

The amount of time required to make a stroke (or cut) when the feed is at an angle to the axis mostly depends on the gear diameter. Table 10.36 gives some representative values for different angles of feed.

10.3.2 Rack Shaving

In rack shaving, the gear to be shaved is rolled back and forth with a rack having serrated teeth. The rack is moved in a direction which is not perpendicular to the gear axis. This gives a crossed-axis effect which makes rack shaving follow the same kind of cutting action as rotary shaving. It is the crosswise sliding that makes the rack cut.

As the work rolls with the reciprocating rack (Figure 10.43), it is also moved across a portion of the rack so as to equalize the wear of the rack. After each stroke of the rack, the gear is fed in a slight amount toward the rack.

External spur and helical gears can be shaved by the rack method. Presently available machines will handle gears up to 200 mm (8 in.) in diameter and 50 mm (2 in.) face width. This method of shaving does not lend itself to shaving large parts. The racks used are quite expensive. The rack must be wider than the gear face width, and it must have a length (and a stroke) longer than the gear circumference. The rack shaving process is very rapid, and a very large number of parts are obtained per sharpening of the rack. Rack shaving is quite economical on high-production jobs.

TABLE 10.37
Average Amount of Time to Rack Shave Gears

Gear Diameter		Time (s)				
mm	in.	1.6 Module (16 Pitch)	2 Module (12 Pitch)	2.5 Module (10 Pitch)	3 Module (8 Pitch)	4 Module (6 Pitch)
25	1	75	70	69	65	60
50	2	73	72	67	63	58
75	3	70	67	64	60	55
100	4	65	62	59	55	50
150	6	58	55	52	48	43
200	8	45	42	39	35	30

The time required to nish steel gears by rack shaving may be estimated from Table 10.37. The table shows an average time based on a moderate amount of stock left for shaving and material with reasonably good machinability. Special conditions or quality requirements would require proportionally more or less time.

10.3.3 Gear Rolling

The gear rolling processes nish the teeth by rolling the gear with a hardened tool that has very precisely ground teeth. This tool is called a *die*. The gear and die are pushed together with a high force.

Figure 10.46 shows the schematic arrangement of single-die rolling. Note the roller steady rests used to transmit the force of the table feed to work.

Figure 10.47 shows a two-die rolling machine. In this case two dies, 180° apart, apply force to the part being rolled. (The part does not need roller steady rests when two dies with equal and opposite forces do the rolling.)

Small parts are generally done with the two-die rolling machine, while large parts are done with the single-die machine. Typical machines roll gears from about 25 mm (1 in.) to 150 mm (6 in.) using two dies. Single-die machines

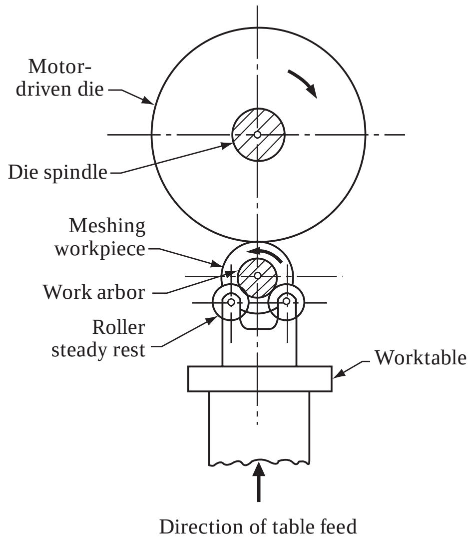


FIGURE 10.46 Operating principle of single-die gear-rolling machine. (From *Modern Methods of Gear Manufacture*, National Broach and Machine Co., a division of Lear Siegler, Inc., Detroit, Michigan. With permission.)

FIGURE 10.47 Two-die gear-rolling machine. (Courtesy of National Broach and Machine Co., a division of Lear Siegler, Inc., Detroit, Michigan.)

are being used to do gears up to 0.5 m (19 in.) in diameter. The face widths are generally quite narrow (from about 20 mm [0.8 in.] to about 70 mm [2¾ in.]).

Gear rolling on high-production jobs is generally done with the axes parallel. It can, of course, be done with the axes crossed by a rather small angle. It takes less force to do the job with axes crossed, but there is less control of helix accuracy. Also, the rolling time is longer.

In rolling teeth, there is some difficulty with the involute accuracy and the quality of the rolled surface. If the gear is too hard or has too much stock left, there is a tendency for slivers of metal to roll over the edges and for there to be folds

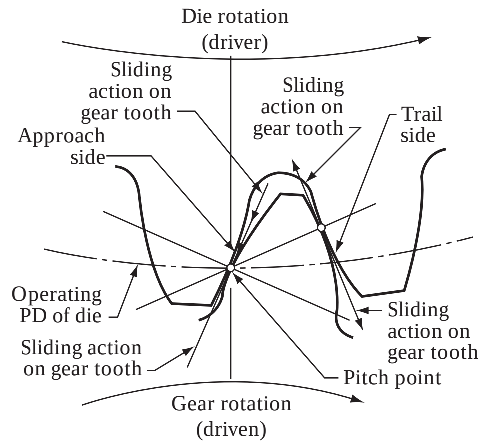


FIGURE 10.48 Different sliding directions as a die rolls with a gear. PD, pitch diameter.

or flaws around the pitch line. The sliding velocities are different on the two sides of the gear being rolled. This fact tends to make the two sides different and to cause trouble at the pitch line, where the sliding velocity reverses. Figure 10.48 shows the characteristic conditions during gear rolling.

Rolling is successful, though, in spite of the problems just mentioned. These factors will help ensure that the process will work:

- The amount of stock left for rolling is small (about one-half of that left for shaving).
- The teeth are cut before rolling with a protuberance cutter so that the die can roll clear of the root fillet.
- The teeth are usually chamfered at the outside diameter corner.
- The gear to be rolled should not be too hard. (A good hardness for rolling is 200 HB [210 HV].)
- Rolling jobs are developed by experimental rolling, involute checking, and then by involute modification of the die to make the involute of the part come out as desired.
- The rolling is often reversal in direction in order to make the two sides of the gear tooth come out alike.

Gear rolling is fast, and it produces a very smooth, burnished surface finish. In the production of small, high-volume vehicle gears, rolling gears that are going to be carburized and hardened before the carburizing process works out well. A low hardness is normal in parts to be carburized, since the final hardness of both case and core material will be established in the carburizing process. Small gears are being made to medium-high-precision accuracy by rolling before carburizing and no grinding after carburizing. (In some cases, such parts may be honed after carburizing.)

To show the relative time involved in shaving, rolling, and honing, the following data from National Broach and Machine Co., Inc. are illustrative. These data are for a 25-tooth helical

gear with 73.66 mm (2.90 in.) pitch diameter and 16 mm ($\frac{5}{8}$ in.) face width. Normal pressure angle was 20°, and helix angle was 32°:

Gear shaving	
Conventional	43 s/piece
Diagonal	22 s/piece
Gear rolling	10 s/piece
Gear honing	21 s/piece

10.3.4 Gear Honing

In the gear honing process, an abrasive tool with gear teeth is rolled with the gear part on a crossed axis. The force holding the honing tool and the work together is very light. The honing tool cuts because of abrasive particles in the composition of the hone or attached to the surface of the hone. The honing is done wet, with an appropriate fluid to serve as a lubricant, a coolant, and a medium to flush away the wear debris from honing.

The gear hone may be a plastic resin material with abrasive grains of silicon carbide. This kind of hone is made by casting in a mold. The hone may be made to medium-precision accuracy and used on gears intended to have medium-high-precision accuracy. The basic accuracy is in the gear before honing. Honing averages the surface and takes off local bumps, scale, etc. Thus, the accuracy of the gear is not primarily derived from the hone.

For high-precision gear work, the hone is made to a relatively high precision. It is often possible to get some improvement in involute, helix (lead), and concentricity by careful honing with a precision hone.

Metal hones with bonded-on abrasives are used for fine-pitch gears and for certain medium-pitch gears where a plastic hone might tend to break.

Vehicle gears are often carburized and then run without grinding. These gears are generally shaved or rolled to finish accuracy before carburizing. The final honing operation does these things:

- Removes heat-treat scale and oxidation
- Removes nicks and bumps from handling (the unhardened gear gets bruised very easily in handling)
- Removes some heat-treat distortion

The honed vehicle gear generally runs more quietly. Its load-carrying capacity is higher because of more uniformity in accuracy and smoother tooth surface.

High-speed gears used in helicopters and in other certain high-speed turbine applications are generally finish ground *after carburizing.* Such gears may still need honing to get a very smooth surface finish, so that they will not fail due to scoring. Frequently, such gears need a surface finish better than the grinding machine will produce.

The better the ground finish, the better the honed finish. Some guideline relations are the following:

- Shave or grind to 0.7–0.9 µm (0.028–0.036 µin.) finish; hone to 0.4–0.5 µm (0.016–0.020 µin.).
- Shave or grind to 0.4–0.6 µm (0.016–0.024 µin.) finish; hone to 0.25–0.35 µm (0.010–0.014 µin.).
- Shave or grind to 0.25–0.35 µm (0.010–0.014 µin.) finish; hone to 0.15–0.2 µm (0.006–0.008 µin.).

When the honed finish must be extra good, the final honing is often done with a rubberlike hone. The hone material is polyurethane. The abrasive is applied as a liquid compound during the honing. The abrasive particles lodge in pores on the hone tooth surface and thereby charge the hone.

The type of honing just described removes almost no stock, and so it has little or no ability to correct involute or helix (lead). The primary purpose of this type of honing is to polish the tooth surface. (In some difficult gear jobs, two-stage honing is used. Normal hones clean up the surface and refine the accuracy. The final finish is obtained by the rubber-like polishing hone.)

Gear honing machines look like gear shaving machines. They are special, though, in that they must handle fluids with abrasives, and they must be able to accurately apply very light forces to the hone. Figure 10.49 shows a close-up of a gear honing operation. The top half of Figure 10.49 shows some typical hones made of steel and carbide coated.

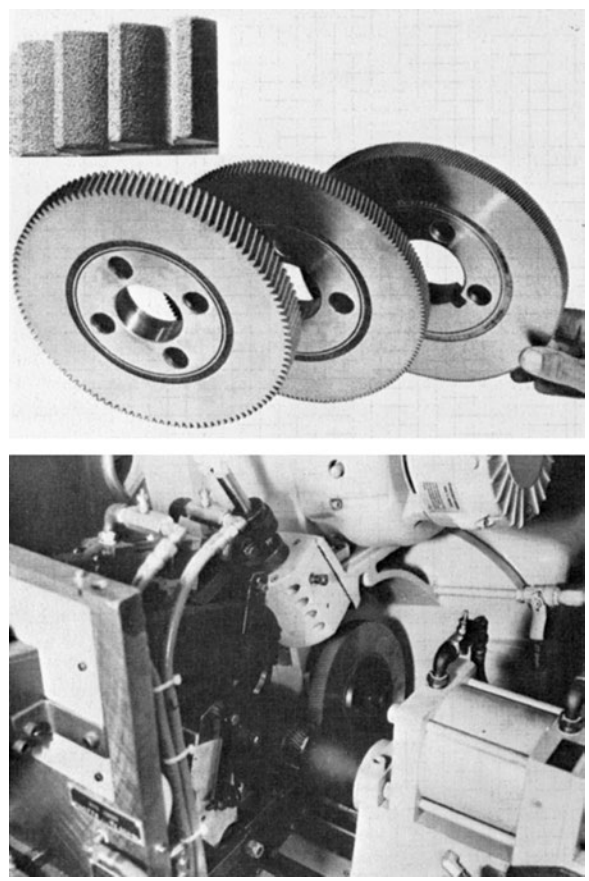


FIGURE 10.49 Gear hones with abrasives bonded to metal and a gear honing arrangement. (Courtesy of National Broach and Machine Co., a division of Lear Siegler, Inc., Detroit, Michigan.)

Gear honing machines for external gears are readily available for gear sizes up to 0.6 m (24 in.) in diameter. Machines for honing internal gears tend to be smaller in capacity, with 0.3 m (12 in.) being typical. The size of teeth commonly honed ranges from 1.2 to 12 module (2 to 20 pitch).

10.4 GEAR MEASUREMENT

Those running gears shops find that they need a considerable capability to measure the geometric accuracy of gear teeth. It is not enough to have machinery to cut, grind, and finish gear teeth. Additional machinery is needed to measure gear tooth spacing, profile, helix, concentricity, and finish.

The gears used in the aerospace field must be rigidly controlled for both geometrical and metallurgical quality. The high-speed gears used with turbine engines are almost as critical. A turbine, for instance, that runs at 20,000 rpm will usually be connected to a pinion. In the oil and gas industry, it is normal for the turbine gear drive to be designed for a life of 40,000 hours—at full-rated power. A pinion meshing with one gear makes 4.8×10^{10} cycles in 40,000 hours at 20,000 rpm. With a high pitch-line speed and full-rated load, it is obvious that a pinion like this will not last this long if its accuracy is deficient. Precise measurements are needed to verify that each and every item of geometric accuracy is within specification limits.

Vehicle gears do not run as long—total life is often no more than 2×10^8 cycles. The vehicle gear, though, is much more heavily loaded than the turbine gear. In addition, noise and vibration requirements have become quite critical. The vehicle gear does not have to be as accurate as the turbine gears, because it does not run so fast or so long. But the vehicle gear has to meet its own level of accuracy. If it does not, vehicle drives that should last a few years will fail in a few months.

In all fields of gearing, the control of gear accuracy is essential. Since this is the case, it is necessary to present some material on accuracy limits and on machines used to measure gear accuracy.

10.4.1 Gear Accuracy Limits

The gear designer faces a dilemma when endeavoring to put accuracy limits on a gear drawing. To get high load-carrying capacity and reliability to meet the gear life requirements, each item must be very closely specified. (Section 5.2.7 shows the rather direct relation between accuracy and load-carrying capacity.)

Besides load-carrying capacity, the designer needs to worry about two other things.

First, the accuracy called for must be *practical* to meet in the gear shop (or shops) available to do the gear work. Specifying impossible accuracy limits is to no avail.

Second, the gear parts need to be made at *reasonable* cost. In a competitive world, it is not the best gear that is needed. What is really needed is the lowest-cost gear that will adequately meet load, life, reliability, and quietness requirements.

To solve the dilemma just given, it is advisable for the gear designer to first consider these things:

- Pitch-line velocity
- Intensity of loading, length of life, and degree of reliability needed
- Requirements as to noise, vibration, or need to run with hot, thin oils

The things just mentioned will guide the designer to the proper level of accuracy. Next, the designer needs to consider the machine tools and the level of operator skill and discipline that are available to make the gears (good trade practice in the product area).

After choosing an appropriate level of accuracy and reviewing manufacturing resources, the designer needs guidelines on the gear trade. Table 10.38 is a brief guide based on practical experience.

In the gear trade, published standards are used extensively. The most important are AGMA 2000-A88 and DIN 3963.

The published standards are periodically revised to cover more quality items and to more clearly define items. Also, the mix may change. For instance, the helix accuracy specified for a high-precision gear may be too low to match the profile and the spacing accuracy achievable with first-rate grinder. For long-range thinking, it is probably best to think in terms of the six levels described in Table 10.38, then try to find (or establish) a set of accuracy limits that is adequate for the job requirements and practical to meet in the gear shop making the parts.

Many major companies* in the United States have found it necessary to set up their own accuracy limits. In this way, they can cover important items not covered in AGMA or DIN standards, and they can adjust the mix of limits to be right for their kind of gear work.

Figure 10.50 illustrates the principal geometric quality items that need control in gearing. For each item, a method of specifying the item is shown. (Other methods are in use, but the ones shown are believed to be the most popular—and practical—for gear manufacturer.)

Table 10.39 shows examples of tolerances for three levels of accuracy and three sizes of gears. This table is based on a survey of key people in the gear trade. It does not exactly agree with any published (or unpublished) accuracy standard. It is hoped that these data will be useful to those studying and setting values for gear accuracy limits.

As a last item, cost needs to be considered. The Fellows Corporation has studied this subject rather thoroughly. They emphasize that the achievement of high accuracy involves several important variables, including the following:

- Machine operator's skill
- Blank accuracy, material, and heat treatment
- Cutting or grinding tool accuracy
- Mounting of cutting tool or grinding wheel

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* These are primarily companies making helicopter gears or high-speed turbine gears for the oil and gas industry.

TABLE 10.38
Accuracy Levels for Gears

		Approximate Relation to Trade Standards	
Designation	**Description of Level**	**AGMA 390.03**	**DIN 3963**
AA Ultrahigh accuracy	Highest possible accuracy, achieved by special tool-room type methods. Used for master gears, unusually critical high-speed gears, or when *both* highest load and highest reliability are needed.	14 or 15	2 or 3
A High accuracy	High accuracy, achieved by grinding, shaving with first-rate machine tools, and skilled operators. Used extensively for turbine gearing and aerospace gearing. Sometimes used for critical industrial gears.	12 or 13	4 or 5
B Medium-high accuracy	A relatively high accuracy, achieved by grinding or shaving with emphasis on production rate rather than highest quality. May be achieved by hobbing or shaping with best equipment and favorable conditions. Used in medium-speed industrial gears and more critical vehicle gears.	10 to 11	6 to 7
C Medium accuracy	A good accuracy, achieved by hobbing or shaping with first-rate machine tools and skilled operators. May be done by high-production grinding or shaving. Typical use is for vehicle gears and electric motor industrial gearing running at slower speeds.	8 to 9	8 to 9
D Low accuracy	A nominal accuracy for hobbing or shaping, can be achieved with older machine tools and less skilled operators. Typical use is for low-speed gears that wear into a reasonable fit. (Lower hardness helps to permit wear-in.)	6 or 7	10 or 11
E Very low accuracy	An accuracy for gears used at slow speed and light load. Teeth may be cast or molded in small sizes. Typical use is in toys and gadgets. May be used for low-hardness power gears with limited life and reliability needs.	4 or 5	12

- Work-holding fixture accuracy
- Accuracy in mounting of work-holding fixture
- Production method
- Distortion
- Inherent capability and condition of machine tool

Fellows has released several charts and tables that show cost versus accuracy trends for fine-pitch and medium-pitch gears and the relative cost of different methods of cutting or finishing gear teeth. Figure 10.51 shows the cost trend as involute accuracy is increased and the range of capability of several gear tooth–making methods. Note that a change from an AGMA 8 quality number (DIN 9) to an AGMA 15 quality number (DIN 3) involves approximately a *tenfold increase* in cost.

Figure 10.52 shows general cost trends for each method of gear tooth making. Note that cutting (hobbing or shaping) is the lowest cost for AGMA 8 quality number (DIN 9). However, for AGMA 12 quality number (DIN 5), grinding or shaving is less expensive. (AGMA 12 is very difficult to achieve by cutting.)

An article in the magazine *Design Engineering*, "The Cost of Gear Accuracy," by Doug McCormick, gives a good analysis of gear accuracy needs and manufacturing considerations and the costs involved in obtaining the desired accuracy.

10.4.2 Machines to Measure Gears

It is possible to make relatively high-accuracy gears without much special measuring equipment. The procedure is along these lines:

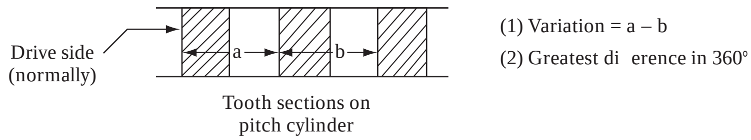
- Make the gear teeth on good-quality machine tools.
- Set up the work accurately and cut or grind with precision cutters or grinding wheels.
- Check the runout of the finished gear by measuring runout over pins. (A precision cylindrical pin is put into each tooth space, and radial runout is measured with a high-accuracy indicator which reads to 0.0025 mm or 0.0001 in.)
- Contact the mating gears and note how the involute profiles fit and how the helices fit across the face width.
- Observe the tooth finish and feel it with your fingernail.

The system just described was widely used in the past and it is still in use by those who make a limited quantity of gears for their own needs. It is possible for skilled mechanics with a general understanding of gear quality to handle things so that generally satisfactory results are obtained.

In general, those who build gears for sale on the open market need a machine or machines to measure the prime variables of involute profile, helix across the face width, tooth spacing, tooth finish, and tooth action by meshing with a master gear. Gears cannot be put in quality grades without accurate measurements. End easement, crown, and profile modification cannot be controlled by contact checks alone. (All these items can result in no contact in certain areas. The gap in a no-contact situation can be determined only by measurement. Use of coordinate measuring machine of modern designs is the best choice to do that.)

In medium-production gear shops making gears up to 0.6 m (24 in.) in diameter, it is usually handiest to have

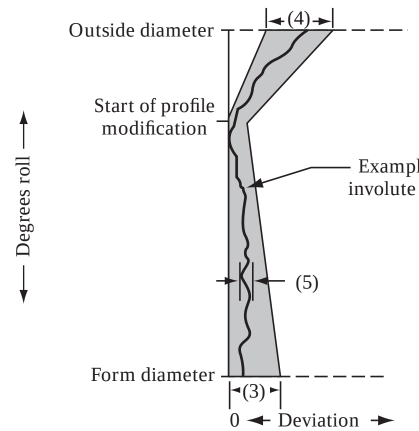
1. Spacing—the variation in circular pitch from one pair of teeth to a pair immediately adjacent.



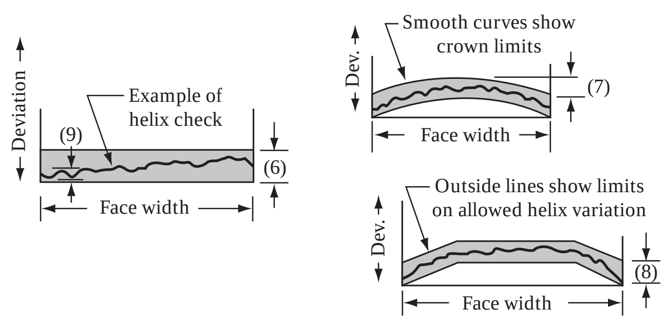
2. Pitch cumulative—the greatest out of position of any tooth side, with respect to any other tooth side, in the gear circumference.

The shaded area of the sample "K chart" shows the range of profile variations allowed by slope and modification tolerances.

3. Slope tolerance
4. Modification tolerance
5. Irregularity



6. Slope tolerance
7. Crown tolerance
8. End easement tolerance
9. Irregularity



10. Composite Tooth-to-Tooth—tooth-to-tooth variation in center distance, rolling with a master.
11. Composite total—total variation in center distance, rolling with a master.
12. Profile finish—arithmetic average finish between form diameter and outside diameter.
13. Root fillet finish—arithmetic average finish in root (below form diameter).
14. Waviness, working surface—contour variations in less than 10% of active profile or in less than 5% of face width.

FIGURE 10.50 Definitions of gear tooth geometry tolerances.

separate machines to check each major variable—for instance, spacing checkers, involute checkers, helix (lead) checkers, master gear rolling checkers, and surface finish checkers. For large gears up to 2 m, it is common practice to have one or more general-purpose checking machines that will check involute, helix, spacing, and finish all on the same machine.

It is desirable to use a checking machine of relatively complete capability or a large gear for these reasons:

- The large gear is too heavy for one person to lift. It takes a relatively long time to hoist the gear onto the checking machine and get it in position so that it is turning on an exactly true axis.
- Gears over 0.6 m are generally not mass produced. With lower quantities, the economics of buying separate machines for each kind of check are generally not attractive; it is usually more cost effective to buy one machine with complete capability.

Figure 10.53 shows a large gear being checked on a machine with complete capability. The equipment in the foreground does the integration to get cumulative tooth spacing from tooth-to-tooth spacing data. Tooth finish data are derived from local parts of involute and helix checks taken at high magnification.

When gears are over 2 m (80 in.) in diameter, putting the gear on a checking machine becomes rather impractical. The

TABLE 10.39
Examples of Tolerances for a Range of Gear Sizes and a Range of Quality Levels

		A. High Precision			B. Medium-High Precision			C. Medium Precision		
	Quality Item	**Small Gear**	**Medium Gear**	**Large Gear**	**Small Gear**	**Medium Gear**	**Large Gear**	**Small Gear**	**Medium Gear**	**Large Gear**
					Spacing					
1	Pitch variation (tooth-to-tooth)	5 µm (0.0002 in.)	8 µm (0.0003 in.)	10 µm (0.0004 in.)	10 µm (0.0004 in.)	12 µm (0.0005 in.)	20 µm (0.0008 in.)	20 µm (0.0008 in.)	25 µm (0.0010 in.)	35 µm (0.0014 in.)
2	Pitch cumulative	17 µm (0.0007 in.)	23 µm (0.0009 in.)	50 µm (0.0020 in.)	30 µm (0.0012 in.)	48 µm (0.0019 in.)	100 µm (0.0040 in.)	50 µm (0.0020 in.)	90 µm (0.0036 in.)	200 µm (0.0080 in.)
					Profile					
3	Slope (total)	7 µm (0.0003 in.)	9 µm (0.00035 in.)	16 µm (0.0006 in.)	13 µm (0.0005 in.)	20 µm (0.0008 in.)	25 µm (0.0010 in.)	25 µm (0.0010 in.)	40 µm (0.0016 in.)	60 µm (0.0024 in.)
4	Modi cation	10 µm (0.0004 in.)	13 µm (0.0005 in.)	20 µm (0.0008 in.)	20 µm (0.0008 in.)	25 µm (0.0010 in.)	36 µm (0.0014 in.)	36 µm (0.0014 in.)	50 µm (0.0020 in.)	75 µm (0.0030 in.)
5	Irregularities	4 µm (0.00016 in.)	5 µm (0.0002 in.)	7 µm (0.0003 in.)	6 µm (0.00024 in.)	8 µm (0.0003 in.)	10 µm (0.0004 in.)	13 µm (0.0005 in.)	20 µm (0.0008 in.)	30 µm (0.0012 in.)
					Helix					
6	Slope	8 µm (0.0003 in.)	12 µm (0.0005 in.)	20 µm (0.0005 in.)	13 µm (0.0005 in.)	20 µm (0.0008 in.)	25 µm (0.0010 in.)	25 µm (0.0010 in.)	40 µm (0.0016 in.)	50 µm (0.0020 in.)
7	Crown	10 µm (0.0004 in.)	– –	– –	18 µm (0.0007 in.)	– –	– –	33 µm (0.0013 in.)	– –	– –
8	End casement	– –	12 µm (0.0005 in.)	12 µm (0.0005 in.)	– –	25 µm (0.0010 in.)	35 µm (0.0014 in.)	– –	50 µm (0.0020 in.)	70 µm (0.0028 in.)
9	Irregularities	4 µm (0.00016 in.)	5 µm (0.0002 in.)	5 µm (0.0002 in.)	6 µm (0.00024 in.)	8 µm (0.0003 in.)	10 µm (0.0004 in.)	10 µm (0.0004 in.)	14 µm (0.0006 in.)	20 µm (0.0008 in.)
					Concentricity					
10	Composite (tooth-to-tooth)	7 µm (0.0003 in.)	9 µm (0.00035 in.)	15 µm (0.0006 in.)	15 µm (0.0006 in.)	20 µm (0.0008 in.)	28 µm (0.0011 in.)	30 µm (0.0012 in.)	45 µm (0.0018 in.)	60 µm (0.0024 in.)
11	Composite, total	15 µm (0.0006 in.)	20 µm (0.0008 in.)	40 µm (0.0016 in.)	30 µm (0.0012 in.)	50 µm (0.0020 in.)	80 µm (0.0032 in.)	60 µm (0.0024 in.)	90 µm (0.0036 in.)	130 µm (0.0050 in.)
					Finish					
12	Pro le, AA	0.5 µm (0.020 µin.)	0.6 µm (0.024 µin.)	0.8 µm (0.032 µin.)	0.8 µm (0.032 µin.)	0.9 µm (0.036 µin.)	1.0 µm (0.040 µin.)	1.6 µm (0.064 µin.)	2.0 µm (0.080 µin.)	2.5 µm (0.100 µin.)
13	Root llet, AA	1.0 µm (0.040 µin.)	1.2 µm (0.028 µin.)	1.6 µm (0.064 µin.)	1.6 µm (0.064 µin.)	1.8 µm (0.070 µin.)	2.0 µm (0.080 µin.)	3.2 µm (0.126 µin.)	4.0 µm (0.160 µin.)	5.0 µm (0.200 µin.)
14	Waviness	1.5 µm (0.060 µin.)	1.8 µm (0.070 µin.)	2.0 µm (0.080 µin.)	2.5 µm (0.100 µin.)	3.0 µm (0.120 µin.)	3.5 µm (0.140 µin.)	5.0 µm (0.200 µin.)	7.0 µm (0.280 µin.)	10 µm (0.400 µin.)

Note: Small gear is 2.5 module (10 pitch), 50 mm (2 in.) face width, 64 mm (2.5 in.) pitch diameter. Medium gear is 5 module (5 pitch), 125 mm (5 in.) face width, 250 mm (10 in.) pitch diameter. Large gear is 12 module (2 pitch), 500 mm (20 in.) face width, 1250 mm (50 in.) pitch diameter.

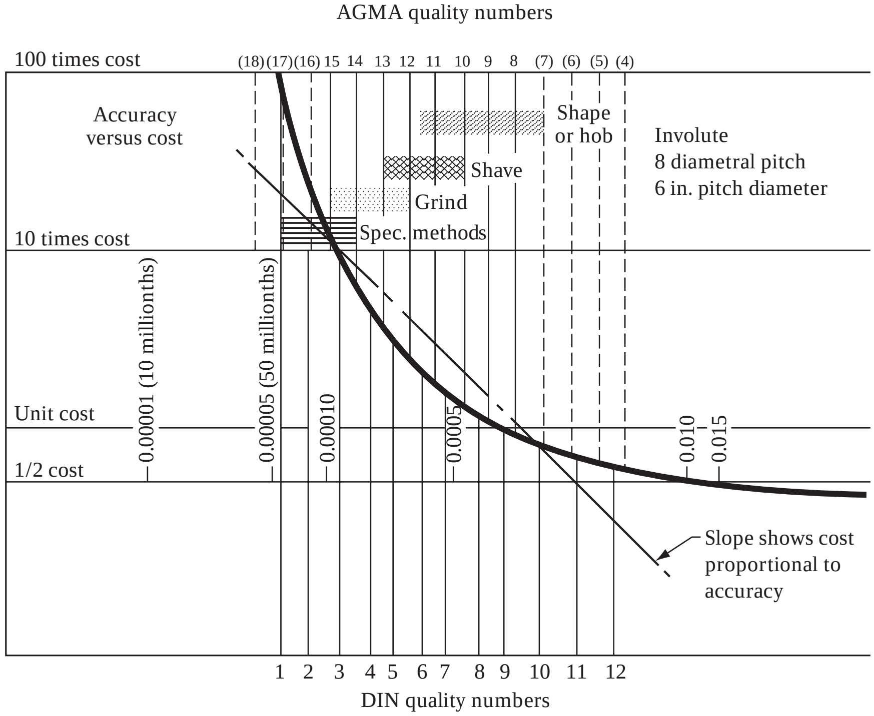


FIGURE 10.51 Approximate change in cost of making teeth on a 3 module (8 pitch), 150 mm (6 in.) in diameter gear for different degrees of involute accuracy. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

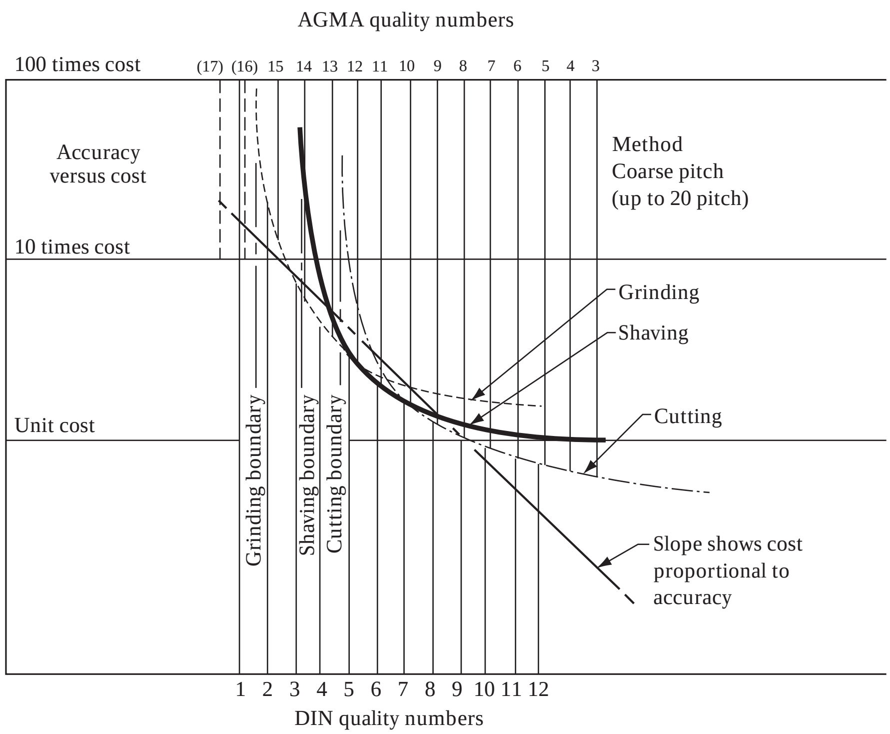


FIGURE 10.52 The general trend of the cost of making gear teeth for different manufacturing methods. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

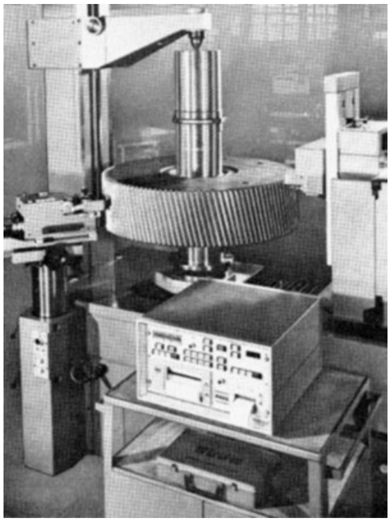


FIGURE 10.53 Gear-checking machine for large gears. Spacing, involute, helix, and surface nish are all measured on machines of this type. (Courtesy of Maag Gear Wheel Co., Zurich, Switzerland.)

solution to the problem is to take the checking machine to the gear. This may be done with portable checking machines that either mount on a machine tool or mount on the gear itself. Figure 10.54 shows an example of portable checking equipment temporarily mounted to the bed of a grinding machine. Figure 10.55 shows a portable involute checker mounted on a large marine gear.

The checking of spacing, involute, helix, and nish does not necessarily determine whether or not a gear is a good gear. Normally, four teeth, 90° apart, are checked. The involute and spacing checks are taken in the center of the face width. The helix is checked at midheight. If just a few teeth are bad, the trouble may be in between the places checked. This possibility is covered by rolling the production gear with a master gear.

If the master is as wide as the gear being checked, every bit of tooth surface will be checked. (If the master gear is narrower, the master gear check can be taken at more than one location across the face width.)

Master gear checks are composite checks. They reveal individual tooth action, and they show the total runout of the gear as a total composite reading (runout plus tooth action effects.)

The machines used for rolling checks are of two types, double anks and single ank.

The double- ank machines force the master gear into tight mesh with the gear being checked. The machine reads center distance variation as the gear revolves. Since both anks touch at all times, the readings show error effects of both sides of the teeth. This mixes up errors, and it becomes troublesome when gears are built to high accuracy on the drive side but are allowed to have considerably less accuracy on the essentially nonworking coast side.

The single- ank rolling machines maintain a constant center distance. A small torque keeps the gear and the master in contact on the side being measured. The machine measures the change in rotation of one part from a theoretical uniform rotation of both parts.

The schematic design of one model of single- ank tester is shown in Figures 10.56 and 10.57. A single- ank checking machine is shown in Figure 10.58.

The machines and methods just described apply to parallel-axis gears (involute-helical). Gears on nonparallel axes (bevel gears, worm gears, and Spiroid gears) do not have as extensive an array of checking machines, but the logic of using machines to check gears sold on the market is the same. It is not possible to cover checking machine types for these gears in this book. A general review of gear inspection machines

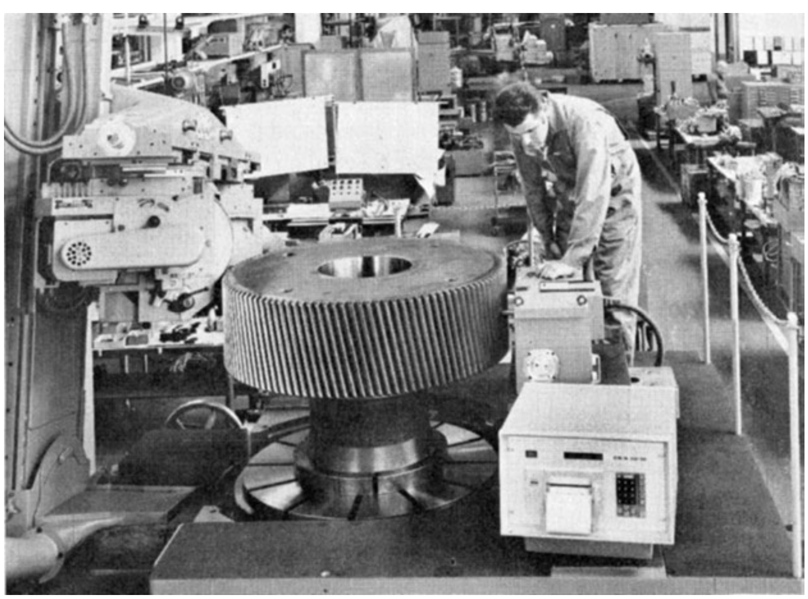


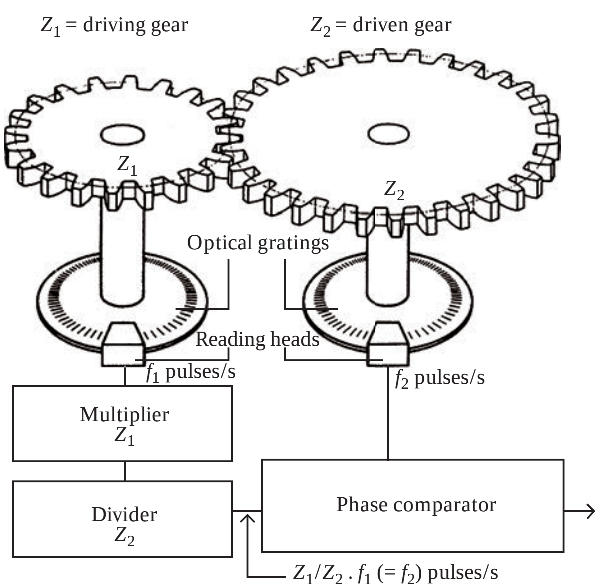
FIGURE 10.54 Portable pro le-checking head mounted on a grinding machine. (Courtesy of Maag Gear Wheel Co., Zurich, Switzerland.)

FIGURE 10.55 Portable pro le-checking head mounted on the rim of a large gear. (Courtesy of General Electric Co., Lynn, Massachusetts.)

and gear inspection practice for all types of gears (any gear on any axis) can be found out from other sources.

10.5 GEAR CASTING AND FORMING

In this part of the chapter, we shall consider some of the ways of making gear teeth other than by metal cutting or grinding.



e two motions which are to be compared, either on a single-flank tester or in a portable application, are monitored by circular optical gratings. e gratings each give a train of pulses whose frequency is a measure of the angular movements of the two shafts and hence of the gears.

FIGURE 10.56 Principle of the Gleason/Goulder single- ank testing machine.

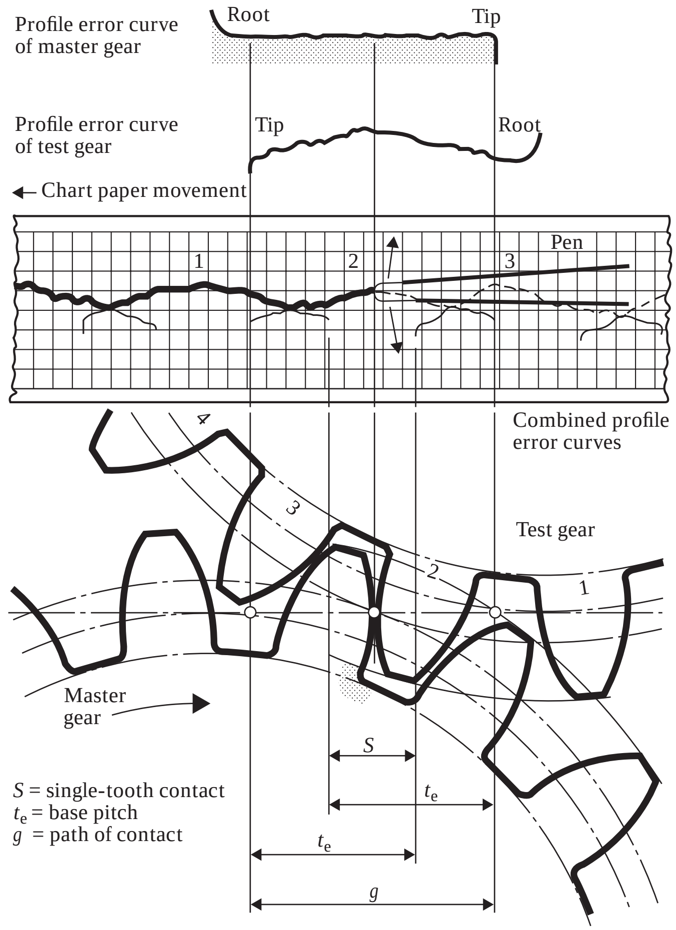


FIGURE 10.57 Single- ank error graph.

10.5.1 Cast and Molded Gears

Although the casting process is most often used to make blanks for gears which will have cut teeth, there are several variations of the casting process that are used to make toothed gears in one operation.

Many years ago, when gear-cutting machines were very limited, it was quite common practice to make a wooden pattern of the complete gear—teeth and all—and then cast the gear in a sand mold. A few old times in the gear trade can still recall the days when a "precision" gear was one with cut teeth and an ordinary gear was one with teeth "as-cast." In recent times, there have been only very limited use of gears with teeth made by sand casting. In some instances, gears for farm machinery, stokers, and some hand-operated devices have used cast teeth. The draft on the pattern and the distortion on cooling make it hard to obtain much accuracy in cast-iron or cast-steel gear teeth.

Large quantities of small gear parts are made by die casting. In the die-casting process, a tool-steel mold is used. The cavity is lled with some low-melting material, such as alloys of zinc, aluminum, or copper. With a precision-machined mold and a blank design that is not vulnerable to irregular shrinkage (such as spoked wheel), accuracy comparable with that of commercial cutting can be obtained. Complicated gear

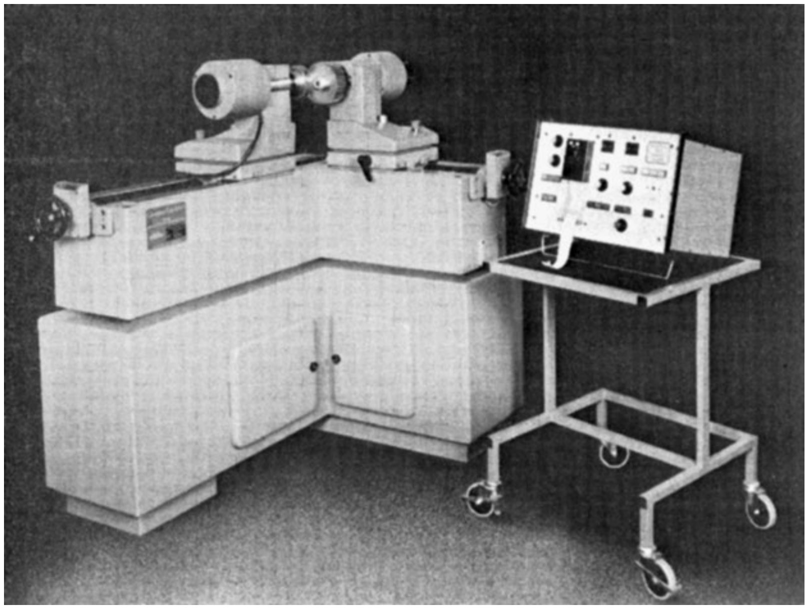


FIGURE 10.58 A single-flank testing machine that rolls a master with a gear. (Courtesy of Gleason/Goulder, Huddersfield, U.K., a subsidiary of Gleason Works.)

shapes which would be quite costly to machine can be made quickly and at low cost by the die-casting process. The main disadvantage of the process is that the low-melting metals do not have enough hardness for high load-carrying capacity. In many applications, though, die-cast gears have sufficient capacity to do the job.

Die-cast gears are usually less than 150 mm (6 in.) in diameter and from 2.5 module (10 pitch) to 0.5 module (48 pitch). There is no particular reason for why larger or smaller gears cannot be made, if dies and casting equipment of suitable size are provided.

A process somewhat similar to die casting used in the making of molded plastic gears. These gears are made in one operation. The raw plastic material is heated in a cylinder to a temperature in the range of 400°F to 600°F (depending on composition). It is forced into a steel mold under pressure as high as 140 N/mm^2 (20,000 psi).

Injection molding machines now on the market range in size from 30 g to 10 kg per shot capacity. The cycle time in injection molding is very fast. Machines of 225 g capacity may make around 100 cycles per hour. One cycle fills the mold. The mold may contain a cavity for one gear, or a half dozen or more parts may be made by filling the cavities in one mold.

The accuracy of injection-molded gears ranges from good to fair. Some plastics are less subject to shrinkage than others. Some—like nylon—absorb water or oil and are subject to distortion through expansion.

Another method of casting gears is the investment-casting process. This process has often been called the *lost-wax* or the *precision* casting process. This process uses a master pattern or a die. The master is filled with some low melting–point metal like a lead–tin–bismuth alloy or some wax or plastic. After this cools, a pattern is formed which is a replica—except for shrinkage allowances—of the pattern to be made. The pattern is used to form a mold called an *investment*. The investment is made by covering the pattern with a couple layers of refractory material. The kind of refractory used mainly depends upon the temperature at which the investment will be heated when it is filled with the molten metal. The investment has to be heated to remove the pattern and leave a cavity for pouring. This heating, however, is not nearly so much as the heating which the investment gets when it is filled with the casting material.

The investment process has had only limited use in gear making. Its most apparent value lies in the making of accurate gear teeth out of materials which are so hard that teeth cannot readily be produced by machining. The process can be used with a wide variety of steels, bronzes, and aluminum alloys. With machinable materials, the process would still be useful if the gear was integral with some complicated shape that was very difficult to produce by machining.

10.5.2 Sintered Gears

Small spur gears may be made by sintering. Small helical gears of simple design may also be made by sintering, provided that the helix angle is not over about 15°.

Machinery presently available will handle parts from about 5 mm ($\frac{3}{16}$ in.) up to about 100 mm (4 in.) in diameter. The sintering process consists of pouring a metal powder into a mold, compressing the powder into a gear-shaped briquette with a broach-like tool which fits the internal teeth of the mold, stripping the mold with another broach-like tool, and then baking the briquette in an oven. It takes about 415 N/mm^2 (60,000 psi) pressure to briquette the larger gears. Presses of 300,000 kg (300 tons) capacity are used for these larger gears.

Pitches in the range of 0.8 module (32 pitch) to 4 module (6 pitch) can be readily sintered. Face widths may range from

about 2.5 mm (3/32 in.) to 38 mm (1.5 in.). Smaller face widths are difficult to strip from the mold. Face widths of over 25 mm (1 in.) may give trouble because of loss briquetting pressure from excessive wall friction.

The sintering process can be used to make a complete-toothed gear in one operation. Splines, keyways, double-D bores, crank arms, and other projections may be made by the same sintering operation that forms the gear teeth. By using different powders in the mold, it is even possible to add a thin bronze clutch face integral with one side of the gear!

Sintered gears favorably compare in accuracy with commercial-cut gears. The surface finish of sintered gears is usually much better than that of cut gears. The tools that are used to make the briquettes must be lapped to a mirror finish to minimize wall friction.

In most cases, a sintered gear goes through the press only once. The pressing time per gear ranges from 2 to about 15 s (depending on the gear size and complexity). The sintering is done in conveyorized furnaces. Since the machinery is almost completely automatic, the operators have to do only such things as fill the powder hopper, transfer trays of green briquettes, gauge pieces for size, and remove finished gears. In spite of the fact that iron powder costs from about 30 cents to as much as $4.00 per pound, gears made by sintering are frequently cheaper than gears of comparable strength and quality made by other processes. The tools required for sinter gears are very costly but are capable of making a large number of parts. Favorable costs for sintered gears are obtained only when there is enough production to adequately liquidate tool costs. The breakeven quantity may vary from about 20,000 to 50,000 pieces (depending on the complexity off the tools).

10.5.3 Cold-Drawn Gears and Rolled Worm Threads

Spur pinion teeth may be formed by cold drawing, and worm threads may be rolled. Both these processes are limited in their application. When they can be used, it is possible to get very low costs and a high-strength part.

Cold-drawn pinion stock comes in long rods. These rods have teeth formed in them for their entire length. A rod may be as long as 2.5 m (8 ft) with a diameter as small as 6 mm (¾ in.). The pinion rod is chucked in automatic or semiautomatic lathes. Short pinions are rapidly cut off the rod. The pinion may be formed with shaft extensions by turning off the teeth for a distance on each end, or the pinion may be drilled and cut off as a disk with a hole in it to permit mounting on a shaft.

Cold-drawn pinions may be made of any material that has good cold-drawing properties. Carbon steel, stainless steel, phosphorous bronze, and many other metals can be used to cold-drawn pinions. The extruding operation of making pinion rod is done hot. Only soft, nonferrous metals like brass, aluminum, and bronze can be used for extruding.

Both the cold-drawing and the extruding processes use dies to form the pinion teeth.

Pinions from about 3 mm (⅛ in.) to about 25 mm (1 in.) pitch diameter can be made by cold drawing or extruding. The number of teeth should not be less than about 15 nor more than about 24. Tooth size may range from 0.25 module (100 pitch) to 1 module (24 pitch). The teeth should be designed with at least 20° pressure angle and enough addenda to avoid undercut. Sharp corners have to be avoided. The tooth contour should have a full-radius root fillet and a full radius at the outside diameter.

The time required for making pinions by the cold-drawing or the extrusion process is essentially the time required to cut off each pinion. This ranges from about 2 to 12 s, depending on the shaft extensions required and the diameter of the part. The time required to make the teeth is the time required to pull the rod stock through the dies.

Worm threads may be made by cold rolling. The process is very rapid, and it produces a very smooth, work-hardened surface which is quite similar to that produced in cold-drawn pinions. Raw stock of 180 HB, for instance, will have a surface hardness equivalent to about 260 HB after rolling. The surface finish may be in the range of 0.25 μm (0.010 μin.) to 0.75 μm (0.030 μin.).

The rolling process exerts great pressure on the worm blank. To get a straight worm, it is necessary to use a tooth depth that is not greater than about one-sixth of the outside diameter. To get a satisfactory profile, the worm should have a normal pressure angle of 20° or more. There should be a generous radius in the root fillet and on the thread crest. Lead angles should not exceed 25°.

There is some springback of metal after rolling, and the die used will have a slight generating action because of its finite diameter. To get a symmetrical worm thread profile of the desired shape, it is often necessary to make the shape of the die slightly different from the shape of the spaces between worm threads.

The time required to roll threads may be estimated from Table 10.40.

TABLE 10.40
Typical Production Time to Roll Worm Threads

Pitch Diameter		Linear Pitch		Threaded Length		Pieces per Minute	
mm	in.	mm	in.	mm	in.	Hand-Fed	Automatic
6.5	0.250	1.2	0.050	9.5	0.375	48	100
9.5	0.375	2.0	0.075	12.5	0.500	48	130
12.5	0.500	2.5	0.100	19.0	0.750	48	150

11 Design of Tools to Make Gear Teeth

The design of gear cutting tools is not necessarily a problem of the gear designer. Usually, gear-manufacturing organizations and gear tool companies will handle the design of tools to make gears.

It often happens, though, that the gear designer has to help out on the tool design. Perhaps a special gear is needed which cannot be readily obtained with conventional tools. Perhaps some work on tool design will show that changes should be made in the gear design to facilitate the tooling. In many cases there is the problem of choosing the proper size or type of tool to best fulfill the requirements of the gear design. Still another problem is the case where tools are on hand from previous job and someone has to determine whether or not a new design can use these tools.

In this chapter we shall take up some of the more common tool design problems. Data and calculation methods will be shown so that—if need be—the gear engineer can calculate the dimensions of the cutting tool.

The tool design data shown in this chapter are, of necessity, limited. Many special problems involved in designing and manufacturing tools are not discussed here. The gear designer should, wherever possible, secure the services of competent tool designers who specialize in gear-cutting tools. The material in this chapter is intended only as an aid to the gear designer, not as a substitute for the services of a tool designer.

11.1 SHAPER-CUTTERS

A variety of kinds and sizes of shaper-cutters are available on the market. The rack-type shaper machines use rack cutters on external work and pinion cutters on internal work. The pinion-type shaper machines use pinion-shaped cutters on both external and internal gears.

External spur gears usually use a disk type of pinion shaper-cutter. Figure 11.1 shows a typical disk-type cutter.

For internal gears, it is frequently necessary to use such small cutters that the disk construction cannot be used. The smallest cutters are usually made shank type. Figure 11.1 shows some typical shaper-cutters and their nomenclature. Table 11.1 shows the largest number of cutter teeth that can normally be used with different numbers of internal gear teeth.

There are no official trade standards on shaper-cutter dimensions. Unofficially, though, all the manufacturers are able and willing to work to essentially the same dimensions and tolerances. Table 11.2 shows the most commonly used dimensions for disk and shank cutters.

Shaper-cutters frequently have small enough numbers of teeth to cause the base circle to come high on the tooth flank. The region below the base circle may be left as a simple radial flank, or it may be filled in. The filled-in design can be used to break the top corner of the gear tooth being cut. Shaper-cutters can be made with a protuberance at the tip. The protuberance cuts an undercut at the root of the gear tooth. This provides a desirable relief for a shaving tool. The protuberance design is also used in some cases to permit the sides of the gear teeth to be ground without having to grind the root fillet.

There are a variety of special features that can be provided—and are frequently needed—in shaper-cutter teeth. Figure 11.2 illustrates six different special features.

Figure 11.3 shows some of the design details of a disk shaper-cutter. Note that the face of the cutter is shown with a rake angle of 5°. Roughing cutters sometimes have a top rake angle* as high as 10°. The sides of the cutter have a side clearance of about 2°. The top of the tooth also has a clearance angle. When the pressure angle is 20°, this angle is made about 1.5 times as much as the side clearance angle. The clearance angles and the top face angle are all necessary to make the shaper-cutter an efficient metal-cutting tool.

Shaper-cutters for finishing work are usually made to a very high degree of precision. Although there are no AGMA standard tolerances for shaper-cutters, the standard tolerances of individual tool companies are generally close to being the same. Table 11.3 shows the tolerances that have been published by Barber Colman for five levels of shaper-cutter quality (radial runout and profile tolerances, spur and helical). Table 11.4 shows the tolerances that have been published by Barber Colman for five levels of shaper-cutter quality (adjacent and nonadjacent indexing tolerances, spur and helical). All values shown are in ten-thousandths of an inch.

Figure 11.4 shows how a shaper-cutter cuts an internal gear. The cutter has external teeth and must be small enough to not destroy the corners of the internal teeth. See Table 11.1 for the maximum number of cutter teeth that can be used.

Shaper-cutters are normally made external. They can, however, be made internal. Figure 11.5 shows a comparison of an external and an internal shaper-cutter. (The internal cutter is often called an *enveloping* cutter.)

When the cutters are small in diameter for the tooth size, they are made integral with a shank. If the face width to be cut is wide, the shank has to be rather long and sturdy. Figure 11.6 shows some shank cutters of rugged design for wider face gears.

Helical gears may be cut with shaper-cutters provided that the cutter has an appropriate helix angle and the shaping machine has a helical guide of appropriate lead to twist the cutter as it strokes back and forth. Ordinarily, helical gears are cut with shaper-cutters, which are sharpened normal to the helix of the cutter tooth. (See Figure 11.7.) If the helix is low, the teeth may be sharpened in the transverse section.

Herringbone gears of the continuous tooth type must be cut with a pair of shaper-cutters working together. To make the cutting match for both sides, it is necessary to use a cutter with the

* Rake angle is sometimes loosely referred to as *face angle*.

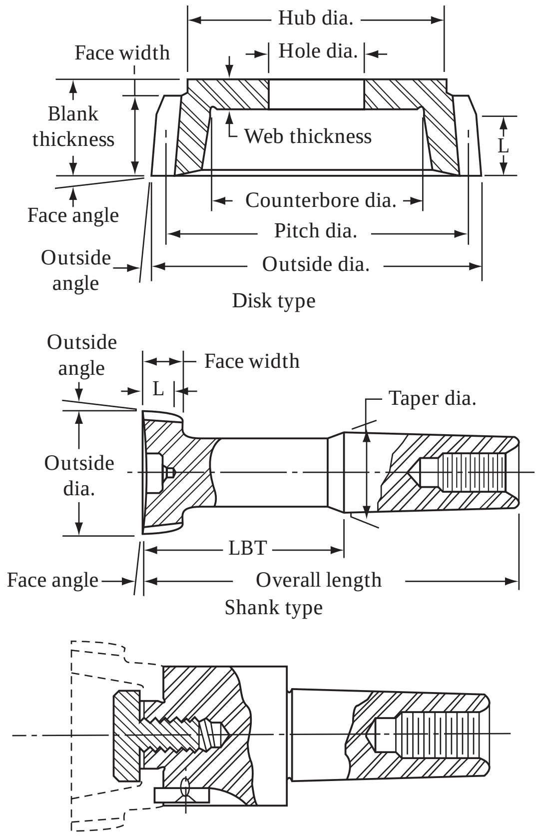


FIGURE 11.1 Typical types of shaper-cutters and their nomenclature. (Courtesy of Fellow Corp., Emhart Machinery Group, Springfield, Vermont.)

top face ground normal to the cutter axis. This makes the top face angle 0°, and it makes one side of the cutter tooth have an acute angle and the other side an obtuse angle. These features do not aid the cutting action of the tool, but they are necessary to produce the continuous tooth. Figure 11.8 shows an example of these cutters. Note that a special sharpening technique has produced a good cutting edge even on the obtuse angle side.

Helical shaper-cutters must have the same normal pitch and the same normal pressure angle as the gear they are cutting. Since the axis of the gear and that of the cutter are parallel, the cutter transverse pitch and transverse pressure angle are also equal to those of the gear. The relation of the hand of helices is as follows:

External gear	
RH cutter	For LH gear
LH cutter	For RH gear
Internal gear	
RH cutter	For RH gear
RH cutter	For LH gear

The helix angle that a shaper-cutter produces depends on both the lead of the guide and the number of cutter teeth. The helix of the cutter must also agree with the helix angle being cut or serious cutter rub will occur. The formula for the relation of cutter teeth number to lead of guide is as follows:

$$\frac{\text{no. of teeth in cutter}}{\text{lead of guide}} = \frac{\text{no. of teeth in gear}}{\text{lead of gear}}. \tag{11.1}$$

TABLE 11.1
Maximum Number of Shaper-Cutter Teeth for Different Internal Gears

	Maximum Number of Teeth in Cutter			
No. of Internal Teeth	14.5° PA, Full Depth	20° PA, Full Depth	20° PA, Stub, 25° PA, Full Depth	30° PA, Fillet Root Splines
16	–	–	–	9
20	–	–	–	13
24	–	–	10	17
28	–	–	11	21
32	–	10	12	25
36	–	13	14	29
40	14	17	18	33
44	16	21	23	37
48	18	25	27	41
52	21	29	32	45
56	24	34	36	49
60	27	38	40	53
64	30	42	45	57
68	33	46	49	61
72	36	50	53	65
80	44	58	62	73

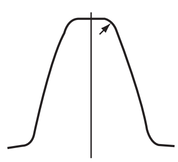
Note: PA: pressure angle.

Shaper-cutters do not usually cut the same whole depth throughout their life. A shaper-cutter can be designed with a front clearance angle that a certain number of teeth may be cut to an exact depth and thickness for the life of the cutter. However, if this cutter is used to cut gears of substantially larger or smaller numbers of teeth, the whole depth cut will slightly vary from the design value. If a shaper-cutter designed to cut an external gear is used to cut an internal gear of the same tooth thickness, the discrepancy in whole depth may be quite appreciable. In many cases, the designer of a shaper-cutter does not know all the numbers of teeth that the cutter may have to cut during its life. This leads the designer to make the front clearance angle large enough so that the cutter usually cuts a little extra on the whole depth. It is usually reasoned that the problem of a little extra depth is less than the problem of having the depth too shallow.

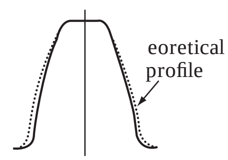
In many high-production jobs, it is desirable to design the shaper-cutter for a particular gear so that the cutter will do exactly what is wanted throughout its life. This is particularly true when cutters are used to preshave cut a gear and leave an undercut. The position of the undercut must remain constant within close limits to tie in with the shaving-cutter design.

TABLE 11.2
Typical Dimensions for Shaper-Cutters

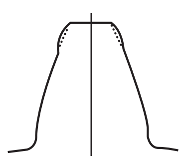
Tooth Size		Approximate Pitch Diameter		Width		Bore		Counterbore	
Module	**Diametral Pitch**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**	**mm**	**in.**
				Disk Type					
9–10	2.5–2.75	150–175	6–7	32	1.25	70	2.75	105	4.125
2.5–6.5	4–10	100	4	22	0.875	45	1.75	65	2.5625
2.5–6.5	4–10	100	4	22	0.875	32	1.25	65	2.5625
2.5–5.0	5–10	75	3	22	0.875	32	1.25	52	2.0625
1.7–5.0	5–14	75	3	17	0.6875	32	1.25	52	2.0625
1.0–1.6	16–24	75	3	22	0.875	32	1.25	52	2.0625
0.5–1.4	18–48	75	3	14	0.5625	32	1.25	52	2.0625
				Shank Type					
				Shank Diameter, Large End					
				mm				**in.**	
2.5–3.5	7–10	38	1.5	17				0.6875	
1.6–2.3	11–16	30	1.1875	14				0.5625	
0.5–1.4	18–48	25	1.0	11				0.4375	



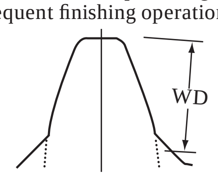
Radius corners
Corners of cutter teeth are radiused to produce a controlled fillet in the root corners of the gear being generated—adds strength to gear and improves tool life.



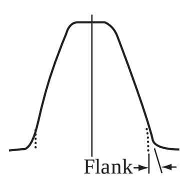
Pressure angle increment
Cutter tooth profile is ground to a slightly lower pressure angle to provide for a constantly increasing amount of stock from root to tip of gear generated—another method of providing relief for subsequent finishing operations.



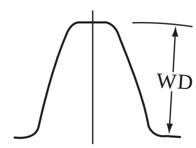
Protuberance tip
Cutter tooth profile is built up on the tip to provide an undercut near the root of the gear being generated—provides relief for subsequent finishing operations.



Chamfering or semitopping
Root of cutter is filled in to generate a sharp corner break or chamfer on the tips of the gear—minimizes tip buildup during heat treatment due to shaving burrs and nicks incurred in handling.



Modifying flank for tip relief
Root or cutter is filled in more gradually than chamfering cutter—removes a small amount of profile from tops of gear teeth—often desirable in high speed gears to minimize noise and heavy tip bearing resulting from tooth deflection under heavy loads.



Topping
Cutter tooth depth is ground equal to the whole depth (WD) of the gear tooth. e outside diameter of the gear is "topped" to size when the teeth are cut; More frequently used for fine pitch gearing.

FIGURE 11.2 Special features that can be provided in shaper-cutter teeth. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

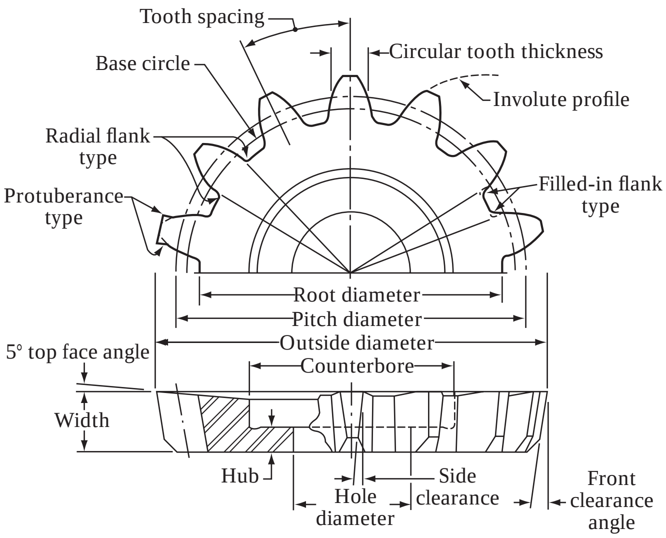


FIGURE 11.3 Design details of disk shaper-cutter. (Courtesy of Illinois Tool Works, Chicago, Illinois.)

11.2 GEAR HOBS

The gear hob is really a cylindrical worm converted into a cutting tool. Cutting edges are formed by gashing the worm with a number of slots. These slots are usually either parallel with the hob axis or perpendicular to the worm thread. The teeth of the hob are relieved back of the cutting edge to make an ef cient cutting tool. The cutting face may be radial, or it may be given a slight hook (rake angle) to improve the cutting action. Usually, nishing hobs are radial.

Figure 11.9 shows the design details of a typical spur gear hob. The involute generating portion of the hob tooth pro le is usually made straight in the normal section. Since the thread angle is usually low, this makes the pro le come out essentially straight even if the hob is gashed axially instead of normally. Theoretically, the hob should have a slight curvature in its pro le to cut a true involute gear. The curvature should correspond to that of an *involute helicoid*. Practically, though, the curvature required is so slight that it is disregarded. Only on multiple-thread hobs of coarse pitch does it become necessary to grind an involute curve. An involute helicoid may be calculated in the normal section by the method shown in Dudley and Poritsky's AGMA paper P241 (1943).

Hobs may be made with straight bores, tapered bores, or integral with hob arbors. The shell type of hob with a straight bore is the most commonly used type. Taper-bore hobs require more wall thickness than shell hobs. Some companies prefer the taper-bore hob because of the more rigid mounting which the taper provides. Integral shank hobs are expensive. They are used when the hob diameter has to be made so small that a big enough hole cannot be put through the hob. Very small hob diameters may be required when the hob runout space or the gap between helices is limited. Worm gear hobs sometimes have to be very small to match the diameter of small worms. See Figure 11.10.

The nominal sizes of shell-type hobs are shown in Tables 11.5 and 11.6. Taper hobs generally require a smaller bore or a larger diameter than the values given in Tables 11.5 and 11.6. Where extra rigidity is required, it is often desirable to put in a larger-diameter hob and support the hob on a larger arbor. For instance, a 10-pitch hob ordinarily has a 1¼ in. bore and a 3 in. outside diameter. In cutting high-speed gears to extreme precision, it is desirable to use a 4 in. hob with 1¾ in bore for 10 pitch.

Tables 11.5 and 11.6 show that multiple-thread hobs require larger bores and diameters than single-thread hobs.

Table 11.7 shows some of the commonly accepted tolerances for different classes of hobs. Although about 20 items may be speci ed in a hob tolerance sheet, the three shown in Table 11.7 are the most important. Lead variation in one turn and hob pro le error both directly affect the pro le accuracy of the gear being cut. Other commonly used hob tolerances only indirectly affect the accuracy of the gear.

Helical gear hobs require a taper when the gear exceeds about 30° helix angle. Even below this angle, a taper is helpful if the gear has over 150 teeth. Figure 11.11 shows a typical hob for a helical gear of 250 teeth and 35° helix angle.

The design of the gap between helices on a double-helical gear and the hob design are tied to each other. The gap must be wide enough to accommodate for both the tapered and full parts of the hob. An exact calculation of gap width is quite difcult, but fortunately, an approximate solution is usually close enough. The following formula is usually accurate within plus and minus 5%:

$$\text{Min. gap} = \sqrt{h_o(d_o - h_o)}\cos\beta_o + \frac{z_o p_n \sin\beta_o}{\cos\gamma} + \frac{x'\sin\beta_o}{\tan\alpha_n} \quad \text{(metric)}, \qquad (11.2)$$

$$\text{Min. gap} = \sqrt{h(D_H - h)}\cos\psi_1 + \frac{n_1 p_n \sin\psi_1}{\cos\lambda} + \frac{x'\sin\psi_1}{\tan\phi_n} \quad \text{(English)}, \qquad (11.3)$$

where

h_o, h—depth of cut (nishing cut may be less depth than whole depth)

h_t—total depth of cut

d_o, D_H—hob outside diameter (at point doing the cutting)

β—helix angle of gear (°) ($\beta_o = \beta - \gamma$)

ψ—helix angle of gear (°) ($\psi_1 = \psi - \lambda$)

γ, λ—lead angle of hob (°)

$$\sin\gamma,\ \sin\lambda = \frac{\text{no. hob threads} \times \text{normal circular pitch}}{\pi \times \text{hob pitch diameter}}$$

p_n—normal circular pitch

z_o, n_1—number of pitches from hobbing center

x'—whichever is larger: gear addendum, or gear dedendum minus clearance

α_n, ϕ_n—normal pro le angle (°)

TABLE 11.3
Shaper-Cutter Table for Sizes and Tolerances (Radial Runout and Profile Tolerances, Spur and Helical)

Quality Number	Normal Diametral Pitch	Pitch Diameter (in.) 0.5–1.999		2–2.999		3–3.999		4–4.999		5–5.999		6–7.999		8–9.999		10–11.9		Sharpening Tolerance Side Clearance		
		R	P	R	P	R	P	R	P	R	P	R	P	R	P	R	P	1°	2°	3°
1	1–1.999											38	31	40	31	41	32	280	140	91
	2–2.999									27	21	29	22	30	23	32	23	170	87	58
	3–4.999					18	15	19	15	20	15	22	15	24	15	25	16	130	65	43
	5–7.999	13	11	15	11	16	12	18	12	19	12	20	13	22	13			120	60	40
	8–13.999	12	9	14	9	15	10	16	10	17	10	18	10					116	58	38
	14–19.999	10	7	11	7.5	13	7.5	14	7.5	14	8							65	33	22
2	1–1.999											27	22	29	23	30	23	150	75	50
	2–2.999									19	15	19	15	20	16	22	16	87	43	29
	3–4.999					12	10	13	10	13	10	14	11	15	11	16	11	70	35	23
	5–7.999	10	8	11	8.5	11	8.5	11	8.5	12	8.5	13	9	13	9			54	27	18
	8–13.999	8	6.5	8.5	7	9	7	9	7	10	7	11	7.5					35	17	12
	14–19.999	6.5	5	7	5	7.5	5.5	8	5.5	8.5	5.5							23	12	8
3	1–1.999											19	16	20	16	21	16	87	44	29
	2–2.999									13	10	13	10	14	11	15	11	64	32	21
	3–4.999					8.5	7	9	7	9.5	7.5	10	7.5	11	7.5	11	8	44	22	15
	5–7.999	7	6	7	6	7.5	6	8	6	8.5	6.5	8.5	6.5	9	6.5			32	16	11
	8–13.999	6	4.5	6	5	6.5	5	7	5	7	5	7.5	5.5					26	13	9
	14–19.999	5	3.5	5	4	5.5	4	5.5	4	6	4							22	11	7
4	1–1.999											14	11	15	12	16	12	52	26	17
	2–2.999									9.5	7.5	10	8	10	8	11	8	35	17	12
	3–4.999					6	5	6	5	6.5	5.5	7	5.5	7	5.5	7.5	5.5	26	13	9
	5–7.999	5	4	5	4	5.5	4	5.5	4.5	6	4.5	6.5	4.5	6.5	5			23	11	8
	8–13.999	4	3.5	4.5	3.5	4.5	3.5	5	4	5	4	5.5	4					17	8	6
	14–19.999	3.5	2.5	4	2.5	4	3	4	3	4.5	3							13	6	4
5	1–1.999											11	8	12	8	13	8.5	46	23	16
	2–2.999									7	5.5	7	5.5	7	5.5	7.5	6	29	15	10
	3–4.999					4.5	3.5	4.5	4	4.5	4	5	4	5	4	5.5	4	15	7	5
	5–7.999	4	3	4	3	4	3	4	3	4	3	4.5	3	4.5	3			11	6	4
	8–13.999	3	2.5	3	2.5	3.5	2.5	3.5	2.5	3.5	2.5	3.5	2.5					10	5	3
	14–19.999	2.5	2	2.5	2	3	2	3	2	3	2							8	4	3

Source: Barber Colman Co., Rockford, Illinois.

Note: This table was furnished by Barber Colman Co. as their recommendation for standard shaper-cutter tolerances. P is pro le tolerance (denominator pitch); R is radial run-out. (All readings are in ten-thousandths of an inch.)

TABLE 11.4
Shaper-Cutter Table for Sizes and Tolerances (Adjacent and Nonadjacent Indexing Tolerances, Spur and Helical)

Quality Number	Normal Diametral Pitch	Pitch Diameter (in.) 0.5–1.999 A	N	2–2.999 A	N	3–3.999 A	N	4–4.999 A	N	5–5.999 A	N	6–7.999 A	N	8–9.999 A	N	10–11.9 A	N
1	1–1.999											7	21	7.5	22	7.5	22
	2–2.999									6.5	16	6.5	17	6.5	17	7	19
	3–4.999					5	12	5	12	5.5	13	5.5	14	5.5	14	5.5	15
	5–7.999	5	9.5	5	11	5	11	5	12	5.5	12	5.5	13	5.5	13		
	8–13.999	4.5	9	4.5	9.5	4.5	10	4.5	10	5	11	5	12				
	14–19.999	4	7.5	4	8	4	9	4	9	4	9	4	9				
2	1–1.999											5	15	5	16	5	16
	2–2.999									4.5	11	4.5	11	4.5	12	4.5	13
	3–4.999					3.5	8	3.5	8	4	8.5	4	9	4	9.5	4	9.5
	5–7.999	3.5	7	3.5	7.5	3.5	7.5	3.5	7.5	4	8	4	8.5	4	9		
	8–13.999	3	6	3	6	3	6.5	3	6.5	3.5	7	3.5	7.5				
	14–19.999	2.5	5	2.5	5	3	5	3	6	3	6						
3	1–1.999											3.5	11	3.5	11	4	11
	2–2.999									3	7.5	3	7.5	3.5	8.5	3.5	9
	3–4.999					3	6	3	6.5	3	6.5	3	6.5	3	7	3	7
	5–7.999	3	5.5	3	5.5	3	5.5	3	6	3	6	3	6	3	6.5		
	8–13.999	2.5	5	2.5	5	2.5	5	2.5	5	2.5	5	2.5	5				
	14–19.999	2	4	2	4	2	4	2	4	2	4						
4	1–1.999											2.5	7.5	3	8.5	3	9
	2–2.999									2.5	6	2.5	6	2.5	6	2.5	6.5
	3–4.999					2	4	2	4	2	4.5	2	4.5	2	4.5	2	4.5
	5–7.999	2	4	2	4	2	4	2	4	2	4	2	4.5	2	4.5		
	8–13.999	1.5	3	1.5	3	1.5	3	1.5	3.5	1.5	3.5	2	4				
	14–19.999	1.5	3	1.5	3	1.5	3	1.5	3	1.5	3						
5	1–1.999											2	6	2	6.5	2	6.5
	2–2.999									2	4.5	2	4.5	2	4.5	2	4.5
	3–4.999					1.5	3	1.5	3	1.5	3	1.5	3.5	1.5	3.5	1.5	3.5
	5–7.999	1.5	3	1.5	3	1.5	3	1.5	3	1.5	3	1.5	3	1.5	3	1.5	3
	8–13.999	1	2	1	2	1	2.5	1	2.5	1	2.5	1	2.5				
	14–19.999	1	2	1	2	1	2	1	2	1	2						

Source: Barber Colman Co., Rockford, Illinois.

Note: This table was furnished by Barber Colman Co. as their recommendation for standard shaper-cutter tolerances. A is adjacent indexing tolerance; N is nonadjacent indexing tolerance (exclusive of runout). (All readings are in ten-thousandths of an inch.)

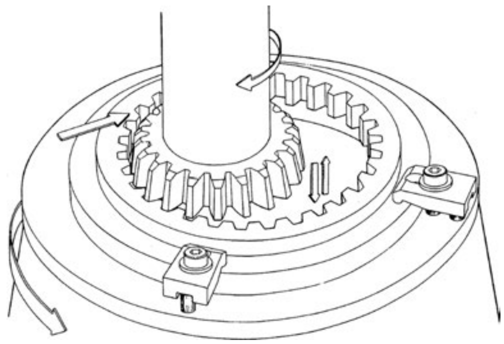


FIGURE 11.4 How a shaper-cutter cuts an internal gear. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

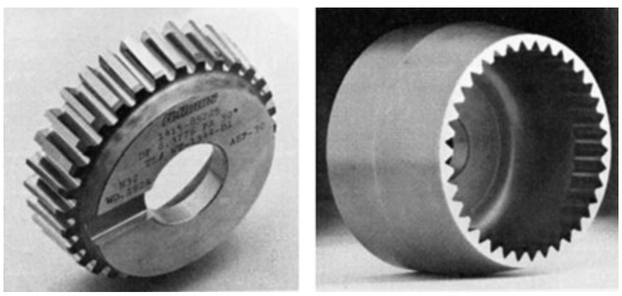


FIGURE 11.5 A comparison of a shaper-cutter with external teeth and one with internal teeth. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

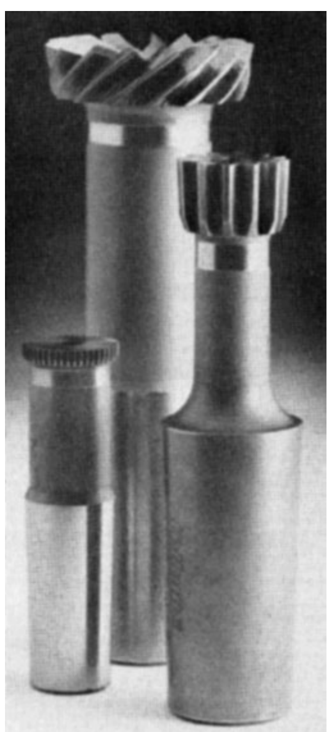


FIGURE 11.6 Shank-type cutters for relatively wide face gears. (Courtesy of Fellows Corp., Emhart Machinery Group, Spring eld, Vermont.)

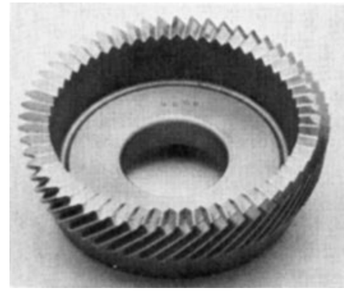


FIGURE 11.7 Helical gear cutters for all but low helix angles are sharpened in the normal section. (Courtesy of Fellows Corp., Emhart machinery Group, Spring eld, Vermont.)

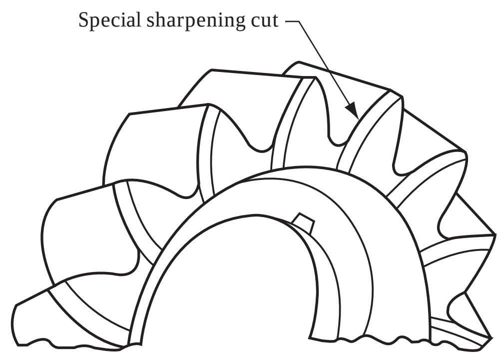


FIGURE 11.8 Diagram of top half of a shaper-cutter for continuous herringbone teeth. Note special sharpening method which changes original obtuse angle at the corner of the cutter to an angle that will cut.

FIGURE 11.9 Typical shell-type hob. (Courtesy of Barber Colman Co., Rockford, Illinois.)

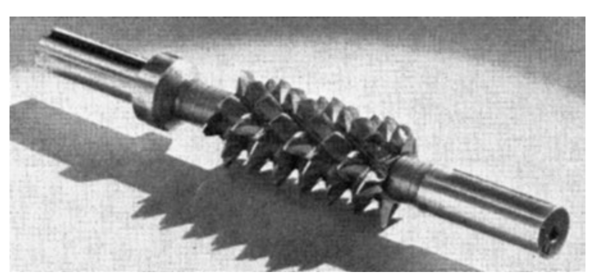


FIGURE 11.10 Worm gear hob with multiple threads and integral with shank. (Courtesy of Barber Colman Co., Rockford, Illinois.)

TABLE 11.5
Typical Dimensions of Shell Hobs, English Dimensions

Diametral Pitch	No. of Threads	Hole (in.)	Outside Diameter (in.)	Length (in.)	Keyway (in.)
1	1 or 2	2.50	10.75	15.00	$\frac{5}{8}\times\frac{5}{16}$
2	1	1.50	5.75	8.00	$\frac{3}{8}\times\frac{3}{16}$
	2	1.50	6.50	8.00	$\frac{3}{8}\times\frac{3}{16}$
4	1	1.25	4.00	4.00	$\frac{1}{4}\times\frac{1}{8}$
	2	1.50	5.00	4.00	$\frac{3}{8}\times\frac{3}{16}$
	3	1.50	5.50	4.00	$\frac{3}{8}\times\frac{3}{16}$
5	1	1.25	3.50	3.50	$\frac{1}{4}\times\frac{1}{8}$
	2	1.50	4.50	3.50	$\frac{3}{8}\times\frac{3}{16}$
	3	1.50	5.00	3.50	$\frac{3}{8}\times\frac{3}{16}$
6	1	1.25	3.50	3.50	$\frac{1}{4}\times\frac{1}{8}$
	2	1.50	4.50	3.50	$\frac{3}{8}\times\frac{3}{16}$
	3	1.50	5.00	3.50	$\frac{3}{8}\times\frac{3}{16}$
8	1	1.25	3.00	3.00	$\frac{1}{4}\times\frac{1}{8}$
	2	1.25	3.75	3.00	$\frac{1}{4}\times\frac{1}{8}$
	3	1.25	4.00	3.00	$\frac{1}{4}\times\frac{1}{8}$
10	1	1.25	3.00	3.00	$\frac{1}{4}\times\frac{1}{8}$
	2	1.25	3.50	3.00	$\frac{1}{4}\times\frac{1}{8}$
	3	1.25	3.75	3.00	$\frac{1}{4}\times\frac{1}{8}$
12	1	1.25	2.75	2.75	$\frac{1}{4}\times\frac{1}{8}$
16	1	1.25	2.50	2.50	$\frac{1}{4}\times\frac{1}{8}$
20	1	0.75	1.875	1.875	$\frac{1}{8}\times\frac{1}{16}$
32	1	0.75	1.50	1.125	$\frac{1}{8}\times\frac{1}{16}$
100	1	0.75	1.375	0.625	–

TABLE 11.6
Typical Dimensions of Shell Hobs, German Dimensions

Module (mm)	Hole (mm)	Outside Diameter (mm)	Length (mm)
1.00	22	50	44
1.25	22	50	44
1.50	22	56	51
1.75	22	56	51
2.00	27	63	60
2.25	27	70	70
2.50	27	70	70
2.75	27	70	70
3.00	32	80	85
3.25	32	80	85
3.50	32	80	85
3.75	32	90	94
4.00	32	90	94
4.50	32	90	94
5.00	32	100	104
5.50	32	100	104
6.00	40	110	126
6.50	40	110	126
7.00	40	110	126
8.00	40	125	156
9.00	40	125	156
10.00	40	140	188
11.00	50	160	200
12.00	50	170	215
13.00	50	180	230
14	50	190	245
15	60	200	258
16	60	210	271
18	60	230	293
20	60	250	319

All linear dimensions above are in millimeters for Equation 11.2 and in inches for Equation 11.3.

Figure 11.12 schematically shows how this formula works. The hob must be fed out into the gap between helices until it stops cutting on the helix it was cutting. If it is a roughing cut, it is not necessary to completely stop cutting—the finish cut can take off a little extra at the tooth ends. Frequently, large gears are rough cut to full depth and then finish cut with a smaller hob and a slightly shallower depth.

As shown in the sketch, the generating zone of the hob must just clear the end of the helix. The minimum gap width is determined by the distance needed to keep either the first full tooth or any of the tapered teeth from hitting the opposite helix. Usually, the hob is centered 1.5 pitches back from the first full tooth. This makes n_1 equal to 1.5 for the first solution. Since the tooth is a full tooth, h equals h_t, and D_H is the full outside diameter. (This paragraph and Figure 11.12 are given in English symbols only. The metric equivalents are easily evident in Equations 11.2 and 11.3.)

After the gap for the first full tooth has been determined, a check should be made to see if any of the tapered teeth require additional gap. As shown in Figure 11.12, the first tapered tooth has a value of $n_1 = 2.5$, and $h = h_t - a_1$. A series of calculations can be made for different tapered teeth to explore the cutting action of all the tapered teeth.

Sometimes it is desirable to lay out the values calculated, as shown in Figure 11.12. In Equations 11.2 and 11.3, the curvature of the gear is not taken into account. An end view will show whether the point at which the hob tends to hit the opposite helix is off-center enough to permit the value of h to be reduced to an appreciable amount to compensate for the curvature of the gear.

In designing a tapered hob, it is usually desirable to use such an amount of taper that the tapered teeth cut a gap width that is either just equal to that of the first full tooth or slightly greater than that of the first full tooth. This ensures that the taper is really doing some work. If the tapered teeth are so short that they do not require as much gap as the first full tooth, they will still do some cutting on the sides of the gear teeth, but they will not be removing the amount of stock that they should.

The end of the hob to be tapered for helical gear cutting is opposite to that which is sometimes used on large spur gears. For helical gears, the rules are the following:

Conventional hobbing

RH hob tapered on LH end, top coming

LH hob tapered on RH end, top coming

Climb hobbing

RH hob tapered on RH end, top coming

LH hob tapered on LH end, top coming

Worm gears can be hobbed on the same machines used to hob spur and helical gears. Even the same kind of hobs can be used, provided that the work is of the same diameter and tooth profile as the hob. Usually, though, special hobs are required for worm gears. Worm gears which mesh with multiple-threaded worms need tangential feed hobbers to eliminate generating flats on the tooth profiles. The tangential hobber has a slow feed in a direction tangent to the gear being cut. The effect of this feed is to continuously shift the center position of the hob during the cut. This shifting makes the hob teeth move into different positions with respect to the gear. The hob cuts as if it had an almost infinite number of cutting edges.

Figure 11.13 shows a worm gear hob which has just finished cutting a single-enveloping worm gear. Note the long taper on the hob. At the start of the cut, only the taper teeth engage the work. At the end of the cut, the gear is engaging only the full-depth hob teeth. This kind of hob is often called (for obvious reason) a *pineapple* hob.

The end of the hob to be tapered depends on the direction of feed used on the hobber. For most hobbers, the direction is such as to require the following:

RH hob to be tapered on RH end
LH hob to be tapered on LH end

TABLE 11.7
Summary of Hob Lead, Profile, and Tooth Thickness Tolerances

			Tooth Size											
			1 Module (25 Pitch)		1.5 Module (16 Pitch)		2.5 Module (10 Pitch)		5 Module (5 Pitch)		8 Module (3 Pitch)		25 Module (1 Pitch)	
Item	**No. of Threads**	**Class**	**μm**	**10^{-4} in.**	**μm**	**10^{-4} in.**	**μm**	**10^{-4} in.**	**μm**	**10^{-4} in.**	**μm**	**10^{-4} in.**	**μm**	**10^{-4} in.**
Lead variation in one axial pitch of the hob	1	AA	5	2	5	2	8	3	10	4	20	8	–	–
		A	10	4	10	4	13	5	15	6	25	0	64	25
		B	15	6	15	6	18	7	23	9	43	17	90	35
		C	20	8	20	8	23	9	28	11	56	22	115	45
		D	40	10	46	18	50	20	64	25	100	40	150	60
	Multiple	A	10	4	10	4	13	5	15	6	25	10	64	25
		B	18	7	18	7	20	8	25	10	43	17	90	35
		C	25	10	25	10	30	12	38	15	56	22	115	45
		D	–	–	46	18	50	20	64	25	100	40	150	60
Profile error in involute-generating portion of hob tooth (tip relief modification excluded)	1	AA	4	1.7	4	1.7	4	1.7	4	1.7	5	2	–	–
		A	5	2	5	2	5	2	5	2	8	3	25	10
		B	8	3	8	3	8	3	10	4	13	5	40	16
		C	8	3	8	3	8	3	10	4	25	10	64	25
		D	13	5	15	6	20	8	30	12	75	30	200	80
	2	A	5	2	5	2	5	2	8	3	13	5	30	12
		B	8	3	8	3	8	3	13	5	18	7	46	18
		C	8	3	8	3	8	3	13	5	28	11	70	27
		D	–	–	18	7	20	8	30	12	75	30	200	80
	3 or 4	A	5	2	5	2	8	3	8	3	13	5	38	15
		B	8	3	8	3	10	4	13	5	18	7	50	20
		C	8	3	8	3	10	4	13	5	28	11	70	27
		D	–	–	18	7	20	8	30	12	75	30	200	80
Tooth thickness error (minus only)	1 or multiple	AA	25	10	25	10	25	10	25	10	38	15	–	–
		A	25	10	25	10	25	10	25	10	38	15	75	30
		B	25	10	25	10	25	10	25	10	38	15	75	30
		C	38	15	38	15	38	15	38	18	50	20	90	35
		D	50	20	50	20	50	20	50	20	75	30	100	40

Note: 10^{-4} in. = ten-thousandths of an inch. Hobs are classified by dimensional tolerances as follows: Classes AA and A: precision ground, Class B: commercial ground, Class C: accurate ground, and Class D: commercial unground.

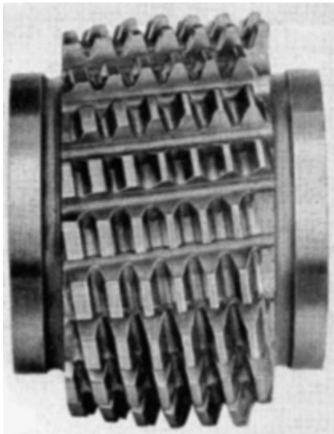


FIGURE 11.11 Multiple-thread hob. (Courtesy of General Electric Co., Lynn, Massachusetts.)

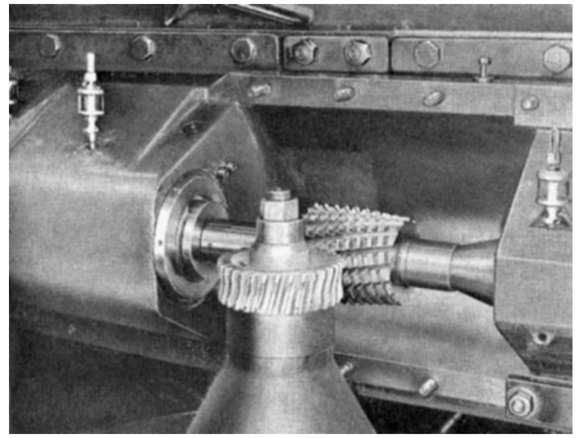


FIGURE 11.13 Worm gear hob and worm gear. (Courtesy of General Electric Co., Lynn, Massachusetts.)

There is no handy way of guring what amount of taper will give the best results on worm gear hobs. A common practice is to make the length of taper about three times the whole depth and the depth of taper about three-fourths of the whole depth.

Pineapple hobs are ef cient cutting tools but rather expensive. Where production requirements are not great, a much simpler hob can be used if the tangential feed is slowed down. In fact, the tangential type of hobber can generate a complete worm gear with only a single cutting tooth. This scheme has been used quite successfully with cemented-carbide y cutters. With high-speed tool-steel y cutters, the single cutting edge often gets dull before it nishes a gear. Figure 11.14 shows such a hob. It has ve teeth, corresponding to ve worm

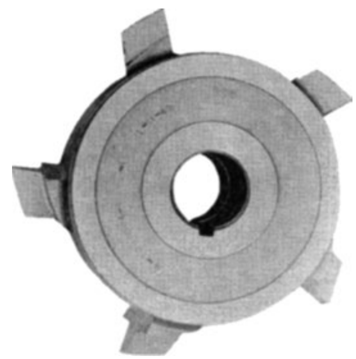


FIGURE 11.14 Pancake worm gear hob.

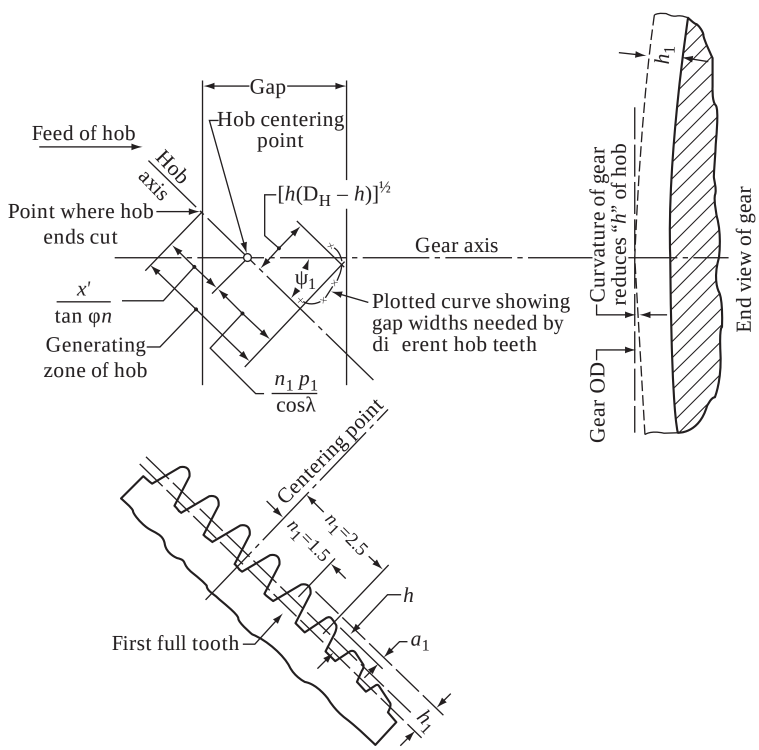


FIGURE 11.12 Diagram of hob in gap between helices.

threads (worm starts). This kind of hob has been called a *pancake* hob.

Worm gear hobs—no matter what kind—must have about the same diameter as the worm they imitate. Since the hob gets smaller in diameter as it is sharpened, it is necessary to make the new worm gear hob slightly larger than the worm. The amount of oversize to use has not yet been standardized. Since any oversize produces error in the gear, the amount of oversize is a function of the amount of error that may be tolerated. If no oversize is used, the hob might cut perfectly when new, but it would have to be scrapped after one sharpening unless the hobbing center distance was held constant and the hob was allowed to cut shallower and thicker teeth after each sharpening. The latter expedient is helpful in some jobs where worms and gears may be sized to t each other, but it is an awkward way of making gears when the job requires all parts to be essentially the same size and quality.

The amount of hob oversize boils down to a compromise between accuracy and hob life. The more oversize, the more hob sharpenings possible before the hob reaches its spent diameter. Usually, a hob is considered spent when its diameter is less than that of the worm. Undersize damages the accuracy of a hob much more than oversize.

The exact calculation of the effects of hob oversize is too complicated a problem to be treated in this book. For general applications where a quick yardstick is needed, the following formula represents a good limit for hob oversize. High precision can usually be obtained when the hob oversize does not exceed the value given by the formula. If the formula is exceeded, the accuracy will probably be in the commercial class (good enough for many jobs, but not good enough for critical jobs). The formula is as follows:

$$\text{Hob oversize} = d_0 - d_{p1}$$
$$= d_{p1}(0.030 - 0.028\tan\gamma)\frac{15.24}{p_x + 7.62} \quad \text{(metric)}, \tag{11.4}$$

$$\text{Hob oversize} = d_H - d$$
$$= d(0.030 - 0.028\tan\lambda)\frac{0.600}{p_x + 0.300} \quad \text{(English)}, \tag{11.5}$$

where

d_0 (d_H)—hob pitch diameter in millimeters (in.)
d_{p1} (d)—worm pitch diameter in millimeters (in.)
γ, λ—worm lead angle in degrees
p_x—axial pitches (of worm and hob are equal), mm (in.)

The cutting edges of the worm gear hob must have a curvature that will permit them to lie on an imaginary worm surface which has the same pro le curvature as the worm. A curvature which corresponds to that which is cut or ground into the worm must be ground into the hob. Since worms used in the United States usually do not have an involute helicoidal shape, calculations of hob pro le must be based on the method used to make the worm (see Section 11.4).

The details of designing a hob can best be understood by studying some design problems. Figure 11.15 shows the English design data for a preshave spur gear hob designed for cutting 10-diametral pitch gears and pinions which will operate at pitch-line speeds up to 10,000 fpm and loads over 1000 lb per in. of face width. The hob has a full-radius tip. It cuts an extra deep tooth of 0.240 in. When cutting standard addendum gears, it produces a tear-tooth thickness of 0.156 in. This thickness and the depth of 0.240 in. remain constant throughout the life of the hob. It will be recalled that shaper-cutters do not produce constant thickness and depth when they are used over a range of tooth numbers. The generating action of the hob is such that tooth numbers do not affect the thickness of the tooth it cuts.

The hob has a protuberance of 0.0014 in. After the gear is shaved down to a design thickness, which is of 0.154 in. to 0.153 in., the undercut caused by the protuberance smoothly blends into the contour of the gear tooth. The location of the protuberance is made high enough on the hob tooth so that only a small amount of shaving will be required to clean up the gear tooth down to the end of the active part of the involute. Note that the hob is straight for a distance of 0.095 in. above pitch line. Only the rst 0.005 in. of the protuberance generates a possible part of the working involute on the gear. The rest of the protuberance cuts in the root llet of the gear.

The amount of protuberance is mainly controlled by changing the angle on the protuberance. Where 16° is used on this hob, about 18° would be used on a 4-pitch hob with a Hi-point* of 0.002 in.

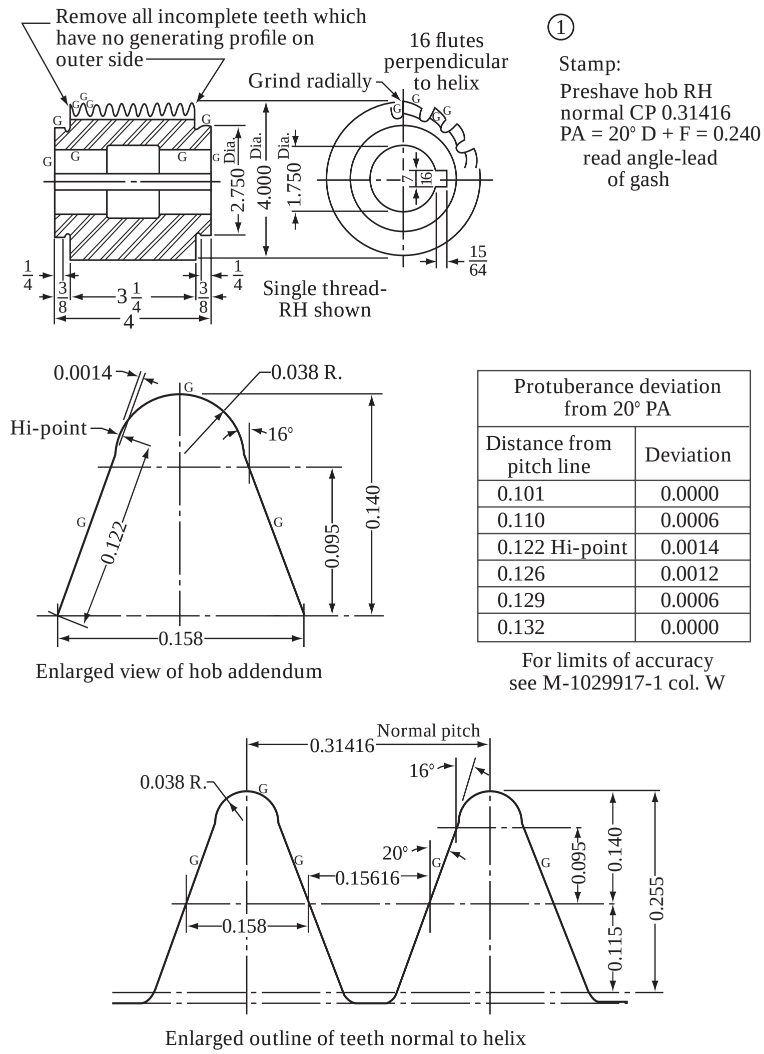
Several important developments in hobs have been made during recent decades. Three developments that are quite important to the gear industry are these:

- *Built-up hobs*—Special hob blades are attached to a body made of less costly material than the blade material.
- *Special roughing hobs*—The special roughing hob is a very ef cient tool to remove stock, but it does not produce gear teeth intended to run together.
- *Skiving hobs*—This hob cans nish cut fully hardened gear teeth.

The built-up hobs may have blades brazed to an alloy steel body, or the blades may be mechanically attached. The blade material may be expensive high-speed steel, or it may even be a carbide material. Figure 11.16 shows an example of a built-up hob.

The larger sizes of built-up hobs may be rebladed. The built-up hob tends to be less expensive (in large sizes) because the body material is not nearly as costly as the blade material.

* Hi-point is the point on the protuberance or a hob that cuts the deepest undercut on the gear (to be shaved or ground).



Protuberance deviation from 20° PA	
Distance from pitch line	Deviation
0.101	0.0000
0.110	0.0006
0.122 Hi-point	0.0014
0.126	0.0012
0.129	0.0006
0.132	0.0000

FIGURE 11.15 Gear hob, preshave.

FIGURE 11.16 Built-up hob with carbide blades. (Courtesy of Barber Colman Co., Rockford, Illinois.)

FIGURE 11.17 K-Kut roughing hob, U.S. Patent No. 3,892,022. (Courtesy of Barber Colman Co., Rockford, Illinois.)

Delivery time may be reduced because a large tool-steel forging is not needed.

The special roughing hobs cut very ef ciently. They remove larger chips, and the ships tend to cut and break away in an ef cient manner. Figure 11.17 shows a patented K-Kut roughing hob developed by Barber Colman Co. Figure 11.18 shows the patented roughing hob developed by Azumi, called the *Dragon hob*. Figure 11.19 shows Dragon hob cutting a large gear.

The K-Kut hob and the Dragon hob were developed and patented at about the same time. One has a U.S. patent and the other a Japanese patent. It is known that Barber Colman Co. and Azumi Mfg. Co. were working independently and did not know details of each other's design until their special roughing hobs were on the market.

Another special patented hob that is very ef cient at metal removal is the E-Z Cut hob. This hob will do both roughing

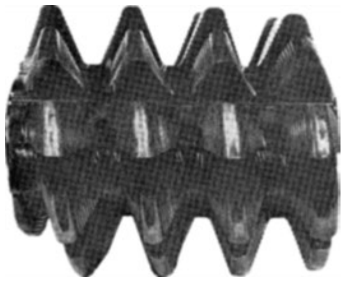


FIGURE 11.18 Dragon roughing hob. (Courtesy of Azumi Manufacturing Co., Osaka, Japan.)

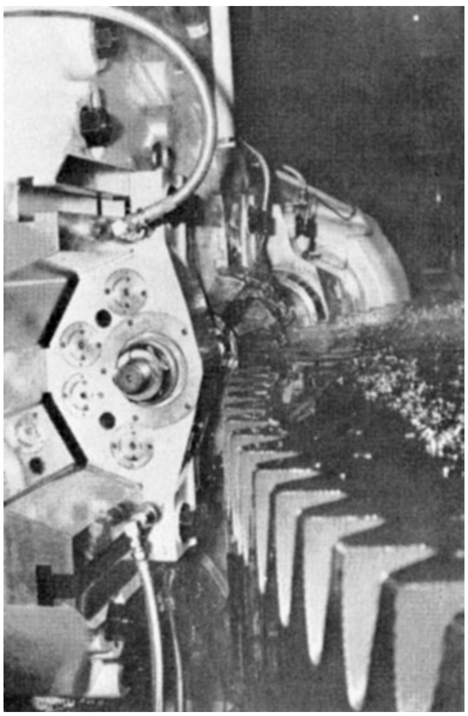


FIGURE 11.19 Dragon hob roughing a large gear. (Courtesy of Azumi Manufacturing Co., Osaka, Japan.)

and finishing. Figure 11.20 shows this hob and a detail of the design. This hob is intended to be used on very large gears where the cutting time is normally of several days' duration.

The skiving hob uses a special carbide blade and a negative rake angle of 30°. Figure 11.21 shows a skiving hob and details of the special rake angle. Figure 11.22 shows a skiving hob being set in position to finish cut a fully hardened gear that was finish hobbed before hardening with a protuberance type of hob. (The pregrinding type of hob can be used as a preskive hob.)

It is a remarkable achievement, of course, to be able to finish a case-hardened gear by hobbing instead of grinding. In the case where very high accuracy and smooth finish are needed, the hard gear—finished by skiving—may be given a further honing operation or a very light final grinding. When a final grind is used, the skiving serves to remove the bulk of the heat-treat distortion, and to remove it in a quicker and more efficient manner than grinding.

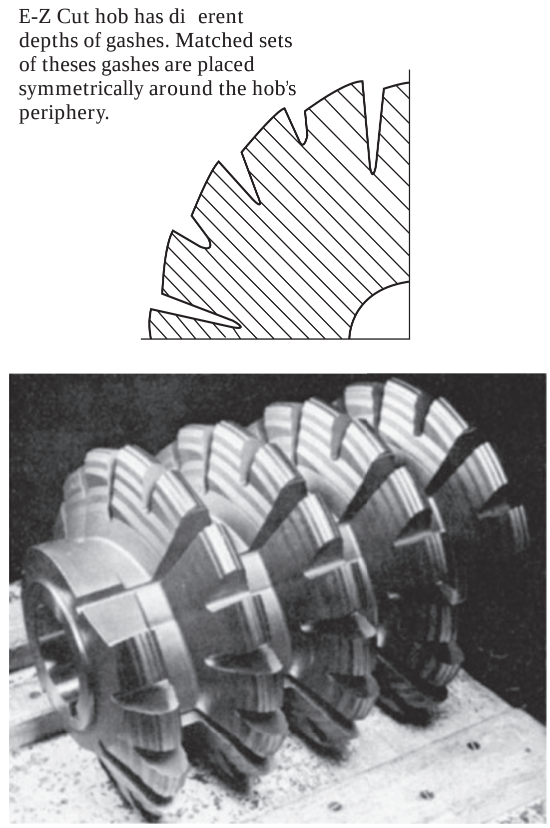


FIGURE 11.20 Detail of E-Z Cut hob design, and the E-Z Cut hob. U.S. Patent No. 3.715,789. (Courtesy of Barber Colman Co., Rockford, Illinois.)

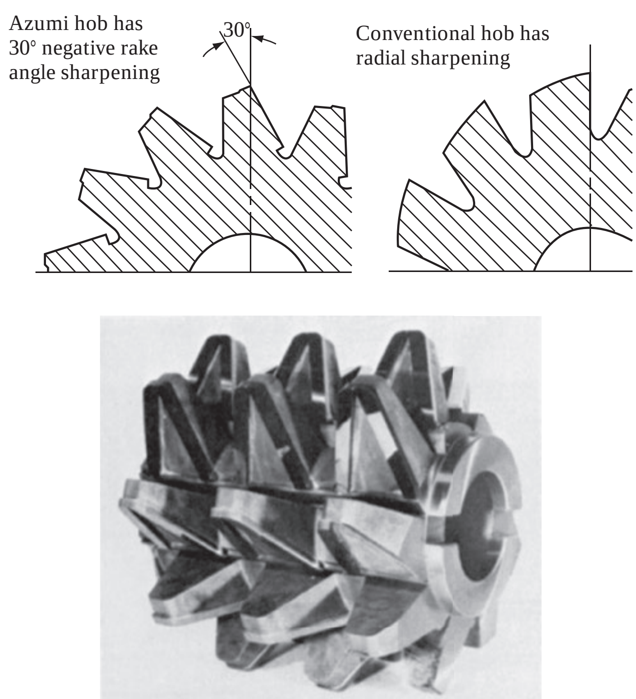


FIGURE 11.21 Skiving hob and detail of negative rake angle. U.S. Patent No. 3,786,719. (Courtesy of Azumi Manufacturing Co., Osaka, Japan.)

FIGURE 11.22 Skiving hob being set to finish a large case-hardened gear. Note undercut in gear teeth. Only the sides of gear teeth are skived, not the root fillet. (Courtesy of Azumi Manufacturing Co., Osaka, Japan.)

11.3 SPUR GEAR MILLING CUTTERS

Spur gears can be formed by milling a slot at a time and indexing to the next slot. The milling cutter is made so that its contour has an involute curve matching that of the gear tooth. A crossing section through the cutter tooth is the same as that through the space between two gear teeth. (See Figure 11.23.)

Standard involute gear cutters are designed to cut a range of gear tooth numbers. Table 11.8 shows the tooth numbers that standard cutters will cut.

Spur gears cut by involute cutters of the same pitch are interchangeable. The form of the cutter is designed to be correct for the lowest number of teeth in the range of the cutter. For instance, a No. 5 cutter has the involute curvature of 21 teeth. If it is used to cut 25 teeth instead of 21 teeth, the curvature will be somewhat too great. However, this is not too serious. Teeth with too much curvature will run together better than teeth with too little curvature.

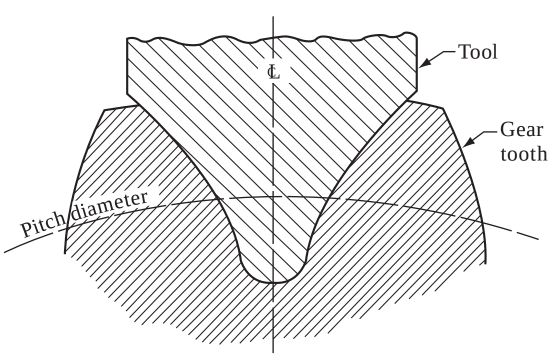


FIGURE 11.23 Milling tool fits space between gear teeth exactly after gear is finished.

TABLE 11.8
Standard Involute Gear Cutters

No. 1 is used to cut	from 135 teeth to a rack
No. 2 is used to cut	from 55 teeth to 134 teeth
No. 3 is used to cut	from 35 teeth to 54 teeth
No. 4 is used to cut	from 26 teeth to 34 teeth
No. 5 is used to cut	from 21 teeth to 25 teeth
No. 6 is used to cut	from 17 teeth to 20 teeth
No. 7 is used to cut	from 14 teeth to 16 teeth
No. 8 is used to cut	from 12 teeth to 13 teeth

If close accuracy is desired in form-milled teeth, it is necessary to get a cutter with a curvature which is right for the exact number of teeth. This can be done by purchasing a special single-purpose cutter.

Standard gear cutters are usually not ground. The highest accuracy can be obtained by using both a single-purpose cutter and a ground cutter.

Some companies build standard gear cutters in half numbers. For example, a No. 3½ cutter has a range of 30 to 34 teeth, whereas a No. 4 cutter has a range of 26 to 34.

Gear designers can help themselves by choosing numbers of teeth that tie in with the design values for standard cutters. If a 4-to-1 ratio is desired, tooth numbers of 14 and 56 would be a good choice. A No. 7 cutter would be just right for the pinion, and a No. 2 cutter would be within one tooth of being just right for the gear. This would give close accuracy without paying extra for single-purpose cutters.

Where close accuracy is desired in form-milled gear teeth, it is often necessary to make precise layouts of gear and cutter teeth so that both the tool and the work may be checked. Points on the involute profile may be laid out by rectangular coordinates.

The calculation of the cutter profile involves two steps:

- The cutter is assumed to be like an internal gear tooth that completely fills the space between two external teeth. A series of arc tooth thicknesses are calculated for a series of assumed diameters.
- The arc tooth thicknesses are converted to rectangular coordinates based on the point at which the gear pitch diameter intersects the center line of the cutter. These coordinates are plotted to give the cutter profile.

Section B.6 of Appendix B shows how to calculate the profile of an internal gear tooth, and Figure B.16 is an example of a calculated internal tooth profile. This method can be adapted for the design of milling cutter profiles.

Gear milling cutters range in size from 8.5 in. in diameter and 2 in. bore for 1 diametral pitch to 1.75 in. in diameter and 0.875 in. bore for 32 pitch. Several kinds of stocking cutters are commonly used to rough cut teeth before finish cutting. Those planning to use gear milling cutters should consult tool vendor's catalogs for further details on the kinds and types of cutters available.

11.4 WORM MILLING CUTTERS AND GRINDING WHEELS

On casual observation, one might assume that a milling cutter would produce on a worm a normal section profile that is of the exact same curvature as the normal section of the cutter. However, there is a slight generating action between the cutter and the work. This is caused by the fact that the thread angle of the worm varies from the top to the bottom of the thread. The inclination of the cutter is set to correspond to the thread angle at the pitch line of the worm. This means that at the top or the bottom of the thread, the cutter will not be tangent to the worm surface in the plane containing the cutter axis (normal section). Since the tangency points do not come in the plane of the normal section, more metal will be removed from the top and the bottom of the worm thread than that corresponding to the cutter normal section profile. A straight-sided conical cutter (see Figure 11.24) will produce a worm thread with a convex curvature in either the normal or the axial sections of the worm. If a straight-sided worm profile is required, the cutter must be formed to a convex curvature which is conjugate with the straight worm.

The amount of curvature produced by the generating action of a milling cutter is rather slightly compared with that produced by a hob or a shaper-cutter. With low thread angles and fine pitches, the curvature is often almost negligible. With coarse pitches and high thread angles, the amount becomes quite significant. For example, a five-thread worm of 1.250 in. axial pitch and 30° thread angle has about 0.006 in. curvature when cut with a straight-sided cutter. A five-thread worm of 0.625 in. axial pitch has only about 0.001 in. curvature when the thread angle is 15°.

Several kinds of worms have been in common use. When worm threads were made on a lathe, it was quite handy to make the worm profile either straight in the axial section or straight in the normal section. This practice carried over into milled worms, with the result that many designs in current production call for a straight-sided worm. This practice puts a burden on the makers of a milling cutter (or the one who dresses a thread-grinding wheel). They must develop the required curvature in the cutting tool.

Most gear engineers in the United States prefer to use a straight-sided milling cutter or grinding wheel. This puts all

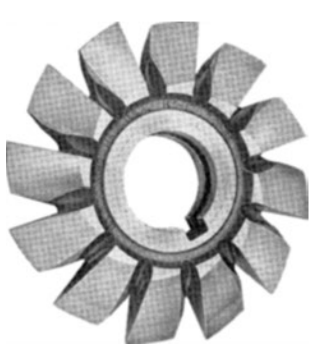


FIGURE 11.24 Worm milling cutter. (Courtesy of General Electric Co., Lynn, Massachusetts.)

the curvature into the worm. This practice makes the job of producing a worm gear hob somewhat more difficult, since it must have a curvature on the cutting edge corresponding to that of the worm thread. It is reasoned, though, that hob makers are better able to take care of this problem than are makers of milling cutters. Worm gear hobs have helical gashes on most cases. The combination of gash angle and relief on the sides of the hob tooth would make it necessary to curve the cutting edge of the hob tooth even if the hob had to match a straight-sided worm. Since hob makers cannot escape curvature problems, it is perhaps reasonable to give them the whole problem.

In England and some other countries, worms of involute helicoidal shape are popular. These worms have an involute curve in a transverse section. This design is not handy to work with unless involute-generating equipment is available to make worm threads. Such equipment is not generally available in the United States.

Worm gear designs frequently need to calculate the curvature produced by milling a worm. If a straight-sided cutter is used, the worm thread profile must be calculated before precise data can be put on the worm drawing to check the thread profile. Also, these data are needed to check the worm gear hob.

Several people have worked on this problem and written technical papers. The references at the end of the book include some of the best work on the worm cutter problem. The problem is so complicated and takes so many equations to get the precise answer that it is not possible to present it in full in this book. As a substitute, a calculation sheet and a sample problem are given. This will give a designer the means of solving for the axial and normal section curvatures of any worm cut by a straight-sided cutter or grinding wheel. The method used is based on Dudley and Poritsky's AGMA paper (1943). More detailed information is available in this reference.

For a sample problem, we shall take a five-threaded worm with 25.00 mm axial pitch. We shall adjust the pitch diameter to give a 25° lead angle. This makes 5° per thread and is about as high a lead angle as should be used with five threads. Using Equations 5.69 through 5.73, the pitch diameter works out to 85.327 mm. Using the proportions of Table 5.29, we get an addendum of 7.21 mm. Cutter pressure angle is 25°. Normal circular pitch is 22.6577 mm. The worm thread thickness will be 10.88 mm.

The worm thread cutter will have an outside diameter of 150 mm. By subtracting twice the worm dedendum (11.65 mm), we get a cutter pitch diameter of 132.70 mm. The cutter thickness at the pitch line will be equal to the normal circular pitch minus the worm thread thickness. This comes out to 11.7777 mm.

The first step in the calculation will be to determine the angle of rotation of the cutter (θ_2) at which various points on the cutter cut the deepest into the worm profile (these are points of tangency with the worm profile). Table 11.9 shows this calculation. Values of Δh may be assumed to give as many points as desired. The five points shown are picked to be one addendum above and below pitch line, at pitch line, and two intermediate points.

TABLE 11.9
Calculation of Angle of Rotation for a Straight-Sided Cutter Cutting a Worm

Given Data

1	Number of worm threads	5
2	Pitch diameter of worm	85.327 mm
3	Pitch diameter of cutter	132.7 mm
4	Axial pitch	25.00 mm
5	Shaft angle[a]	25°
6	Cutter pressure angle	25°
7	Cutter tooth thickness	11.78 mm

Calculation Steps

8	Distance *a* (assume)	7	3.5	0	−3.5	−7
9	[(2) + (3)] × 0.50	109.0135				
10	Lead = (1) × (4)	125.00				
11	(10) ÷ 2π	19.89437				
12	sin(5)	0.422618				
13	cos(5)	0.906308				
14	tan(6)	0.466308				
15	−1 ÷ (14)	−2.14451				
16	(15) × (12)	−0.906308				
17	(15) × (13)	−1.94358				
18	(9) × (13)	98.79978				
19	(9) × (16)	−98.7998				
20	(11) × (12)	8.407723				
21	(11) × (17)	−38.6664				
22	(18) + (20)	107.2075				
23	(22) × (22)	11493.45				
24	(19) − (21)	−60.1334				
25	(24) × (24)	3616.029				
26	(22) × (24)	−6446.75				
27	(7) ÷ 2	5.89				
28	(3) ÷ 2	66.35				
29	(8) × (14)	3.264154	1.632077	0	−1.63208	−3.26415
30	(27) − (29)	2.625846	4.257923	5.89	7.522077	9.154154
31	r_2 = (28) + (8)	73.35	69.85	66.35	62.85	59.35
32	(30) × (12)	1.109731	1.799476	2.489222	3.178967	3.868712
33	(31) × (16)	−66.4777	−63.3056	−60.1335	−56.9614	−53.7894
34	(32) + (33)	−65.3679	−61.5061	−57.6443	−53.7825	−49.9207
35	(34) × (34)	4272.968	3783.003	3322.865	2892.555	2492.072
36	(35) − (25)	656.9398	166.9747	−293.163	−723.474	−1123.96
37	(23) + (35)	15766.42	15276.45	14816.31	14386.00	13985.52
38	(36) ÷ (37)	0.041667	0.010930	−0.019787	−0.050290	−0.080366
39	(26) ÷ (37)	−0.408892	−0.422006	−0.453112	−0.448127	−0.460959
40	(39) × (39)	0.167192	0.178089	0.189322	0.200818	0.212483
41	(38) + (40)	0.208859	0.189019	0.169536	0.150528	0.132118
42	$\sqrt{(41)}$	0.457011	0.434763	0.411747	0.387979	0.363480
43	(39) + (42)	0.048120	0.012758	−0.023365	−0.060148	−0.097479
44	Rotation angle = $\sin^{-1}$(43)	2.758124	0.730968	−1.33881	−3.44831	−5.59404

[a] The shaft angle is usually set to be the same as the worm lead angle.

After θ_2 has been determined, a second calculation sheet is worked through to get the axial section of the worm. This calculation literally determines the position of a point on the cutter with respect to the axial section of the worm. If angles of θ_2 other than the critical value were used, it would be possible to plot the entire cutting curve of a point on the cutter. In fact, a solution can be obtained by assuming a series of θ_2 values for each line (30) value from Table 11.9 and then plotting individual curves to see which value of θ_2 does the deepest cutting on the worm. In some cases, it may not be possible to get a

solution for θ_2 directly from the first calculation sheet. If this happens, a solution can still be obtained by plotting curves for each line (30) value from Table 11.9 and reading values at the point of deepest cutting. The Dudley and Poritsky (1943) paper gives further information on this alternative method.

Table 11.10 shows the calculation of the worm axial section. Items 6, 7, and 8 for this calculation are taken from Table 11.9. Items 26 and 32 are the answers. They are rectangular coordinates of points on the axial section profile. Item 26 is a radial dimension, while 32 is an axial dimension—with 0 at the approximate pitch diameter of the worm. The approximate pitch diameter is used because the point used in the beginning for the pitch point of the cutter does not reach in quite deep enough to give a point on the pitch line of the worm in this calculation.

In general, it is best to check hobs and worms in the normal section. This permits the indicator to read normal to the surface being checked instead of at an angle. Table 11.11 shows the calculation of the normal section from the coordinates just obtained for the worm axial section. Items 19 and 23 are rectangular coordinates of the worm normal section.

Machines customarily used to check the profiles of worms are set at an inclination corresponding to the normal pressure angle. Since the curvature of the profile is not great, it is handy to measure the profile deviations from a straight line. This makes it desirable to continue the calculation from a straight

TABLE 11.10
Calculation of Axial Section of Worm Cut with a Straight-Sided Cutter

			Given Data			
1	Number of worm threads				5	
2	Pitch diameter of worm				85.327 mm	
3	Pitch diameter of cutter				132.7 mm	
4	Axial pitch				25.00 mm	
5	Shaft angle				25°	
			Calculation Steps			
6	r_2 (line 31, Table 11.9)	73.35	69.85	66.35	62.85	59.35
7	(line 30, Table 11.9)	2.625846	4.257923	5.89	7.522077	9.154154
8	Rotation (line 44, Table 11.9)	2.758124	0.730968	−1.33881	−3.44831	−5.59404
9	[(2) + (3)] × 0.50	109.0135				
10	Lead = (1) × (4)	125.00				
11	(10) ÷ 2π	19.89437				
12	sin(5)	0.422618				
13	cos(5)	0.906308				
14	sin(8)	0.048120	0.012758	−0.023365	−0.060148	−0.097479
15	cos(8)	0.998842	0.999919	0.999727	0.998190	0.995238
16	(6) × (14)	3.529584	0.891108	−1.55024	−3.78030	−5.78540
17	(6) × (15)	73.26503	69.84432	66.33189	62.73621	59.06735
18	(12) × (16)	1.491667	0.376599	−0.655160	−1.59763	−2.44501
19	(12) × (7)	1.109731	1.799476	2.489222	3.178967	3.868712
20	(13) × (16)	3.198890	0.807618	−1.40499	−3.42612	−5.24335
21	(13) × (7)	2.379825	3.858989	5.338153	6.817317	8.296481
22	(9) − (17)	35.74847	39.16918	42.68161	46.27729	49.94615
23	(20) + (19)	4.308620	2.607095	1.084227	−0.247151	−1.37464
24	(21) − (18)	0.888158	3.482390	5.993313	8.414942	10.74149
25	[(22) × (22)] + [(23) × (23)]	1296.517	1541.022	1822.896	2141.649	2496.508
26	$\sqrt{(25)}$	36.00719	39.25585	42.69538	46.27795	49.96506
27	(23) ÷ (26)	0.119660	0.066413	0.025395	−0.005341	−0.027512
28	$\sin^{-1}(27)$	6.872482	3.807981	1.455153	−0.305994	−1.57652
29	(28) × π ÷ 180	0.119947	0.066462	0.025397	−0.005341	−0.027515
30	(29) × (11)	2.386278	1.322216	0.505261	−0.106248	−0.547402
31	(30) + (24)	3.274437	4.804606	6.498574	8.308694	10.19409
32	(31) − Z_p [a]	−3.22414	−1.69397	0	1.810120	3.695519

[a] Z_p is the value from line 31 in the column in which r_2 is equal to half of the cutter pitch diameter. In this case, it is the idle column, and Z_p is 6.498574.

TABLE 11.11
Calculation of Normal Section Deviation from Known Axial Section

	Calculation Steps					
1	(line 32, Table 11.10)	−3.22414	−1.69397	0	1.810120	3.695519
2	Worm pitch diameter × π	268.0627				
3	Lead (line 10, Table 11.10)	125.00				
4	(3) ÷ (2)	0.466309				
5	Lead angle = $\tan^{-1}$(4)	25.00006				
6	(line 26, Table 11.10)	36.00719	39.25585	42.69538	46.27795	49.96506
7	(6) ÷ (4)	77.21746	84.18423	91.56031	99.24313	107.1502
8	(3) ÷ 2π	19.89437				
9	(7) + (8)	97.11183	104.0.786	111.4547	119.1375	127.0445
10	(1) ÷ (9)	−0.033200	−0.016276	0	0.015194	0.029088
11	$\sin^{-1}$(10) (°)	−1.90258	−0.932579	0	0.870559	1.666876
12	(11) × π ÷ 180	−0.033206	−0.016277	0	0.015194	0.029092
13	(8) × (12)	−0.660620	−0.323812	0	0.302277	0.578776
14	(7) × (10)	−2.56364	−1.37017	0	1.507854	3.116824
15	(13) + (14)	−3.22426	−1.69398	0	1.810132	3.695600
16	cos(11)	0.999449	0.999868	1.00	0.999885	0.999577
17	(6) × (16)	35.98734	39.25065	42.69538	46.27261	49.94392
18	(6) × (10)	−1.19545	−0.638923	0	0.703126	1.453403
19	sin(5)	0.422619				
20	−(18) ÷ (19)	2.828664	1.511816	0	−1.66373	−3.43904
21	Normal pressure angle (from Equations 5.63 and 5.64)	24.40341				
22	tan(21)	0.453692				
23	(2) ÷ 2π	42.66350				
24	(17) − (23)	−6.67617	−3.41285	0.031882	3.609110	7.280421
25	(24) × (22)	−3.02892	−1.54838	0.014464	1.637424	3.303069
26	(25) + (20)	−0.200258	−0.036565	0.014464	−0.026310	−0.135967
27	cos(21)	0.910659				
28	Deviation = (26) × (27)	−0.182367	−0.033298	0.013172	−0.023959	−0.123820
29	Distance *A* = (24) ÷ (27)	−7.33114	−3.74767	0.035010	3.963185	7.994673

Note: If (15) does not equal (1), this approximation method is not good enough, and a trial–error procedure must be used. Assume a new value for (10) and repeat steps (11) to (15), until (15) equals (1). For derivation curve, plot (28) against (29).

line. Items 30 and 31 show the deviations from a straight line set at a normal pressure angle of 24.4072°. This angle was calculated from the cutter pressure angle using Equations 5.63 and 5.64.

Figure 11.25 shows a plot of the normal section deviations. The solid curve is a plot of items 31 and 32. It will be noted that this curve does not read 0 at the point where *A* is 0. This is caused by the fact that the axial section was based on an approximate pitch diameter (see earlier). The dashed line curve has been moved over by 0.013 mm to correct for this approximation. The dashed line curve is the one that should be used for checking. The other curve could be used, but it would confuse the inspectors to have the checking line intersect the surface instead of passing tangent to the surface.

Figure 11.25 shows that the normal section profile has a curvature of about 0.1 mm. Although this is not much distance, it is a significant amount when it comes to meshing of precision gear teeth. To get a worm that will smoothly operate under high speed and load, it will be necessary to hob its mating gear with a hob which has the same normal section curvature as the worm. The calculation just given is a method of getting both data to check the finished worm and data to check the hob used to finish the worm gear.

11.5 GEAR SHAVING CUTTERS

When gears are made by the shaving process, the gear engineer may have a frequent need to study the shaving tool. Essentially, this tool represents a gear which has a conjugate tooth action with the part being shaved. The surfaces of the shaving tool are serrated with small rectangular grooves. The cutter is purposely designed to run on a shaft which lacks a few degrees of being parallel with that of the work. The out-of-parallel condition causes the cutter teeth to have a sliding motion across the gear teeth even at the pitch line. It is this sliding motion, together with the serrations and the fact that the tool and the work are meshed together at a tight

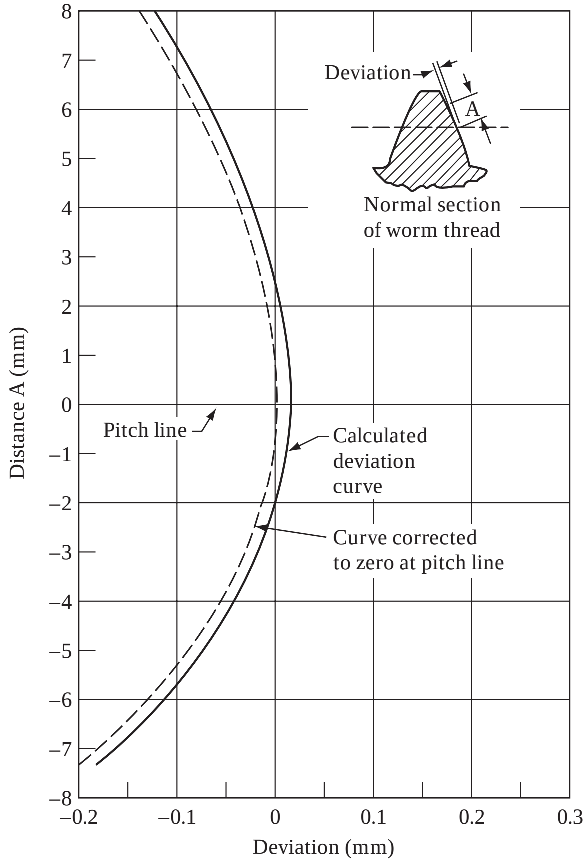


FIGURE 11.25 Curvature of worm normal section.

center distance, that causes the tool to shave off tiny slivers of material. (See Figure 11.26 for shaving cutter examples and Figure 11.27 for how a shaving cutter works.)

Since the shaving cutter is mounted on a shaft that is not parallel to the gear axis, the teeth of the shaving cutter and the teeth of the work run together like a pair of crossed-helical gears. The choice of shaft angle governs the cutting action of the cutter. In general, the higher the shaft angle, the faster the cutter cuts. The best control over helix angle, though, is gained with a low shaft angle.

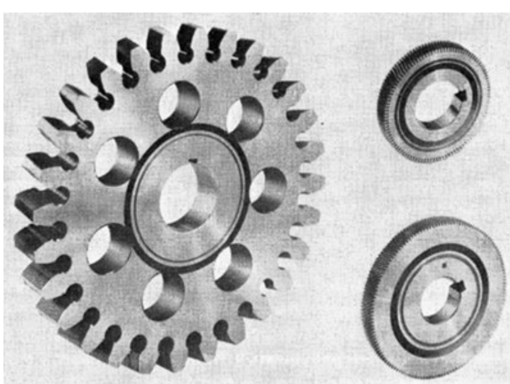


FIGURE 11.26 Shaving cutters of different pitches. (Courtesy of National Broach and Machine Co., a division of Lear Siegler, Inc., Detroit, Michigan.)

Since the choice of shaft angle is largely a matter of judgment in weighing different variables, there are no fixed rules. The general practice is outlined in Table 11.12.

Presently, there are no general trade standards for shaving cutter diameters, bore sizes, or face widths. Each manufacturer develops a practice which may agree in some respects and disagree in others with those of other cutter manufacturers. Light-duty shaving machines for fine-pitch gears up to 100 mm (4 in.) in diameter use cutters from about 50 to 75 mm (2 to 3 in.) in diameter. Medium-duty shaving machines for gears up to 450 mm (18 in.) use 150 to 280 mm (6 to 11 in.) cutters. Heavy-duty machines for gears up to 1.2 m (48 in.) or more use 175 to 300 mm (7 to 12 in.) cutters. Many shaving cutter users and manufacturers prefer a 200 to 225 mm (8 to 9 in.) cutter whenever the design permits.

In general, it is desirable to have a hunting ratio between the number of cutter teeth and the number of gear teeth. This tends to make it desirable to design shaving cutters with prime numbers of teeth. A prime number will hunt with all numbers of teeth except itself. Thus, numbers of teeth like 37, 41, 43, 47, 53, 59, 61, 67, 73, are popular for shaving cutters.

The normal circular pitch of a shaving cutter must be equal to the normal circular pitch of the work. Likewise, the normal pressure angle of the cutter must be equal to that of the work. Ordinarily, the helix angle of the cutter is opposite in hand to that of the work. When the hands of helix are opposite and the gear is external, the shaft angle is the difference between the helix angles. With spur gears or low-helical angle gears, the cutter helix might be designed with either hand for a given hand on the workpiece. If the hands of the helices are the same and the gear is external, the shaft angle is the sum of the two helix angles.

The conditions just mentioned usually make it impossible to have the pitch diameter of the cutter come out to some even figure like 200.0 mm or 8.000 in. The designer just usually chooses teeth and shaft angles so that the cutter diameter is very close to the desired diameter.

Shaving cutters are sharpened on their involute surfaces. This makes the cutter change quite appreciably in outside diameter during its life. Like shaper-cutters, shaving cutters do not produce the same thickness on the work throughout their life. If the thickness of the work is held constant, the depth that the shaving cutter is fed into the work will vary.

The tendency of the shaving cutter to cut a deeper depth as the teeth are made thinner in sharpening may be controlled by reducing the cutter outside diameter a proper amount each time the cutter is sharpened.

To properly function, the shaving cutter should finish the involute at least as deep as the form diameter. Preferably, the cutter should finish the involute close to the diameter that corresponds to the deepest point of undercut left by the preshave tool. Perfect blending of root fillet with shaved profile can be obtained only when the shaving action stops in the middle of the undercut. As the shaving cutter tip rolls out of mesh, it follows a path which takes it closer to the root diameter of the gear. This movement may cause trouble. The cutter tip may foul either the root diameter or the root fillet of the gear. It

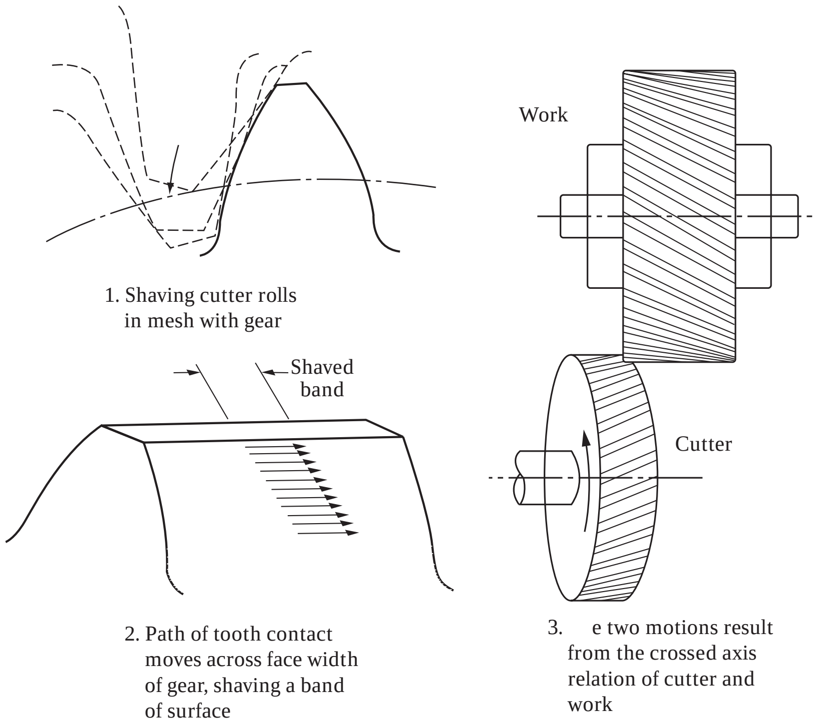


FIGURE 11.27 Shaving cutter action. (Courtesy of Ex-Cell-O Corp., Tool Products Division, Detroit, Michigan.)

TABLE 11.12
General Practice for Setting Value of the Shaft Angle

Application	Shaft Angle, Degrees
Spur pinions, under 20 teeth	8–12
Spur pinions, 20 to 35 teeth	10–15
Spur gears	10–15
Helical pinions, narrow face	8–12
Helical gears, narrow face	10–15
Helical pinions, wide face	5–10
Helical gears, wide face	10–15
Internal gears	4–8

is necessary to size the cutter outside diameter so that it will clear the root fillet even when it is at its deepest point of penetration with the gear. This means that the working depth of the cutter must always be a certain amount less than the whole depth of the gear.

The calculations necessary to design a shaving cutter are fairly complicated. They will not be given here.

11.6 PUNCHING TOOLS

The tool used to punch gears usually has three working parts: the punch, the die, and the knockout piece. Sheet metal is placed between the die and the punch. The punch has the shape of the gear to be made. For punching an external gear, an external-toothed punch is used. The die has the shape of the stock that is left after the punching is removed from the sheet. An internally toothed die is used for making an external gear.

In punching a gear, the first operation is to feed the sheet metal into the tool. The punch is driven into the die by the ram of the punching machine. This cuts out the gear and leaves it stuck in the die. The punching is removed from the die by a movement of the knockout tool. This tool has a shape very similar to that of the punch. The punching-tool assembly may be so built that spring action both retracts the punching tool and actuates the knockout piece. In this case, the punching machine has to furnish power only for the punching stroke (during which the springs are compressed).

Only thin gears can be punched. The thickness of the sheet stock should not exceed the gear tooth thickness at the pitch line. As a rule of thumb, the stock should not be thicker than $1.5m_t$ ($1.5/P_d$). This means that a 1-module (25-pitch) gear could be made up to about 1.5 mm (0.06 in.) face width.

The punching operation upsets the metal so that the tooth corners are rounded some on the die end. The face of the tooth is formed by a shearing operation. This shearing is not exactly at 90° to the axis of the gear. This means that a pair of newly punched gears cannot be expected to have uniform contact across even their narrow face width. The accuracy of punched gears may be improved by secondary operations. A shaving type of punching operation may be used to take a small amount of metal off the gear tooth faces. A coining operation may also be used to improve accuracy. In coining, the punching is rammed into the bottom of a die with so much pressure that the metal flows to completely conform to the shape of the cavity. It is also conceivable that conventional crossed-axis shaving might be used on punched

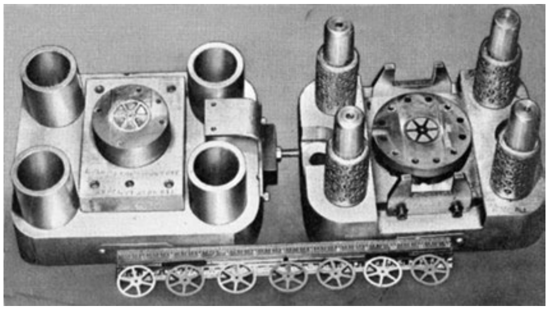


FIGURE 11.28 Gear punching tool.

parts. The fact that punched gears are essentially thin wafers pressed on a small shaft is the greatest drawback to shaving. The reactions of the shaving tool would tend to cock the gear on its shaft.

The power required to punch gears primarily depends on the cross-sectional area to be sheared. As a rule of thumb, the tonnage capacity of the punching machine should be equal to at least twice the force calculated as the product of the shear area times the ultimate shear strength. For instance, a 1 in. gear of $\frac{1}{32}$ in. thickness would have a shear area at the teeth of about $2.5 \times n \times 1$ in. $\times \frac{1}{32}$ in., or 0.245 in.^{2}. If the material was steel with 60,000 psi shear strength, a press of about 15-ton capacity would be needed to cut the teeth. Additional power would be required if the gear had a large bore to be cut. All sizes of punching machines up to as large as 2000 tons are on the market. Figure 11.28 shows an example of a gear punching tool.

11.7 SINTERING TOOLS

Before gears are sintered, the metal powder is compressed into a gear-shaped briquette. A complicated and expensive set of tools is required for the briquetting operation. After the briquette is made, it is sintered in an oven. The sintering consists of heating the briquette to a temperature almost up to the melting point. Carefully controlled atmosphere furnaces are used for this operation.

Sintered gears are rather porous. This is an advantage from the lubrication standpoint, but a detriment to the strength of the part. Higher-strength gears can be obtained by filling most of the voids in the part by metal infiltration. This is done by placing a slug of lower-melting metal (such as copper) on top of the briquette when it is placed in the oven. Upon heating, the metal slug melts and soaks through the briquette. After heating and cooling, the briquette becomes an impregnated gear.

In making sintered gears, the special tools are used in the briquetting part of the process rather than in the actual sintering.

Figure 11.29 shows in schematic fashion the three tools used to make a briquette. The metal powder is poured into the cavity of a die barrel. The floor of the die barrel is the stripper. The powder is pressed against the stripper by a ram. The die barrel has internal teeth, while the ram and the stripper have external teeth.

After the powder is compressed, the briquette is ejected by withdrawing the ram, then pushing the briquette out with an upward stroke of the stripper.

The die barrel is made by broaching internal teeth into a special die steel. After broaching, the die is hardened by furnace heat treatment. The die steel used must be one which will have very little dimensional change upon hardening. Air-hardening types of steel are often used for the die barrel. The ram and the stripper have ground external teeth. Both these pieces are ground slightly larger than the expected size of the die barrel after heat treatment. The die barrel is lapped with a series of about three external toothed laps. This removes

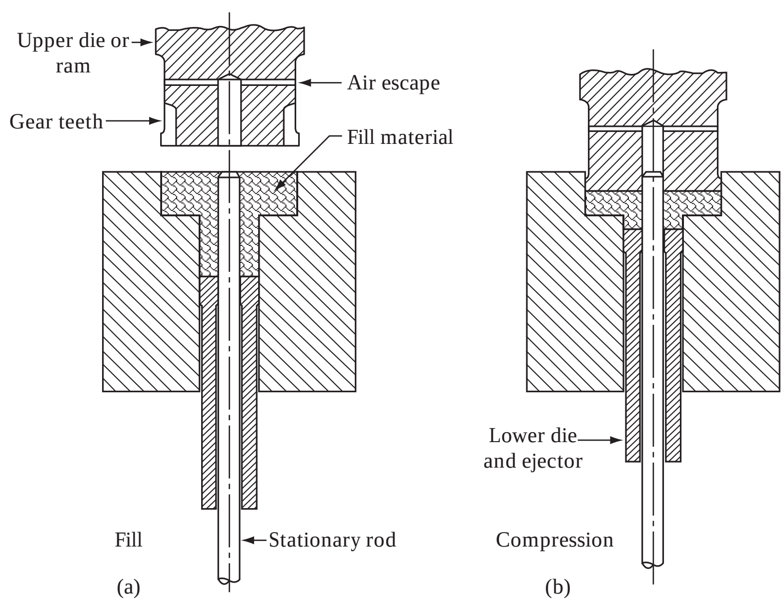


FIGURE 11.29 Configuration of the tools (a) at the beginning and (b) at the end of the sintering process.

slight errors, polishes the die teeth to a mirror-like nish, and enlarges the die so that the ram and the stripper will t with an almost perfect size-to-size t.

The teeth of the ram and stripper are designed so that all surfaces can be ground and measured with great precision. The root llets are often made as true arcs of circles. This facilitates measurement of root size. Also, it is easy to dress a true circular arc on the tip of a form-grinding wheel. The sides of the teeth are involute curves. No corner radii or chamfers are used at the tips of the teeth. Figure 11.30 shows the way the teeth of the ram and the die t with each other. To keep the powder from leaking, it is necessary that the tooth surfaces t so well that no clearance of even as little as 0.01 mm (0.0005 in.) exists anywhere between the contacting tooth surfaces. Great skill and care are required to build a set of sintering tools that will t on all surfaces with this kind of precision.

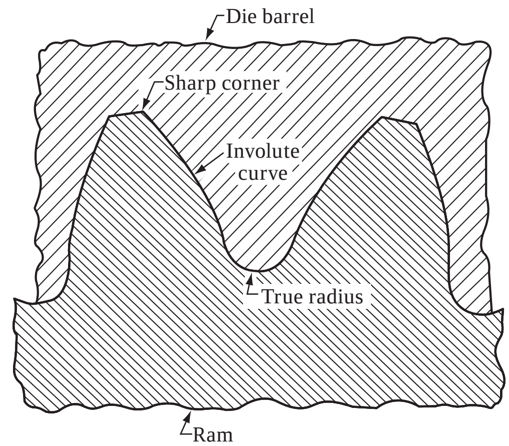


FIGURE 11.30 Surfaces of ram and die are ground true and t with almost no clearance.

12 The Kinds and Causes of Gear Failures

The previous chapters have covered the selection and the design of a gearset. If gear engineering were an exact science, information of this type would be all that the designer needed to know. But it is not an exact science. It is both an art and a science. Those who design gears are constantly surprised that some gears run better and last longer than would be expected from the design formulas, while others prematurely fail even when they are operated well within the design limits of the transmitted load. The gear designer needs to be able to evaluate the various causes of gear wear and failure. New gear designs must be based on both textbook logic and practical field experience if the best possible job is to be done.

In studying cases of gear wear and failure, it is very important that the correct analysis be made. Frequently, the cause of failure will be something quite different from the amount of transmitted load. An incorrect analysis can lead a designer to make a new gearset larger than it ought to be, and yet the new set may still fail because the real cause of trouble is still uncorrected.

In this chapter, we will study the kinds of gear failure and the causes of gear failure. In a power package, the gear unit is a basic element, taking power from a prime mover and delivering the power to a power-absorbing unit. Troubles in the gear unit may stem from power system troubles, or they may be localized entirely in the gear unit. The investigator needs to be alert to a wide variety of things that may cause failure and then carefully analyze the evidence to sort out what improper thing—or combination of things—led to a failure sequence.

12.1 ANALYSIS OF GEAR SYSTEM PROBLEMS

No matter what the failure is, it is desirable to look at the whole power package and the history of the particular power package in question before proceeding with a detailed study of gear teeth or gear bearings. In this part of the chapter, we will analyze the troubles that are essentially gear system or geared power package troubles. The next part will analyze gear tooth and gear bearing failures inside the gear unit. The last part will cover some examples of failures.

12.1.1 Determining the Problem

A malfunction in a geared power system can take many forms. In general terms, one or more of the following may apply (see Table 12.1).

Any of the problems listed in the table may cause a geared system to be investigated for failure. In a general sense, failure should not be thought of as just inability to operate. Rather, failure should be thought of as some unsatisfactory condition which either carries a threat that a geared system will become inoperable or poses an environmental disturbance that is considered improper. Of course, there can easily be situations in which one party may consider that the geared machinery is failing to properly operate, and yet that claim is not justified by the generally accepted norms for that particular kind of equipment.

When a gear system breaks down, the real failure may be quite different from the reported failure. For instance, when gear teeth break, investigation may reveal that they were seriously worn long before they broke. The problem then is to discover why the abnormal wear occurred. Figure 12.1 shows a good example of this.

Vibration is another example. A high-speed gear unit may run for months with all modes of vibration within reasonable limits. Then there is an onset of high vibration. Close investigation may reveal small amounts of gear tooth wear and the development of excessive tooth spacing error. The problem may then be a tooth wear problem rather than a vibration problem. (The vibration problem is secondary to the tooth wear problem.) Figure 12.2 shows a tooth spacing situation caused by wear which resulted in severe high-frequency vibration.

In solving gear failures, the available evidence must be sifted to find out what went wrong first. Generally, *one thing* started a chain of events. The problem has been defined when the primary cause of failure has been found and other failure evidence can be considered secondary.

12.1.2 Possible Causes of Gear System Failures

When a gear system is not working properly, the cause of the trouble may come from any one of several areas. A good investigator has no preconceived idea of the cause, knowing that error or wrong action in any one of five major areas can lead to a gear system failure. Table 12.2 shows many items in these areas (design, manufacture, installation, environment, and operation) that can cause failure when done improperly.

Some of the things in Table 12.2 are quite obvious, but others tend to be somewhat subtle. Let us take a few examples to illustrate the subtle.

1. Design example	A two-half gear casing has inadequate dowels. At overhaul, worn dowel holes permit two halves to shift. When the casing is bolted, tight bearings are cramped, and gear teeth are misaligned, bearing and/or gear-tooth failure results.
2. Manufacturing example	In a lightweight casing, bolts are tightened all on one side, then all on the other side. The casing takes a bow, and high-performance (aerospace) gears are misaligned. Scoring occurs immediately in first full load test.
3. Installation example	Oil pipes from the filter to the gear unit pick up blowing sand at a desert installation. Failure to carefully clean the pipes just before assembly results in high-speed pinion bearing failures even though a hot-oil flush was used just before the new equipment started up. (The sand was downstream from the oil filter.)

TABLE 12.1
Kinds of Malfunction in a Geared Power System

Problem	Consequences
Broken part	Drive system probably inoperable
Abnormally worn part	Drive system probably operable; may be unfit to continue operation for any appreciable time
Abnormal vibration	Drive system probably operable; may be unfit to continue operation for any appreciable time
Abnormal noise	Drive system operable; may be environmental hazard or prone to early failure
Abnormal part temperature; abnormal oil temperature	Drive system may be operable, but in danger of early failure
Serious oil leakage	Drive system operable, but an environmental hazard; may be in danger of early failure
Part seized in its bearings; part moved out of position and jammed with another part	Drive system will not transmit torque; inoperable condition may be more apparent than defect which caused it

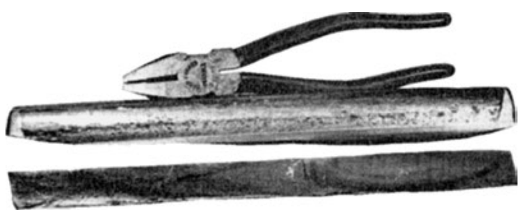


FIGURE 12.1 A broken out helical gear tooth. Note serious pitting and wear of tooth flank. (The actual size of the tooth can be judged from the standard pliers laid next to the tooth.)

4. Environment example	Gear unit becomes too hot. Gear teeth and bearings wear. Unit runs roughly, and then pinion bearing fails. Fly ash had settled on the gearbox. The unit had a splash system and "OK" thermal rating—provided that the gear casing is clean enough to properly radiate heat.
5. Operation example	Gear unit drives a compactor that is rolling chunks of processed mineral ore. Operators feed ore to the conveyor belt in an erratic manner, with batches of excess material alternating with lean feed areas. The gear unit pits as a result of frequent momentary overloads. Training operators to properly run the loading machine solves the problem, but new gears are needed to replace those damaged by green operators.

12.1.3 Incompatibility in Gear Systems

Perhaps the hardest thing to diagnose in a gear system failure is an incompatibility situation. In this situation, there is nothing particularly wrong with any of the units or subassemblies that are bolted together. Something tends to fail not because it was made wrong, but because of the behavior of another part of the power package. And the part that causes the trouble is standing up all right (or at least is runable)!

A good example of this is a coupling between a turbine and a gearbox. The coupling is designed and built to take relatively high misalignment. When it is misaligned, though, the coupling puts varying moments and axial force reactions into

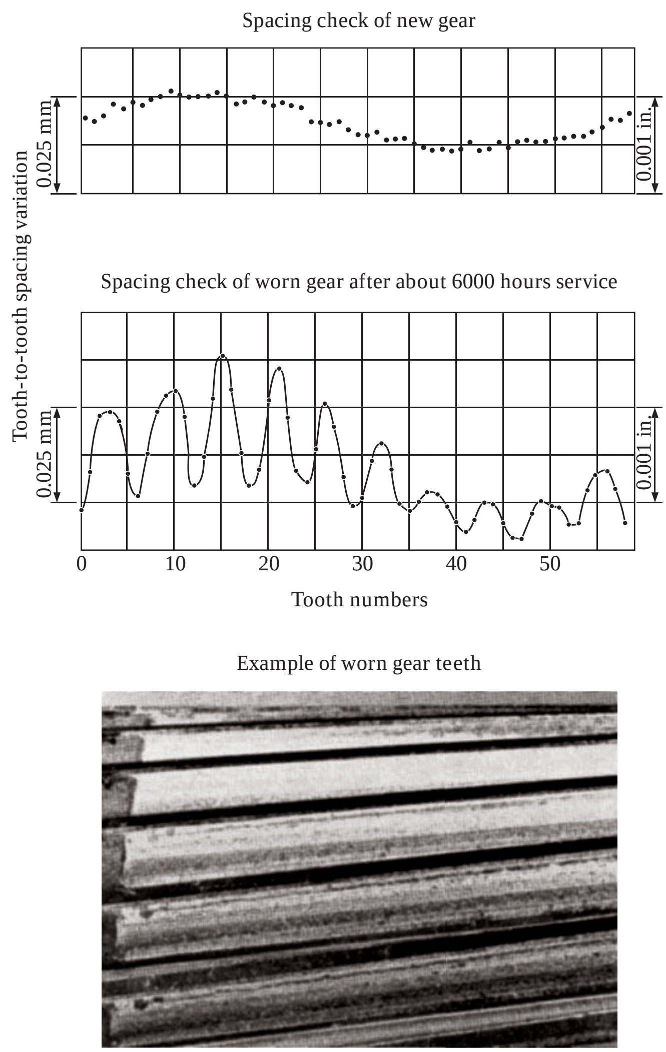


FIGURE 12.2 A very high–speed gear pits and wears. This results in circular pitch variation. Then the result of the several bumps per turn of the gear causes severe vibration and shock. The gear unit then fails due to either tooth breakage or bearing failure.

TABLE 12.2
Major Areas of Concern When a Gear System Fails

Design	Manufacture	Installation	Environment	Operation
1. Kind of gear (spur, helical, bevel, worm, Spiroid) 2. Design arrangement 3. Tooth design 4. Gear body design 5. Shaft design 6. Bearing design 7. Casing design 8. Seal design 9. Bolting design 10. Lubrication system design 11. Vibration criticals known and tolerable	1. Tooth accuracy (profile, spacing, lead, concentricity) 2. Tooth material (hardness, composition, cleanliness) 3. Gears (weld quality or casting quality, fit to shaft, balance, etc.) 4. Casings (bore sizes, bore position accuracy; joint flatness and squareness, oil tightness) 5. Assembly (right parts, correct bolt tightening, fits checked)	1. Foundation (adequate rigidity) 2. Alignment (with driving and driven units) 3. Oil system (clean, all connections made, oil filled properly) 4. Instrumentation (temperature, pressure, flow, vibration OK) 5. Bolting (to foundation, flanges, shaft couplings)	1. Air (adequate for cooling, not polluted with fly ash or chemicals) 2. Temperature (not adjacent to hot equipment, extremes of heat and cold within specification limits) 3. Water (adequate protection against rain water, swamp water) 4. Housekeeping (spare parts or disassembled parts can be cleaned and protected from rust and corrosion)	1. Break-in (may require oil change after short initial run; may require some running at reduced load before maximum load is applied) 2. Operation (meet specification limits on temperature, oil flow, power rating, etc.) 3. Overload (operate without undue extra loading) 4. Misapplication (high-speed idling, run backward, stall at excess torque, etc.) 5. Starting (no undue starting torque or vibration in starting interval)

the end of the pinion shaft. The pinion radial bearing, adjacent to the coupling, tends to fail because of the coupling disturbance, but the coupling itself survives. The most practical solution may be a better-quality coupling rather than a revised design pinion bearing or an attempt to run the power package with misalignment held to uncomfortably close limits.

Table 12.3 shows some of the more common ways in which incompatibility can lead to trouble or failure in a geared power package.

Table 12.3 shows situations in which the gear unit is in trouble because something is wrong with another part of the drive system. The reverse situation, in which the gear unit gets something else into trouble, is also shown, because this is still a gear problem.

In gear systems, it is necessary that each unit be able to run alone under its rated conditions, but each unit must also be a good neighbor that does not cause neighboring equipment to get into trouble. Thus, the things connected to the gear unit

TABLE 12.3
Examples of Incompatibility Situations in a Geared Power System

Vibration	Misalignment	Reactions	Temperatures
1. Gear tooth meshing frequency breaks turbine blades. 2. Reciprocating engine or compressor in trouble with valves or timing. Serious torque pulsations and low-frequency vibration. 3. Propeller changed. New vibration mode not tolerable to gear unit. 4. Imbalance in heavy gearing shakes light section of turbine.	1. Turbine or diesel engine shifts position as it heats up. Drive to gear unit seriously misaligned or out of position axially. 2. Foundation shift under heavy driven equipment. Gear casing becomes twisted. 3. Thrust bearing wears in prime mover. Axial shift overloads gear thrust bearing. 4. Hull of ship or frame of airplane shifts as cargo load changes. Movement too much for gearbox (beyond design allowance).	1. Coupling reactions aggravate gearbox bearings. 2. Overhung load on gear output shaft beyond design allowance. 3. Imbalance of high-speed impeller overhung from pinion prematurely fails pinion bearings. 4. Heavy vibration-absorbing coupling with multiple pads has a pad in trouble. Severe moment applied to gear shaft, leading to gear bearing and/or gear tooth failure.	1. Heat of engine exhaust puts thermal distortion into gearbox. 2. Hot exhaust of engine too close to air-cooled heat exchanger for package oil system. 3. Package start up too quick for thermal equilibrium to be established in gearbox. Bearing seizure due to lack of clearance and/or gear tooth seizure due to lack of backlash. 4. Common oil drain from gearbox overloaded with hot, frothy oil from other system equipment. High-speed gears overheat due to gearbox flooding.

need to be tolerable to the gear unit, and the gear unit needs to be tolerable to the other things in the system.

The statement just made is an engineering viewpoint relative to troubleshooting problems in the field. From a business standpoint, the technical responsibility for making a compatible package may lie with the package builder, or it may lie with the builder of a power plant who buys individual pieces of equipment and assembles them into an operating system.

The things in Table 12.3 may be somewhat subtle, just like the things in Table 12.2. As an example, impellers are often overhung on pinion or gear shafts. At a speed of 60,000 rpm, the centrifugal force becomes terrific. The pinion–impeller assembly may be balanced just fine at 10,000 rpm. Unfortunately, at 60,000 rpm, the centrifugal force is 36 times greater. The impeller will tend to stretch and shift where it is fitted to the pinion. This movement may change the balance enough to create an unsatisfactory running condition for the pinion bearings and the pinion teeth. In cases like this, special design features and special balancing techniques are necessary to achieve satisfactory operation of the high-speed system.

One more example will show how subtle things can happen even in relatively slow-speed equipment. Most large power packages will have a common oil sump and a lubricating oil system supplying oil under pressure to all pieces of equipment. The gearbox oil drain may connect into a long drain line that is draining oil from all the other equipment. Quick trouble can develop if a malfunction like worn seals puts too much oil and foam into the main drain. Oil backs up into the gearbox. Rapid gear overheating and failure can result if large gears start churning oil in a closely fitted gear casing designed for dry sump operation. A gear unit with a rating over 1000 hp and output speed of only 1200 rpm can be quite vulnerable to this sort of trouble. (Small gear units, around 100 hp with output speed around 500 rpm, may run just fine under wet sump conditions—in fact, wet sump design is often used in low-horsepower units.)

12.1.4 Investigation of Gear Systems

Since there is a rather wide variety of things that may lead to failure in a gear system (or any other system, for that matter), it is desirable to be alert to all the factual circumstances that surround the power system being investigated. Table 12.4 outlines the kinds of information that should be considered.

Table 12.4 covers the many things that would be of concern in investigating the failure of a helicopter gear drive or the gear drive for a piece of major equipment in a petrochemical plant. Of course, not as much information would be available for a small drive used in a minor installation, and hopefully a problem could be more easily solved.

For comparison, the manufacturer of critical gears on a major installation might have involute, spacing, and lead charts for each gear element stored in permanent files under the serial number of the gear unit. If the problem involved small amounts of wear, original inspection charts showing the exact condition of the teeth when new would be very helpful. Hence, the existence of these charts should be ascertained from a review of the quality plan for geometric accuracy. Then, of course, copies of the charts should be obtained.

In low-cost gearing on a minor installation, the manufacturer would probably work to a general quality level rather than routinely making inspection charts. Each production unit might be primarily passed on a visual inspection that the teeth had a reasonably good fit when rolled together with marking colors on the teeth.

The significance of the history has to do with design capability and manufactured quality versus application hazards. If a unit is in a very early stage of development, there may be particular reason to make an in-depth review of the suitability of the design. Perhaps the design is inadequate.

On the other hand, if the design has been proven by many units in service with an established time between overhaul (TBO), one must look for something unusual in the particular

TABLE 12.4
Checklist of Factual Information That May Be Useful in Investigating Gear Failure

Identification	History	Design and Manufacture	Environment
1. Kind of drive system (identify driving and driven equipment)	1. Who designed gear unit and when	1. Compliance with AGMA specifications	1. Weather (typical and extremes)
2. Serial numbers (of package, of gear unit, of gear or pinion involved)	2. Extent and nature of development test work	2. Compliance with other specifications (bearings, couplings, API, ISO, etc.)	2. Possible pollutants in air
3. Drawing numbers or catalog numbers (of gear or pinion involved, of bearings, of couplings to gear unit)	3. Number of similar units built	3. Kind of heat treatment for gears	3. Possible exposure to water, mud, wear debris, etc.
4. Geographic (address of site, identification of building or vehicle for each gear unit)	4. Number of units in service	4. Method of finishing gear teeth	4. Proximity of other machinery (consider vibration, heat, and fumes from nearby machines or process equipment)
5. Company names (owner, package builder, gear unit builder, gear parts makers)	5. Expected or operating TBO	5. Kind of oil and operating temperature limits	5. Nature of foundation or vehicle supporting gear unit
	6. Expected or proven design life	6. Quality plan for geometric accuracy	6. Possible damage in storage, in handling, or in cleaning parts
	7. When unit was built, and when put into service	7. Quality plan for metallurgical control	
	8. Load histogram data	8. Design drawings of each gear and pinion involved	
	9. Frequency of starts	9. Bearing and coupling design data	
	10. Condition of gears at last overhaul or inspection before failure		

application or in the environment. Perhaps the load histogram is more severe, or perhaps some environment factor is out of control. Of course, there is always the risk of random manufacturing errors or the risk of an incompatibility problem resulting from a change in the power package configuration outside the gear unit.

Table 12.4 is primarily valuable as a guide to the many kinds of information that may be helpful in conducting investigations of failures in geared power packages.

12.2 ANALYSIS OF TOOTH FAILURES AND GEAR BEARING FAILURES

Section 12.1 has been concerned with the whole drive system of prime mover, gear unit, and driven unit. This section will take up the gear unit alone and get into the details of gear tooth and gear bearing failures.

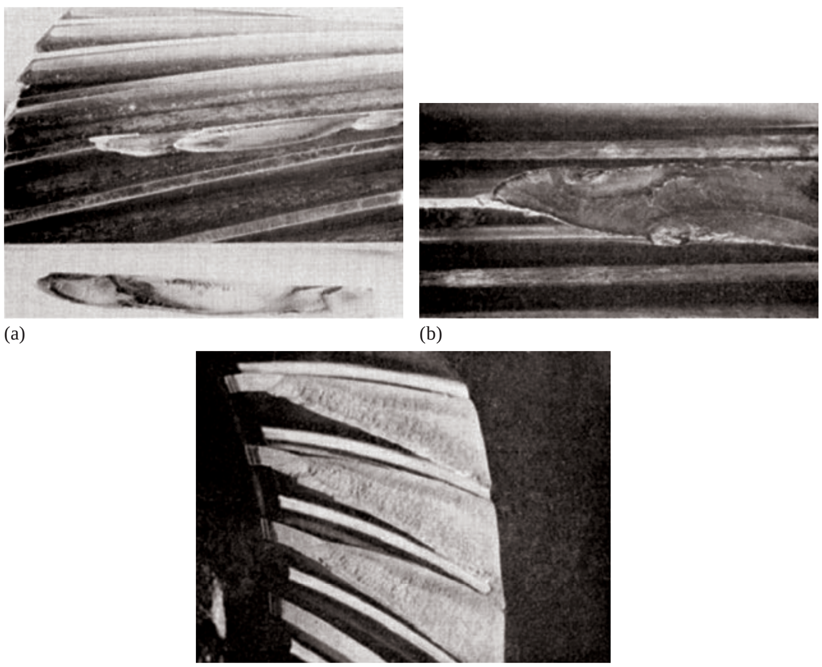
12.2.1 Nomenclature of Gear Failure

The various kinds of gear failures and the nomenclature to describe these failures have been standardized by AGMA.

Although the early work of AGMA described gear tooth failures rather well, further study and use of AGMA Standard 1010-E95 led to several refinements and additions.

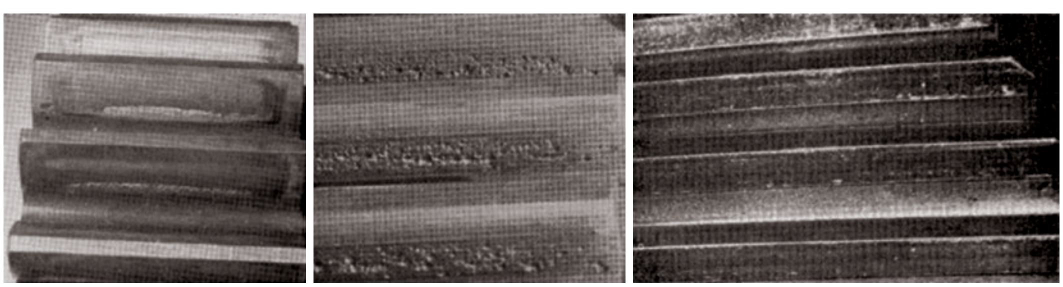
There are 18 recognized ways in which the surface of a gear tooth may be damaged. There are three kinds of tooth breakage. Since several kinds of tooth damage may occur at the same time, it is obvious that the gear engineer who analyzes a set of failed gears will have to carefully note all the items of evidence and then use just the right words to describe the findings. It takes a good detective to properly unravel gear failures.

Figures 12.3 through 12.7 show examples of gear tooth failures. These photos of actual gear teeth match some of the many kinds of failure shown in the standard on failure nomenclature. Since there are many possible variations in the



(a) (b) (c)

FIGURE 12.3 Examples of tooth breakage: (a) helical, 38 HRC; (b) spur, 55 HRC; and (c) bevel, 58 HRC. Note the clear-cut eye of the break on the spur tooth.



(a) (b) (c)

FIGURE 12.4 Examples of macropitting: (a) spur, 60 HRC; (b) helical, 35 HRC; and (c) helical, 35 HRC. In general, gears that have initial pitting have macrosized pits. Lower view c shows ledge wear as a result of macropitting for thousands of hours.

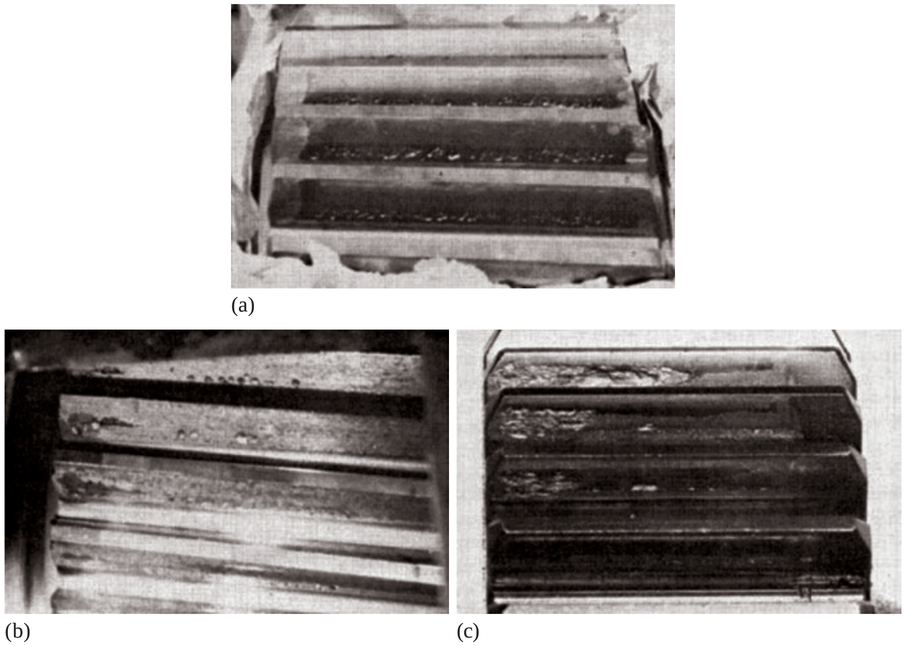


FIGURE 12.5 Examples of gross pitting: (a) helical, 35 HRC; (b) helical, 35 HRC; and (c) spur, 60 HRC. This kind of pitting is clearly destructive pitting. Lower view shows both gross pitting and spalling.

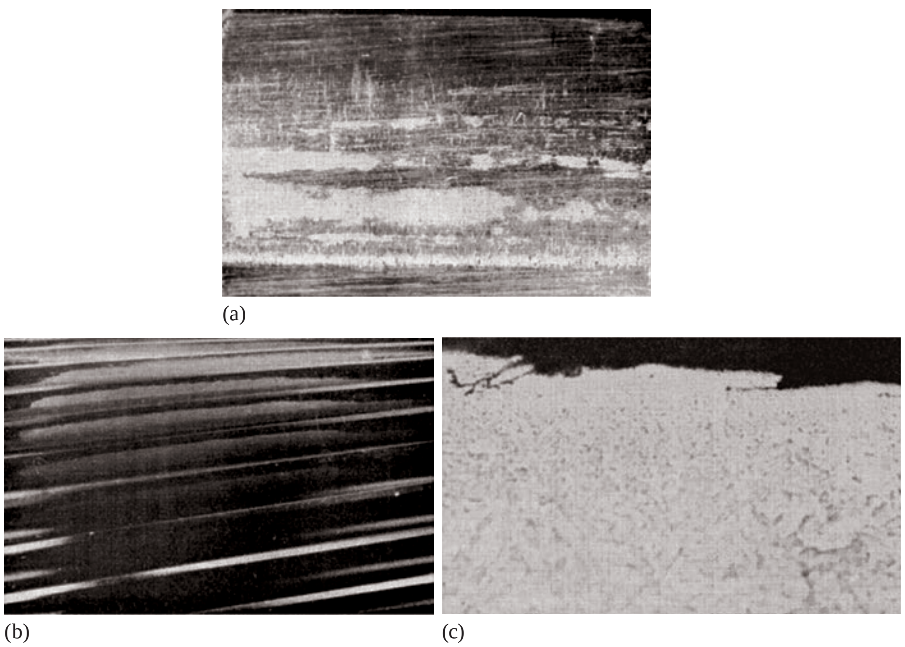


FIGURE 12.6 Examples of micropitting: (a) spur, 60 HRC; (b) helical, 60 HRC; and (c) ×1000 enlargement through pits. Until the late 1970s, micropitting was thought of as erosion or etching. Micropitting often precedes macropitting.

kinds of tooth failure, it is advisable to keep a copy of the standard handy as a complete reference on this subject.

12.2.2 Tooth Breakage

When a gear tooth breaks from fatigue, the surface of the fracture is quite smooth. The slow progress of the fracture apparently causes the metal to break like a brittle material. Many years ago, it was customary to say that a piece that had failed in fatigue had crystallized. No doubt, the appearance of the break led to this incorrect explanation of fatigue failure. When a broken tooth appears to have had a fatigue failure, there are several things to look for.

Focal point: There may be evidence of a focal point, or eye. This is the point where the break started. If the focal point can be found, some local defect may be located to explain the break. Sometimes a tear or a notch in the root fillet may coincide with the focal point. An inclusion or a heat-treat crack

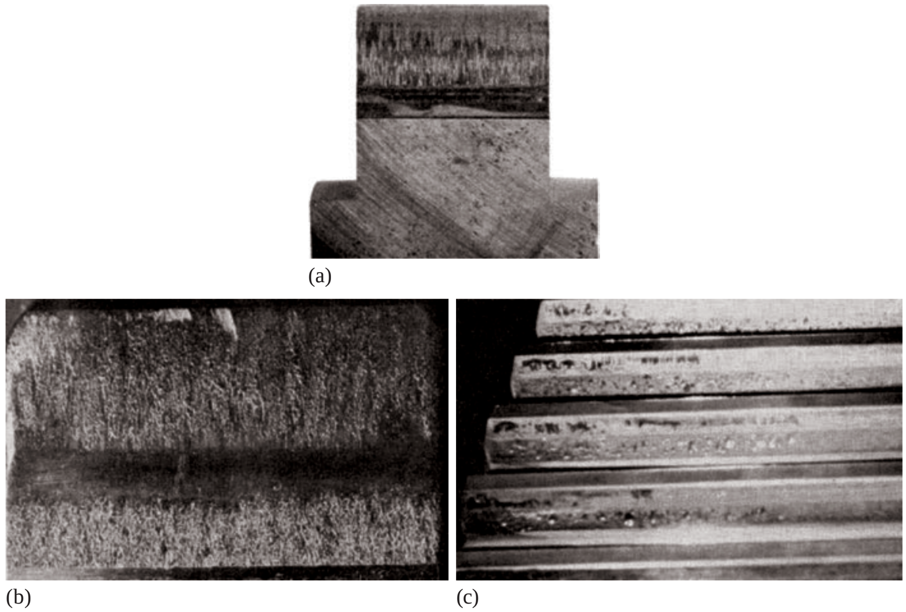


FIGURE 12.7 Examples of scoring: (a) spur, 60 HRC; (b) spur, 60 HRC; and (c) helical, 35 HRC. In the lower view, scoring, macropitting, gross pitting, and misalignment are all evident.

may be found at the focal point. If any defect is found at the focal point, it is quite likely that this defect is at least partly to blame for the failure.

Fretting corrosion: During the time that a fatigue break is growing, oil seeps into the crack and is compressed each time the gear tooth goes through the mesh. The slight motion of the tooth, coupled with oil and high pressure, will often set up fretting corrosion in the crack. Since it takes many hours to set up fretting corrosion, the red stain in a fatigue break roughly indicates the length of time involved in the fracture. When one tooth of a gear breaks by fatigue, it is desirable to closely examine the teeth that are not broken. Frequently red stains can be found seeping out of almost invisible cracks in other teeth. Such evidence indicates that other teeth are about to break.

Evidence of overload: When a gear tooth breaks from sudden shock or overload, the fracture usually has a stringy appearance. Even though the tooth may be fully hardened, the break will look like fibers of a plastic material which has been wrenched apart. When several consecutive teeth are broken from a gear, usually one or two teeth broke by fatigue. Then, as the gear continued to rotate under torque, the shock of the mating gear jumping across the gap left by the fatigued teeth broke additional teeth. The investigator can often look at a series of broken teeth and find the tooth that failed first.

Teeth that fail by overload will frequently have evidence of surface failure, such as plastic yielding. Since it takes two to four times as much force to fail a tooth by overload as it does to fatigue out a tooth, it is very likely that evidence of the force will show up on the tooth surface. All the teeth in the gear should be studied to see if all or only one or two have evidence of severe surface loading. This will often show the manner in which the severe overload was applied.

Break location: Gear tooth fractures ordinarily start in the root fillet. A cantilever beam is weakest at its base. When gear tooth breaks start at other locations, there is something unusual. Pitch-line pitting will sometimes be severe enough to cause a tooth fracture to start at the pitch line. Sometimes shrink fits or residual heat-treat stresses will cause a break to start in the root midway between two teeth. Case-hardened teeth with too deep a case or too weak a core will sometimes shatter from the pitch line up. The whole upper part of the tooth breaks loose like a cap. Subsurface stresses may cause teeth to break anywhere on the tooth flank.

Break pattern: Sometimes a tooth will break in an irregular and seemingly unexplainable manner. Often such breaks can be solved by studying the macrostructure of the part. Defects in forging procedure may create lines of weakness which a fatigue crack will follow.

When an end breaks off a gear tooth, this may have some special significance. The most obvious thing to look for is misalignment. Contact markings may show that the entire load was being carried on one end of the tooth.

Another likely cause of end breakage is accidental damage to the gear tooth during assembly. A gear dropped on the floor or banged by a crane hook will have some metal which is upset above the tooth surface as well as an indentation. If all the upset metal is not carefully scraped or honed away, severe overload will result. Many people think that low-hardness gears will rapidly wear away any bumps caused by careless handling. Unfortunately, this is often not the case. Teeth of steel gears of 200 HB have been known to break when the damaged area was only 1.5 mm ($\frac{1}{32}$ in.) in diameter on a tooth 20 mm (¾ in.) deep. A raised spot only 0.025 mm (0.001 in.) high will often cause unbearable root stresses.

Notches caused by the unskilled filing of burrs from the ends of teeth may cause end breaks. If the end of the tooth is properly finished, the end is actually slightly stronger than the middle of the tooth. Gears that are well made and well mounted usually—when tested to destruction—break from the middle of the tooth.

Broken gear teeth should always be examined and, if possible, measured for wear. When appreciable wear is present, it is always possible that the wear caused the breakage. In hardened spur gears of high precision, an amount of wear as small as 0.025 mm (0.001 in.) may be enough to double the root-fillet stresses! Helical gears can stand more wear than spur gears because of their overlapping action and the tendency for uniform wear. Sometimes as much as 0.8 mm ($\frac{1}{32}$ in.) can be worn from the surface of medium-hard helical gears without increasing the root stresses too much. Wear on case-hardened gears is doubly damaging to tooth strength. The load on the root fillet is increased, and the thickness of the thin, hard case (which supports most of the root stress) is appreciably reduced.

12.2.3 Pitting of Gear Teeth

Pitting, like tooth breakage, is a fatigue failure. It is almost impossible to make a gear pit without about 10,000 cycles of contact or more. If even more load is applied than the load that will cause pitting in the range of 10,000 to 20,000 cycles, the usual result is that the surface is rolled or peened.

There are several locations where pitting is apt to occur. Helical pinions of medium hardness with 20 or more teeth frequently pit along the pitch line. The mating gear may also pit, but if it is about the same hardness as the pinion and has been heat treated in the same way as the pinion, it is likely that most of the pitting will be on the pinion.

Pinions are more apt to pit than gears for two reasons.

First, the pinion is ordinarily the driver. The directions of sliding are such that sliding is away from the pitch line on the driver and toward the pitch line on the driven member. Figure 12.8 shows how the sliding motion on the driver tends to pull metal away from the pitch line. This leaves the pitch line high and also tends to stretch the metal at the pitch line. On the gear, the sliding tends to compress the metal at the pitch line. The cracks that form when a surface is severely loaded have a tendency to intersect at the pitch line of the driver, while on the driven member they do not.

Second, the pinion, being smaller, has more cycles of operation than the gear. The slope of the fatigue curve makes the part with the most cycles the most apt to fail.

When the gear drives the pinion, the cycles favor the pinion failing first, but the sliding action is worst on the gear. Tests made by the author on 4-to-1 ratio helical gears of medium hardness showed that the gear would usually pit first when it was driving the pinion and made of the same hardness material as the pinion!

Spur gears of conventional design have a region of contact where only a single pair of teeth carries the entire load. This region usually includes the pitch line, about one-third of the upper part of the dedendum, and about one-third of the lower

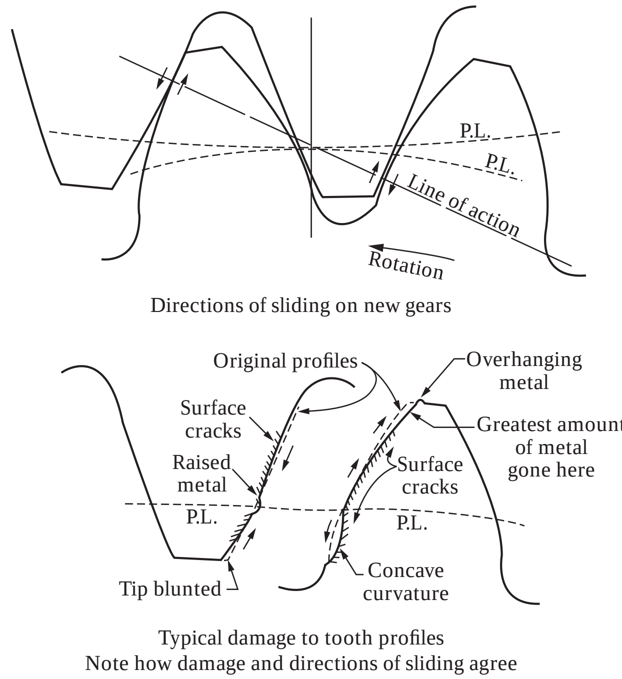


FIGURE 12.8 Effects of direction of sliding on gear tooth surfaces when they begin to fail by pitting, wear, or plastic flow of metal.

part of the addendum. At the tips and the roots of the teeth, there are two pairs of teeth in position to share the load. In most cases, the highest calculated Hertz stress (contact stress) will occur at the lowest position on the pinion at which one pair of teeth carries full load. If this stress is appreciably higher than that at the pitch line, pitting is apt to occur in this region. Because the effects of sliding do not particularly aggravate conditions at this point, pitting will not ordinarily start here unless the stress differential between this point and the pitch line is fairly significant.

The highest Hertz stress on the gear will occur in the gear addendum, since it is the gear addendum that contacts the pinion dedendum. For some reason, the pinion usually has more tendency to pit in the dedendum than the gear has a tendency to pit in the addendum. Way (1935, 1939) and others have explained this on the basis that oil is trapped in surface cracks on the pinion dedendum but is not trapped on the gear addendum. This is based on the pinion's being the driver.

The tests made earlier have shown that the surface of rolls was more apt to fail when the sliding was in a negative direction than when it was in a positive direction. When negative sliding occurs, the sliding velocity is opposite to the rolling velocity. Negative sliding occurs in the dedendum regions of both the gear and the pinion; while in the addendum regions, the sliding is always positive.

When pinions have small numbers of teeth, there is still another danger point on the pinion. At the base circle of the pinion, the radius of curvature of the involute profile is reduced to zero. This means that the stress on the surface tends to approach infinity, even though two pairs of teeth may be sharing the load. The net result of carrying heavy load near the base circle is a rapid peening and pitting away of the metal

until this critical region stops carrying any appreciable load. On medium- and low-hardness pinions, base-circle pitting may remove enough metal to correct the abnormal stresses, and then the damage may stop. On fully hardened parts, more serious effects may result. The base-circle pitting may leave a bunch of tiny cracks at a critically stressed region on a brittle material. It must be remembered that the tensile stress due to the beam loading of the tooth is high at the bottom of the tooth.

When gears are loaded heavily enough or when local errors are present, pitting may break out almost anywhere on the tooth. Even the addendum region of a driven gear can be made to pit if the applied load is severe enough.

In all the locations discussed earlier, the pitting may be of the initial pitting type, or it may be destructive pitting. Pits of the initial pitting type are usually quite small in diameter. On medium-hard parts with 2.5-module (10-pitch) teeth, they are apt to be about 0.4 mm ($\frac{1}{64}$ in.) in diameter. On 5-module (5-pitch) parts, they may be around 0.8 mm ($\frac{1}{32}$ in.) in diameter. Destructive pits are usually much larger.

It frequently happens that gears which are in trouble with pitting are also in trouble with lubrication. The oil may be too thin or the surface too rough to support a good oil film. There may also be abrasives in the oil. Wear may occur up to the point at which the profile accuracy is appreciably damaged. Then the Hertz stresses will be increased, and the tensile stresses will be high due to friction. The combination of effects will make a gear pit under much lighter load than would ordinarily be expected. When investigating pitting, it is always desirable to make accurate measurements of tooth thickness and involute profile. These may be compared with the checks made when the set was new. If serious surface wear is occurring, it is obvious that the control of pitting will require control of the lubrication problem which is causing the wear. (See Sections 12.2.5 and 12.3.4.)

Helical gears that are through-hardened (low to medium hardness) will often extensively pit but still be quite runable. The whole dedendum may become covered with relatively small and uniform pits. The removal of metal by pitting tends to create a ledge condition. The dedendum will tend to keep its involute shape, but a layer of metal is gone. If this layer is not too thick, the gears may still have relatively good load-carrying capacity.

A helical gearset may start with some misalignment. The pitting just described will start at one end. Curiously, the pitting of the misaligned gear does not stop in the middle of the face width, but will tend to continue right across the face width. Figure 12.9 shows a good example of this situation.

The general rating practice for through-hardened gears will allow a high enough surface loading to cause pitting when tooth errors, metal quality, or lubricant are somewhat less than what might be desired. Fortunately, it is often possible to get several years of service out of gears that have pitted rather extensively. The user of gears who is confronted with such a situation should be alert to the severity of the pitting and the rate at which it is progressing. Both items may be either tolerable or intolerable.

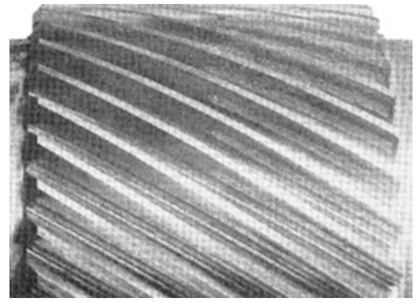


FIGURE 12.9 A misaligned helical gear has pitting which is progressive across the face width. Ledge wear is present at RH end. The leading edge of the pitted area shows typical macropits.

Quite often a gearset that has pitting early in its life can be helped. Changes in alignment may be accomplished by bearing adjustment or by realigning the gear unit to driving and driven equipment. A change to a heavier or better type of oil may also be beneficial. Under improved running conditions, the pitted surface may wear and polish a slight amount and thereby heal; a year or so later, the pitting may actually not look as bad as it did when the corrective action was first taken.

12.2.4 Scoring Failures

Scoring is essentially a lubrication failure. Tears and scratches appear on the rubbing surfaces of the teeth. Unlike fatigue failures, which occur after many cycles of operation, scoring is apt to occur as soon as new gears are first brought up to full load and speed.

Although scoring is a lubrication failure, it cannot be blamed on the lubricant in many cases. Frequently, the design of the gears or the workmanship in finishing the teeth is such that no lubricant could be expected to make up for the defects in the gears. The gear designer should avoid both high Hertz stresses and high sliding velocities at the tips of the teeth. This can be done by proper choice of pitch, addendums, and modification of the involute profile (see Section 13.2). Errors in manufacture, such as poor surface finish, waves on the tooth surface caused by erratic action of gear-finishing machines, poor involute profiles, or irregular tooth spacing, all tend to promote scoring.

There are four places at which scoring is apt to start:

1. Where tip of gear contacts root of pinion
2. At lowest point of single-tooth contact on pinion
3. At highest point of single-tooth contact on pinion
4. Where tip of pinion contacts root of gear

In helical gears where the axial overlap amounts to two or more axial pitches, there is no clearly defined position corresponding to the point where single-tooth contact starts and ends on spur gears. However, there may still be two intermediate points that are critical. If tip relief or modification is used, the most critical points may be where the relief ends on the pinion tooth and where it ends on the gear tooth.

After a gear has scored for a short time, the damaged area will frequently extend all the way from the pitch line to the tip and

from the pitch line to the root. Along the pitch line itself, there will be a narrow island of metal which will not score. However, this island will soon pit away if the gear is left in service.

Scoring failures are rather hard to analyze. The damage usually spreads so fast that it is impossible to tell just where it started or why it started. When running laboratory tests on scoring, it is quite handy to test a ratio at which the number of pinion teeth divides evenly into the number of gear teeth. This makes one particular gear tooth contact with only one pinion tooth. Ordinarily, with a hunting ratio, each gear tooth will contact each pinion tooth. With an even ratio, pitting will break out between some pairs of teeth and not break out on other pairs. This makes it possible to closely study the effect of tooth errors on the tendency to score.

Scoring seems to be considerably influenced by the affinity of one metal for another. It appears that some metals will bond together much more easily than others. Fully hardened steels will generally resist scoring better than medium-hard steels. Low-hardness steels score much more easily than medium-hard steels. In many cases, some benefit has been noted from having the pinion harder than the gear and made of a different kind of steel than the gear. For instance, case-hardened pinions that run with a medium-hard gear will frequently make a more score-resistant set than a set in which the pinion and the gear are both of the same steel and medium hard.

There are two degrees of scoring, by definition: initial scoring and severe scoring. The main difference is that initial scoring occurs for a while and then stops. Severe scoring keeps going until so much metal is removed that the gear teeth finally break off. When scoring first starts, it is difficult to tell whether the scoring is of the initial or the severe variety. If the score marks are quite fine and the scored surface has a kind of smooth, etched appearance, there is a good chance that the scoring is of the initial variety. If the surface is rough, with ragged tear marks, there is no doubt that the scoring is of the severe variety.

Scored gears which are stopped before the scoring has progressed too far can often be put back into successful service. The scored areas should be dressed down with a stone or fine abrasive paper to restore some degree of surface finish. When the gears are put back into service, it is desirable to use a higher-film-strength oil—such as one with a good extreme pressure additive or just one of heavier viscosity. Care should be taken to see that the oil-distribution system is wetting down all the teeth before they go into the mesh. In rare cases, it has been found that the oil was too thick to spread out over the whole tooth surface. The solution to this kind of problem may be to use a thinner oil which will spread more rapidly. Changes in oil nozzle designs or in oil pumping systems are often required to overcome gear-scoring problems.

Scoring can often be prevented by careful breaking in of gearsets. It is desirable to run the teeth at light loads and low speeds until the tooth surfaces have polished up. This is particularly beneficial on the lower-hardness steels and on bronzes meshing with steel. Fully hardened steel gears benefit some from running in, but not nearly so much as the softer materials. Tests made on gears of 200 HB have shown that careful running in can sometimes reduce an 0.8 μm (32 μin.) machined surface to a 0.25 μm (10 μin.) finish and work-hardened surface equivalent to 250 HB. Obviously, changes like this greatly improve a gear's ability to resist scoring.

Scoring is particularly troublesome with high-speed gears running with thin, hot oil. A change to thicker oil or cooler running can be very helpful. If this cannot be done, an improvement in the character and the smoothness of the finish may solve the problem. Helicopter gears are quite often honed after precision grinding to get a kind of finish that will adequately resist scoring under rather difficult service conditions.

Scoring may be a problem with slow-running final drive gears in trucks, tractors, and earth-moving equipment. In these vehicle gears, the problem is a very heavy tooth load and pitch-line speeds so slow that a proper EHD oil film is not obtained. This kind of problem is generally solved by using heavy oils and strong EP additives. The additives tend to create a chemical film on the tooth surface and/or a chemical reaction with the surface that protects the surface from scoring. (See Sections 5.2.8 and 12.3.3 for more details on lubrication.)

12.2.5 Wear Failures

According to the dictionary, *wear* means damage caused by use. Under this kind of definition, all kinds of tooth damage, such as pitting, scoring, tooth breakage, and abrasion, might be considered as just different kinds of tooth wear. Most gear engineers consider a worn gear tooth to be one that has had a layer of metal more or less uniformly removed from the surface. Damage which does not remove the entire surface but instead makes craters or scratches in the surface is generally not called wear. Such damage, of course, is best described as pitting and scoring.

In this section, we shall consider the kinds of wear that, by abrasion or other means, remove layer after layer of material from the surface of the tooth. This is the kind of wear that reduces tooth thickness and frequently severely changes the contour of the tooth.

Tests and experiences indicate that all kinds of gear wear fit a logical pattern. Figure 12.10 shows a schematic plot of torque versus speed for a gearset. The total area is divided into five different regions. Each of these regions is bounded by a failure line of one kind or another.

In region 1, the gear is not running fast enough to develop a hydrodynamic oil film. The wear is rapid when measured in terms of metal removed per million contacts of the tooth. Since the gear is running slowly, the wear may not be particularly fast in terms of wear per hour or per day. The size of region 1 can be reduced in most cases by using thicker oil or by using oil with a special additive that will make the fluid cling to the gear teeth more tenaciously. Too thin oil or too rough a surface finish will make region 1 larger.

The lower part of region 2 is the ideal place to run a gearset. The speed is high enough to develop a good film. If the oil is free of abrasive foreign material, is noncorrosive, and properly adheres to the surface, a gear can almost indefinitely run in region 2 without measurable wear.

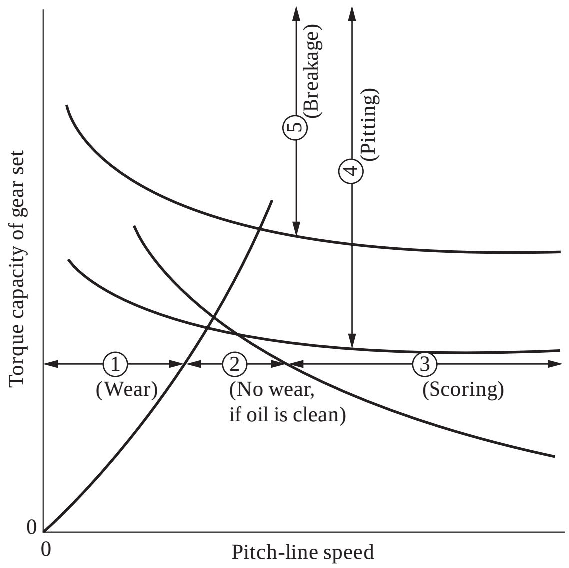


FIGURE 12.10 The regions of gear failure.

In region 3, there is rapid failure. The speed is high enough to produce an oil lm, but it shears the oil lm so fast that too much heat is developed. The lm breaks down, and scoring or welding occurs.

Region 4 is the region in which pitting is apt to occur. Since pitting is a fatigue failure, the size of region 4 tends to increase with time—more time means more cycles, and more cycles mean a lower load that can be carried at a given speed. Region 4 is also enlarged by poor lubrication conditions. This seems to stem from two causes. Wear damages the tooth pro le and causes load to be concentrated near the pitch line. This is one of the regions that is vulnerable to pitting damage. Wear also tends to result in higher local coef cients of friction. A higher friction force means that there is more tendency for the surface to be ruptured with the cracks that cause pits.

Region 4 changes rather surprisingly with a substantial change in the regime of lubrication. The stress-versus-cycles curve for pitting failure changes both slope and location. Figure 12.11 illustrates this tendency in aerospace gearing lubricated with light synthetic oil. The same general tendency can be noted in slow-speed vehicle gearing when changing from a lighter weight mineral oil to heavier weight oil and when changing from mild EP additives to strong EP additives. (Table 4.7 in Section 4.1.4 presents more information on regimes of lubrication.)

A survey of the gear wear characteristics of most types of industrial and vehicle gears shows that the most the common cause of a breakdown of the gear tooth surface is pitting related. To say it another way, a worn gear tooth—when closely studied—is very apt to have worn primarily because of some mode of pitting. The successful designer of power

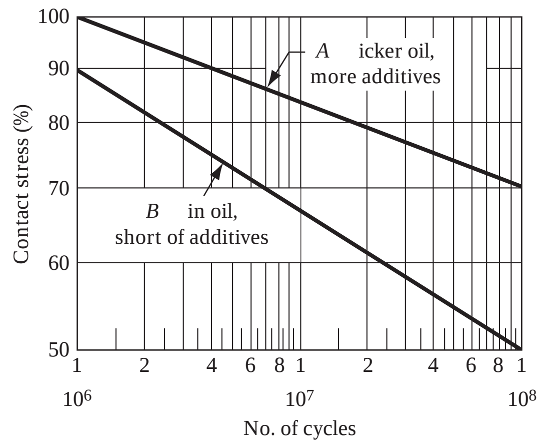


FIGURE 12.11 A substantial change in the lubricant may shift the allowable contact stress from curve *B* to curve *A*.

gearing will develop a very keen appreciation for all the subtle things involved with the region 4 boundary.

Region 5 is the region in which tooth breakage is apt to occur. The size of this area is also increased as a function of the length of time the set operates. Breakage is a fatigue failure. The size of region 5 is increased when wear occurs. Wear makes gears run with more shock and vibration. This is just like increasing the torque. Wear also weakens the tooth by removing metal from the base of the tooth and by acting as a factor to increase the stress concentration in the root llet.

From the foregoing, it can be seen that as soon as one kind of gear damage occurs, continued running will make the gear less and less able to resist all the other kinds of gear damage. Frequently, a wrecked set of gears will show all kinds of damage, such as abrasive wear, pitting, scoring, peening, wire edging, and tooth breakage. The investigator may be hard pressed to deduce which kind of damage started the fatal chain of events.

The existence of region 1 can be demonstrated in two different ways. One is to measure the mesh loss of gear teeth. When appreciable torque is applied and held constant over a wide speed range, a curve like Figure 12.12 will result. The tail at the left-hand side indicates that the oil lm has failed and the

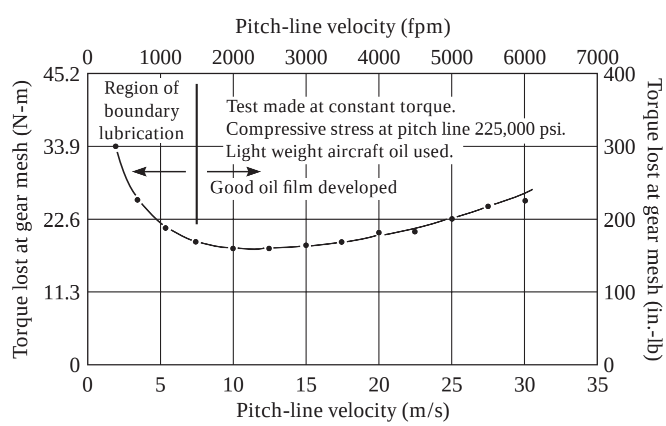


FIGURE 12.12 Friction torque as a function of speed.

coefficient of friction increases while speed decreases. Another way is to measure wear. In one test, a set of precision spur gears was operated for 20 million cycles at a pitch-line speed of 30 m/s (6000 fpm). Involute measurements showed wear to be less than 2.5 μm (0.0001 in.). Then the set was run at 5 m/s (1000 fpm) with the same torque for 5 million cycles. After this shorter, slower run, wear had increased to 13 μm (0.0005 in.).

In high-speed marine and power plant equipment, it is quite likely that after a year or more of operation, the wear will be so slight that machining marks a few microinches deep will still be visible on the tooth surface. In slow-speed gears such as drive gears on freight locomotives or on winches or hoists, it is not uncommon to see about 1 mm ($\frac{1}{16}$ in.) of metal worn away in 6 months. It is true that high-speed gears generally have cleaner oils and lighter loads than slow-speed gears. In spite of this, though, the weight of evidence seems to show that well-designed fast-running gears wear less than equally well-designed slow-running gears when wear is judged on the amount per million cycles of operation.

Foreign material will, of course, cause wear at any speed. Dirt, sand, and oxides may get into the oil stream. These can cause rapid wear or lapping of the gear teeth. Great care must be taken to keep the oil system of a gearset from becoming contaminated. In some kinds of transportation equipment, gear wear is frequently caused by dirt getting into the oil. On the other hand, power plant gears seldom have this kind of trouble. They are protected from wind, weather, and contacts with soil.

12.2.6 Gearbox Bearings

If the bearings in a gear unit fail, the gear unit has, of course, failed. From a responsibility standpoint, those technically responsible for gear units are responsible for the gear teeth, the gear bearings, the seals, the oil breather device, the shaft extension splines or keyways, the oil circulation system, etc.

Records that have been kept of the frequency and the kind of gear unit troubles invariably show that about two-thirds of the incidents involve problems with bearing failures or bearing damage. (From a practical standpoint, the gear designer and builder may have to worry more about bearings than about gear teeth!)

The high incidence of bearing problems should not be blamed on the inadequacy of bearings as a mechanical element or on the lack of technical knowledge available in the bearing field. Bearings are frequently involved for two reasons. First, the geometry of the bearing and its fit-up in the gearbox are more complex than gear-tooth geometry and gear-tooth fit-up. Second, the bearings are generally more sensitive to some wrong condition in the application, and they will start to fail sooner than the gear teeth.

Some of the kinds of wrong things that may lead to early bearing troubles are the following:

- The unit is running with foreign material in the oil (iron filings, sand, dirt, water, acid, and so forth).
- The unit is running too hot.
- The unit traveled a rough road to the site and incurred damage through vibration and fretting corrosion.
- The unit has high vibration (in running).
- The unit is misaligned or out of position in an axial direction.
- Driving or driven equipment is running poorly (turbine out of balance, reciprocating compressor has some valves stuck, and so on).
- Coupling devices between gearbox and connected apparatus are not functioning properly (severe moments or thrusts are being developed).

12.2.7 Rolling-Element Bearings

Rolling-element bearings like ball bearings, cylindrical roller bearings, tapered roller bearings, and spherical roller bearings are frequently used in gearboxes. These bearings work well at slow speeds. They have low friction losses, particularly at slow speeds, and they position a gear part rather precisely.

At high speeds, the friction losses are not so low, and the bearing may have difficulty handling the speed. Figure 12.13 shows a general guide for speed limits on rolling-element bearings. With good lubrication, rolling-element bearings can generally live up to the limits in Figure 12.13. If the rolling-element bearing is used beyond these limits, the gear designer would be well advised to consider it a special application which a bearing specialist needs to study in depth to make sure that the bearing design features, the accuracy and quality of the bearing, and the lubrication scheme are adequate for the high-speed situation.

Rolling-element bearings are designed against fatigue limits. If a 2000-hour life is needed, a size of bearing may be picked for which calculations—on a surface fatigue basis—predict a 90% probability of running this long without serious pitting. This is called a *B-10 life*. A bearing with a B-5 life has a 95% probability of running without pitting for the time used in the calculations.

Generally speaking, gear designers know how to make the calculations to pick the right size of bearing. (Frequently, it is desirable to have the specialists at a major bearing company make the calculations and recommend the correct size and style of bearing for a critical gear application.)

When there are troubles in a gearbox with rolling-element bearings, the cause of the trouble is usually something other than that too small a bearing was used for the job. It is usually good practice, though, to review the bearing rating calculations to see if some error might have been made, or if perhaps the real load on a job in production might be higher than that anticipated back in the beginning of a project.

General areas of concern in investigating rolling-element bearings are the following:

- Fit of bearings on shaft or in housing inappropriate
- Bearings not adequately cooled and lubricated
- Bearing internal clearance wrong for the application (see Figure 12.14)
- Bearings damaged in shipment, in assembly, or in overhaul work

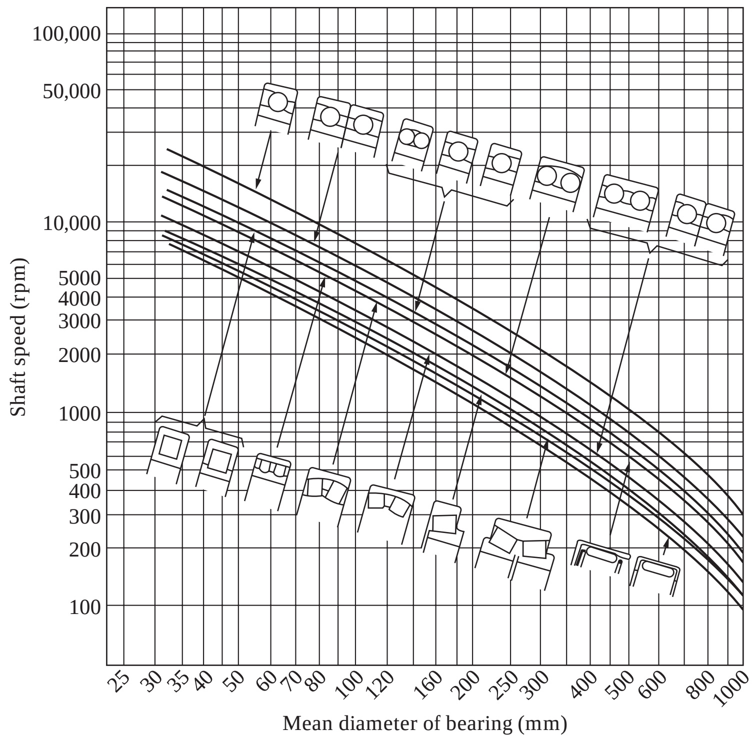


FIGURE 12.13 A general guide to when speed becomes critical in rolling-element gear bearing. With special lubrication, very high accuracy, and special materials in cages, rollers, and races, rolling-element bearings can be operated at speeds substantially higher than the limits shown. (Courtesy of SKF Industries, Inc., King of Prussia, Pennsylvania.)

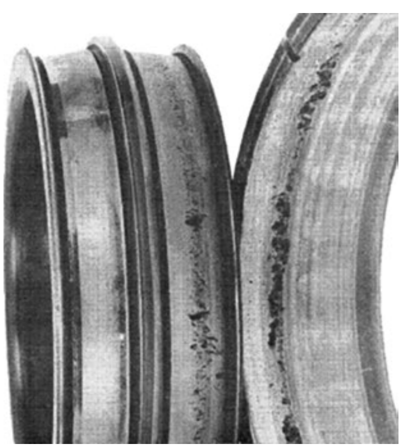


FIGURE 12.14 A rolling-element bearing failure in a gearbox due to improper internal clearance. (Failure occurred in less than 100 hours of running time.)

- Bearings not made right at the bearing factory
- Foreign material gets into the bearing
- Application of the bearing in the gearbox improper for that style of rolling-element bearing
- Accuracy of the bearing or the materials in the bearing not suitable for the high speed of the application (see Figure 12.13)

Many kinds of gears will have both a thrust load and a radial load to be carried at a given shaft end. Ball bearings, spherical roller bearings, and tapered roller bearings can all carry both thrust load and radial load. Unfortunately, there are often situations in which the combination of those loads will get bearings into trouble as wear occurs or as thermal changes shift the bearing t or clearance. Frequently, the best solution may be to use two bearings at a shaft end, with one bearing taking only thrust and one bearing taking only radial load. The common design that pairs a ball bearing for thrust with a cylindrical roller bearing for radial load is a good example of this technique.

Table 12.5 shows some of the possible faults that may exist in new bearings bought from a bearing company. The purchaser of rolling-element bearings, in general, receives a mechanical hardware item with amazing accuracy in roundness and perfection in nish for a surprisingly low price. Usually, the bearings are made correctly. Once in a while, they are not. If there is a manufacturing fault, it may be present in all bearings in a shipment of a hundred or more, because bearings are usually made on automated or semiautomated machines.

When there is trouble with rolling-element bearings, the investigator should be alert to all the areas of concern mentioned earlier. One of these is defect in manufacture of the bearings.

Table 12.5 shows some of the manufacturing defects that tend to show up once in a while in new bearings. Roundness, concentricity, and race nish might have been added to the table. Generally, these basics are all right. Table 12.5 shows some of the other things that may be wrong, and that often require a special investigation to nd.

TABLE 12.5
Defects in Manufacture of Rolling-Element Bearings

	Kind of Defect		
Kind of Bearing	**Races**	**Rolling Elements**	**Cages**
Ball bearing	• Curvature not held closely to design • Hardness too low or too high • Grain size too large • Cleanness of steel not up to specifications	• Balls not all round and of good finish • Missing ball • Nicked ball • Hardness too low • Grain size too large • Cleanness of steel not up to specifications	• Broached with a dull or worn broach • Plating defective • Composition of material out of limits
Roller bearing[a]	• Guide rail at wrong angle • Finish of guide rail inadequate • Clearance of roller in guide rails too small or too large	• Roller end out of square with roller axis • Roller crown wrong or out of position on roller • Roller corners not rounded properly • Roller ends not finished smoothly	• Shape of pocket wrong to fit roller • Pocket clearance too small or too large • Cage blocks oil entry • Cage not guided adequately

[a] All the items for ball bearings apply to roller bearings.

12.2.8 Sliding-Element Bearings

Sliding-element bearings are used for both radial and thrust loads. In some cases, the thrust bearing is integral with the radial bearing; the typical construction is a cylinder with a flange on one end. In many cases, though, the two kinds of bearings are separate. High-speed turbine gearsets used in the oil and gas industry will often have a tilting-pad thrust bearing and split-cylindrical radial bearings.

Sliding-element bearings need a certain amount of speed to run acceptably. In gear applications, the load on the bearing is steady, so there is not a fatigue life problem as with rolling-element bearings. This means that a good sleeve bearing in a gear unit will generally be able to run for many years at high speed without failure. Sliding-element bearings are quite commonly used in turbine applications because of their ability to handle heavy loads and run with high reliability for many years.

If there is going to be trouble with sliding-element bearings in a gear application, the trouble will generally show up early. In fact, a damaged or failed sleeve bearing has often been found on the very first full-speed, full-load test of a new gear design.

Some of the things that can go wrong and cause early sliding-element bearing failures are the following:

- Foreign material in oil lines or in gearbox passageways gets flushed into the bearings. (The most dangerous locations are downstream from the oil filter, but upstream from the bearing. The debris gets a chance to lodge in the bearing before it has a chance to lodge in the filter.)
- Oil nozzle is plugged, or oil passageway is not drilled.
- Clearance in bearing is too small.
- Bearing fit to casing or to shafts is improper.
- Bearings are not made correctly at the bearing factory.
- The finish, roundness, or cylindricalness of a shaft is inadequate for a radial bearing; the finish, flatness, or squareness of a thrust runner is inadequate for a thrust bearing.
- Application of bearing to gearbox is improper for that style of sliding-element bearing.
- The accuracy of the bearing or materials in the bearing is not suitable for the high speed of the bearing.

Like rolling-element bearings, sliding-element bearings are generally made correctly at the factory. Except for those used in automotive gearing, sliding-element bearings are generally not made on highly automated machinery. If something is wrong, it may be limited to a few bearings and represent some random human error or machine malfunction. Table 12.6 shows some of the defects that may occur in new bearings.

One of the problems in Table 12.6 is very simple—a bearing with the wrong direction of rotation. It would seem that this problem would be easy so avoid, but it is really quite troublesome. Bearings at opposite ends of the pinion may need to be made differently, since the thrust faces generally point toward the gear teeth. By simply exchanging the bearings from one end of the pinion to the other end, it is possible—in some gearsets—to end up with the unfortunate situation of two bearings running in the wrong direction.

Presumably, the bearings are marked with part numbers, and the assembly drawing shows which part number goes on which end. The bearing maker may put the wrong part number on a bearing, or the mechanic may pick up a bearing of the wrong part number at assembly. Either mistake can lead to a bearing being run in the wrong direction.

Sometimes, sliding-element bearings fail after thousands of hours of operation. If the bearing was made correctly and everything fit, what could lead to a delayed failure in a part not subject to fatigue failures?

TABLE 12.6
Defects in Manufacture of Sliding-Element Bearings

Kind of Bearing	Kind of Defect
Full-circle cylindrical	• Bore not concentric to outside diameter within tolerance
	• Bore or outside diameter sizes out of tolerance
	• Bonding of babbitt to backing not sound
	• Composition of babbitt, bronze, or other bearing overlay material not within speci cation limits
	• Feed grooves or bleeder grooves not accurately made
	• Edges of feed grooves not rounded properly
Split cylindrical	• All the above items apply, plus the following:
	• Bearing made for clockwise rotation, should have been made for counterclockwise rotation or vice versa
	• Curvature of bearing half wrong, so half does not t casing properly
Tapered land thrust bearing	• Lands not alike or leveled
	• Ramp curvature out of speci cation
	• Size and shape of plateau at ramp end too big or too small or misshapen
	• Ramps put in for the wrong direction of rotation
	• Feed grooves or bleeder grooves not accurately made
	• Edges of grooves not rounded properly
Tilting pad bearing	• Pads not equal height
	• Pad support pins (or other support mechanism) not tted properly

Some common things that tend to cause service failures of sliding-element gear bearings include the following:

- Wear in gear teeth or couplings cause vibration. A high vibration level fatigues out a bearing that normally would have no fatigue problem. (See Figure 12.15.)
- Shafts and gear teeth rust when a unit is shut down for a long time in a wet climate (or outdoors in a dry climate). Bearings fail due to rusted shafts. Rusted gear teeth may also cause failure due to rough running and wear debris in the oil system.
- Oil lters are inadequate or become lled with dirt and then bypass un ltered and dirty oil. Bearings become contaminated with dirt due to in-service conditions.
- The wrong oil is used, or the right oil is used too long. Tarry deposits may plug oil ori ces. The lack of antifoam protection may ood a high-speed unit that would normally drain properly and may lead to rapid overheating of bearings and gear teeth.

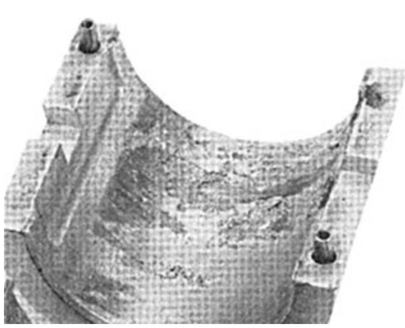


FIGURE 12.15 A sliding-element bearing failure in a high-speed gearbox. Excessive system vibration failed the bearing. Gear teeth did not fail, but they were close to failure.

- Careless overhaul of a unit may lead to nicks in journal surfaces, bearings installed with the wrong thrust shims, metering ori ces left out of oil feed lines, and so forth.

12.3 SOME CAUSES OF GEAR FAILURE OTHER THAN EXCESS TRANSMISSION LOAD

If a gear unit fails, there is a somewhat natural inclination to believe that it must have been too small to handle the nameplate rating. This may be true, and, of course, a rst step in an investigation can be a careful recheck of the power capacity for the size of gear unit involved.

Quite often, though, there is some problem not related to transmitted power. In this part of the chapter, some of the more common things that can go wrong in a gear application will be reviewed.

12.3.1 Overload Gear Failures

The designer of a gearset usually bases the gear size on a maximum torque and a continuous torque. These are ordinarily based on the amount of power that is being transmitted by the gearset under different conditions of operation. It may happen that there are momentary torque uctuations that are far in excess of the transmitted power.

On ships, there is a torque uctuation each time a propeller blade passes the rudder post. The gearset may have a high-speed turbine driving it, with a large amount of kinetic energy in the turbine rotor. If the ship's power system is not carefully designed, a very nasty torsional vibration may be excited by the slow propeller reacting against the high-speed turbine. Since the gearset is in between, it may be the victim of the con ict between the turbine trying to run at constant speed and the propeller trying to run at a variable speed.

Reciprocating engines driving generators, steam turbines driving air compressors, electric motors driving fans, and a host of other applications are subject to possible trouble from torsional vibration. In all these cases, a good overall engineering design can usually hold the torsional vibration down to reasonable values. The application factors used in gear rating formulas (see Section 5.2.4) assume that torsional vibration has been reasonably well controlled. If this has not been done, then there is the chance that the gears will be rapidly pounded to pieces by torsional vibrations far in excess of the torsional vibrations contemplated by the application factor.

Several things may be done to eliminate torsional vibrations. Changes in shaft stiffness and moments of inertia of rotating elements can change the amplitude and frequency of vibration. The stimulus may be reduced by adding blades or pistons or by making other changes to the device connected to the gearset. Going into the subject of torsional vibrations is beyond the scope of this book.

Another kind of overload is the overload due to tooth errors. Tooth errors prevent the masses of the driven and the driving apparatus from rotating at uniform velocities. The changes in velocity due to tooth errors cause momentary overloads. These loads have been commonly called the *dynamic* load. The dynamic load is made up of the transmitted load plus a load increment caused by a tooth error going through the mesh. As an error goes through the mesh, there is ordinarily an acceleration period and a deceleration period. If the impulse produced is great enough, the teeth may separate on their driving faces, and then close back together with an impact. When separation occurs, the greatest overload happens when the teeth come together again.

When gears are run at high enough speeds, the kinetic energy becomes so great that tooth errors cannot appreciably change the velocity. Under this condition, the overload is simply whatever load is required to bend the tooth out of the way (that is, the static load that would cause a deflection corresponding to the tooth error).

Earle Buckingham has published books which give methods for estimating dynamic loads on all kinds of gears. His work shows that it is quite easy to get a combination of tooth errors, moments of inertia of gears and connected apparatus, and shaft stiffnesses which results in dynamic loads five or six times as great as the transmitted load.

Dynamic load factors used in standard design equations assume relatively low dynamic loads. If designers have enough field experience with a given application, they can determine dynamic load factors which will be quite accurate for the application. It may be hard to calculate dynamic load and get a close check on field experience. The calculator has to assume an amount of error and then can quite accurately calculate shaft stiffnesses and moments of inertia. It becomes difficult, though, to follow the error through mesh and calculate the precise value of the instantaneous load at the worst point. Effects of friction, elastic dampening of the system, and wear are hard to predict. (See Section B.2 for more details on dynamic load.)

When a dynamic overload more than twice the transmitted load is calculated, the designer should try to make changes to reduce the dynamic load. This may be done by using more accurate teeth, reducing masses, using more elastic materials, or changing shaft stiffnesses. One of the greatest values of dynamic load calculations is that the designer can see which items in the design ought to be changed to reduce the dynamic load.

Another source of gear overloads is imbalance. Heavy parts running at high speed must be dynamically balanced very closely to run properly. In addition, they must be mounted on good foundations. It sometimes happens that a gearset will be tested in the shop on a heavy concrete base. Its balance will be adjusted and it will operate very smoothly. Out in the field, the set may be mounted on a rather weak support. On ships, airplanes, and locomotives, it is always hard to design a good foundation for high-speed apparatus. Vibrations may occur which in time will destroy the tooth accuracy. Then the lack of accuracy can lead to severe dynamic overloads.

12.3.2 Gear Casing Problems

Many gear failures are caused by the improper functioning of the gear casing. A gearset depends upon the gear casing, the gear bearings, and the flexible couplings to properly support it, to protect it from dirt and moisture, to protect it from reactions of misaligned driving or driven shafts, and to provide nozzles and reservoir capacity to handle a proper circulation of the lubricant.

Providing support for parallel-axis gearing is usually not difficult. Gears on right angle axes are often hard to support. If one or more members are overhung, deflection of the gear support and shafting may be quite critical. Frequently, an overhung gear part will have good contact under average load conditions, but a sudden peak load will cause so much deflection as to throw the entire load on one end of the tooth. This can cause a quick failure whenever overhung gears show damage at the ends of the teeth; it is desirable to investigate how tooth contact changes as load is increased. It may be necessary to change the tooth design so that the entire load is at one end of the tooth under light-load conditions to favor the contact when maximum load occurs. Sometimes even this will not work. When new, the set may have modifications that compensate for deflection. Long running at light load may wear away these modifications. Then, when the severe peak load occurs, there is no modification to protect the teeth from end loading. In this kind of situation, there is no alternative but to design the casing and the shafting to provide a stiffer support for the gear teeth.

The gear casing is usually made in at least two halves and bolted together. Dowels are used to properly position the casings before the bolts are tightened. There may be trouble because the dowels are too small or too few or they become worn. If the dowels do not function properly, serious misalignments of gear teeth or bearings can be the result.

Jackscrews should be provided to disassemble a gear casing. Without jackscrews, a gear casing may become bent or

damaged at the joint by pounding and prying on the casing to get it apart. Casings damaged in disassembly are apt to be misaligned when reassembled. They are also apt to have oil leaks.

Bearing wear is another cause of gear failure. Sleeve bearings may gradually increase their clearance. Ball or roller bearings may have inner races that turn on their shafts or outer races that turn in their seats. A gear supported by a pair of either type of bearings may gradually become misaligned by unequal wear of the bearings supporting it. In some configurations, even an equal amount of wear on two bearings will permit misalignment! Slight amounts of bearing wear can be quite serious. A 2.5-module (10-pitch) tooth which is of medium hardness may be seriously distressed by a misalignment of as little as 0.025 mm (0.001 in.) in the face width.

Gear rating formulas (such as those in Section 5.2.4) allow for misalignment in calculating load-carrying capacity. The gear designer tends to assume that all misalignment is due to errors in machining either the gears or the cases and seldom allows anything for further misalignment due to wear of bearings. In many cases, bearings do not wear enough to cause appreciable misalignment. In these cases wear of the gear teeth will improve contact, and there will probably be no trouble due to misalignment. However, if the misalignment increases because of bearing wear, there is often a good chance for gear failure.

When gears fail, the gear bearings are also likely to fail. In such cases, the designer should try to deduce whether the bearings failed or wore first, precipitating a gear failure, or whether the gear failure caused the bearings to fail because of overload or because bits of metal from the gear teeth got into the bearing. When both the gear teeth and the bearings have failed, it is unlikely that they both failed by coincidence at the same time. One or the other is apt to be to blame for the failure of both.

Gear failures may be caused by malfunctioning of flexible couplings. When a gear with two bearings is connected to a shaft from another device that has two bearings, a flexible coupling is needed to connect the two. In most cases, the designer cannot expect that four bearings will be so precisely in line that no flexible joint is required.

Flexible couplings are also needed to allow differential expansions between different pieces of apparatus. Steam turbines, electric motors, generators, pumps, and many other devices are commonly connected to gearsets by flexible couplings.

Flexible couplings may fail just like any other machine element. A failed coupling is not necessarily one that is broken in two. It may have become jammed so that it is no longer flexible. This can lead to the shaft supporting the gear being made to run in a misaligned position because of a reaction being transmitted through the coupling. The gear may also be jammed out of position in an endwise direction. This can upset the tooth contact on worm gears, bevel gears, or double-helical gears. Spur gears, however, usually do not suffer from endwise movement unless a difference in face width between the gear and the pinion has caused an indentation to be worn into the part with the wider face width.

In the larger sizes and at higher speeds, worn-out couplings will run with quite severe torsional shock and vibration. These things can also cause gear failures.

High-speed gears sometimes get into trouble because the gear tends to churn or pump the oil in the casing. Many oils will badly foam when churned too severely by a toothed gear wheel. Oil churning can also rapidly overheat a unit that does not have an oil cooler. The gear casing must provide a means to adequately lubricate the gears, but in addition, it must allow the oil to get out of the way of fast-moving gear teeth. Frequently, it is necessary to put a sheet metal shroud around a gear which dips below the oil level. This shroud will keep most of the oil away from the gear when it is running.

12.3.3 Lubrication Failures

Many gear failures are blamed on the lubricant, when in fact the failure is caused by a faulty mechanical design. Inaccurate teeth, poor surface finishes, lack of profile modification, and overly heavy loads for the strength of the materials may cause failures of the tooth surfaces. (Obviously such failures are not really lubrication failures but rather are design failures [or manufacturing failures].) It is true that many design failures may be remedied to a certain extent by special lubrication. However, the gear designer can get the best results only with both a good mechanical design and a good lubricant and lubricating system.

True lubrication failures usually result from one of the following troubles:

1. The lubricating oil does not have the right additives or a strong enough additive package to handle the loads, speed conditions, or temperature of the particular application. A high coefficient of friction and rapid wear are generally the results of inadequate additives.
2. The lubricating oil does not have enough viscosity to develop a suitable oil film between the contacting surfaces. In general, slow-speed gears require rather viscous oil with good chemical additives. Gears above 10 m/s (2000 fpm) can use the thinner oils quite well, and a strong additive package is usually not needed.
3. Heat developed by the gears is not removed quickly enough by the lubricating medium.
4. Wear products and corrosion on the tooth surfaces are not flushed away by the lubricant.
5. The lubricant becomes contaminated with dirt, sand, metal particles, sludge, or acids.
6. The lubricating system does not wet all the tooth surfaces before each surface goes through mesh.
7. Lubricant is not contained; it leaks out, or vaporizes and escapes. (Loss of oil may result from inadequate vents or seals, leaks in casing joints, inattention to refilling the gearbox with oil, etc.)

TABLE 12.7
Fluids Used to Lubricate Gears

Fluids	Oiliness	Where Used
Petroleum oils	Good	All types of gears except under unusual temperature conditions
Diester	Good	Aircraft and military gears with wide temperature ranges
Polyglycol	Good	Some bronze gears, steel gears at very high temperatures
Silicone	Poor	Some EP cases, light load
Water	Very poor	Some nonmetallic gears
Phosphates	Good	Aircraft hydraulic equipment

In early days, when wooden-toothed gears were common, animal fats were used as lubricants. In modern times, petroleum or mineral oils are the most widely used gear lubricants. Modern research has shown that many substances have the ability to lubricate gears. Likewise, many fluids which have the appearance and the viscosity of a lubricant may actually have little or no lubricating ability.

Table 12.7 shows some of the fluids that have demonstrated their ability to lubricate gear teeth. This table shows that several kinds of liquids may be used to lubricate gears. To date, petroleum oils have been by far the most widely used gear lubricants.

When there is a suspicion of a lubrication failure, basic lubrication practice for gear units should be reviewed. The next few paragraphs and tables give a brief review of normal practice for industrial, aircraft, and vehicle gears.

AGMA Standard 9005-E02 does a good job of specifying petroleum oils for use on various kinds of gears. This standard classifies the oils according to viscosity as measured by a Saybolt universal viscosimeter. The classifications are shown in Table 12.8.

Table 12.9 shows the recommended lubricants for industrial gears, while Table 12.10 shows the data for worm gears. Those concerned with gear lubrication problems should study the considerable amount of detailed information given in AGMA 9005-E02.

In many cases, it is not possible to follow AGMA lubrication recommendations. Frequently, the oil has to lubricate several other things besides the gears in a piece of machinery. The lubrication requirements of these other things may force the choice of lubricant to be something other than that which is best for the gear. For instance, aircraft gas turbine engines have to be able to start under arctic conditions. This makes it necessary to use much thinner oil than would ordinarily be used. Whenever thin oils are used, it is necessary to have better tooth accuracy and finish than would otherwise be required. Tooth profile modifications to compensate for bending also become more important.

Aircraft and helicopter gears usually use a synthetic (diester) type of oil. Although the oil is thin, the load-carrying capacity is quite good. Generally, the same oil is used for the engine as the gears. This dictates special properties for the oil. These are the more common oils used:

- Mil-L-6086 oil, mineral oil, about 8 cSt at 210°F
- Mil-L-7808 oil, synthetic, about 3.6 cSt at 210°F
- Mil-L-23699 oil, synthetic, about 5 cSt at 210°F

In vehicle gears, the transmission gears (used to change speed) are normally lubricated with motor oils. The wheels are driven by axle gears (also called differential gears or final drive gears when these names are appropriate). These latter units are lubricated with quite heavy oils, generally called gear oils.

TABLE 12.8
Viscosity of AGMA Lubricants

Rust and Oxidation Inhibited Gear Oils, AGMA Lubricant No.	Viscosity Range,[a] mm²/s (cSt) at 40°C	Equivalent ISO Grade[b]	EP Gear Lubricants,[c] AGMA Lubricant No.	Viscosities of Former AGMA System[d] at 100°F (SSU)
1	41.4 to 50.6	46	–	193 to 235
2	61.2 to 74.8	68	2 EP	284 to 347
3	90 to 110	100	3 EP	417 to 510
4	135 to 165	150	4 EP	626 to 765
5	198 to 242	220	5 EP	918 to 1122
6	288 to 352	320	6 EP	1335 to 1632
7 Comp[e]	414 to 506	460	7 EP	1919 to 2346
8 Comp[e]	612 to 748	680	8 EP	2837 to 3467
8A Comp[e]	900 to 1100	1000	8A EP	4171 to 5098

Note: Viscosity ranges for AGMA lubricant numbers will henceforth be identical to those of ASTM 2422.

[a] Viscosity system for industrial fluid lubricants, ASTM 2422. Also British Standards Institute, BS 4231.

[b] Industrial liquid lubricants ISO viscosity classification, International Standard, ISO 3448.

[c] EP lubricants should be used only when recommended by the gear drive manufacturer.

[d] AGMA 9005-E02.

[e] Oils marked "Comp" are compounded with 3% to 10% fatty or synthetic fatty oils.

TABLE 12.9
AGMA Lubricant Number Recommendations for Enclosed Helical, Herringbone, Straight Bevel, Spiral Bevel, and Spur Gear Drives

Type of Unit[a] and Low-Speed Center Distance	Ambient Temperature[b,c] −10°C to + 10°C (15°F to 50°F)	Ambient Temperature[b,c] 10°C to 50°C (50°F to 125°F)
Parallel Shaft (Single Reduction)		
Up to 200 mm (to 8 in.)	2–3	3–4
Over 200 mm to 500 mm (8 to 20 in.)	2–3	4–5
Over 500 mm (over 20 in.)	3–4	4–5
Parallel Shaft (Double Reduction)		
Up to 200 mm (to 8 in.)	2–3	3–4
Over 200 mm (over 8 in.)	3–4	4–5
Parallel Shaft (Triple Reduction)		
Up to 200 mm (to 8 in.)	2–3	3–4
Over 200 mm to 500 mm (8 to 20 in.)	3–4	4–5
Over 500 mm (over 20 in.)	4–5	5–6
Planetary Gear Units (Housing Diameter)		
Up to 400 mm (to 16 in.) outside diameter	2–3	3–4
Over 400 mm (over 16 in.) outside diameter	3–4	4–5
Straight or Spiral Bevel Gear Units		
Cone distance to 300 mm (to 12 in.)	2–3	4–5
Cone distance over 300 mm (over 12 in.)	3–4	5–6
Gear motors and shaft-mounted units	2–3	4–5
High-speed units[d]	1	2

Note: Ranges are provided to allow for variations in operating conditions such as surface n-ish, temperature rise, loading, speed, etc. AGMA viscosity number recommendations listed infer to rust-and-oxidation gear oils shown in Table 12.8. EP gear lubricants in the corresponding viscosity grades may be substituted where deemed necessary by the gear drive manufacturer.

[a] Drives incorporating overrunning clutches as backstopping devices should be referred to the gear drive manufacturer, as certain types of lubricants may adversely affect clutch performance.

[b] For ambient temperatures outside the ranges shown, consult the gear manufacturer. Some synthetic oils have been successfully used for high- or low-temperature applications.

[c] Pour point of lubricant selected should be at least 5°C (9°F) lower than the expected minimum ambient starting temperature. If the ambient starting temperature approaches lubricant pour point, oil sump heaters may be required to facilitate starting and ensure proper lubrication.

[d] High-speed units are operating at speeds above 3600 rpm or pitch-line velocities above 25 m/s (5000 fpm) or both. Refer to Standard AGMA 6011-I03, Practice for high-speed helical and herringbone gear units, for detailed lubrication recommendations.

Table 12.11 shows the general relation between the viscosity of SAE types of motor oils and gear oils. The motor oils have moderately strong additives. The gear oils—used in high-load, slow-speed gears—tend to have very strong additives of the EP type. Different vehicle builders have studied their special lubricant needs and in many cases have established proprietary speci cations for their products.

When thicker than normal oils are used, there is danger of oil-churning trouble or trouble in getting the oil spread across all the contacting surfaces. Also, friction losses and cooling problems become greater.

The problem of removing heat from the gears is well recognized, but there is little exact knowledge to guide either the designer or the troubleshooter. As a general rule of thumb, it takes about 4 L (1 gal) of oil per minute to remove the heat developed when 400 hp is transmitted through a gear mesh. This rule assumes that the oil circulates through the oil cooler and that the removal of heat by radiation and conduction is not

TABLE 12.10
AGMA Lubricant Number Recommendations for Enclosed Cylindrical and Double-Enveloping Worm Gear Drives

Type, Worm Gear Drive	Worm Speed[b] (rpm) Ceiling	Ambient Temperature[a] −10°C to + 10°C (15° to 50°F)	Ambient Temperature[a] 10°C to 50°C (50° to 125°F)	Worm Speed[b] (rpm) Floor	Ambient Temperature[a] −10°C to + 10°C (15° to 50°F)	Ambient Temperature[a] 10°C to 50°C (50° to 125°F)
Cylindrical Worm[c]						
Up to 150 mm (to 6 in.)	700	7 Comp, 7 EP	8 Comp, 8 EP	700	7 Comp, 7 EP	8 Comp, 8 EP
Over 150 mm to 300 mm (6 to 12 in.)	450	7 Comp, 7 EP	8 Comp, 8 EP	450	7 Comp, 7 EP	7 Comp, 7 EP
Over 300 mm to 450 mm (12 to 18 in.)	300	7 Comp, 7 EP	8 Comp, 8 EP	300	7 Comp, 7 EP	7 Comp, 7 EP
Over 450 mm to 600 mm (18 to 24 in.)	250	7 Comp, 7 EP	8 Comp, 8 EP	250	7 Comp, 7 EP	7 Comp, 7 EP
Over 600 mm (over 24 in.)	200	7 Comp, 7 EP	8 Comp, 8 EP	200	7 Comp, 7 EP	7 Comp, 7 EP
Double-Enveloping Worm[c]						
Up to 150 mm (to 6 in.)	700	8 Comp	8A Comp	700	8 Comp	8 Comp
Over 150 mm to 300 mm (6 to 12 in.)	450	8 Comp	8A Comp	450	8 Comp	8 Comp
Over 300 mm to 450 mm (12 to 18 in.)	300	8 Comp	8A Comp	300	8 Comp	8 Comp
Over 450 mm to 600 mm (18 to 24 in.)	250	8 Comp	8A Comp	250	8 Comp	8 Comp
Over 600 mm (over 24 in.)	200	8 Comp	8A Comp	200	8 Comp	8 Comp

Source: AGMA, Lubrication of industrial enclosed gear drives, AGMA 250.04, AGMA, Arlington, Virginia, 1983. With permission.
Note: Both EP and compounded oils are considered suitable for cylindrical worm gear service. Equivalent grades of both are listed in the table. For double-enveloping worm gearing, EP oils in the corresponding viscosity grades may be substituted only where deemed necessary by the worm gear manufacturer.
[a] Pour point of the oil used should be less than the minimum ambient temperature expected. Consult gear manufacturer on lube recommendations for ambient temperatures below −10°C (14°F).
[b] Worm gears of either type operating at speeds above 2400 rpm or 10 m/s (200 fpm) rubbing speed may require force-feed lubrication. In general, a lubricant of lower viscosity than recommended in the table shall be used with a force-feed system.
[c] Worm gear drives may also satisfactorily operate using other types of oils. Such oils should be used, however, only upon approval by the manufacturer.

appreciable. In sets of large size transmitting several thousand horsepower, low friction losses resulting from favorable oil-lm conditions may make it OK to use 4 L (1 gal) per minute for each 800 hp going through a gear mesh.

Many small gearsets have no oil coolers. All the heat must be removed from the casing by radiation and conduction. Thermal ratings have been developed by the AGMA for many of these applications. A thermal rating restricts the power of a gearset so that the temperature rise of the set will not exceed a safe operating temperature. Some gear casings are made with ns to help the heat transfer and thereby permit a higher thermal rating.

TABLE 12.11
Typical Vehicle Gear Lubricants

Designation of Oil	Viscosity Range (SSU) At 0°F (−18°C) Min.	Viscosity Range (SSU) At 0°F (−18°C) Max.	Viscosity Range (SSU) At 210°F (99°C) Min.	Viscosity Range (SSU) At 210°F (99°C) Max.
Motor oil SAE no.				
5W	–	4000	–	–
10W	6000	12,000	–	–
20W	12,000	48,000	–	–
20	–	–	45	58
30	–	–	58	70
40	–	–	70	85
50	–	–	85	110
Gear oil SAE no.				
75	–	15,000	–	–
80	15,000	100,000	–	–
90	–	–	75	120
140	–	–	120	200
250	–	–	200	–

Many cases of gear failure result from overheating. Gearsets may become covered with dust or ashes which insulate the set so that it cannot dissipate the heat that it should to meet its thermal rating. Even when oil coolers are used, the tubes may foul with time, and eventually the gearset may be getting oil that is not properly cooled. Poor oil nozzle design can also cause trouble. The oil nozzle must spread the oil across the face width, and it must allow the oil to contact the gear surface for a slight interval of time before it is thrown off. Some designers claim that the best cooling is achieved when oil is fed into the outgoing side of the mesh. With careful nozzle design and testing under full-speed conditions, it is usually possible to put the nozzles on the incoming side and achieve satisfactory cooling. In high-speed sets, the incoming side is often preferred because it is more certain that the teeth will be wet when they go into the mesh. In the most critical high-speed gear units, nozzles put oil on both sides of the mesh, with the largest amount going to the outgoing side.

One of the problems of lubricating gears with grease is that wear products or oxides formed by corrosion are not ushed away from the working surfaces. Gears with rough surfaces will tend to wear off the high spots and polish up. This is very desirable, except that if the lubricant becomes full of metal

particles, it will start acting as a lapping compound. This sort of trouble may be avoided by running the gears in and then changing to a clean lubricant before shipping the unit.

Sometimes, there will be appreciable wear and oxidation even after a unit is run in. Where high pressure and vibration are present, the phenomenon of fretting corrosion may occur. Fretting corrosion is ordinarily found more often in ball-bearing ts, spline joints, and hub-on-shaft connections than it is on gear teeth. However, if vibration and pressure are high enough and rubbing velocity is low, this nasty trouble can attack gear teeth. When the lubricant does not ush away the oxides formed by fretting corrosion, there can be very rapid wear. Fretting corrosion is particularly apt to attack a gearset that holds a load for a long period of time in one spot without turning. If oil, moisture, oxygen, and vibration are present, a gearset which is locked under load is very apt to fret a deep groove across the gear teeth at the spot where the teeth are in contact.

It is quite obvious that lubricants must be kept clean to prevent wear. In tractors, trucks, and locomotives, it is very hard to keep wind-blown sand or dirt out of the gear casings. In internal combustion engines, there may be trouble with the gears because the high temperatures in the cylinders keep breaking down the oil. If the oil is not changed often enough, there may be considerable acid and sludge in the gear lubricant. Gear teeth with high contact pressures cannot tolerate much deterioration of the oil, even though other parts of the engine may still be able to get by when oil is pretty well worn out.

Wetting down all the contact surfaces is often a problem. High-speed worm gears, in particular, require that all the tooth surfaces be well lubricated before they go into mesh. An oil nozzle may seem to be splashing oil all over the gear teeth. However, when the teeth are moving at high speed, the relative motion of the oil may not hit all teeth. Rapid wear and blackening of the teeth may occur.

When it appears that gear failure is the result of lack of oil wetting, several things may be done. Relocation of oil nozzles or more jets may be the answer. Sometimes it is necessary to use more pressure, so that the oil jet has more kinetic energy to overcome gear windage. A thinner oil which spreads more rapidly may be the answer.

The gear designer can best avoid lubrication failures by studying lubrication schemes that have been successfully used on a similar kind of apparatus in the past. Then if it is necessary to use something different, it is appropriate to test it before it is released for production. Experience is the best teacher when it comes to something as tricky as gear lubrication.

12.3.4 Thermal Problems in Fast-Running Gears

With high-speed gears, friction with the surrounding air can cause serious overheating. Gear teeth are projections on the surface of a cylinder or cone. When the gear body is turning fast enough, these projections tend to pump air much like the blades on a centrifugal compressor. In meshing gears, the further complication arises that the air that is between teeth must be expelled as the teeth go through mesh. A tooth entering a tooth space comes in and out of the space somewhat like a piston going in and out of a cylinder. The air between gear teeth must be expelled each time a pair of teeth meshes.

Spur teeth in parallel-axis gearing and straight bevel teeth in gearing with intersecting axes do the poorest job of ef - ciently expelling air. Helical teeth and spiral bevel teeth expel the air in an axial direction as the contact lines move axially across the face width. This is more effective.

As a general guide, spur and straight bevel teeth should not be used when the pitch-line velocity exceeds 50 m/s (10,000 fpm). The air trapping problem would become too troublesome, unless the face width is quite narrow for the module of the teeth. From the standpoints of accuracy and smoothness of meshing, spur teeth and straight bevel teeth are generally used at speeds that do not exceed about 10 m/s (2000 fpm), so air trapping is not usually a problem.

For helical teeth, the severity of the air trapping and related disturbances can be judged quite well by the axial meshing velocity. This velocity is calculated by

Axial meshing velocity = tangential meshing velocity ÷ tan(helix angle)

$$= \frac{\pi d_{p1} \times n_1}{60,000 \tan\beta} \text{ (m/s; metric)} \tag{9.1}$$

$$= \frac{\pi \times d \times n_P}{12 \tan\psi} \text{ (fpm; English),} \tag{9.2}$$

where

V_t—tangential meshing velocity (V_t = pitch circle circumference × rpm)
d_{p1}, d—pitch diameter of pinion (mm or inches)
n_1, n_P—pinion revolutions per minute
β, ψ—helix angle (°)

When the axial velocity is high, there is a thermal heating problem. The gear teeth get hot and the temperature across the face width is nonuniform. Figure 12.16 illustrates an example of this situation.

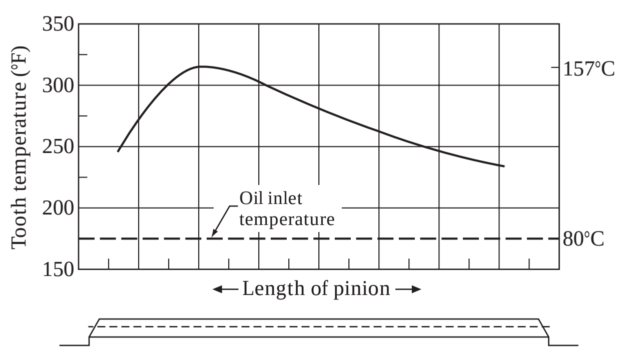


FIGURE 12.16 Temperature of pinion teeth across the face width. This is a random example of unpublished test data. Axial meshing velocity exceeded 800 m/s (160,000 fpm).

At one end of the face width, the fast-moving air will strike the wall of the gear casing and overheat the casing wall. In a bad thermal situation, the gear teeth may turn blue and the paint on the gear casing may be burned by the casing hot spot.

Gearbox failure can result from thermal trouble by any one of these things:

- Carburized gear teeth are overheated enough to soften.
- Gear teeth score or pit due to oil-film breakdown on overheated teeth.
- The thermal pattern causes enough thermal distortion to seriously alter the tooth contact pattern. Gear teeth fail as a result of nonuniform load distribution (local overloading).
- Thermal distortion of the casing misaligns gears and/or bearings.
- The gear bearing which is in the overheated area of the casing fails from overheating.

The severity of the possible thermal troubles can be judged from Table 12.12.

The thermal disturbance is a function of the square of the pitch-line velocity. This explains why a 2-to-1 change in speed means going all the way from almost no trouble to almost unmanageable trouble. (The thermal trouble has increased fourfold.)

TABLE 12.12
Severity of Thermal Problems with Helical Gears

Axial Meshing Velocity V_x		
m/s	**fpm**	**Severity**
400	80,000	No trouble except in very large units where thermal distortion may be enough to require correction. (Need good oil jet system and generous size casing)
500	100,000	Probably no serious trouble. (Need very good oil jet system and generous size casing for gear sizes)
700	140,000	Probably have some trouble. May be manageable if gears are not too large and thermal distortions are handled by compensations in tooth fit
850	170,000	Usually difficult to handle. Much skill in tooth compensations needed plus special quality of lubricant
1000	200,000	Probably impractical to handle even with utmost design skill

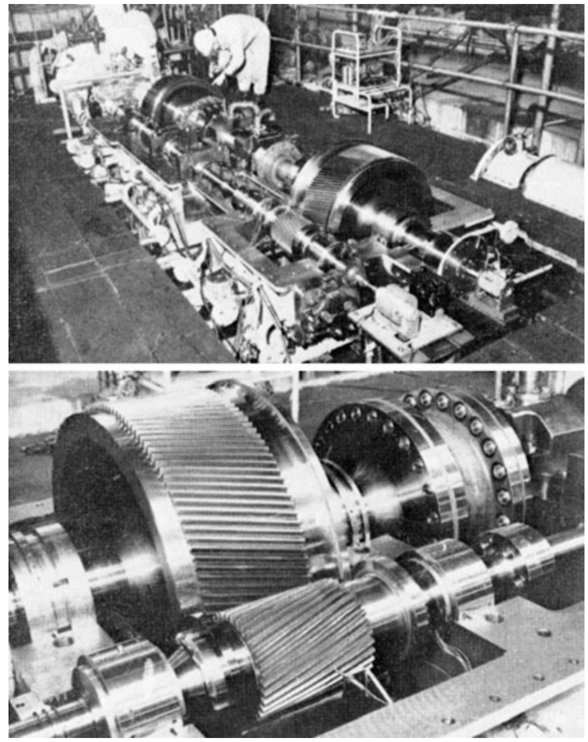


FIGURE 12.17 Test gears and test bed arrangement for ultra high–speed gear testing. (Courtesy of Ishikawajuma Harima Heavy Industries Co., Ltd., Tokyo, Japan.)

A speed of 700 m/s (140,000 fpm) is equivalent to around Mach 2! This is a very high speed. Airplane designers are very aware of the thermal heating at Mach 2. Gear designers have their own thermal problems to contend with when meshing gear teeth tend to create high Mach number air velocities.

The axial meshing velocity is, of course, a direct function of the helix angle. Table 12.13 shows the relation of the helix angle to the tangential and axial meshing velocities.

Work in the United States and Europe in the 1960s developed some initial understanding of the thermal problems in fast-running gears. In the 1970s, much more data were obtained. Figure 12.16 shows a random example of test data on a single-helical gear drive where the axial meshing velocity was high enough to make the situation difficult. Note that gear tooth surface temperatures are about 77°C (140°F)

TABLE 12.13
Relation of Helix Angle to Meshing Velocities

Tangential Meshing Velocity		Axial Meshing Velocity							
		10° Helix		15° Helix		30° Helix		35° Helix	
m/s	**fpm**	**m/s**	**fpm**	**m/s**	**fpm**	**m/s**	**fpm**	**m/s**	**fpm**
100	20,000	567	113,000	373	75,000	173	35,000	143	29,000
125	25,000	709	142,000	467	93,000	217	43,000	179	36,000
150	30,000	851	170,000	560	112,000	260	52,000	214	43,000
175	35,000	992	198,000	653	131,000	303	61,000	250	50,000

higher than the inlet oil temperature. The hot spot on the pinion causes the tooth-scoring situation to become critical. The uneven temperature across the face width leads to thermal distortion of the pinion and very poor load distribution, unless the pinion is so made that special helix modifications compensate for the potential thermal mismatch between pinion and gear.

Much of the data on thermal behavior remain unpublished. The subject is very complex, and many gear manufacturers want to get more test data and field experience before publishing papers. Some references are shown, though, at the end of the book. The IFToMM-ASME-AGMA paper by Martinaglia (1972) and the ASME paper by Akazawa et al. (1980) are probably the best papers available now to show test results and to explain some design considerations. Figure 12.17 shows a photograph of the test arrangement at Ishikawajima-Harima Heavy Industries Co., Ltd., where data for the Akazawa et al. (1980) paper were obtained.

13 Special Design Problems

The work of gear design involves the solving of special design problems. The designer may learn much about standard gear design practices and then find that most applications tend to need something special. In this chapter, we shall consider some sample problems where special gear design work is needed. The solutions to these problems will illustrate some of the many calculations that a gear specialist may need to make from time to time.

13.1 CENTER DISTANCE PROBLEMS

It often happens that spur or helical involute gears operate (or should operate) on a center distance that is different from the theoretical center distance.

The first problem is a problem that is old in the gear trade. It is called the *drop-tooth* problem.

We can imagine a simple spur gear design of a 16-tooth pinion, 20° pressure angle meshing with a 62-tooth gear. The 5-module pinion and gear were made with a standard addendum of 5 mm. The pinion was slightly undercut. Many units are in service. The pinion seriously pits in the first few hundred hours. If it is not replaced, the teeth break. The gears show very little surface distress and appear capable of running three to five times as long as the pinion. What can be done to fix this design without having to make all new gears?

The solution is to cut 15 pinion teeth on the same blank used to cut 16 teeth. One tooth is dropped. Problem 13.1 shows the calculations.

PROBLEM 13.1
Drop-Tooth Design, Fixed Center Distance

GIVEN

16 pinion teeth, 62 gear teeth.
5 module, 20° pressure angle spur gears.
Center distance is 195 mm.
Addendum of pinion and gear are both 5 mm.

REQUIRED

A redesign of the pinion to increase its load-carrying capacity. The new pinions should be able to run with gears on hand with no change in center distance.

SOLUTION

The number of pinion teeth will be changed to 15 by dropping one tooth.

The new pinion will have the same outside diameter as the original pinion.

The original pinion pitch diameter (PD), from Equation 3.3, is

$$\text{Pinion PD} = 16 \times 5 = 80 \text{ mm.} \tag{13.1}$$

The new tooth ratio, from Equation 3.7, is

$$\text{Ratio} = 62 \div 15 = 4.133333. \tag{13.2}$$

The operating pitch diameter of the new pinion, from Equation 3.5, is

$$\begin{aligned}\text{Operating pinion PD} &= (2 \times 195) \div (4.133333 + 1.0)\\ &= 75.974031 \text{ mm.}\end{aligned} \tag{13.3}$$

The theoretical pitch diameter of the 15-tooth pinion, from Equation 3.3, is

$$\text{Pinion PD} = 15 \times 5 = 75 \text{ mm.} \tag{13.4}$$

The 20° pressure angle (PA) of the original pinion came at 80 mm. Now the 20° pressure angle comes at 75 mm. The spread ratio for comparing cutting (theoretical) and operating pitch diameters for the 15-tooth pinion is

$$\text{spread ratio } m' = 75.974031 \div 75 = 1.012987. \tag{13.5}$$

The operating pressure angle can be obtained from these relations:

$$\cos(\text{operating PA}) = \cos\alpha \div m' \text{ (metric)}, \tag{13.6}$$

$$\cos(\text{operating PA}) = \cos\phi \div m' \text{ (English)}, \tag{13.7}$$

where

$$\text{Spread ratio } m' = \frac{\text{operating pitch dia.}}{\text{theoretical pitch dia.}}. \tag{13.8}$$

The calculation is

$$\cos(\text{operating PA}) = \cos 20° \div 1.012987 = 0.927645 \tag{13.9}$$

$$\text{Operating PA} = 21.929316°. \tag{13.10}$$

The outside diameter (OD) of the 16-tooth pinion is

$$\text{Pinion OD} = 80 \text{ mm} + 2(5 \text{ mm}) = 90.00 \text{ mm.} \tag{13.11}$$

TABLE 13.1
Comparison of the New Drive with Original Drive

Item	Original	Drop-Tooth Replacement
Teeth	16/62	15/62
Ratio	3.875	4.133
Operating pressure angle	20°	21.9293°
Addendum of pinion	5 mm	7.01 mm
Addendum of gear	5 mm	2.99 mm

We need to use this same outside diameter for the 15-tooth pinion, since we will mesh it with the same gear on the same center distance. The operating addendum of the 15-tooth pinion is

$$\text{Pinion addendum} = (90 - 75.9740) \div 2 = 7.01 \text{ mm.} \tag{13.12}$$

CONCLUSION AND COMMENT

The new 15/62 drive will compare with the original drive in this manner (see Table 13.1).

The new 15-tooth pinion will have a substantial increase over the old pinion in load-carrying capacity (around 20% improvement). A 3-to-1 increase in pinion life is quite certain, and the increase could be more than 5 to 1. The change in ratio is not great.

Figure 13.1 shows a layout comparison of the 15-tooth pinion and the 16-tooth pinion.

PROBLEM 13.2
Fixed Center Distance, Standard Tools

This problem is a common one of a fixed center distance and standard cutting tools that cut numbers of teeth that do not match the center distance. It is solved by cutting the parts on pitch diameters that are different from the operating pitch diameters.

GIVEN

Center distance = 154 mm
Ratio = 2 to 1

REQUIRED

A spur tooth design that can be cut with standard 20° hobs or shaper-cutters.

SOLUTION

We will find the module that will allow cutting good gears to run at 154 mm center distance.

The first step is to get the operating pitch diameters:

$$\text{Operating pinion PD} = \frac{2 \times 154}{2+1} = 102.666666, \tag{13.13}$$

$$\text{Operating gear PD} = \frac{2 \times 154}{2+1} \times 2 = 205.333333. \tag{13.14}$$

From other considerations (see Section 5.1), the number of pinion teeth ought to be in the range of 24 to 28. The shop has old 6-diametral-pitch hobs on hand and new 4-module metric hobs. (They are in the process of changing over from English standard cutting tools to metric standard cutting tools.) The 6-diametral-pitch hobs would cut 24 teeth at a pitch diameter of 4.000 in. or 101.600 mm. The 4-module hobs would cut 25 teeth at a pitch diameter of 100.000 mm. We will use the new metric hobs and 25 teeth.

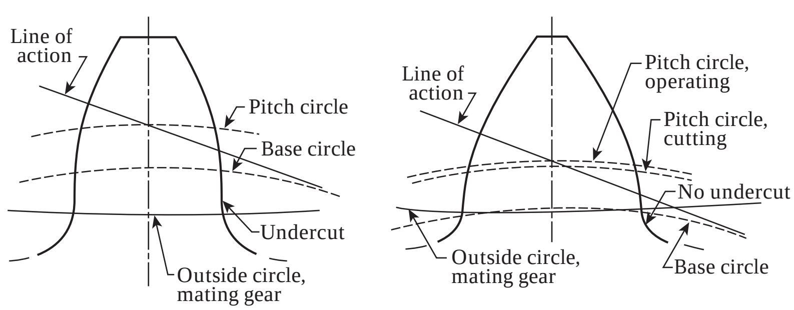
The theoretical pitch diameters become

$$\text{Pinion PD} = 25 \times 4 = 100 \text{ mm}, \tag{13.15}$$

$$\text{Gear PD} = 50 \times 4 = 200 \text{ mm}. \tag{13.16}$$

The spread ratio becomes

$$m' = 02.666666 \div 100 = 1.026666. \tag{13.17}$$



16-tooth pinion 15-tooth pinion

FIGURE 13.1 Comparison of the original, undercut, 16-tooth pinion with replacement 15-tooth pinion. Note the absence of undercut and the sturdy shape of the 15-tooth pinion.

The operating pressure angle becomes

$$\alpha' = \cos^{-1}(\cos 20° \div 1.026666) = 23.753679°. \quad (13.18)$$

We will choose an addendum ratio of 1.1 for the pinion and 0.9 for the gear. (See Table 5.8.) The operating addenda for the pinion the and gear are

$$\text{Operating pinion addendum} = 1.1 \times 4 = 4.40 \text{ mm}, \quad (13.19)$$

$$\text{Operating gear addendum} = 0.9 \times 4 = 3.60 \text{ mm}. \quad (13.20)$$

The outside diameters now become

$$\text{Pinion OD} = (2 \times 4.40) + 102.666 = 111.47 \text{ mm}, \quad (13.21)$$

$$\text{Gear OD} = (2 \times 3.60) + 205.333 = 212.53 \text{ mm}. \quad (13.22)$$

The addendum values for the pinion and the gear when hobbing are

$$\text{Theoretical pinion addendum} = (111.47 - 100) \div 2 = 5.735, \quad (13.23)$$

$$\text{Theoretical gear addendum} = (212.53 - 200) \div 2 = 6.265. \quad (13.24)$$

The nishing 4-module hob is designed slightly oversize so that when it is hobbing at full depth, the nished gear will have a reasonable amount of backlash on standard center distance. The hob produces gear teeth with an arc tooth thickness at the 20° pitch line of 6.20 mm (for 0.166 backlash).

The standard 4 module addendum is 4 mm. To cut our spread center gears, the hob is held at these amounts:

$$\text{Pinion } \Delta h = 5.735 - 4.00 = 1.735 \text{ mm}, \quad (13.25)$$

$$\text{Gear } \Delta h = 6.265 - 4.00 = 2.265 \text{ mm}. \quad (13.26)$$

The gear teeth will have larger than standard tooth thicknesses (TT) at the 20° pitch line:

$$\text{Pinion hobbing TT} = 6.20 + (2 \tan 20° \times 1.735) = 7.463, \quad (13.27)$$

$$\text{Gear hobbing TT} = 6.20 + (2 \tan 20° \times 2.265) = 7.849. \quad (13.28)$$

At the operating pitch diameters, the involute teeth will be thinner. We now need to calculate tooth thicknesses for the operating pitch line (at 23.753679°).

The tooth thicknesses at the operating pitch diameters were calculated by the use of Table B.4. In step 1, the pitch diameters for cutting were entered. The assumed pressure angle for step 10 was the operating pressure angle 23.7537°. In step 15, the tooth thicknesses at the operating pitch diameters were obtained. They are

$$\text{Arc TT pinion, operating} = 6.573 \text{ mm}, \quad (13.29)$$

$$\text{Arc TT gear, operating} = 5.881 \text{ mm}. \quad (13.30)$$

The circular pitch (CP) at 102.6666 mm is

$$\text{CP} = (102.6666 \times 3.1415926) \div 25 = 12.90147 \text{ mm}. \quad (13.31)$$

The backlash (BL) is

$$\text{BL} = 12.90147 - (6.573 + 5.881) = 0.447 \text{ mm}. \quad (13.32)$$

For 4 module, the backlash on standard pitch diameters that went with 6.20 standard TT is

$$(4 \times 3.14159) - (2 \times 6.20) = 0.166 \text{ mm}. \quad (13.33)$$

The increase in backlash is

$$\Delta\text{BL} = 0.447 = 0.166 = 0.281. \quad (13.34)$$

If this backlash change is not wanted, it can be controlled by hobbing shallow:

$$\Delta\text{depth} = \Delta\text{BL} \div 2 \tan 20° = 0.281 \div 0.728 = 0.385 \text{ mm}. \quad (13.35)$$

The gear can be hobbed shallow by ⅔ of Δ depth, 0.256 mm, and the pinion can be hobbed shallow by ⅓ of Δ depth, 0.128 mm. This will slightly increase the tooth thicknesses and keep the backlash approximately constant. (A small reduction in hobbing depth is tolerable.)

The following tabulation summarizes the results of these calculations are described in Table 13.2.

PROBLEM 13.3
Fixed Center Distance, Adjust Helix Angle, Standard Tools

Our next center-distance problem has to do with designing some double-helical gears with 30° to 35° helix angle to go on a center distance of 308 mm. The desired ratio is between 2.8 and 2.9. Again we want to use standard hobs on hand in the shop.

GIVEN

Center distance = 308 mm
Ratio = 2.8 to 2.9
Helix angle 30° to 35°

REQUIRED

A tooth design that can be cut with standard 20° pressure angle hobs.

TABLE 13.2
The Summarized Results of the Computations

	Cutting		Operating	
Item	**Pinion**	**Gear**	**Pinion**	**Gear**
Number of teeth	25	50	25	50
Pitch diameter	100	200	102.6667	205.3333
Module	4	4	4.1066	4.1066
Pressure angle	20°	20°	23.7537°	23.7537°
Outside diameter	111.47	212.53	111.47	212.53
Whole depth[a]	9.27	9.15	9.27	9.15
Arc tooth thickness at pitch line	7.463	7.849	6.573	5.881
Arc tooth thickness after whole-depth adjustment	7.554	8.031	6.667	6.068

[a] Standard whole depth for 4 module is 9.4 mm.

SOLUTION

We will nd a tooth design that can be cut with standard tools. By adjusting the helix angle, we will not need to use spread centers.

The rst step is to pick tooth numbers. These are going to be high-speed gears, hobbed and shaved at medium hardness—about 320 HV or 300 HB minimum gear hardness. This leads us to aim for 33 to 37 pinion teeth. (See Section 5.1.)

If we had 32° helix and 35 teeth with a ratio of 2.85, we would have these approximate values:

$$\text{Pinion PD} = \frac{2 \times 308}{2.85 + 1} = 160 \text{ mm}, \tag{13.36}$$

$$\text{Pinion module, transverse} = \frac{160 \text{ mm}}{35} = 4.57, \tag{13.37}$$

$$\text{Pinion module, normal} = 4.57 \times 32° = 3.875. \tag{13.38}$$

The results just obtained indicate that a hob of 4 normal module and 20° normal pressure angle will probably be OK. We will proceed on this basis.

With 35 pinion teeth, a ratio of 2.85 would require 99.75 gear teeth. We could use either 99 or 100 gear teeth. Our choice will be 99.

The ratio now becomes

$$u = \frac{99}{35} = 2.828571. \tag{13.39}$$

The pinion and gear pitch diameters are

$$\text{Pinion PD} = \frac{2 \times 308}{2.828571 + 1} = 160.89554 \text{ mm}, \tag{13.40}$$

$$\text{Gear PD} = \frac{2 \times 308}{2.828571 + 1} \times 2.828571 = 455.10446 \text{ mm}. \tag{13.41}$$

The normal circular pitch is

$$p_n = 4 \times 3.14159265 = 12.566371. \tag{13.42}$$

The transverse circular pitch is

$$\frac{160.89554 \times 3.14159265}{35} = 14.44195. \tag{13.43}$$

This will give a helix angle of

$$\cos(\text{helix angle}) = \frac{12.566371}{14.441950} = 0.8701298$$

$$\text{helix angle} = 29.5262°.$$

Our results gave too low a helix angle. Double-helical gears should have at least a 30° helix angle. We will repeat the steps using 33 pinion teeth and 94 gear teeth. (Because 97 is a prime number, 34/97 is out—we cannot factor 97 to set up the hobbing machine.)

The ratio now becomes

$$u = \frac{94}{33} = 2.848485.$$

The pitch diameters are

$$\text{Pinion PD} = \frac{616}{3.848485} = 160.06299 \text{ mm}, \tag{13.44}$$

$$\text{Gear PD} = \frac{616}{3.848485} \times 2.848485 = 455.93701 \text{ mm}. \tag{13.45}$$

The transverse circular pitch now becomes

$$p_t = \frac{160.06299 \times 3.14159265}{33} = 15.23796 \text{ mm}. \tag{13.46}$$

The helix angle is

$$\cos(\text{helix}) = \frac{12.566371}{15.23796} = 0.824675 \quad (13.47)$$

$$\text{Helix angle} = 34.444403°. \quad (13.48)$$

All design objectives have now been met. The tooth numbers of 33/94 have a common factor of 3. This will be OK, since high-speed gears do not wear with regime III conditions and reasonable tooth loading. (See Section 5.1.2 for a discussion of the hunting tooth issue.)

The transverse pressure angle is

$$\begin{aligned}\text{Transverse PA} &= \tan^{-1}[\tan(\text{normal PA}) \div \cos(\text{helix angle})]\\ &= \tan^{-1}(\tan 20° \div \cos 34.444403°) = 23.81425°.\end{aligned} \quad (13.49)$$

The results of this problem can be summarized as described in Table 13.3.

The ratio is 2.8485.

The center distance is 308 mm.

PROBLEM 13.4
Fixed Design, Standard Tools, Adjust Center Distance for Tooth Proportions

In this problem, we consider the situation in which the center distance is not fixed. The desired tooth design is what is fixed. In addition, there is a desire to use standard hobs that are on hand.

GIVEN

Tooth ratio = 2 to 1

Tooth numbers = 24 and 48

The parts are to be pregrind hobbed with 5 normal module, 20° normal pressure angle hobs. The desired helix angle is 12°, and the desired transverse pressure angle is 22.500°. After carburizing, the parts will be finish ground.

TABLE 13.3
The Results of the Computations

	Cutting and Operating Data	
Item	**Pinion**	**Gear**
Number of teeth	33	94
Pitch diameter	160.06299	455.93701
Module, normal	4	4
Module, transverse	4.850394	4.850394
Helix angle	34.44440	34.44440
Pressure angle, normal	20°	20°
Pressure angle, transverse	23.81425°	23.81425°

REQUIRED

A design with a spread center distance to give the desired operating, transverse pressure angle.

SOLUTION

The gears will be cut 12° helix angle. The first steps are to find the transverse pressure angle for the hobbing and the transverse module (or pitch) at hobbing:

$$\text{Transverse PA} = \tan^{-1}\left(\frac{\tan 20°}{\cos 12°}\right) = 20.410311°, \quad (13.50)$$

$$\begin{aligned}\text{Transverse module} &= \text{normal module} \div \cos(\text{helix angle})\\ &= 5 \div \cos 12° = 5.111702°.\end{aligned} \quad (13.51)$$

Next the spread ratio to get 22.500° operating transverse pressure angle is determined:

$$m' = \frac{\cos 20.410311}{\cos 22.500} = 1.014439. \quad (13.52)$$

The pitch diameters for manufacture (cutting and grinding) are

$$\text{Pinion PD} = 24 \times 5.111702 = 122.68086 \text{ mm},$$

$$\text{Gear PD} = 48 \times 5.111702 = 245.36172 \text{ mm}.$$

The manufacturing center distance (or theoretical center distance) is

$$\begin{aligned}\text{Theoretical center distance} &= (122.680860 + 245.361720)\\ &\div 2.0 = 184.02129 \text{ mm}.\end{aligned} \quad (13.53)$$

The spread center is the theoretical center distance multiplied by the spread ratio:

$$184.02129 \times 1.014439 = 186.67837 \text{ mm}.$$

The pinion base-circle diameter (BD) is

$$\text{Pinion BD} = 122.68086 \times \cos 20.410311 = 114.97886 \text{ mm}. \quad (13.54)$$

The pinion operating pitch diameter is

$$\text{Operating pinion PD} = \frac{2 \times \text{center distance}}{(\text{ration} + 1)} \quad \text{(metric or English)} \quad (13.55)$$

$$\text{Operating pinion PD} = \frac{2 \times 186.67837}{(48 \div 24) + 1} = 124.45224 \text{ mm}. \quad (13.56)$$

We can now check whether or not everything was done right by computing the operating transverse pressure angle. It should come out 22.500°. The calculation is

$$\text{Operating transverse PA} = \cos^{-1}\left(\frac{\text{pinion BD}}{\text{pinion operating PD}}\right) \quad \text{(metric or English)} \tag{13.57}$$

$$\text{Operating transverse PA} = \cos^{-1}\left(\frac{114.97886}{124.45224}\right) = 22.500021°. \tag{13.58}$$

The result checks OK (within the limits of four-place accuracy). As a final step in defining the gearing, we should get the lead and axial pitch of the pinion.

Pinion lead

$$\text{Pinion lead} = \frac{\pi \times \text{normal module} \times \text{no. pinion teeth}}{\sin \text{helix angle}} \quad \text{(metric or English)} \tag{13.59}$$

$$\text{Pinion lead} = \frac{3.14159265 \times 5 \times 24}{\sin 12°} = 1813.227 \text{ mm.} \tag{13.60}$$

Pinion axial pitch

$$\text{Pinion axial pitch} = \text{lead} \div \text{no. teeth (metric or English)} \tag{13.61}$$

$$\text{Pinion axial pitch} = 1813.227 \div 24 = 75.55112 \text{ mm.} \tag{13.62}$$

The pinion lead, pinion axial pitch, and pinion base diameter do not change when going from theoretical for manufacture to operating data. (The same, of course, holds true for the mating gear data).

We will now summarize the results of this problem (Table 13.4).

13.2 PROFILE MODIFICATION PROBLEMS

Profile modifications can make spur or helical gears run more quietly and carry more load. Bevel gears are generally cut or ground to achieve a localized contact pattern. The localized contact of the bevel teeth provides some easing of load at the tips of the pinion and gear teeth, which tends to accomplish the same thing as an involute profile modification in spur or helical teeth.

If spacing errors of some magnitude are present, proper profile modification will give the teeth a little clearance at the first point of contact. Figure 13.2 shows how a plus involute on the pinion will accomplish this.

As shown in Figure 13.2, a true involute design provides no clearance at the first point of contact. If a pair of teeth is spaced too close together, there is a bump as the tooth comes into mesh. With the modification, there is a little relief at the first point of contact. This makes the teeth smoothly come into mesh even if an occasional pair of teeth is too close together. The tooth action (with modification) is analogous to sliding a brick down a shingled roof. Without modification, the action is like sliding a brick over an uneven cobblestone street.

When gears are loaded heavily enough, there is appreciable bending. Even if the accuracy is perfect, bending creates interference at the first point of contact. Figure 13.3 shows how this happens.

With heavily loaded teeth, there is also trouble at the last point of contact. This comes from the fact that the Hertz stress jumps to an extreme value when the full band of contact is broken by the discontinuity of the tooth tip (see Figure 13.4).

To illustrate the kinds of modification, we shall work out two sample problems. Problem 13.5 will be for teeth of medium-precision accuracy. Problem 13.6 will be for teeth of precision accuracy used at high speed.

TABLE 13.4
The Results of the Computations

	Cutting and Grinding		Operating	
Item	**Pinion**	**Gear**	**Pinion**	**Gear**
Number of teeth	24	48	24	48
Helix angle	12° RH	12° LH	12.16814°	12.16814°
Module, normal	5	5	5.06900	5.06900
Module, transverse	5.111702	5.111702	5.18551	5.18551
Pressure angle, normal	20°	20°	22.04339°	22.04339°
Pressure angle, transverse	20.41031°	20.41031°	22.5000°	22.5000°
Pitch diameter	122.68086	245.36172	124.45224	248.90449
Base diameter	114.97886	229.95773	114.97886	229.95773
Lead	1813.227	3626.454	1813.227	3626.454
Axial pitch	75.5511	75.5511	75.5511	75.5511
Center distance	184.02129	184.02129	186.67837	186.67837

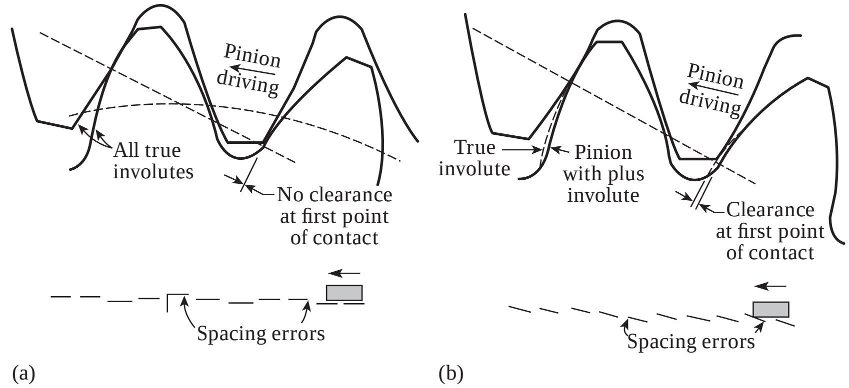


FIGURE 13.2 Plus involute provides tip clearance. (a) No clearance in true involute design and (b) a positive clearance when modified tooth profiles are engaged in mesh.

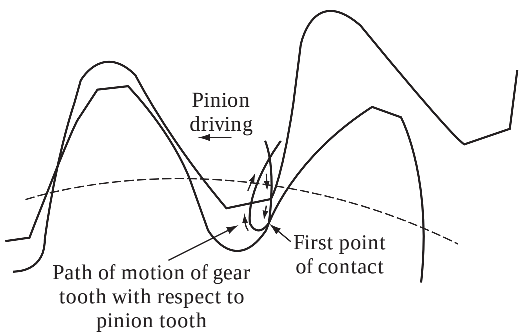


FIGURE 13.3 Interference at first point of contact due to tooth bending.

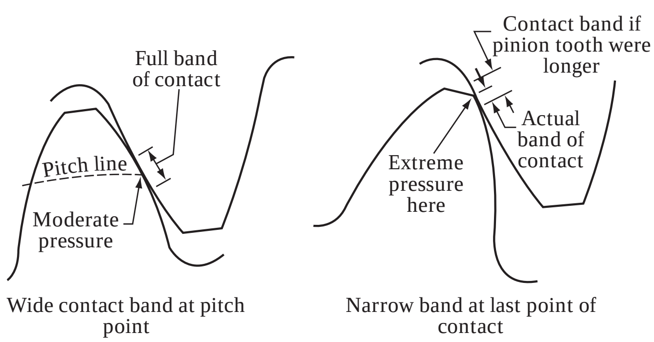


FIGURE 13.4 Severe stress caused by discontinuity of tooth tip.

Problem 13.5 will consider a single helical set of gears having large teeth and running at a typical industrial gear speed. The teeth need long life and high reliability, so they are case carburized and ground.

PROBLEM 13.5
Standard Profile Modification

GIVEN

Single helical gearset, 26 and 51 teeth.
The teeth are 20 module normal and 20° normal pressure angle.
The cutting and operating tooth data are the same.
The gearset has a center distance of 787.2 mm (30.9922 in.).
The design is for 700 mm (27.56 in.) face width.

The gear unit is designed to handle 4897 kW (6566 hp) at a pinion speed of 150 rpm. This makes the input torque 203,273.6 N · m (1,799,120 in.-lb). This load is relatively constant. A life of 30,000 hours is needed.

REQUIRED

Profile modifications that are reasonably practical to obtain in grinding and that will compensate for tooth errors and tooth deflection. The overall aim is for a smooth-running unit with no tendency for premature distress at the tip of the gear or the tip of the pinion.

SOLUTION

The modifications required are of medium criticalness. With good lubrication and good metallurgy in the gears, the design is reasonably conservative, provided that the gears, the casing, and the foundation for this unit are accurate enough to provide medium precision. A relatively standard system of profile modification will be used.

The first step is to calculate additional data items. This can be done using the methods illustrated in the preceding sample problems and the data in Sections 5.1.5, 5.1.8, 5.1.12, 5.1.13, and 5.2.2. The results are described in Table 13.5.

The next step is to estimate tooth deflection. At the design load of 1092.5 N/mm (6238 lb/in.), the approximate deflection will be (see Section. 5.2.4.3)

$$\text{Mesh deflection} = \frac{\text{tangential load}}{\text{face width}} \times \frac{1}{\text{tooth modulus of elasticity}}, \tag{13.63}$$

$$\Delta x = \frac{W_t}{b} \times \frac{1}{x_{EG}} \ \text{(mm; metric)}, \tag{13.64}$$

$$\Delta x = \frac{W_t}{F} \times \frac{1}{E_G} \ \text{(in.; English)}, \tag{13.65}$$

TABLE 13.5
The Results of the Computations

Item	Metric Pinion	Metric Gear	English Pinion	English Gear
Number of teeth	26	51	26	51
Pitch diameter	531.62 mm	1042.79 mm	20.9296 in.	41.0546 in.
Module, normal	20	20	–	–
Module, transverse	20.4468	20.4468	–	–
Normal diametral pitch	–	–	1.270	1.270
Pressure angle, normal	20°		20°	
Pressure angle, transverse	20.41031°		20.41031°	
Addendum	22 mm	18 mm	0.866 in.	0.709 in.
Whole depth	47 mm	47 mm	1.850 in.	1.850 in.
Helix angle	12° RH	12° LH	12° RH	12° LH
Face width	710 mm	700 mm	27.95 in.	27.56 in.
Torque on pinion	203,274.6 N m		1,799,120 in. lb	
Tangential force	764,733 N		171,921 lb	
Load per unit of width	1092.5 N/mm		6238 lb/in.	
K-factor	3.10 N/mm²		450 psi	
Unit load	54.62 N/mm²		7922.3 psi	
Revolutions per minute	150	76.5	150	76.5
Pitch-line velocity	4.17 m/s		822 fpm	

where

Δx—total mesh deflection in the transverse plane, tangent to the pitch circle

x_{EG}—modulus of elasticity for the teeth in mesh; x_{EG} = 20,000 N/mm² (approx. value 20° teeth, low helix angle)

E_G—modulus of elasticity for the teeth in mesh; E_G = 2,900,000 psi (20,000 N/mm² = 2,900,000 psi)

The mesh deflection calculates

$$\Delta x = \frac{1092.5}{20{,}000} = 0.056 \text{ mm } (0.00215 \text{ in.}).$$

A word of explanation on mesh deflection is in order. If we imagine the gear locked against rotation, the mesh deflection comes from the angle that the pinion will rotate when the tooth load is applied. Thus,

$$\tan(\text{rotation angle}) = \Delta x \div \text{pitch radius of pinion.} \tag{13.66}$$

The stiffness constants for mesh deflection of the teeth have never been known with certainty. Some tests are reported in the technical literature, but the data are still rather limited. The author has had some experience in making these tests. It is difficult to get good data because the gear teeth are very stiff. The value of 20,000 N/mm² is a reasonable estimate for design purposes when the teeth are cut to full depth with a generous allowance on the whole depth to accommodate shaving or grinding and there is a generous backlash allowance.

If the 20,000 N/mm² is in error, it is probably a little too small. Some data (unpublished) would indicate that this value could be as high as 25,000 N/mm² for the design problem under consideration.

In our design problem, the gears are designed with an overall load-distribution factor of 1.85. These are medium-accuracy gears. The total helix mismatch can be expected to be around 100 μm (0.004 in.). This much helix misfit makes it necessary to use a fairly high load-distribution factor. (See Figure 5.30.)

With a 1.85 load-distribution factor, the highest loaded teeth in mesh will deflect about 1.85 times as much as the average deflection. This means that a gear tooth entering mesh (first point of contact) may be out of position by about

$$1.85 \times 0.0546 = 0.101 \text{ mm } (0.004 \text{ in.}).$$

We now have a basis for setting the profile modification. At the first point of contact, we want to come fairly close to taking all the load off the tip of the gear. The tip of the gear enters the mesh at the pinion form diameter. This is a highly critical area for pitting, scoring, and noise generation. Without relief, the gear tip would actually tend to cut into the lower pinion flank. (Note Figure 13.3.)

At the last point of contact, the conditions are not so critical. At this point, we only need to partially relieve load.

The depth to the start of profile modification may be set by the general design rules of Table 5.54. (If the job is highly critical, the technique of Problem 13.6 should be followed.)

The diameter for the start of modification on the gear or the pinion is

$$\text{Start modification diameter} = \text{OD} - 2(\text{depth to start}). \tag{13.67}$$

For our case,

$$\text{Gear OD} = 1042.79 + 2(18) = 1078.79 \text{ mm},$$

$$\text{Depth to start} = 0.450 \times 20 = 9 \text{ mm},$$

$$\text{Start modification diameter, gear} = 1078.79 - 2(9) = 1060.79 \text{ mm},$$

$$\text{Pinion OD} = 531.62 + 2(22) = 575.62 \text{ mm},$$

$$\text{Depth to start} = 0.400 \times 20 = 8 \text{ mm},$$

$$\text{Start modification diameter, pinion} = 575.62 - 2(8) = 559.62 \text{ mm}.$$

The amounts of profile modification can now be set as shown in Table 13.6).

The first point modifications apply to the gear.

The last point modifications apply to the pinion.

The next problem is for a set of high-speed gears that might be used in an aerospace gear application. The example is similar to Problem 13.5 with a 26/51 tooth combination and 12°. However, the size of the tooth is 4 module instead of 20 (one-fifth the size). The aspect ratio is much smaller, 0.6 instead of 1.31. The gears and the casing will be built to high precision.

PROBLEM 13.6
Special Profile Modification to Reduce Scoring Risk

GIVEN

Single helical gearset, 26 and 51 teeth.

The teeth are 4 module, normal, and 20° normal pressure angle.

The gearset has a center distance of 157.440 mm (6.1984 in.).

The face width is 63.6 mm (2.50 in.).

The unit is designed to handle 1802.5 kW (2417 hp) at a pinion speed of 10,000 rpm. This makes the input torque 1147.5 N · m (10,156 in.-lb). At this maximum rating, 2000 hours are required. Other ratings are much lower in power.

REQUIRED

Profile modification data. This unit has a high risk of scoring, so the modifications are primarily designed to reduce the scoring hazard.

SOLUTION

Since scoring is critical, the modifications will be worked out on a somewhat different basis from that used in Problem 13.5.

The first step is to make out a tabulation of tooth data. (This tabulation is like Problem 13.5 in many respects, since the teeth are scale models of each other. See Table 13.7.)

The approximate deflection at the design load of 339.4 N/mm (1941 lb/in.) is

$$\Delta x = \frac{339.4}{20{,}000} = 0.017 \text{ mm } (0.00067 \text{ in.}).$$

These gears will be built to precision accuracy. The low aspect ratio means no serious trouble from bending or twisting. The teeth are small, though, and the load per unit of face width is not high. A load distribution of 1.35 is reasonable to expect. (See Figure 5.30.)

The tooth modification should be based on a deflection which allows for the local overload resulting from nonuniform load distribution (see Table 13.7).

At the first point of contact, the modification makes the load quite light. Tooth-to-tooth spacing errors of up to 0.0075 mm (0.0003 in.) will be allowed. The minimum modification needed can be rationalized as

$$0.80 \times (0.023 + 0.0075) = 0.024 = 0.024 \text{ mm } (0.0010 \text{ in.}).$$

TABLE 13.6
The Amounts of Profile Modification

	First Point of Contact		Last Point of Contact	
Item	**Metric**	**English**	**Metric**	**English**
Design modification	0.101 mm	0.004 in.	0.063 mm	0.0025 in.
Drawing tolerance limits	−0.084 mm	−0.0033 in.	−0.046 mm	−0.0018 in.
	−0.119 mm	−0.0047 in.	−0.081 mm	−0.0032 in.
Diameter at start of modification	1060.8 mm	41.76 in.	559.6 mm	22.03 in.
Diameter at end of modification	1078.79 mm	42.47 in.	575.62 mm	22.66 in.

TABLE 13.7
Tooth Modification

	Metric		English	
Item	**Pinion**	**Gear**	**Pinion**	**Gear**
Number of teeth	26	51	26	51
Pitch diameter	106.32342 mm	208.5575 mm	4.1859 in.	8.2109 in.
Module, normal	4	4	–	–
Module, transverse	4.089362	4.089362	–	–
Normal diametral pitch	–	–	6.35	6.35
Pressure angle, normal	20°		20°	
Pressure angle, transverse	20.4031°		20.4031°	
Addendum	4.4 mm	3.6 mm	0.173 in.	0.142 in.
Whole depth	9.6 mm	9.6 mm	0.378 in.	0.378 in.
Helix angle	12° RH	12° LH	12° RH	12° LH
Face width	65.6 mm	63.6 mm	2.58 in.	2.50 in.
Torque, on pinion	1147.5 N m		10,156 in. lb	
Tangential force	21,585 N		4852.5 lb	
Load per unit of width	339.4 N/mm		1941 lb/inch	
K-factor	4.82 N/mm^2		700 psi	
Unit load	84.85 N/mm^2		12,325 psi	
Revolutions per minute	15,000	7647.09	15,000	7647.09
Pitch-line velocity	81.58 m/s		16.059 fpm	

The maximum modi cation at the rst point will have to allow for the tolerance in grinding. This would be about 0.010 mm (0.0004 in.) for precision accuracy.

The range is then 0.024 to 0.034 mm pro le modi cation for the tip of the gear. This means that only a very light load is carried at the rst point of contact, even when the worst spacing errors go through the mesh and the worst helix mismatch permitted by the tolerances is present.

At the last point of contact, conditions are slightly less critical. A constant of 0.60 is probably OK:

$$0.60(0.023 + 0.0075) = 0.018 \text{ mm } (0.0007 \text{ in.}).$$

The range of modi cation for the last point is 0.018 to 0.028 mm. This modi cation is at the tip of the pinion.

For smooth tooth action at high speed, the contact zone should have an arc corresponding to one tooth interval that is unmodi ed. This means that a distance corresponding to one base pitch (BP) should be unmodi ed. The base pitch is

$$BP = 3.14159 \times 4 \times \cos 20.41031 = 11.777 \text{ mm } (0.4637 \text{ in.}).$$

To nd out where to start the modi cation, a layout can be made or calculations can be made using the method shown in Table 5.11. For this problem, we will make a layout that is ve times the size. (Since the teeth of Problem 13.6 are ve times to one time smaller than the teeth of Problem 13.5, a layout ve times the size is the same size as the teeth in Problem 13.5.)

Figure 13.5 shows the layout just described. All dimensions are laid out ve times the size. The zone of action (ZA) scales 97.2 mm. The base pitch at ve times the size is 58.88 mm.

For this problem, we will put the unmodi ed region in the center of the zone of action. This makes the modi ed zone (MZ) at each end become

$$MZ = (97.2 - 58.9) \div 2 = 19.15 \text{ mm.}$$

We can now scale the radial distance from the diameter at start of modi cation (SD) to the outside diameter. The results are

$$\text{Depth to start of modi cation, gear} = 8 \text{ mm } (8 \div 5 = 1.6),$$

$$\text{Depth to start of modi cation, pinion} = 9 \text{ mm } (9 \div 5 = 1.8).$$

The outside diameters for our problem are

$$\text{OD gear} = 208.557 + 2\,(3.6) = 215.757 \text{ mm},$$

$$\text{OD pinion} = 106.323 + 2\,(4.4) = 115.123 \text{ mm}.$$

The diameters at the start of modi cation are

$$\text{SD gear} = 215.757 - 2(1.6) = 212.557 \text{ mm},$$

$$\text{SD pinion} = 115.123 - 2(1.8) = 111.523 \text{ mm}.$$

In this problem, we will do the extra work to convert the pro le modi cation data to an involute pro le diagram, commonly called a *K-chart*.

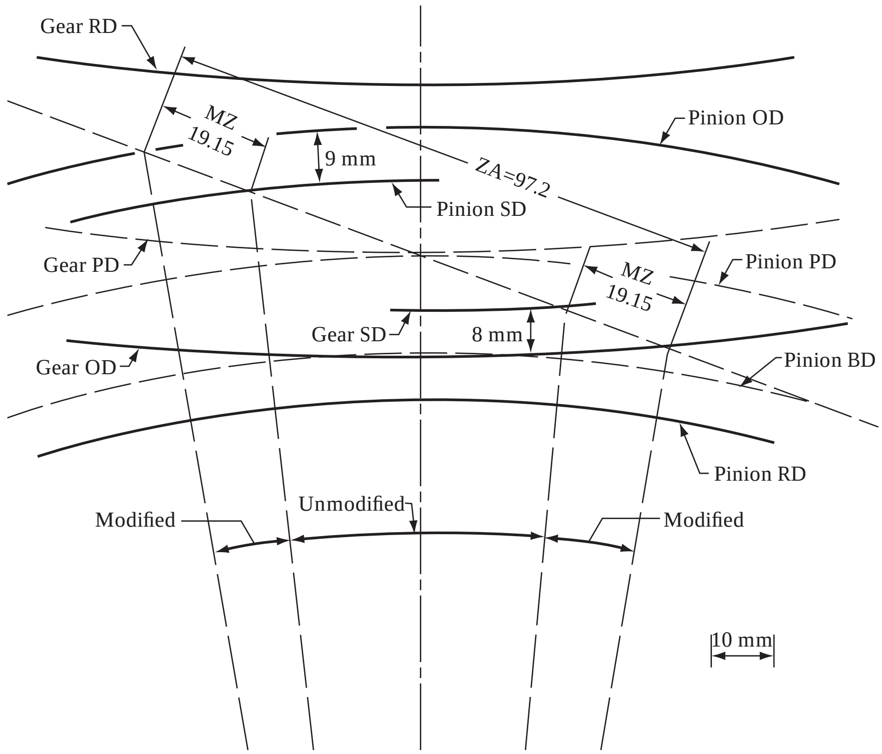


FIGURE 13.5 Layout of Problem 13.6 at ve times the size. One base pitch is unmodi ed and located in the center of the zone of action.

We now need roll angles to go with the outside diameters, the diameters at the start of modi cation, and the form diameter for the pair. Table 13.8 has been worked through to give these values. (See Sections 5.1.8 and 5.1.9 for the equations used in Table 13.8.)

Now that all the data are available, the K-charts can be made for the pinion and the gear. Figure 13.6 shows these charts. The modi cations are between the start modi cation diameter SD and the outside diameter. The region from the form diameter (FD) to the start diameter has a slope tolerance. (For more on tolerances, see Section 10.4.1.)

13.3 LOAD RATING PROBLEM

Our next problem will involve load rating a set of vehicle gears. The gears in a drive system with a standard engine have been satisfactory. Now a newly developed engine with 40% more power capacity is available. It is hoped that the present gear drive can be made somewhat better and that it can be used with little or no change with the new engine.

PROBLEM 13.7
Load Rating Vehicle Gears

GIVEN

A spur gearset, 18 and 85 teeth.
The teeth are 4 module and 25° pressure angle.
The cutting and operating data are the same.
The center distance is 206 mm.
The pinion addendum is 4.6 mm. Gear addendum is 3.4 mm.
The face width is 40 mm.
The teeth are case-carburized.

REQUIRED

This gearset needs to handle the following load schedule with the new engine. The problem is to evaluate the suitability of the given gearset.

SOLUTION

This kind of problem is best solved by a load histogram approach that compares calculated tooth stresses with a family of curves showing the estimated capability of the gear material to carry stress. (See Table 13.9.)

Table 13.10 shows a load-rating analysis made for the second-gear torque of 800 N · m (7080.6 in.-lb). The geometry factors are obtained from Sections 5.2.3 and 5.2.5. The derating factors are based on vehicle practice. (See Section 5.2.10.)

After the contact stress and the bending stress are obtained, it is easy to obtain the stresses for other torque values. The contact stress is proportional to the square root of torque. The bending stress is proportional to torque. Of course, the derating factors must stay constant if these proportions are to work.

The values of stress for the range of loads are described in Table 13.11.

The next step is to plot the load histograms. Stress is plotted against cycles, starling with the highest stress condition.

TABLE 13.8
Calculation of Form Diameter, Roll Angles, and Contact Ratio

Data Item or Operation		1 Pinion	2 Gear
1	No. of teeth	26	51
2	Pitch diameter (PD)	106.3234	208.5575
3	Addendum	4.40	3.60
4	Outside diameter (OD), (2) + 2.0 × (3)	115.123	215.757
5	Pressure angle, transverse	20.41031	20.41031
6	Base diameter, cos(5) × (2)	99.6483	195.4641
7	Gear ratio, $(1)_2 \div (1)_1$	1.96154	1.96154
8	Inverse ratio, 10 ÷ (7)	0.509804	0.509804
9	$\cos^{-1}[(6) \div (4)]$	30.050867	25.049061
10	OD roll, tan(9) × 57.29578	33.147592	26.777214
11	PD roll, tan(5) × 57.29578	21.319846	21.319846
12	Addendum roll, (10) − (11)	11.827746	5.457368
13	Pinion roll, $(12)_1 + [(12)_2 \times (7)]$	22.532592	–
14	Gear roll, $(12)_2 + [(12)_1 \times (8)]$	–	11.487200
15	LD roll, pinion, (10) − (13)	10.61500	–
16	LD roll, gear, (10) − (14)	–	15.290014
17	$\tan^{-1}[(15) \div 57.29578]$	10.49599	–
18	LD pinion (6) ÷ cos(17)	101.34402	–
19	$\tan^{-1}[(16) \div 57.29578]$	–	14.941817
20	LD gear (6) ÷ cos(19)	–	202.30438
21	Circular pitch, π × (2) ÷ (1)	12.8471	12.8471
22	Extra involute, 0.016 × (21)	0.2055	0.2055
23	Form diameter (FD), pinion (18) − (22)	101.1385	–
24	$\cos^{-1}[(6) \div (23)]$	9.847735	–
25	FD roll, pinion = tan(24) × 57.29578	9.945866	–
26	Form diameter, gear (20) − (22)	–	202.0989
27	$\cos^{-1}[(6) \div (26)]$	–	14.721915
28	FD roll, gear = tan(27) × 57.29578	–	15.054692
29	Roll per tooth, 360° ÷ (1)	13.84615	–
30	Contact ratio, $(13) \div (29)_1$	1.6273	–
31	Start modi cation diameter (SD)	111.523	212.557
32	$\cos^{-1}[(6) \div (31)]$	26.68073	23.13461
33	SD roll, tan(32) × 57.29578	28.7926	24.4796

Note: For metric calculations use millimeter dimensions, and for English calculations use inches. See Section 5.9 for cases when (22) may be too much extra involute.

For our problem, stall is the highest, low gear is second highest, and second gear is third highest. The cycles to plot then become the following:

Stall	1000 cycles
Low gear	1000 + 400,000 = 401,000 cycles
Second gear	401,000 + 2,000,000 = 2,401,000 cycles
Third gear	2,401,000 + 2,900,000 = 5,301,000 cycles

Figure 13.7 shows the resulting load histogram for bending, and Figure 13.8 shows the load histogram for contact stress.

The histograms now reveal which load conditions are most critical. For bending, low gear and second gear are the ones to worry about. For contact stress, it is second gear and third gear.

For our problem, we can assume that it was necessary to have L3 reliability against breakage. This means that any gear in the train of gears would have at least a 97% probability of surviving the full load schedule without tooth breakage.

Figure 13.7 shows that grade 2 material is needed to get the desired reliability in the pinion. If a load histogram had been made for the gear, it would have shown that the lower cycles and lower bending stresses for the gear made it able to get by with grade 1 material.

In regard to the hazard of serious pitting, we will assume that L10 reliability is satisfactory. (If 10% of the gears are badly pitted, they can be replaced during the life of the unit

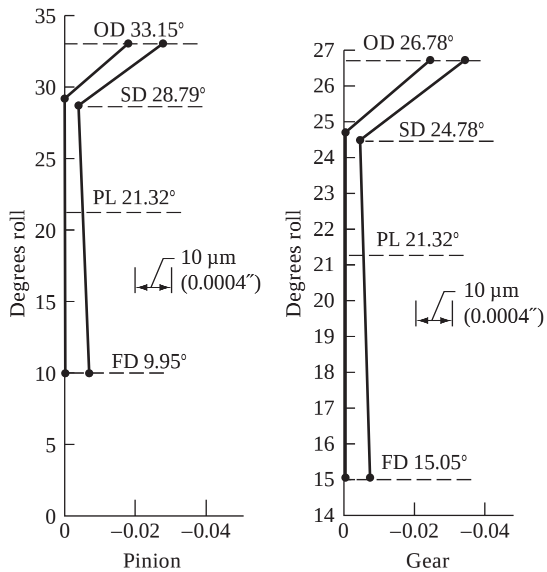


FIGURE 13.6 Involute charts for Problem 13.6.

or continued in service with a considerably increased risk of tooth breakage.)

Figure 13.8 shows that grade 1 material might be acceptable. Again, a load histogram for the gear would show that the lower cycles helped the gear even though the contact stresses are the same for pinion and gear.

Before deciding what to do, we need to review the probable accuracy of our calculations. There may be trouble from several sources:

- The assumed derating factors may be too low (poor mechanical design).
- The lubricant may be inadequate.
- It may not be possible to get the speci ed material quality. (Grade 1 was speci ed, but lack of metallurgical skill and controls yield only grade 0.)

At the outset, it is quite certain that a low derating factor like 1.4 or 1.3 can be obtained only in heavily loaded vehicle gears where tooth de ections and wear-in of the surface will give a good load distribution across the face width. Our histograms are in some error for the fourth gear, light loads. We can see, though, that fourth gear is not close to governing our design decision, so we will not take the time to redo the histograms for a more accurate derating of the fourth-gear load condition.

In regard to lubrication, low-gear and second-gear conditions have such a low pitch-line velocity that they are close to being out of regime II operation and into regime I. Our contact–stress curves in Figure 13.8 are drawn for regime II. If we really got into regime I, the contact–stress curves would have a steeper slope and this size of gear would not be adequate.

In regard to material, the gear trade is still learning how to control and rate gear quality (see Section 6.2.2 for a discussion of gear quality grades). Some manufacturers are making a bona de grade 2 quality, but they are apt to be short of knowledge concerning the detrimental effects of either accidental or intentional changes in the details of process procedures. (At any time, grade 2 in production might lapse into grade 1.)

With all this in mind, the solution to Problem 13.7 adds up to the following:

1. The load schedule of the new engine requires a design for grade 2 quality. (Even if the product is not as good as the design, it will still probably be somewhat better than grade l.)
2. The mechanical design for casings, shafts, bearings, and gear mounting must follow the best proven practice for vehicle design. A derating factor of 1.3 for strength will not be obtained at heavy load with a poor design. (Was the old design a good one or marginal with the old engine?)
3. A strong EP lubricant with high viscosity must be used. Lubrication studies may show that an SAE grade 140 gear oil is needed at the top operating temperature. (If really good oil cannot be used, it is probably best to make new gears that are large enough to survive under regime I conditions and the increased torque of the new engine.)

TABLE 13.9
The Load Schedule

Load Condition	Pinion Speed (rpm)	Pinion Torque		Hours	Cycles
		Metric (N m)	English (in. lb)		
Stall engine	(slow)	1200	10,620.9	–	1000
Low[a] gear	254	920	8142.7	26	4×10^5
Second gear	300	800	7080.6	111	2×10^6
Third gear	382	630	5576.0	127	2.9×10^6
Fourth gear, maximum	573	430	3805.8	600	2×10^7
Fourth gear, cruise	620	260	2301.2	2000	7.4×10^7

[a] A shifting transmission is ahead of this nal drive set of gears.

TABLE 13.10
Gear Rating Analysis

		Metric		English	
	Given Data				
1	Identi cation of mesh	Second gear		Second gear	
2	Calculation dimensions	Metric		English	
3	Column	1	2	1	2
4	Number of teeth	18	85	18	85
5	Module, normal (cutting)	4.000		–	
6	Diametral pitch, normal (cutting)	–		635	
7	Pressure angle, normal (cutting)	25°		25°	
8	Helix angle (cutting)	0°		0°	
	Operating Data				
9	Center distance	206 mm		8.11024 in.	
10	Face width	42 mm	40 mm	1.6535 in.	1.5748 in.
11	Pinion and gear (rpm)	200	42.35	200	42.35
12	Input power	16.755 kW		22.469 hp	
	Calculation of Dimensions				
13	Gear ratio, $(4)_2 \div (4)_1$	4.722222		4.722222	
14	Pitch diameter (cutting)	72 mm	340 mm	2.8346 in.	13.3858 in.
15	Pitch diameter, (operating)	72 mm	340 mm	2.8346 in.	13.3858 in.
16	Spread ratio, $m' = (15) \div (14)$	1.000	1.000	1.000	1.000
17	tan PA (cutting), tan(7) ÷ cos(8)	0.4663077		0.4663077	
18	PA (cutting), $\tan^{-1}(17)$	25°		25°	
19	cos PA (operating), cos(18) ÷ (16)	0.9063078		0.9063078	
20	PA (operating), $\cos^{-1}(19)$	25°		25°	
21	Addendum	4.60	3.40	0.181	0.134
22	Whole depth	9.12	9.12	0.359	0.359
23	Outside diameter, 2.00 × (21) 4 + (15)	81.20	346.80	3.1966	13.6535
	Calculation of Rating Data				
24	Pinion torque	800 N m		7080.6 in. lb	
25	Tangential driving force	22222 N		4995.8 lb	
26	Load per unit face width	555.5 N/mm^2		3172.4 psi	
27	[(13) + 1.000] ÷ (13)	1.2118		1.2118	
28	*K*-factor	9.35		1356.2	
29	C_d (overall derating)	1.4		1.4	
30	C_k (geometry factor)	473.3		5700	
31	s_C (contact stress)	1712.3 N/mm^2		248,370 psi	
32	Unit load	138.87		20144	
33	K_d (overall derating)	1.3		1.3	
34	K_t (geometry factor)	2.29	2.05	2.29	2.05
35	s_t (bending stress)	413.43	370.09	59,970	53,680
36	Pitch-line velocity	0.754 m/s		148.4 fpm	

TABLE 13.11
Stresses for the Range of Loads

	Contact Stress		Root Stress	
Loading Condition	**N/mm²**	**psi**	**N/mm²**	**psi**
Start-up	2097.2	304,200	620.1	90,000
Low gear	1836.3	266,350	475.4	68,960
Second gear	1712.3	248,370	413.4	59,970
Third gear	1519.6	220,400	325.6	47,230
Fourth gear, maximum	1255.4	182,090	222.2	32,230
Fourth gear, cruise	976.2	141,590	134.4	19,490

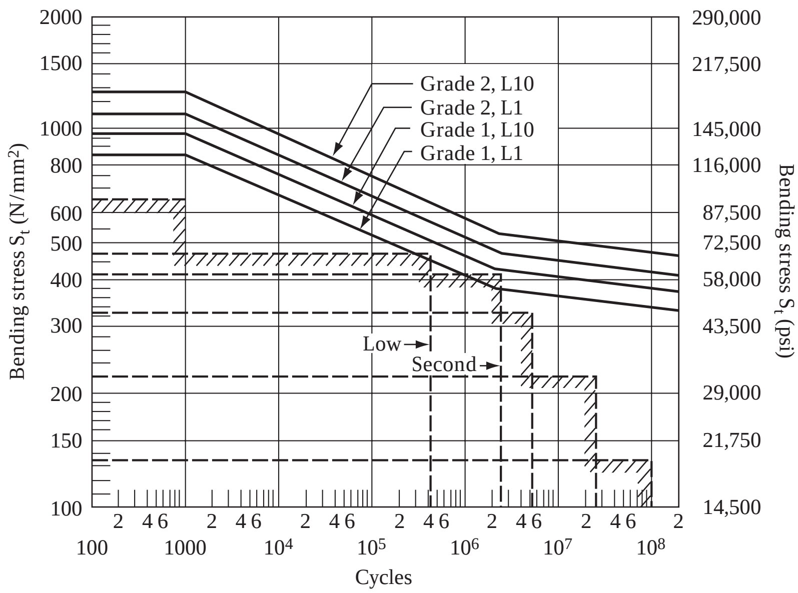


FIGURE 13.7 Histogram plot of pinion bending stress for Problem 13.7. Low gear and second gear are the most critical.

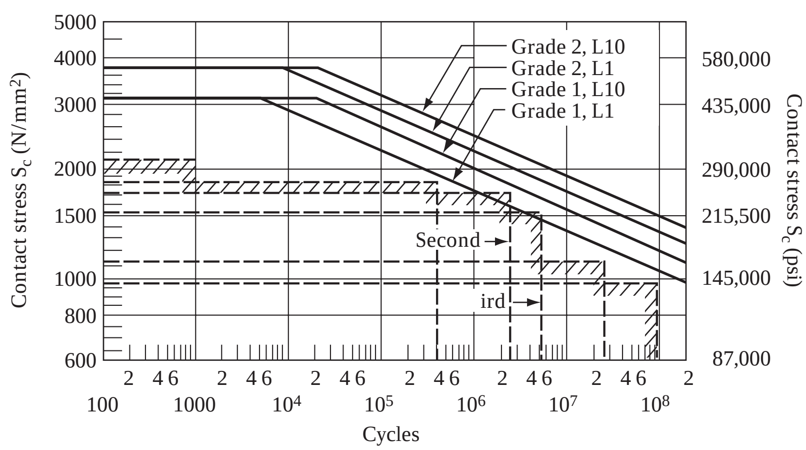


FIGURE 13.8 Histogram plot of pinion contact stress for Problem 13.7. Second gear and third gear are the most critical.

14 Gear Reactions and Mountings

It is often assumed that gear engineers are primarily concerned with toothed wheels that mesh with each other. The gear engineer, of course, has to understand and deal with the design application of gear teeth. In a broad sense, though, the gear engineer has to be concerned with the whole gearbox. For instance, the bearings that support the toothed gear wheels and the mountings of gears on shafts are a very important part of the technology of gear units. This chapter covers how to determine the reaction forces that come from typical loaded gears and it also covers typical gear mounting arrangements.

14.1 MECHANICS OF GEAR REACTIONS

The main function of a gear is to transmit motion and/or power. The main function of the supporting body is to neutralize or create a state of equilibrium. Since a gear is a rotating or moving body, a state of dynamic equilibrium must be obtained.

To be in dynamic equilibrium, all the reactions from the rotating gear must be neutralized by equal and opposite forces supporting the gear shaft. All couples and moments from the gear or the power source must also be contained. In essence, the total work forces and moments must be equal the total forces and work out. Only the most basic form of dynamics is considered. It is realized that shaft unbalance, accelerating and decelerating changes, and other various dynamic forces exist. Normally these are not an important factor. Gears generally rotate about a near-uniform center axis which always tends to pass through its center of gravity and percussion; gears and connecting bodies reach a state of relative constant velocity. Only short periods of extreme acceleration or deceleration exist. However, these conditions must be considered and taken into account when they form the major loading conditions. They will not be considered further here.

In this section, a brief review is made to the basic mechanics used in gear reaction calculations. The units to be used may be either English units or metric units. Table 14.1 shows the units normally used.

To have dynamic equilibrium, all the laws of dynamics must be obeyed and all forces must have direction, magnitude, and point of application. This is shown by a graphical or a pictorial representation by means of a line with an arrowhead. The arrow denotes direction; the length of the line, magnitude; and the tail, the point of origin.

14.1.1 Summation of Forces and Moments

Any number of lines of force on three planes can be resolved into one result.

Figure 14.1 shows two forces on an *XY* plane and their resultant:

$$L = \sqrt{X^2 + Y^2}, \tag{14.1}$$

$$\tan\theta_x = \frac{Y}{X}. \tag{14.2}$$

Conversely,

$$X = L\cos\theta_x, \tag{14.3}$$

$$Y = L\sin\theta_x. \tag{14.4}$$

A moment of a force is equivalent to the sum of the individual moments of its components. This is true in either coplanar or three planes.

In Figure 14.2, a moment is shown on the *XY* plane:

$$M = L \cdot d \tag{14.5}$$

$$M = X \cdot dy + Y \cdot dx. \tag{14.6}$$

These same laws hold for three planes, and for dynamic equilibrium, their total sum must equal zero except for the power transmitted or lost.

In Figure 14.3 for forces

$$L = \sqrt{X^2 + Y^2 + Z^2}, \tag{14.7}$$

where

$$\begin{aligned} X &= L\cos\theta_x, \\ Y &= L\cos\theta_y, \\ Z &= L\cos\theta_z. \end{aligned} \tag{14.8}$$

TABLE 14.1
Units Used for Gear Reactions

Item	Metric	English
Force	Newtons	Pounds
Distance	Meters or millimeters	Inches or feet
Time	Minutes or seconds	Minutes or seconds
Work	Newton-meters	Inch-pounds
Power	Kilowatts	Horsepower
Velocity	Meters per second	Feet per minute

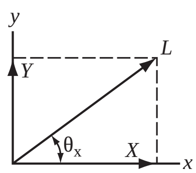


FIGURE 14.1 Vector diagram.

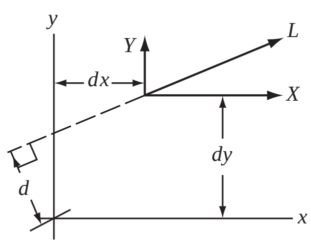


FIGURE 14.2 Moment diagram ($M = L \cdot d = X \cdot dy + Y \cdot dx$).

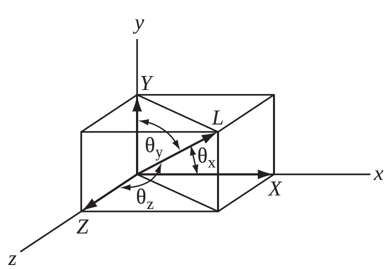


FIGURE 14.3 Three-dimensional vector diagram.

Summation of moments is shown in Figure 14.4:

$$\boldsymbol{M} = \boldsymbol{M}_x + \boldsymbol{M}_y + \boldsymbol{M}_z \tag{14.9}$$

or

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}, \tag{14.10}$$

$$\cos\theta_x = \frac{M_x}{M},$$
$$\cos\theta_y = \frac{M_y}{M}, \tag{14.11}$$
$$\cos\theta_z = \frac{M_z}{M}.$$

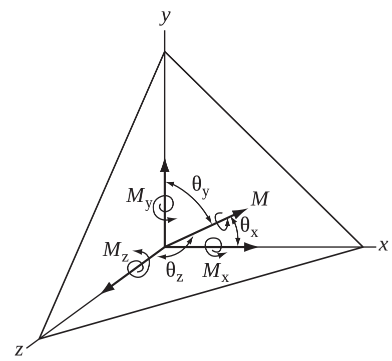


FIGURE 14.4 Three-dimensional moment diagram.

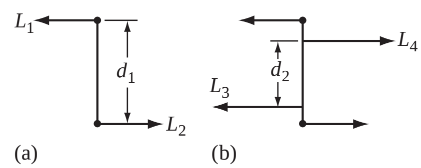


FIGURE 14.5 A couple: (a) couple and (b) couple in equilibrium.

It is also important to know two other facts pertaining to couples and moments:

1. Figure 14.5a shows a couple that has no resultant; the two forces are equal but act in opposite directions. This produces rotation, and the moment is the product of one force multiplied by the entire distance between them. Only an equal and opposite couple can balance another couple. It cannot be balanced or held stationary by a single force or reaction (see Figure 14.5b).
2. All gears rotate about an axis, and it is here that the application of moments in space can be simpli ed. No force has a moment about a parallel axis.

14.1.2 Application to Gearing

Figure 14.6a and b shows how the moments in space are made coplanar and the *X*-axis is a point. Therefore, the moment of force about the *X*-axis is due only to the sum of the *Y*- and *Z*-forces and arms.

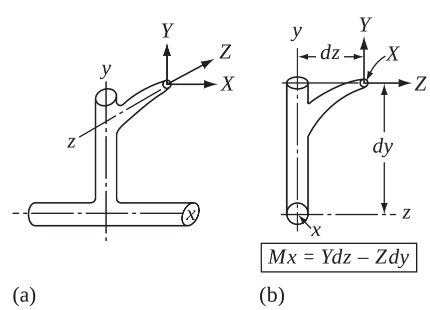


FIGURE 14.6 Force vectors at a point in space. (a) A local reference system and (b) a schematic for the torque calculation.

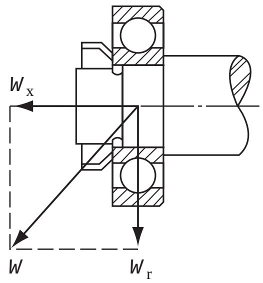


FIGURE 14.7 Load reactions at a bearing: W—combined load, W_r—radial load at 90° to axis of reaction, W_x—thrust load directly along axis of rotation.

To have equilibrium,

$$\Sigma X = 0, \quad \Sigma X = 0, \quad \Sigma Z = 0, \tag{14.12}$$

$$\Sigma M_x = 0, \quad \Sigma M_y = 0, \quad \Sigma M_z = 0. \tag{14.13}$$

Since this shows only three independent variables, there can exist a maximum of three unknown values. To have dynamic equilibrium, there cannot be more than six unknown quantities to be solved, and both sets of Equations 14.12 and 14.13 must be completely satis ed. These rules always hold true for gears rotating about any axis.

To reduce gear reactions and their resultant calculations to the simplest form, no matter how many moments or forces there are on a gear, always resolve them into two basic bearing loads, one of which is axial and the other radial, as shown in Figure 14.7.

The one basic rule that must be applied to all gear mountings and mechanics analysis is the summation of all forces must be equal to zero and the summation of all moments must equal zero.

This rule is valid for gears, those rotating steadily, with no acceleration/deceleration.

14.2 BASIC GEAR REACTIONS, BEARING LOADS, AND MOUNTING TYPES

There are various sources of gear reactions and bearing loads. These are further complicated by the manner in which the gear and its shaft are mounted.

14.2.1 The Main Source of Load

The main sources of loads are torque, reactions, weight, centrifugal forces, and vibrations. In most cases, the gear torque is the main applied load and is usually caused by power input and work being done at the output.

To determine torque or twisting moment and loads from horsepower, the amount of horsepower, the revolutions per minute, the pitch diameters of gear and pinion, and the reduction ratio must be known:

$$T_1 = \frac{P \times 9549.3}{n_1}, \tag{14.14}$$

$$T_P = \frac{P_h \times 63.025}{n_P}, \tag{14.15}$$

where
P (P_h)—power (metric [English]), kW (hp)
T_1 (T_P)—pinion torque, N · m (in. lb)
n_1 (n_P)—speed, rpm
d_1 (d)—pitch diameter, mm (in.)

The gear torque is

$$T_2 = \frac{T_1 \times \text{number of gear teeth}}{\text{number of pinion teeth}}, \tag{14.16}$$

$$T_G = \frac{T_P \times \text{number of gear teeth}}{\text{number of pinion teeth}}. \tag{14.17}$$

The tangential load at the pinion pitch diameter is designated for both metric and English units:

$$W_t = \frac{2 \times \text{pinion torque}}{\text{pitch diameter of pinion}}. \tag{14.18}$$

14.2.2 Gear Reactions to Bearing

With W_t known, it is possible to calculate the separating force W_r' and the axial force W_x. Once these are determined, the values may be vectorially added so that W_t and W_r' determine the total radial load W_r on the gear and the bearing and the axial force W_x.

As stated in Section 14.1 and shown in Figure 14.8, the total radial (W_r) and axial loads (W_x) determined in space are all that is necessary to determine total loads W on the gear teeth and reactions on bearings:

$$\mathbf{W}_r = \mathbf{W}_r' + \mathbf{W}_t \tag{14.19}$$

$$\mathbf{W} = \mathbf{W}_r + \mathbf{W}_x \tag{14.20}$$

Here, the following equalities take place: $\mathbf{W}_r = |W_r|$, $|\mathbf{W}_r'| = W_r'$, $|\mathbf{W}_t| = W_t$, and $|\mathbf{W}_x| = W_x$.

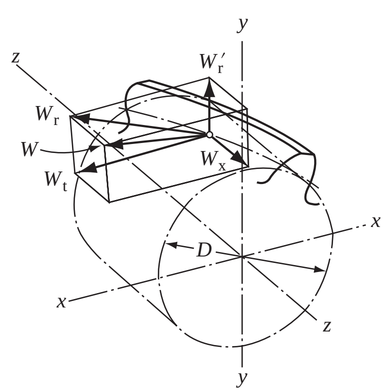


FIGURE 14.8 Tooth reactions on a gear loaded on three planes.

14.2.3 Directions of Loads

The method of calculating W_r', W_r, and W_x is found later on and is shown for spur and helical gears. There are also general rules that hold true for directions of loads on driving and driven members:

- The radial load on the bearing supporting a gear tooth that is driving is always opposite to its direction of rotation.
- The radial load on the bearing supporting a gear tooth that is being driven is always the same as its direction of rotation.

The rules for determining axial loads tend to be more complicated:

- When viewed from the axis of rotation and when the hand of spiral is left on a counterclockwise-rotating driving gear, its axial bearing thrust is away from the viewer.
- When viewed from the axis of rotation and when the hand of spiral is right on a counterclockwise-rotating driving gear, its axial bearing thrust is toward the viewer.
- When viewed from the axis of rotation and when hand of spiral is left on a clockwise-rotating driving gear, its axial bearing thrust is toward the viewer.
- When viewed from the axis of rotation and when hand of spiral is right on a clockwise-rotating driving gear, the direction of axial bearing thrust is away from the viewer.
- The direction of the driven gears' axial thrust is always directly opposite the axial thrust of the driving gear.

This can be best shown in tabular form in Table 14.2.

It should always be kept in mind that the tangential, radial, axial, and total vectorial sums of any of these same components on driving gear are always directly opposite to those on the driven gear.

TABLE 14.2
Direction of Axial Thrusts on Driving and Driven Gears

Hand of Spiral	Direction of Rotation	Driving	Driven
Left	Clockwise	Toward viewer	Away from viewer
	Counterclockwise	Away from viewer	Toward viewer
Right	Clockwise	Away from viewer	Toward viewer
	Counterclockwise	Toward viewer	Away from viewer

Note: For thrust directions on bevel and hypoid gears, see Section 14.7.

14.2.4 Additional Considerations

Consideration must be given, not only for the axial and radial bearing loads generated from the geometry of the gears, but also for the other external forces that must be supported by the shaft and the bearings. For example, spur gears should not normally generate any axial thrust, but, on a common shaft, a pump, a fan, or a propeller could be attached, generating substantial axial or centrifugal forces, and this would have to be restrained as well as the gear forces.

Transfer of power in any mechanism always results in losses due to inef ciency. In the calculations presented here, 100% ef ciency is assumed. In types of gearing with high rubbing rates, such as worms, the output power should be reduced because of the higher losses incurred.

Weights of actual gears are usually small that they are of no consequence. Direction of gravity and weight must be considered when large gears or when large masses are attached to the gear shaft. Care must also be taken in allowing for the effect of *g* or gravity loadings. In some cases values as high as 15 *g* are encountered.

Forces caused by the rotation of a mass around a center different from its own center of gravity are called centrifugal forces. Pars that are out of balance, have large external masses, or rotate in a planetary eld have centrifugal forces that must be considered. The most common formula for calculations is

$$W_{cf} = \frac{\bar{r}wn^2}{36{,}128}, \tag{14.21}$$

where

W_{cf}—centrifugal force (lb)
$\bar{r}$—radius from rotational center to center of gravity (in.)
n—rotation per minute
w—weight (lb)

or, where inches per ounce of unbalance are known,

$$W_{cf} = 1.73\left(\frac{n}{1000}\right) \times \text{inches per ounce of unbalance.} \tag{14.22}$$

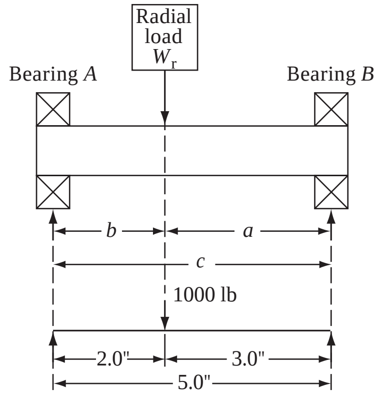
For those needing an answer in kilograms of force, the simple English formula shown above can be used, and the answer in pounds can be converted to kilogram-force by dividing by 2.20462.

All gear reactions can be resolved into tangential force, separating force, and axial thrust. The relative values of these vary in intensity depending on the type of gears, and the loads applied to the bearings are extremely dependent on the type of mounting used.

14.2.5 Types of Mountings

There are really two basic types of mountings: straddle and overhung.

In straddle mountings, as shown in Figure 14.9, the radial load is divided in inverse proportion to the distance from



Radial load on $A = \dfrac{W_r a}{c}$

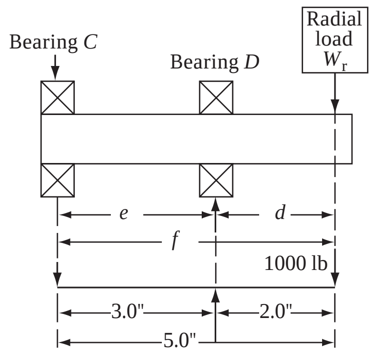
$= \dfrac{1000 \times 3.0}{5.0} = 600$ lb

Radial load on $B = \dfrac{W_r b}{c}$

$= \dfrac{1000 \times 2.0}{5.0} = 400$ lb

(600 + 400 = 1000. Check.)

FIGURE 14.9 Example of straddle mounting.



Radial load on $C = \dfrac{W_r d}{e}$

$= \dfrac{1000 \times 2.0}{3.0} = 667$ lb

Radial load on $D = \dfrac{W_r f}{e}$

$= \dfrac{1000 \times 5.0}{3.0} = 1667$ lb

(1667 – 667 = 1000. Check.)

FIGURE 14.10 Example of overhung mounting.

point of application to the total distance between shaft supports. The bearing reactions act opposite the direction of the load from the gearing.

In overhung mountings as in Figure 14.10, the load is applied outside the two supports. This produces a greater reaction or load on the bearing nearest the applied load. The load on the bearing farther away is smaller. The bearing loads are not in the same directions. The bearing reaction nearest the applied load is opposite the direction of the load. The reaction on the other bearing, farthest away, is in the same directions as the applied load.

Since, in an overhung-mounted gear, the largest bearing load is nearest the mesh, this assumes greater importance in heavily loaded optimum-designed bearing mountings. To compensate for this, generally, the bearing nearest the mesh is of a type that has the greatest radial load-carrying ability. The bearing farthest away is often a ball or an axial resistance bearing and constrains both the axial and the smaller radial loads. This method is used when there is no great temperature or deflection problems. In some cases, two different-sized bearings are used, with the greater-capacity bearing nearest the overhung member. It is wise to locate the axial thrust–retaining bearing next to the gear when large temperature variations or deflections exist. Change of rotation and potential thrust direction changes must be taken into account.

While it is necessary to balance out reactions loads, from now on, whenever a load is calculated, it will be the actual load applied to the bearing or the support in the correct direction. It will not be the bearing reaction, which opposes the load and is the opposite direction of the bearing load.

There are relative merits between the overhung- and straddle-mounted gears and shafts. The most commonly used and preferred method is the straddle mounting. It balances loads, more efficiently utilizes the bearings, and in most cases, reduces misalignment and deflection problems.

When there are extreme or unusual axial-thrust loads or special mounting problems, the overhung mounting has its merits.

14.2.6 Efficiencies

Gear efficiencies and resultant power losses can cause great variation in the actual power delivered and the load transmitted. These loss values vary from 0.5% per mesh to values as high as 80% per mesh, depending on the type of gear, lubrication, bearings, and degree of accuracy of manufacturing. It is a poor gear design or application that has efficiency of less than 50%. It is generally a nonplanar application that has great losses. Some designers allow a set value for each mesh. While 100% efficiency is assumed in all these calculations, it is always necessary at least to consider the effect of efficiency. If the power loss is low, generally, a few percentage points or less, it is best to ignore this effect on bearings and gears. Coplanar gears can be from 97% to 80% efficient. When critical, experience or actual test data should be used to determine losses.

TABLE 14.3
Approximate Gear Losses for Various Types

Different Types of Gears	Total Range of Losses per Mesh (%)
Spur and helical external	½–3
Spur and helical internal	½–3
Bevel gears	½–3
Worms	2–50
Spiroids[a]	2–50
Hypoids	2–50
Crossed helicals	2–50

Note: All nonplanar gears have great variation due to ratios, sizes, materials, lubricants, and relative sliding.

[a] Registered trademark of Spiroid Division, Illinois Tool Works, Chicago, Illinois.

As a quick guide for efficiencies, or if useful data are not available, the values in Table 14.3 are suggested for good conditions.

14.3 BASIC MOUNTING ARRANGEMENTS AND RECOMMENDATIONS

Once the bearing loads are calculated for any gear, experience, art, and common sense give some pointed recommendations and rules. These rules apply to all simply mounted gears no matter what type or shape they take. Various other rules for each specific type of gear are shown later under their appropriate heading.

Keep bearing mountings for gears as close as possible to their faces, allowing reasonable space for lubrication and arrangements. This eliminates large moments and reduces vibration problems.

Whenever possible, use only two bearings for each gear shaft. When making a center-distance layout for gear proportions, also consider shaft, spline, and bearing sizes and load-carrying abilities. These often are limiting conditions.

Whenever possible, straddle mount both members of a mesh. For straddle-mounted gears, a spread between bearings of approximately 70% of the pitch diameter is a minimum. When only one gear can be straddle mounted, it should be the gear with the highest radial load.

Overhung gears should have a spread between bearings of 70% of the pitch diameter, and the spread between bearings should be at least twice the overhang. The shaft supporting the overhung gears should be greater in diameter than the overhang (see Figure 14.11).

Idlers can often be positioned to offset and minimize bearing loads. Consideration must be given to the potential for the two meshes on the gear train setting up undesirable torsional vibration and overloads.

In general, for all types of gears, the tangential forces applied to the bearing are opposite the direction of rotation of the driving member. The separating forces are away from the tooth surfaces. The driven gear has its tangential forces and separating forces always opposite the driving pinion or the driving force. Reverse the direction of rotation, and there is a reversal of tangential driving loads, but the separating tends to remain in the same direction away from the teeth (toward the center).

Axial thrusts vary for all types of gears, but they also reverse direction with reversal of rotation except in the case of Zerol* and straight bevel gears.

If the total force on any pinion or any gear of a single mesh is found, the force on the mating gear of that mesh is directly opposite to it. When calculations are laborious, this fact can be used to establish the direction and the magnitude of the forces on the mating gear for that individual meshing point in space.

All shifting should be checked for torsional and moment load-carrying abilities. The distance between supports and shaft diameter should preferably be designed so that the operating speed is below the first critical speed. The shaft should not operate near any critical vibration frequency.

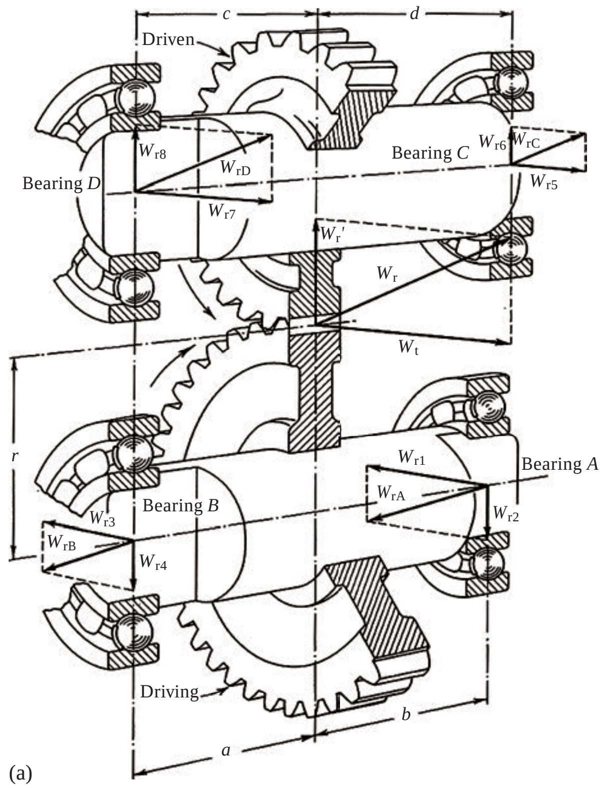
14.3.1 Bearing and Shaft Alignment

It is extremely important to minimize gear tooth mounting displacements. The bearing sizes and the shaft dimensions are determined only in part by strength and life considerations. The above-mentioned rules are good examples that should be followed in addition to utilizing strength and stress knowledge. The bearing spacing and the shaft stiffness, as well as the housing and the methods of attachments between gear, shaft, bearing, and housing, must be considered. In most applications, the deflections should not exceed 0.001 at the working surfaces between the gear and the pinion. The total amount of allowable misalignment between gear surfaces should be further investigated and calculated. The method of calculating increase stresses in spur and helical gears due to misalignment should be investigated. The total contributing factors to misalignment should not exceed the calculated or known allowable value.

14.3.2 Bearings

From the tentative gear layout, calculate the bearing loads and establish the preferred types and sizes of bearings. Actual calculations should be based on data that follow in detail each type of gearing and mounting that is involved. It is an accepted practice to preload the bearings carrying the thrust loads in order to obtain mountings with minimum total axial displacements under operating loads. The amount of preload depends on the mounting, the load, and the operating speed, and should be established in collaboration with the bearing manufacturer. At this time, the design of the gearbox with complete speed, load, and operating data should be submitted to the bearing manufacturer for approval of bearing sizes, types, bearing mountings, lubrication, and bearing life. The calculation of the bearing load capacities and the life is not included here, but such data and information are readily available from any first-rate bearing manufacturer. Many bearing manufacturers have engineering design books that are given

* Registered trademark of Gleason Works, Rochester, New York.



(a)

Forces		
	Bearing *A*	Bearing *B*
Tan force W_t	$W_{r1} = \dfrac{W_t a}{a+b}$	$W_{r3} = \dfrac{W_t b}{a+b}$
Separating force W_r'	$W_{r2} = \dfrac{W_r' a}{a+b}$	$W_{r4} = \dfrac{W_r' b}{a+b}$
Total load	$W_{rA} = \sqrt{(W_{r1})^2 + (W_{r2})^2}$	$W_{rB} = \sqrt{(W_{r3})^2 + (W_{r4})^2}$
	Bearing *C*	Bearing *D*
Tan force W_t'	$W_{r5} = \dfrac{W_t c}{c+d}$	$W_{r7} = \dfrac{W_t d}{c+d}$
Separating force W_r	$W_{r6} = \dfrac{W_r' c}{c+d}$	$W_{r8} = \dfrac{W_r' d}{c+d}$
Total load	$W_{rC} = \sqrt{(W_{r5})^2 + (W_{r6})^2}$	$W_{rD} = \sqrt{(W_{r7})^2 + (W_{r8})^2}$

(b)

FIGURE 14.11 Internal spur gear bearing loads. (a) Acting loads and (b) equation used for the calculations.

to users of their products. Selection of dimensional fits can also be learned. Generally, the most important rule is for the tight or the interference fit to be between the inner race and the shaft, and a loose or controlled slippage to be between the outer race and the housing or bearing liner.

14.3.3 Mounting Gears to Shaft

Whenever possible, the gear should be made integral with the shaft. However, because of complex shapes and bearing and mounting arrangements, and with several gears and components mounted on a single shaft, this is not always possible. When gears must be assembled onto the shaft, involute splines are often used. In mounting gears with splined bores, a cylindrical centering fit is recommended to avoid excessive eccentricity in the final mounting. The most satisfactory method is to provide a suitable length of cylindrical fit at one or both ends of the gear hub for centering purposes. The splines are then used for driving only. This method is particularly applicable in the case of involute side fit splines

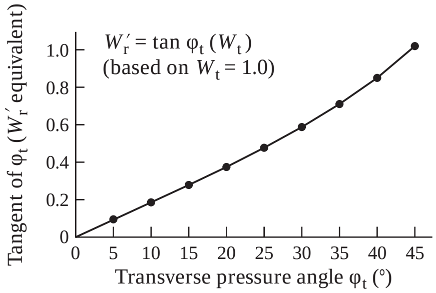


FIGURE 14.12 Gear blank designed to take care of inward thrust. Also a preferred mounting with spline drive.

that cannot be adequately centered from the major or minor diameters (see Figure 14.12). When space or length allowable is critical, the major-diameter- t involute spline is often used. The internal spline is often broached and the major diameter of the external member is ground to t the broached part. When used this way, the spline is stressed from both tangential driving torques and possible tip loading from interference ts caused by assembly or temperature contractional effects.

The present practice is to avoid the use of minor-diameter- t involute splines whenever possible. In many applications, straight-sided splines and serrations are used. Serrations and straight-sided splines are generally treated in the same manner and their rules of usage are interchangeable. Hardened gears with straight-sided splines in the bore should be centered in assembly by the bore or the minor diameter of the splines, which, after hardening, should be ground concentric with the teeth. Unhardened gears with straight-sided splines should be centered in cutting and in assembly by the major diameter of the splines. Since heat treatment may introduce distortion in all splines, it is important that the splines be of no greater length than is actually required for load transmission. In blanks with long hubs, the splines should be located as nearly as possible to the gear teeth and, if possible, directly under the gear teeth.

Keys, pins, and press t diameters are used in many gear applications. In data gearing, the loads are normally light and the mounting method is picked to give the least amount of running-index error. The integral gear and shaft is preferred, with the cylinder t of gear to the shaft next. When heavy loads and extreme accuracy requirements are needed, ingenious combinations of various basic mounting methods must be employed.

14.3.4 Housing

The function of the housing is to form a strong base in which to mount the bearings and support the gears and the shafts and to create an environment and space where a satisfactory lubricant may be introduced to lubricate and cool the gears. The housing is also used to mount and support various other components such as accessories and parts near or common to the gearing. In many illustrations, the lubrication system is entirely contained with the housing; in some oil systems, lines into and out of the housing must be supplied.

The actual design and calculation of most gearboxes can be so complicated that a general practice is to employ experience or test data or actually to submit the gear housing to loads and measure the results by dial or strain gauges. In simple-type shapes, basic stress analysis is used. With the bearings selected and the general arrangement established, it is possible to proceed with the design of the supporting structure. The design should ensure, whenever possible, that loads from external sources such as belts, propellers, and pups are carried entirely independently of the gear mountings; otherwise, the design must be made suf ciently rigid to carry these external loads without affecting the operating position of the gears. Flexible-type couplings are preferred to connect the assembly to the input and output loads in order to protect the gears from outside forces resulting from shaft de ections or misalignments.

The housing and the supports should be of adequate section and of suitable shape, with ribbing to support the bearings rigidly against the forces, as indicated by the analysis of their magnitude and directions. The choice of material for the housing depends, to a great extent, on the application. If weight is not a factor, ferrous castings or a combination of ferrous castings with weldments may be used. Where weight is very important, aluminum or magnesium castings may be used for the housing. When light metal housings are used, special design considerations must be given to minimize the effect of temperature changes on the mountings and to provide adequate rigidity under all operating conditions. Best practice calls for mounting the bearings in steel liners, either tted and pinned in the housing bores or cast in and machined in place. Also, the thrust bearings that control the position of the gears on their axes should be located close to the gear teeth. Two opposed bearings should be arranged so that temperature changes will not seriously affect the preloading. It should be noted that in order to obtain equal rigidity in a light metal casting it may be necessary to use sections of two to four times the thickness of those used in steel or iron.

14.3.5 Inspection Hole

As an aid to assembly and for periodic inspection in service, an opening should be provided in the housing to permit the inspection of the tooth surfaces of at least one member of the pair without disassembly. This is especially desirous in nonplanar axis gears or in bevel and hypoid gears.

14.3.6 Break-In

While gears and bearings are designed, mounted, and manufactured to carry their rated loads without trouble, the initial period of operation is most critical, and preferably, a break-in run at lighter loads or slower speed should be made. The load should gradually be increased until full operating load and speed are reached to check the complete gearbox.

14.4 BEARING LOAD CALCULATIONS FOR SPUR GEARS

The most common gear used is the spur type. This is true not only because of the simplicity of stress calculations, design, and manufacturing but also largely because of the ease of mounting, calculations, and retention of bearing loads. Spur gears are almost always devoid of any large self-generated axial thrusts. Their only components are the tangential driving load and its separating components, which results in a radial load only, which is the vectorial sum of these two forces (see Equation 14.19).

14.4.1 Spur Gears

In a simple spur gear, only a radial load is present and W_r can be interchangeably used with the total load *W*. Because this fact assumes greater importance in other types of gears such as helical, W_r is used here.

The tangential driving load can be calculated by Equation 14.18. The separating load is designated as W_r'. For universal usage, the separating load is a function of the transverse pressure angle or that pressure angle formed by a plane slicing through the gear tooth at 90° to the axis of rotation. In a spur gear, the normal pressure angle ϕ and the transverse pressure angle ϕ_t are one and the same. To simplify matters later on, the transverse pressure angle ϕ_t will always be used in separating and radial load calculations (for metric calculations, use $\tan\phi_t$ and use newtons for W_t instead of pounds):

$$W_r' = W_t \tan\phi_t. \tag{14.23}$$

Let W_r designates the total radial load:

$$W_r = \sqrt{W_t^2 + W_r'^2}. \tag{14.24}$$

If W_t is designated on the *X* plane and W_r on the *Y* plane, Equation 14.24 exactly agrees with Equations 14.1, which is for forces on one plane.

The values W_r, W_t, and W_r' will all be considered as originating from the pitch diameter of the pinion or the gear and in the center of its contact face, as will *W* and W_x later on.

Figure 14.13 shows a perspective of two spur gears and their respective driving or driven load.

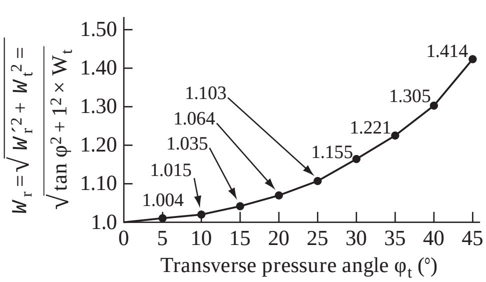


FIGURE 14.13 Spur gear bearing loads.

The pressure angle of the gear controls the effect of the separating component. The smaller pressure angle gives lesser separating and total forces, as shown in Figure 14.14. Equation 14.24 is plotted against the tangent of the pressure angle to calculate W_r proportions shown in Figure 14.15. A pressure angle of 0° to 45° represents one unit of tangential driving load in pounds.

Figures 14.9 and 14.10 and sample problems show how to calculate overhung- and straddle-mounted gear resultant bearing reactions.

Spur gears have no calculable axial thrust; however, they tend to walk or to be displaced by torque and other components. It is wise to allow for small amounts of axial movements or to restrain the gear. In heavily loaded spurs, or when spurs are common to shafts carrying other axial loads, it is a necessity to restrain or position them.

14.4.2 Helical Gears

All data shown so far have applied to external gears. However, internal gears are used in many applications. In external gears, the direction of rotation is changed in the mesh. In internal gears, the rotation between the two gears is in the same direction. In most cases, one or both of the gears are overhung. While it is possible to arrange straddle mounting, normally, space and length do not permit it. Internal gears offer the advantage of larger reduction or step-up changes in smaller center distances, but at the expense of allowable space for bearing mounting arrangements.

The procedure for calculating bearing loads is shown in Figure 14.11; W_r', W_t, and W_r are calculated before.

14.4.3 Gears in Trains

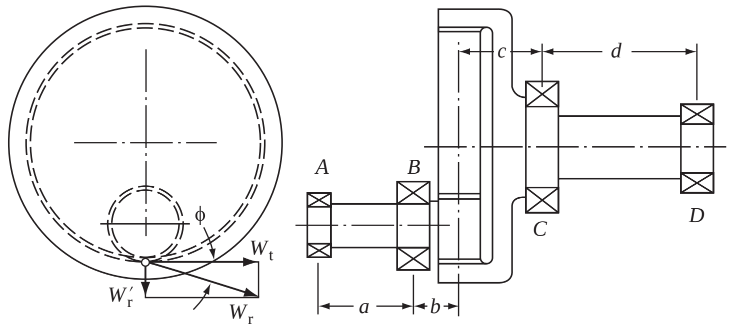
Many times, gears are in trains, either on one continuous plane with idlers or on offset planes on intermediate shafts and in planetary drives. Planetary gears are considered as a separate type of gear train.

14.4.4 Idlers

The idlers may be used to change the direction of rotation, to gain additional distance between centers with smaller and greater quantity of parts, or to provide additional mounting pads at various revolutions per minute for driven components, shafts, or accessories.

If only the input and output shafts are used and a constant horsepower is driven through the train, the load on all the gear teeth in the train is the same. When the idlers are all arranged in one straight line, the two separating components W_r' cancel the output, but the tangential driving loads are directly added. Under the aforementioned conditions, the resultant bearing load is twice W_t.

When the gear train is offset at an angle between centers, the total forces tend to combine or cancel out. For accuracy of results, a graphical or a vector analysis should be made.



(a)

Forces	Pinion loads	
	Bearing *A*	Bearing *B*
Tan force W_t	$W_{r1} = \dfrac{W_r b}{a}$	$W_{r3} = \dfrac{W_t(a+b)}{a}$
Separating force W'_r	$W_{r2} = \dfrac{W'_r b}{a}$	$W_{r4} = \dfrac{W'_r(a+b)}{a}$
Total load	$W_{rA} = \sqrt{(W_{r1})^2 + (W_{r2})^2}$	$W_{rB} = \sqrt{(W_{r3})^2 + (W_{r4})^2}$
	Gear loads	
	Bearing *C*	Bearing *D*
Tan force W_t	$W_{r5} = \dfrac{W_t(c+d)}{d}$	$W_{r7} = \dfrac{W_t c}{d}$
Separating force W'_r	$W_{r6} = \dfrac{W'_r(c+d)}{d}$	$W_{r8} = \dfrac{W'_r c}{d}$
Total load	$W_{rC} = \sqrt{(W_{r5})^2 + (W_{r6})^2}$	$W_{rD} = \sqrt{(W_{r7})^2 + (W_{r8})^2}$

(b)

FIGURE 14.14 Chart of relation W'_r for $W_t = 1.0$. (a) Acting loads and (b) equation used for the calculations.

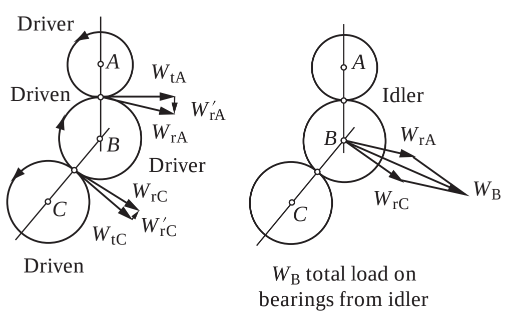


FIGURE 14.15 Chart of total radial load versus ϕ_t.

Figure 14.16 shows a vector analysis of idler loads. If in Figure 14.16 the angle on *BC* in relation to *AB* has been acute instead of obtuse, the reaction *WB* would have been much smaller in magnitude. If it is possible, proper selection of rotation and angles of intersection can be adjusted to reduce the total bearing loads generated in an idler gear train.

14.4.5 Intermediate Gears

The other most common gear train in spur gears is the intermediate shaft arrangement. Its main advantage is the combination of greater gear ratio reduction and a gain in center distances for mounting various combinations.

There are basically two gear meshes but only three shafts instead of four. Also, the planes of the mesh are parallel but separated (see Figure 14.17).

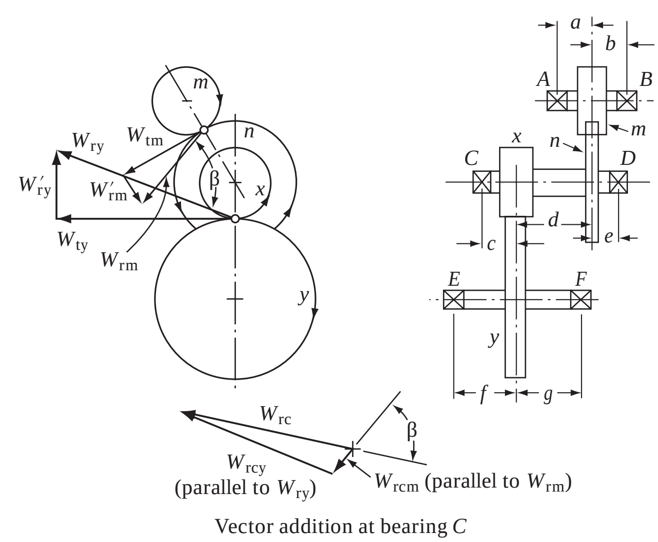


FIGURE 14.16 Vector analysis of idler loads.

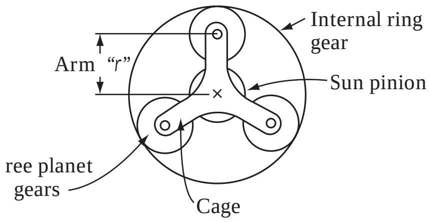


FIGURE 14.17 Intermediate gear trains.

The total loads that are applied to and must be reacted against by bearing *D* are found by the summation of moments about bearing *C*. Summation of forces, however, is taken about bearing *D*. When contact is at an angle or several meshes are concerned, both vertical and horizontal planes of moments must be taken. The bearings must resist making the total moment equal to zero. The proportions of spacing between bearings are very critical. The distances or the moment arms should be selected to favor the *F* bearing in Figure 14.17 would, in most gearboxes, be smaller because of lack of space. Therefore, the distance *g* on the *y* gear was intentionally made longer to reduce the bearing load on *F* and put the greater load on *E*, as it should have more space available for a larger bearing with greater load-carrying capacity. Also, whenever possible, the distance between gears and bearings is as small as feasible to eliminate large moment loads in the shafts. The shaft on $x - n$ gears will have a coupling tendency to cock or walk, and consideration should be given to axial-thrust bearings to resist the coupling effect. To aid the positioning of planes of intersection on the gear teeth, avoid large axial moments and overhangs.

14.4.6 Planetary Gears

Planetary gears are used most often for maximum horsepower transmitted in the smallest space or for when large changes in speed ratios are needed. The increased carrying capacity is generally possible because the gear reactions tend to equalize each other, reducing bearing loads, size required, and total weight. Helical, bevel, and spur gears are used in planetary drives, but spurs are often preferred because of ease of manufacturing and elimination or reduction of axial-thrust loads.

For simplicity's sake, spur gears have been assumed in these discussions, but helical and other types of gears can just as easily be used.

In a simple planetary arrangement, there is generally a sun gear, a planet gear, an internal ring gear, and a cage or an arm (see Figure 14.18).

14.4.6.1 One Planet

If the planetary drive has one planet gear, load reactions are different from those in the case where there are two or more planet gears. The one-planet-gear calculations are basically the same as the combination of an external idler and an internal gear and external pinion. For example, the sun-and-one-mesh planetary drive is the same as two external gears meshing. The reaction on the internal gear is the same as in a

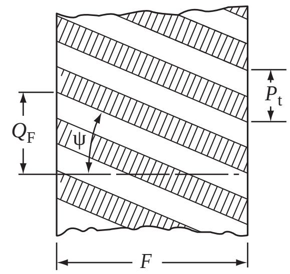


FIGURE 14.18 Simple planetary arrangement.

simple one meshing with an external pinion. However, on the planet gear, consideration must be given for the reactions on its center of rotation from the cage or the arm.

14.4.6.2 Several Planets

If two or more planets are used, preferably three or more, the reactions are different because three planets mate with one sun gear and one ring gear, and the bearing reactions tend to balance out. There are basically zero resultant separating bearing forces on the sun pinion and the internal ring gear. The planet gear bearing has the loads from the two meshes and the cage-driving arm, plus the weight and the centrifugal force from the planet and its bearings. When there are high-speed planets, the effect of centrifugal force must be considered. If speeds and weights are low, centrifugal force may be ignored. If they are high, compute them by means of Equation 1.21 or 1.22.

Centrifugal force may become the prime factor in the nal decision of bearing design or type of planetary gear drive used.

When the total gear reaction *W* or W_{cf} (centrifugal force) are found, they are vectorially added or computed as follows:

$$L = \sqrt{W_{cf}^2 + W^2}, \tag{14.25}$$

where
L—total load of all forces
W_{cf}—centrifugal force
W—total gear reactions

However, helix forces, unbalance, and driven propeller or fan reactions must be considered when they are tied into the planetary drive system. When two or three planets are used, the loads should be considered as equally shared by all planets. When there are more than three planets, it is not always possible to equally share the load, and some allowance should be made for unequal loading of all the individual planets.

The following special recommendations apply to idlers. Preferably, use two or more planets to carry the load. When possible, use three planets. If more than three are needed, many designers feel that it is wise to go to ve or more because of the lack of load-sharing tendencies.

Always mount the entire system as exibly as possible. For example, always mount the planets on spherical oating

bearings or mounts. Make all mountings or actual holding surfaces as light in weight as can be utilized to reduce centrifugal forces.

14.5 BEARING LOAD CALCULATIONS FOR HELICALS

So far, all data have been based on spur gears and all radial values calculated are directly applicable to helical gears if the transverse pressure angle is considered.

14.5.1 Single-Helical Gears

Helical gears have thrust and reactions on all three planes. Besides the tangential driving and separating forces, there is also an axial-thrust load. This thrust is due to the helix angle and is normally considered emanating from the tooth contact at the pitch diameter.

Since thrust is applied at the distance of the pitch radius from the axis of rotation, a radial moment is produced. However, in helicals, the moment is produced within the gear.

This information on helicals can be transcribed into the laws of mechanics. Three forces are shown in Figure 14.8.

W_t—tangential driving load
W_r'—separating force
W_x—axial thrust
W—total load
M_a—moment produced by axial thrust
W_t and W_r' are found as before. It is important, though, to consider a new effect on the pressure angle:

$$\tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta} \text{ (metric)}, \qquad (14.26)$$

$$\tan\phi_t = \frac{\tan\phi_n}{\cos\psi} \text{ (English)}, \qquad (14.27)$$

where
α_t (ϕ_t)—transverse pressure angle
α_n (ϕ_n)—normal pressure angle
β (ψ)—helix angle

The separating load is a function of the transverse pressure angle, which in a helical gear is always greater than ϕ_n, the normal pressure angle. Therefore, W_r' will always be a slight amount larger than in a spur gear but is calculated as before in Equation 14.23.

The helical angle ψ assumes great importance in calculating the axial and the separating load, and in the equations presented, the helix angle will be used for all calculations:

$$\tan\psi = \frac{\pi \times \text{pitch diameter}}{\text{lead}}. \qquad (14.28)$$

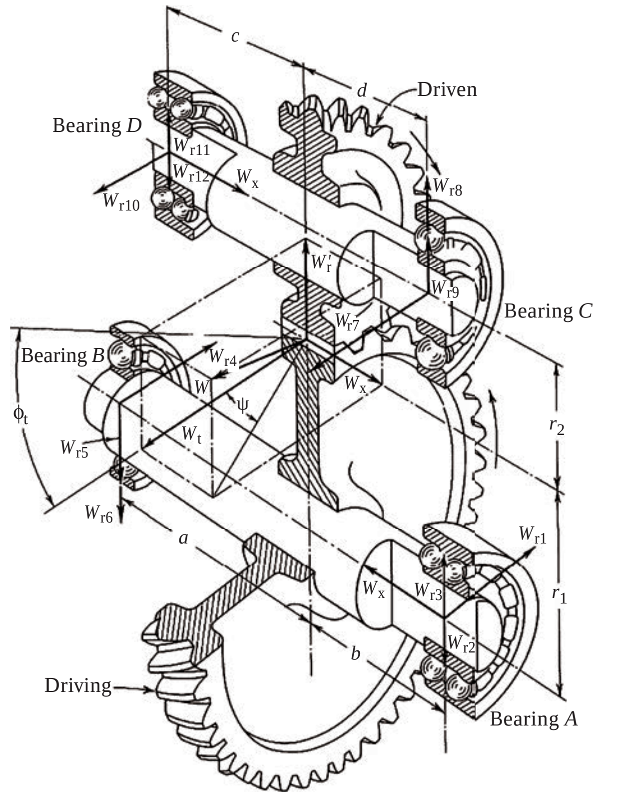


FIGURE 14.19 Helical gear face advance. Face advance = $Q_F + F \tan\psi$. Contact ratio of faces $Q_F/P_t \geq 1$.

Lead is the distance a point would axially advance across the face of the gear for one revolution measured at the pitch diameter. A small helix angle has a high lead and a large helix angle has a low lead.

W_x increases with higher helix angle and, whenever possible, should be kept to the smallest angle. However, for a helix angle to be of any value, it should have a face overlap of a little over 1. This means that the contacting distance of the face advance along the pitch-diameter periphery of a helical tooth is equal to or greater than the transverse circular pitch (see Figure 14.19).

Normally, for high-speed gears, a face advance of over 2 is preferred.

$$\text{Face contact ratio} = \frac{\text{face advance}}{\text{circular pitch}}. \qquad (14.29)$$

As gears become more heavily loaded with higher speeds, and quietness is a requirement, a greater overlap ratio is needed. Therefore, the selection of the helix angle becomes a direct function of face advance.

$$\tan \text{helix angle} = \frac{\text{face advance}}{\text{face width}}. \qquad (14.30)$$

With the selection of the helix angle, the helical gear forces can be determined as shown in Figure 14.8. In its simplest form,

$$W_x = W_t \times W_t \text{ tan helix angle.} \quad (14.31)$$

The total load W is the vector summation of W_t, W_x, and W_r',

$$W = \sqrt{W_t^2 + W_r'^2 + W_x^2}, \quad (14.32)$$

for X-, Y-, and Z-axes.

More consideration must now be given to the summation of forces on various planes. In some cases, values are additive and in other cases they are subtractive.

The moment produced from the helix is now calculated:

$$M_a = W_x \times \text{pitch radius of gear,} \quad (14.33)$$

where M_a is the axial moment, and W_x is the axial thrust.

M_a produces an additional moment and, in turn, an additional radial load, which must be contained by the bearings and supports. Also notice that, if the vectors representing W_t, W_r', and W_x are assumed to be, respectively, the X-, Y-, and Z-axis; Equation 14.32 conforms to Equation 14.7.

From the standpoint of load axially produced, it is always desired to keep it to the absolute minimum value. Since a helical gear always requires a bearing to resist an axial force, this can be a disadvantage in some installations. The forces can become as large as to be determined in large helix angles. See Figure 14.20 and Table 14.4 for details of bearing load calculations.

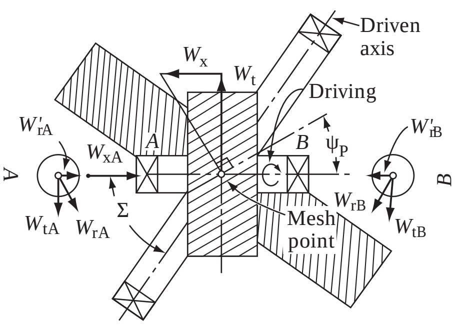


FIGURE 14.20 Helical gear bearing loads.

14.5.2 Double-Helical Gears

To overcome axial thrust, the forces are sometimes counteracted or nulli ed by double-helical gears. The total face is made into two equal halves, both with the same helix angle but opposite hands. The axial thrusts oppose one another and the forces are contained in the gear and not transmitted to the bearing. The double-helical gears are often made of two separate halves of helical gears bolted or joined together, or else a space between the gears is allowed in the middle for tool clearances. There are some gear machine tools that permit a continuous herringbone when face width is limited.

Only one gear in a double-helical train is positioned or restrained axially by a bearing, and the rest of the gears then get their location from this one part. For speed-increasing gears, it is often wise to use or consider either helical or double-helical gears. As axial leads and helix angles increase, so also do the merits of double-helical gears.

14.5.3 Skewed or Crossed-Helical Dears

Besides helical gears on parallel axes, it is possible to have the axes of rotation skewed or at an angle with each other. The latter gears are called *crossed-helical* or *skewed helical gears*. When the individual gear reactions are found, the same methods are used. In helical gears, the total reaction for the pinion is equal to and opposite the mating gear. In a pair of crossed helicals, each gear and pinion must be considered and calculated separately.

The direction of W_x is generally obvious.

In crossed-helical gears, one of the mating parts could be a helical or a spur gear and it would mate with a helical gear. All spiral helicals tend to have only point contact, are used to transmit light loads, and have small bearing reactions. Allowance must be made for the helix angle in relation

TABLE 14.4
Equations for Helical Gear Bearing Loads with Straddle Mounting

	Gear Bearing Loads		Pinion Bearing Loads	
Force	**Bearing *A***	**Bearing *B***	**Bearing *C***	**Bearing *D***
tan force W_r	$W_{r1} = \frac{W_t a}{a+b}$	$W_{r4} = \frac{W_t b}{a+b}$	$W_{r7} = \frac{W_t c}{c+d}$	$W_{r10} = \frac{W_t d}{c+d}$
Separating force W_r'	$W_{r2} = \frac{W_r' a}{a+b}$	$W_{r5} = \frac{W_r' b}{a+b}$	$W_{r8} = \frac{W_r' c}{c+d}$	$W_{r11} = \frac{W_r' d}{c+d}$
Thrust W_x	$W_{r3} = \frac{W_x r_1}{a+b}$	$W_{r6} = \frac{W_x r_1}{a+b} = W_{r3}$	$W_{r9} = \frac{W_x r_2}{c+d} = W_{r3}$	$W_{r12} = \frac{W_x r_2}{c+d} = W_{r9}$
Total radial load	$W_{rA} = \sqrt{W_{r1}^2 + (W_{r2} - W_{r3})^2}$	$W_{rB} = \sqrt{W_{r4}^2 + (W_{r5} + W_{r6})^2}$	$W_{rC} = \sqrt{W_{r7}^2 + (W_{r8} + W_{r9})^2}$	$W_{rD} = \sqrt{W_{r10}^2 + (W_{r11} - W_{r12})^2}$
Total thrust	W_x (may be applied to either bearing *A* or *B*)		W_x (may be applied to either bearing *C* or *D*)	

Note: See Figure 14.20 for de nition of terms and bearing locations.

to the axis of rotation of each gear. In external helical gears, the hands of the mating parts are equal and opposite, but in crossed-helical gears, all types of hands and angles of helix can and must be considered. Calculations of the bearing loads from the gear reactions are the same as for helical gears. More care and consideration must be given to the angles of helix in relation to the shaft angles. The hand of the helices and the shaft angles can be varied so that direction of rotation is reversed. Once the true helix angle and the actual direction of rotation between the driver and driven gears are determined, the reactions are easily found.

Figure 14.21 shows a pair of crossed-helical gears. The light lines on the driving gear show the teeth, as they would appear to the viewer. The meshing point considered in the diagram is underneath the driver. After allowing for the direction of rotation, shaft angle, and hand of spirals, the direction of the forces can be found using Table 14.1. The angles ψ_P and ψ_G can be found, and, knowing W_t, the axial thrust for the gear and the pinion are determined as in any helical gear. See Figure 14.20 for details on bearing load calculations. In the metric system, the symbol for pinion helix angle is β_1 instead of ψ_P. For the gear, the symbol is β_2 instead of ψ_G.

14.6 MOUNTING PRACTICE FOR BEVEL AND HYPOID GEARS

All gears operate best when their axes are maintained in correct alignment, and, although bevel and hypoid gears have the ability to absorb reasonable displacements without detriment to the tooth action, excessive misalignments reduce the load capacity and complicate the manufacture and the assembly of gears. Accurate alignment of the gears under all operating conditions may be accomplished only by good design, accompanied by accurate manufacture and assembly of both mountings and gears. The following recommendations are given to aid in the design of bevel and hypoid gearbox assemblies in order to obtain the best performance of the gears.

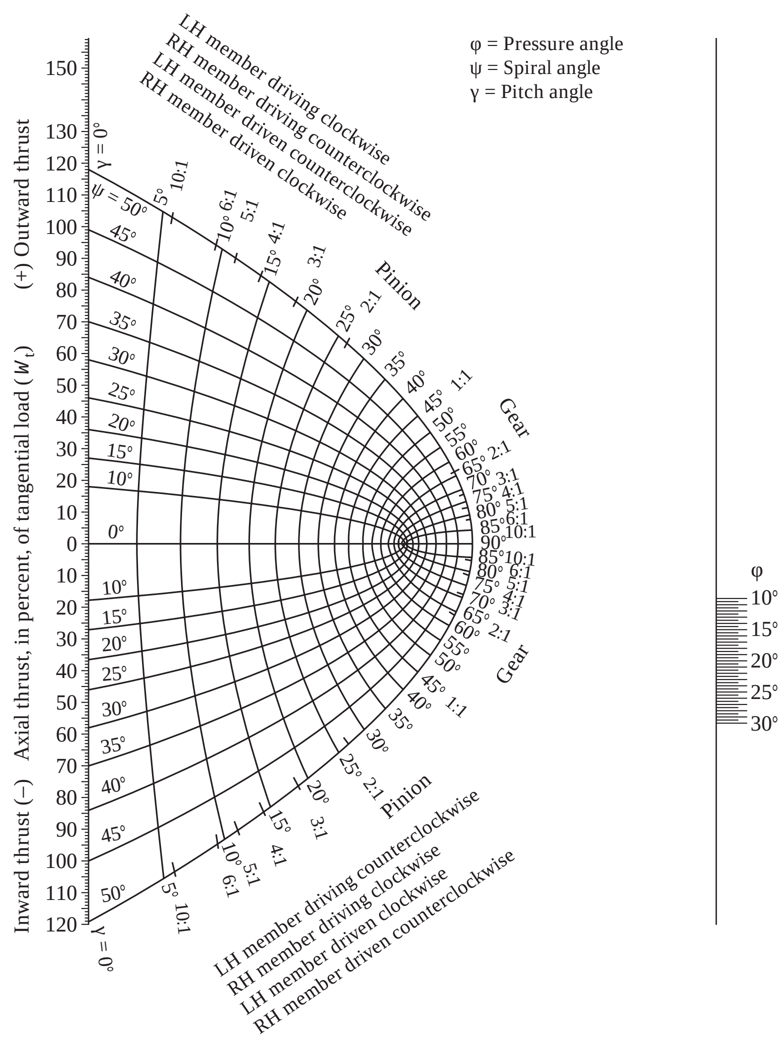


FIGURE 14.21 Bearing loads on crossed-helical gears. Regardless of shaft angle between crossed-helical pinion and crossed-helical gear, the directions of the pinion tangential force W_t and the pinion thrust force W_x are as shown for the mesh point.

14.6.1 Analysis of Forces

In the design of the mountings for bevel and hypoid gears, the first step is to establish the magnitude and the directions of the axial and separating forces, from Figures 14.22 and 14.23, and draw a diagram of the resultant force on the gears as shown in Figures 14.24 through 14.27. These provide the basis for the design of the gear blank and the mountings and for the selection and the arrangement of the bearings. In case the gear must operate in both directions of rotation or if there is reversal of torque, the force diagram should include both conditions of operation. Clockwise and counterclockwise rotations are as viewed from the back of the gear looking toward the cone center. Values shown are for shaft angles at right angles, but values and determination of directions also apply to shaft angles other than 90°.

14.6.2 Rigid Mountings

Gear mountings should be designed for maximum rigidity since the problems involved in producing satisfactory gears rapidly multiply when deflections in the mountings cause excessive gear displacements. It is necessary to modify the standard cutting in order to narrow and shorten the tooth contacts to suit the flexible mounting. The decrease in bearing area raises the unit tooth pressure and reduces the number of teeth in contact, which results in problems of noise and increases the danger of surface failure and tooth breakage.

14.6.3 Maximum Displacements

For gears from 6 to 15 in. diameter (150 to 375 mm), the desirable limits of gear and pinion displacements at maximum load are listed as follows:

- The pinion offset should not exceed ±0.003 in. from correct position (±0.076 mm).
- The pinion should not axially yield more than 0.003 in. in either direction (±0.076 mm).
- The gear offset should not exceed ±0.003 in. from correct position (±0.076 mm).
- The gear should not axially yield more than 0.003 in. in either direction (±0.076 mm in either direction) on miters or near miters, or more than 0.010 in. (0.25 mm) away from the pinion on higher ratios.

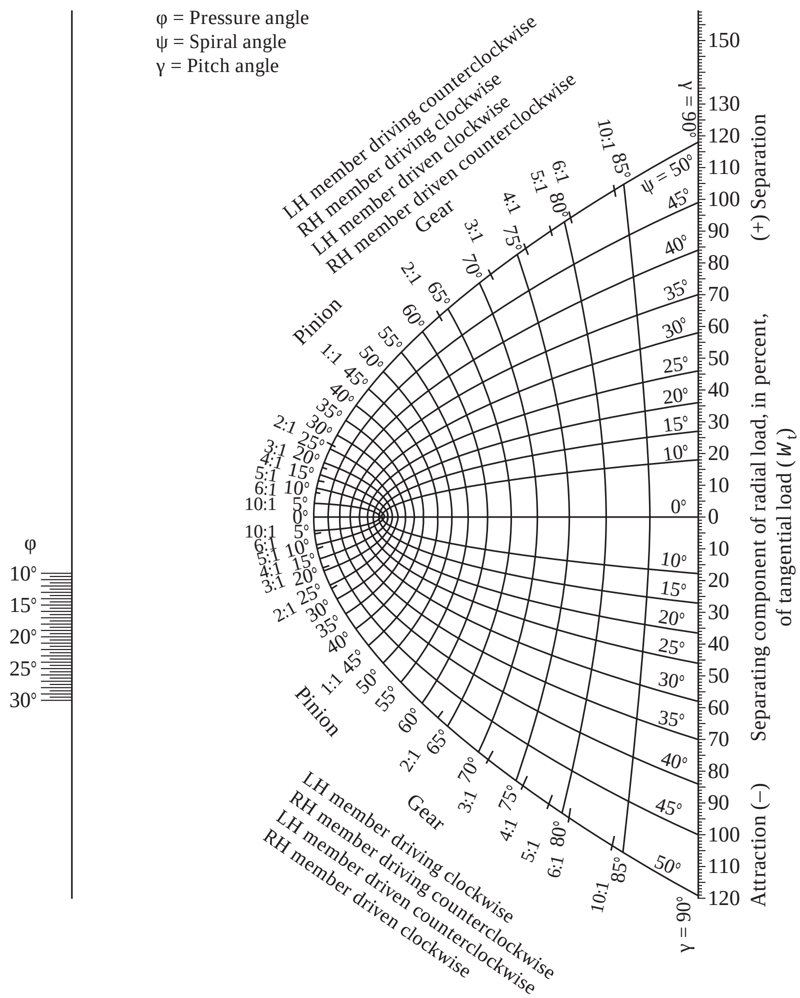


FIGURE 14.22 Design chart for axial thrust of bevel gears.

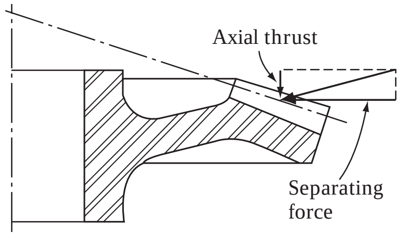


FIGURE 14.23 Design chart for separating component of radial load for bevel gears.

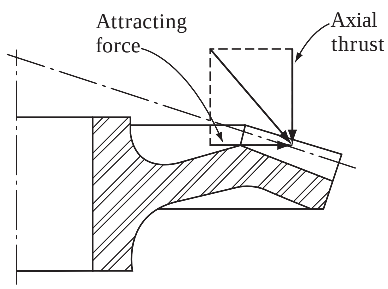


FIGURE 14.24 Resultant force on axial plane due to RH gear being driven counterclockwise or driving clockwise; also LH gear being driven clockwise or driving counterclockwise.

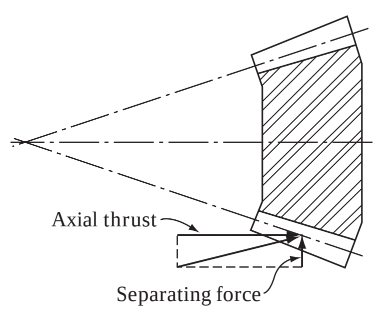


FIGURE 14.25 Resultant force on axial plane due to RH gear being driven clockwise or driving counterclockwise; also LH gear being driven counterclockwise or driving clockwise.

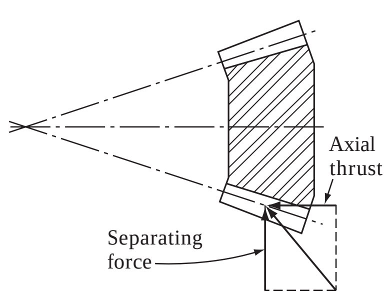


FIGURE 14.26 Resultant force on axial plane due to LH pinion driving clockwise or being driven counterclockwise; also RH pinion driving counterclockwise or being driven clockwise.

The aforementioned limits are for average applications in which the gears operate over a range of loads and at maximum load approximately 10% of the time. When maximum load occurs for longer periods, reduce the limits accordingly. Somewhat wider tolerances are allowable for large gears.

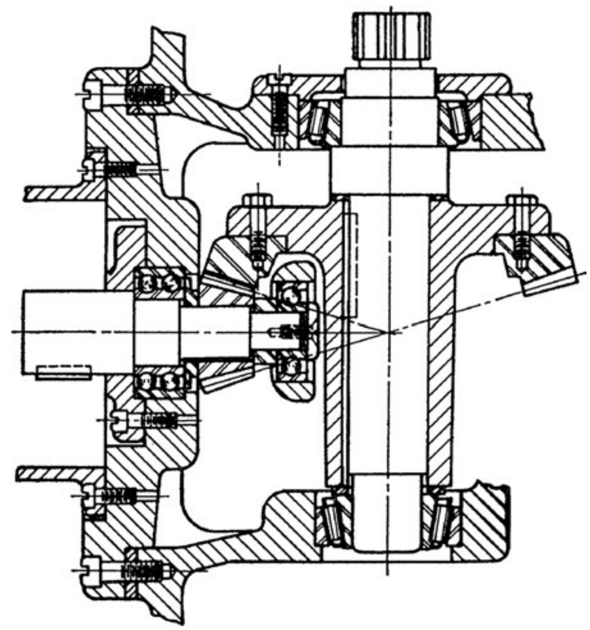


FIGURE 14.27 Resultant force on axial plane due to LH pinion driving counterclockwise or being driven clockwise; also RH pinion driving clockwise or being driven counterclockwise.

14.6.4 Rolling-Element Bearings

While plain bearings may be used in bevel or hypoid gear mountings, it is usually much easier to maintain the gears in satisfactory alignment with rolling-element bearings. Either ball or roller bearings are satisfactory. However, it is important that the type and the size of bearings be carefully selected to suit the particular application, especially considering the operating speeds, the loads, the desired life of the bearings, and the rigidity required for best operation of the gears.

14.6.5 Straddle Mounting

The preferred design of a bevel or a hypoid gearbox provides straddle mountings for both gear and pinion, and this arrangement is generally used for industrial and other heavily loaded applications. Usually, the desired bearing life and mounting rigidity may be more economically obtained with a straddle mounting. When it is not feasible to use this arrangement for both members of a pair, the gear member that usually has the higher radial load should be straddle mounted.

In straddle mounting the gear or the larger member of a pair, usually two tapered roller bearings or angular-contact ball bearings are mounted opposed, with suf cient spacing between them to provide control of the gear (Figure 14.28).

A second straddle-mounting arrangement (Figure 14.29), used mostly for pinions, provides an inboard or a pilot bearing for pure radial support of one end of the pinion shaft; and, on the opposite end, bearings suitable for carrying both radial and thrust loads. In this location, several different types of bearings are used: a single double-row, deep-groove

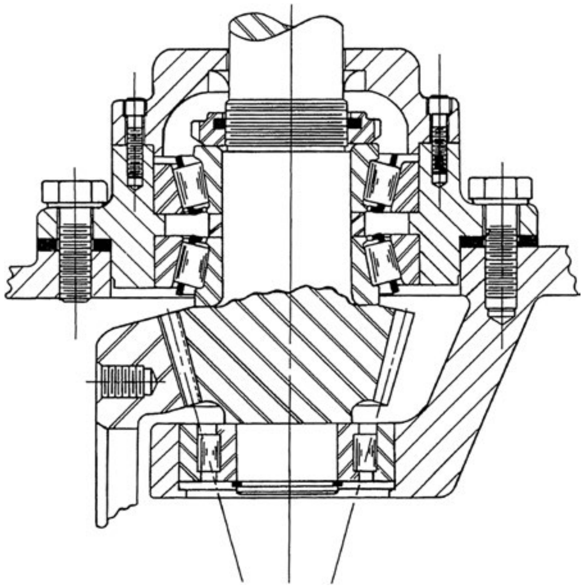


FIGURE 14.28 Webless-type gear member, straddle mounted using two opposed tapered roller bearings widely spaced to provide good control of gear.

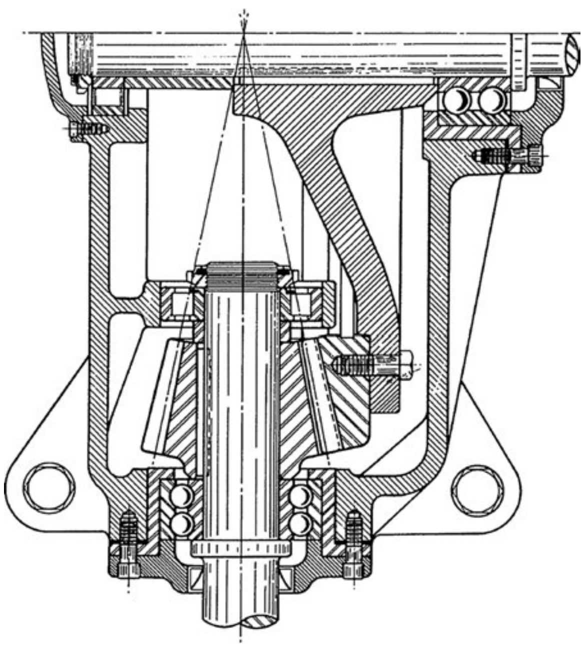


FIGURE 14.29 Straddle pinion mounting for short shafts, showing use of combined thrust and radial bearings.

angular-contact ball bearings; two single-row angular-contact bearings mounted *DB* or *DF*; or two tapered roller bearings, either directly or indirectly mounted.

A similar arrangement is also used for straddle mounting the gear or the larger member of a pair in large applications, and in an aluminum or a magnesium housing, where temperature changes would seriously affect the preloading or thrust bearings if spaced on either end of the gear shaft (Figure 14.30).

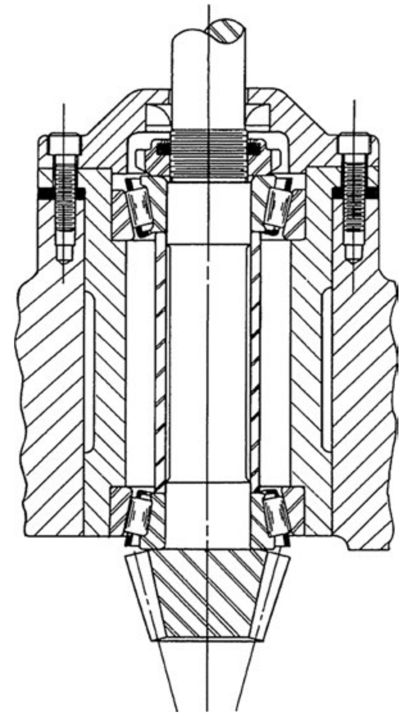


FIGURE 14.30 Straddle mounting for both members of a spiral bevel pair, using a straight roller radial bearing on one end of the shaft and combined thrust and radial bearing on other end. Shim-type adjustments for positioning both gear and pinion.

14.6.6 Overhung Mounting

When the pinion is mounted overhung on tapered roller bearings, the indirect mounting should be used (Figure 14.31). This arrangement provides greater stability to the mounting for a given spacing of the bearings, and the thrust load for normal operation is thus carried by the bearing adjacent to the pinion, adding further to its stability.

The bearing adjacent to the pinion should be located as closely as possible to the pinion teeth to reduce the overhang. See Figure 14.32.

All spiral bevel and hypoid gears should be located against the thrust in both directions. Provision against inward thrust on straight and Zerol bevel gears is often omitted, provided the conditions of operating are such that inward thrust cannot occur. One of the advantages of Zerol and straight bevel gears is that the change of rotation does not change direction of axial thrust.

Since the contact pattern of spiral bevel and hypoid gears is controlled by their axial position, special attention must be given to the selection of the type and to the preloading of the bearings.

The lubrication of bevel and hypoid gears is most important; for speci c data, refer to the instructions of Gleason Works.

14.6.7 Gear Blank Design

The gear blank should be designed to avoid excessive localized stresses and serious de ections within it. The direction of the resultant force should be considered in the design of the

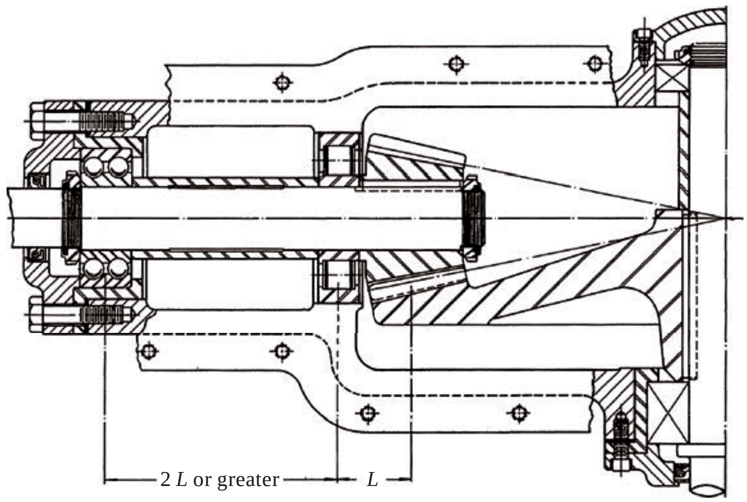


FIGURE 14.31 Typical overhung pinion mounting with shim adjustment for positioning, and selected spacer for preloading bearings.

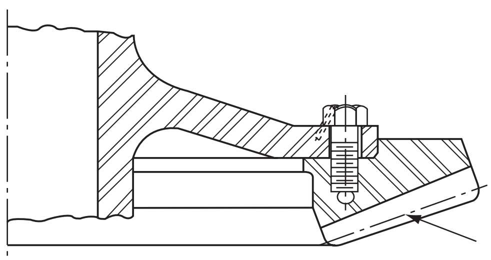


FIGURE 14.32 Typical overhung pinion mounting and straddle gear mounting with shim adjustment for positioning both gear and pinion.

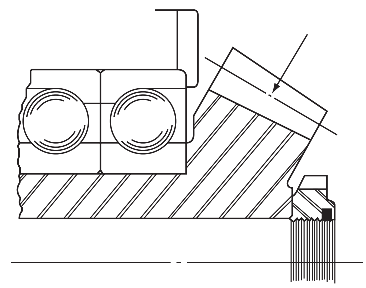


FIGURE 14.34 Method of mounting gear when thrust is inward.

blank and in the method employed in mounting it on its shaft or centering hub (Figures 14.12 and 14.33 through 14.37).

Ring gears are made in three general types: the webless ring, the web-type ring, and the counterbored type. Of these, the bolted-on webless ring design shown in Figure 14.28 is best for hardened gear larger than 7 in. (180 mm) in diameter. The t of the gear on its centering hub should be either size-to-size or slight interference. These gears should be clamped to the centering hub with ne-thread cap screws as shown in Figures 14.28, 14.34, and 14.37, or with bolts. Several methods

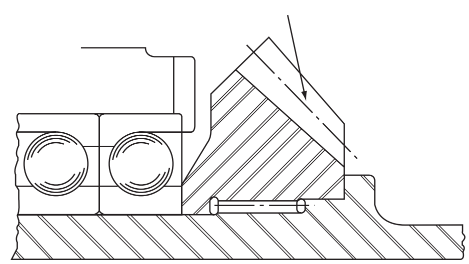


FIGURE 14.35 Blank sections beneath the teeth should be suf- ciently rigid in the direction of thrust so that de ections will be minimized.

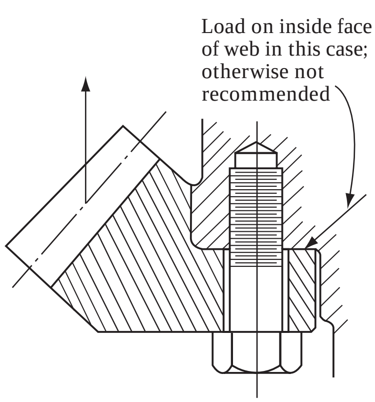


FIGURE 14.33 Method of mounting gear to absorb the thrust with minimum amount of de ection.

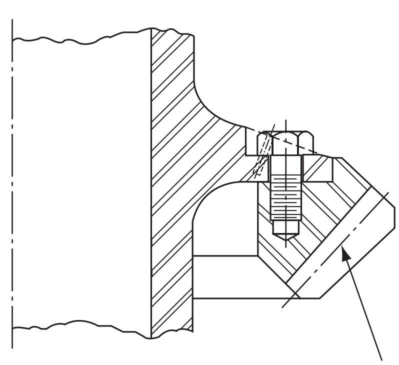


FIGURE 14.36 Method of mounting gear to hub to absorb high outward thrust.

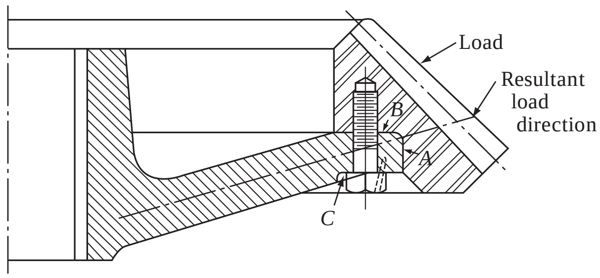


FIGURE 14.37 Webless-type miter gear—counter-bored type.

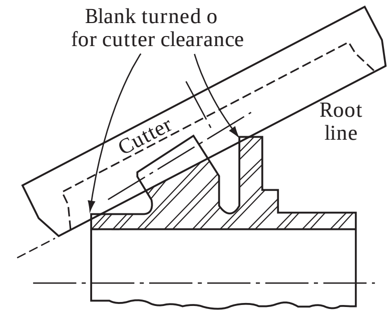


FIGURE 14.39 When using Zerol bevel, spiral bevel, and hypoid gears, clearance must be provided for the cutter as shown.

of locking screws and nuts in place are indicated in the illustrations. Self-locking-type nuts and cap screws are widely used for this purpose. For severe operating conditions, cap screws should be locked more positively than is possible by wiring.

The method of centering the gear on the counterbore, shown in Figure 14.37, is recommended for large gears, especially near miters. Figure 14.34 illustrates mounting for a gear that operates with an inward thrust. Designs in which screws or bolts are subjected to added tension from gear forces should be avoided. On severe reversing or vibration installations, separate dowel drives may be used with the designs or the splines as in Figures 14.12 and 14.38.

The use of dowels or dowel- t bolts has been found unnecessary in most automotive and industrial drives because, with bolts or cap screws tightly drawn, the friction of the ring gear in its hub prevents shearing of the screws.

Hardened gears smaller than 7 in. (less than 180 mm) in diameter may best be designed with integral hubs. A front hub on a Zerol bevel, spiral bevel, or hypoid gear should in no case intersect the extended root line so as to interfere with the path of the cutter toward the root cone apex as shown in Figure 14.39. On hub-type gears, the length of the hub should be at least equal to the bore diameter, and the end of the hub should be securely clamped against a shoulder on the shaft.

Whether the gear is mounted on a anged hub or is made integral with the hub, the supporting ange should be made suf cient rigid to prevent serious de ections in the direction of the gear axis at the mesh point. For larger gears, the web preferably should be made conical and without ribbing to permit machining for balance, to eliminate oil churning when dip lubrication is used and to lessen the danger of stress concentration being set up with the castings. The gear hub and the bearing arrangement should be designed with suf cient rigidity that the use of a thrust button behind the gear mesh is unnecessary. The area of a thrust button is too small to provide durable support.

For hardened gears, the blanks should be designed with sections and shapes suitable to hardening with minimum distortion.

14.6.8 Gear and Pinion Adjustments

Provision should be made for axially adjusting both gear and pinion at assembly.

Shim-type adjustments as shown in Figures 14.29 and 14.30 or threads and nuts with locks as shown in Figure 14.40 may be used. Shims less than 0.015 in. (0.38 mm) thick may *pound out* in service. When used adjacent to a bearing, the shims should be placed next to the stationary member. If an assortment of shims varying in thickness in steps of 0.003 in.

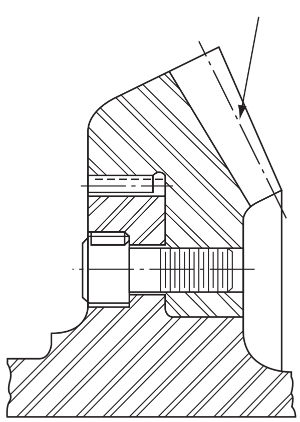


FIGURE 14.38 Positive drive for severe reversing or shock loads.

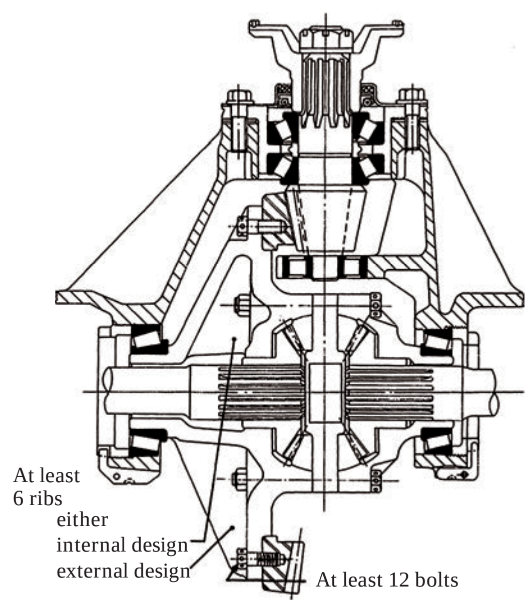


FIGURE 14.40 Truck axle—straddle-mounted pinion and with threaded-nut adjustments for the gear bearing.

(0.076 mm) is used (such as 0.020 in. [0.50 mm], 0.023 in. [0.60 mm], and 0.026 in. [0.65 mm]), the member to be adjusted may be positioned within ±0.0015 in. (±0.04 mm).

14.6.9 Assembly Procedure

The following is a recommended procedure for the assembly of bevel and hypoid gearboxes:

- Thoroughly clean housing and parts, following the bearing manufacturer's instructions in handling the rolling-element bearings.
- Keep lapped or matched gears in their original sets.
- Assemble even-ratio pairs with the teeth mating as lapped. Tooth and mating spaces are usually designated with X markings.
- Set pinion on its correct axial position by measurement.
- Adjust gear along its axis until the specified backlash is obtained.
- Securely lock gears and thrust bearings in this position. Note, for spiral bevel and hypoid gears, both members should be locked against thrust in both directions.
- As a final check, paint the teeth with marking compound and observe the tooth contact pattern under light load. This may indicate the need for further adjustment. Check to ensure that there is an adequate supply of the specified lubricant to the mesh and to the bearings.

14.7 CALCULATION OF BEVEL AND HYPOID BEARING LOADS

The normal load on the tooth surfaces of bevel or hypoid gears may be resolved into two components: one in the direction along the axis of the gear, and the other perpendicular to the axis. The axial force produces an axial thrust on the bearings, while the force perpendicular to the axis produces a radial load on the bearings. The direction and magnitude of the normal load depends on the ratio, the pressure angle, the spiral angle, the hand of spiral, the direction of rotation, and whether the gear is the driving or driven members.

14.7.1 Hand of Spiral

The hand of spiral on spiral bevel and hypoid gears is donated by the direction in which the teeth curve; that is, the LH teeth incline away from the axis in a counterclockwise direction when an observer looks at the face of the gear, and the RH teeth incline away from the axis in the clockwise direction. The hand of spiral of one member of a pair is always opposite to that of its mate. It is customary to use the hand of spiral of the pinion to identify the combination; that is, a LH combination is one with the LH spiral on the pinion and a RH spiral on the gear. The hand of spiral has no effect on the smoothness and the quietness of operation or on the efficiency. Attention, however, is called to the difference in the effect of the thrust loads as stated in the following paragraph.

A LH spiral pinion driving clockwise (viewed from the back) tends to move axially away from the cone center, while a RH pinion tends to move toward the center because of the oblique direction on the curved teeth. If there is excessive end play in the pinion shaft because of faulty assembly or bearing failure, the movement of a RH pinion driving clockwise will take up the backlash under load, and the teeth of the gear and the pinion may wedge together, while a LH spiral pinion under the same conditions would back away and introduce additional backlash between the teeth, a condition that would not prevent the gears from fluctuating. When the ratio, the pressure angle, and the spiral angle are such that doing so is possible, the hand of spiral should be selected to give an axial thrust that tends to move both the gear and the pinion out of the mesh. Otherwise, the hand of spiral should be selected to give an axial thrust that tends to move the pinion out of the mesh. Often the mounting conditions dictate the hand of spiral to be selected.

In a reversible drive, there is, of course, no choice unless the pair performs a heavier duty in one direction a greater part of the time.

In the calculation of reactions, Zerol and straight bevel gears may be treated as a special case of spiral bevel gears in which the spiral angle is 0°. In these, the direction of the axial thrust is always outward on both members regardless of the direction of rotation and whether the pinion is the driving or the driven member.

On hypoids, when the pinion is below center and to the right when facing the front of the gear, the pinion hand of spiral should always be left. When the pinion is above center and to the right, the pinion hand should always be right (see Figure 14.41).

Specification of hand of Zerol gears is illustrated in Figure 14.42.

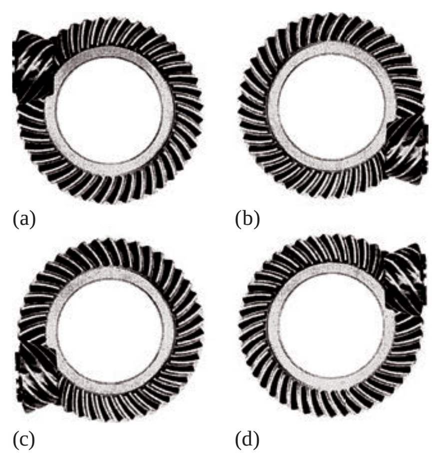


FIGURE 14.41 Both pinions shown in (a) and (b) are referred to as having an offset below center, while those in (c) and (d) have an offset above center. In determining the direction of offset, it is customary to look at the face of the gear with the pinion at the right.

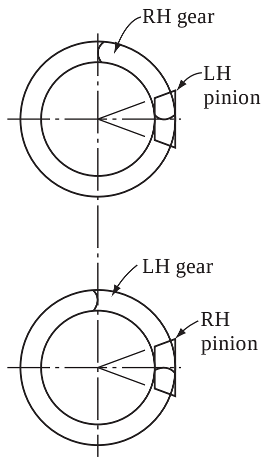


FIGURE 14.42 Diagram illustrating hand of Zerol. The upper drawing shows an LH, and the lower, an RH Zerol bevel gear combination.

14.7.2 Spiral Angle

The spiral angle, if possible, should be selected to give a face contact ratio of at least 1.25. For maximum smoothness, the face contact ratio should be between 1.50 and 2.0. On straight or Zerol bevel gears, the spiral angle will, of course, be zero.

14.7.3 Tangential Load

The tangential load on a bevel or a hypoid gear is given by

$$W_t = \frac{19{,}098{,}600P}{d_{2m} \times n_2}, \tag{14.34}$$

$$W_t = \frac{126{,}050P_h}{d_{mG} \times n_G}, \tag{14.35}$$

where
W_t (W_t)—tangential load, N (metric) (lb [English])
P (P_h)—power, kW (hp)
d_{2m} (d_{mG})—mean pitch diameter of gear, mm (in.)
n_2 (n_G)—speed, rpm

The mean pitch diameter of a bevel gear may be determined by subtracting the term (bevel gear face width times the sin of the bevel gear pitch cone angle) from the bevel gear pitch diameter at the large end of the bevel gear.

The tangential load on the mating bevel pinion will be equal to the tangential load on the gear. The tangential load on the mating hypoid pinion, however, must be determined as follows:

$$W_{tP} = W_{tG} \frac{\text{cos spiral angle of pinion}}{\text{cos spiral angle of gear}}. \tag{14.36}$$

14.7.4 Axial Thrust

The value of the axial thrust is dependent on the tangential tooth load, the spiral angle, the pressure angle, and the pitch angle. This may be determined with the aid of Figure 14.22. Figure 14.22 is symmetrical about a horizontal centerline; therefore, there are two points of intersection of the pitch-angle and the spiral-angle curves. The selection of the proper point is dependent on the hand of spiral, the direction of rotation, and whether the member is the driving or the driven one. This is given in Figure 14.22. Note that the intersection points for the pinion and its mating gear are always on opposite sides of the horizontal centerline. The pressure angle ϕ is given on the scale at the right. A straight line connecting the pressure angle with the intersection point of the spiral-angle and pitch-angle curves can now be extended to the scale on the left. This scale gives the axial thrust W_x, in percent, of the tangential tooth load W_t.

The spiral angle on straight and Zerol gears is zero.

For determining the axial thrust in hypoid gears, the pressure angle of the driving side should be used. Instead of using the pitch angles, the face angle of the pinion and the root angle of the gear should be used.

Note that the spiral angle, the pitch angle, and the pressure angle for the corresponding member must be selected in each case.

The actual axial-thrust load may be determined by multiplying the tangential load in pounds for the corresponding member by the percentage given in Figure 14.21:

$$W_x = (\text{percentage from Figure 14.21}) \times W_t. \tag{14.37}$$

Figure 14.43 illustrates the direction of rotation and the direction of thrust.

14.7.5 Radial Load

The radial load caused by the separating force between the two members may be determined in a similar manner to that used for the axial thrust with the aid of Figure 14.23. The same

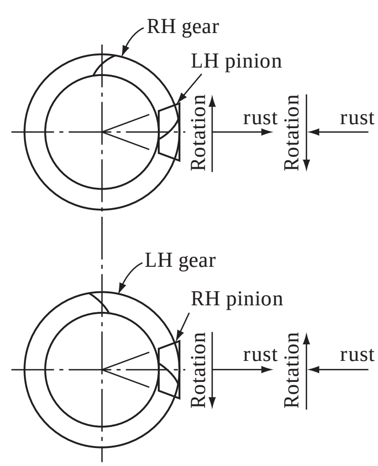


FIGURE 14.43 Diagram illustrating hand of spiral and its effect on the direction of axial thrust in spiral bevel gears. The upper drawing shows an LH; and the lower, an RH spiral bevel combination.

procedure is followed here as outlined in Section 14.7.4. The scale on the right gives the component part of the radial load W_r, in percent, of the tangential tooth load W_t.

This component of the actual radial load may be determined by multiplying the tangential load in pounds for the corresponding member by the percentage given in Figure 14.23:

$$W_r' = (\text{percentage from Figure 14.23}) \times W_t. \tag{14.38}$$

In addition to the foregoing element, there is a radial component caused by the tangential load itself, and a third component caused by the couple which the axial thrust on the given member produces.

The total radial load produced on the bearings will therefore depend on the resultant of these three components.

For the overhung mounting, the radial components of bearing A will be (see Figure 14.44)

$$W_{r1} = W_r' \frac{L+M}{2}, \tag{14.39}$$

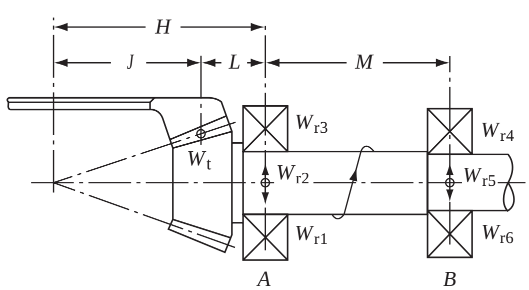
$$W_{r2} = W_t \frac{L+M}{2}, \tag{14.40}$$

where W_r' is the separating component from Figure 14.23, and W_t is the tangential load.

$$W_{r3} = W_x \frac{d_m}{2M}, \tag{14.41}$$

where

L—distance along the axis from center of the face width to center of bearing A (for hypoid gearing, see Equation 14.58)
M—spacing between centers of bearings A and B
W_x—axial-thrust load
d_m—mean pitch diameter ($d_m = d - F \sin \gamma$). (for a hypoid pair, use Equation 14.55 for d_m and N by directions given for Equation 14.58)
d—pitch diameter
F—face width
γ—pitch angle



Overhung

FIGURE 14.44 Diagram showing load components on a pinion and bearings—overhung mounting.

The total radial component on bearing A will be

$$W_{rA} = \sqrt{W_{r2}^2 + (W_{r1} - W_{r3})^2}. \tag{14.42}$$

In like manner, the radial components on bearing B will be

$$W_{r4} = W_r' \frac{L}{M}, \tag{14.43}$$

$$W_{r5} = W_t \frac{L}{M}, \tag{14.44}$$

$$W_{r6} = W_x \frac{d_m}{2M}. \tag{14.45}$$

The total radial component on bearing B will be

$$W_{rB} = \sqrt{W_{r5}^2 + (W_{r4} - W_{r6})^2}. \tag{14.46}$$

For the straddle mounting shown in Figure 14.45, the radial components on bearing C will be

$$W_{r7} = W_r' \frac{K}{N+K}, \tag{14.47}$$

$$W_{r8} = W_t \frac{K}{N+K}, \tag{14.48}$$

$$W_{r9} = W_x \frac{d_m}{2(N+K)}, \tag{14.49}$$

where N is the distance* along the axis from the center of face width to the center of bearing C, and K is the distance† along the axis from the center of face width to the center of bearing D.

The total radial component on bearing C will be

$$W_{rC} = \sqrt{W_{r8}^2 + (W_{r7} - W_{r8})^2}. \tag{14.50}$$

In like manner, the radial components on bearing D will be

$$W_{r10} = W_r' \frac{K}{N+K}, \tag{14.51}$$

$$W_{r11} = W_t \frac{K}{N+K}, \tag{14.52}$$

$$W_{r12} = W_x \frac{d_m}{2(N+K)}. \tag{14.53}$$

* For hypoid pair, use Equation 14.55 for d_m and nd N by directions given for Equation 14.58.
† For hypoid pair, nd K by Equation 14.57; see Figure 14.45.

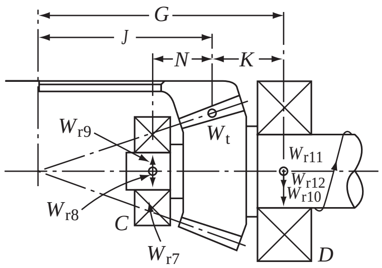


FIGURE 14.45 Diagram showing load components on a pinion and bearings—straddle mounting.

The total radial component on bearing D will be

$$W_{rD} = \sqrt{W_{r11}^2 + (W_{r10} + W_{r12})^2}. \tag{14.54}$$

For a hypoid gear pair, change the preceding equations as follows:

$$d_{mG} = \text{gear pitch diameter} - \text{face width} \times \sin \text{gear root angle}, \tag{14.55}$$

$$d_{mP} = d_{mG} \times \frac{\text{number of pinion teeth}}{\text{number of gear teeth}} \times \frac{\text{cos gear spiral angle}}{\text{cos pinion spiral angle}}, \tag{14.56}$$

$$K = G - J, \tag{14.57}$$

$$L = H - J, \tag{14.58}$$

$$J_P \frac{d_{mG}}{2} \cos\varepsilon \ \text{ for hypoid pinion,} \tag{14.59}$$

$$J_G \frac{d_{mP}}{2} \cos\varepsilon \ \text{ for hypoid gear,} \tag{14.60}$$

where

G—distance along axis from center line of mate to center of bearing D
H—distance along axis from center line of mate to center of bearing A
ε—$\tan^{-1}$[tan(spiral angle pinion − spiral angle gear) × sin gear root angle]
M—spacing between centers of bearings A and B
N—spacing between centers of bearings C and D minus dimension K

See Figures 14.44 and 14.45.

PROBLEM 14.1
Example of Spiral Bevel Gear Bearing Load Calculation

A sample problem for a 16-tooth pinion driving a 49-tooth gear demonstrates how the equations given in this section are used.

GIVEN

A 16/49 combination, 5 diametral pitch, 1.500 in. face width, 20° pressure angle, and 35° spiral angle. (See Table 14.5 for further data relative to this problem.)

LOAD DATA

71 hp at 1800 r/min of the pinion; LH pinion driving clockwise; gear is straddle mounted; pinion, overhung.

TABLE 14.5
Spiral Bevel Gear Sample Data and Bearing Arrangement (English Units)

	Pinion		Gear	
Item	**Symbol**	**English Units**	**Symbol**	**English Units**
Pitch diameter	d	3.200″	D	9.800″
Pitch angle	γ	18°50′	Γ	77°55′
	$\sin\gamma$	0.3104	$\sin\Gamma$	0.9506
Mean pitch diameter	$d_{mP} = d - F\sin\gamma$	2.734″	$d_{mG} = D - F\sin\Gamma$	8.374″
	Overhung Mounting			
	L	1.500″	–	–
	M	5.000″	–	–
	Straddle Mounting			
	K	–	–	2.500″
	N	–	–	3.500″

TABLE 14.6
Solution to Sample Problem on Spiral Bevel Gear Bearing Loads (English Units)

		Bearing Loads			
		Overhung Mounting		Straddle Mounting	
Force	**Load Calculations**	**Bearing *A***	**Bearing *B***	**Bearing *C***	**Bearing *D***
Tangential, W_t	$W_t = \frac{126{,}050P}{d_{mG}n}$	$W_{r2} = W_t\left(\frac{L+M}{M}\right)$ $W_{r2} = 1818\left(\frac{1.5+5.0}{5.0}\right)$ $= 2363$	$W_{r5} = W_t\left(\frac{L}{M}\right)$ $W_{r5} = 1818\left(\frac{1.5}{5.0}\right) = 545$	$W_{r8} = W_t\left(\frac{K}{N+K}\right)$ $W_{r8} = 1818\left(\frac{2.5}{3.5+2.5}\right)$ $= 758$	$W_{r11} = W_t\left(\frac{N}{N+K}\right)$ $W_{r11} = 1818\left(\frac{3.5}{3.5+2.5}\right)$ $= 1060$
Thrust, W_x	$W_x = W_t \times$ thrust factor See Figure 14.22	–	$W_x =$ thrust factor $\times W_t$ $W_x = +0.80 \times 1818 = 1454$	–	$W_x =$ thrust factor $\times W_t$ $W_x = +0.20 \times 1818 = 364$
Separating, W'_r	$W'_r = W_t \times$ separating factor See Figure 14.23	$W_{r1} = W'_r\left(\frac{L+M}{M}\right)$ $W_{r1} = 364\left(\frac{1.5+5.0}{5.0}\right) = 473$	$W_{r4} = W'_r\left(\frac{L}{M}\right)$ $W_{r4} = 364\left(\frac{1.5}{5.0}\right) = 109$	$W_{r7} = W'_r\left(\frac{K}{N+K}\right)$ $W_{r7} = 1454\left(\frac{2.5}{3.5+2.5}\right) = 606$	$W_{r10} = W'_r\left(\frac{N}{N+K}\right)$ $W_{r10} = 1454\left(\frac{3.5}{3.5+2.5}\right) = 848$
Thrust couple	$W_x\left(\frac{d_m}{2}\right)$	$W_{r3} = W_x\left(\frac{d_m}{2M}\right)$ $W_{r3} = 1454\left(\frac{2.734}{25.0}\right) = 398$	$W_{r6} = W_x\left(\frac{d_m}{2M}\right)$ $W_{r6} = 1454\left(\frac{2.734}{25.0}\right) = 398$	$W_{r9} = W_x\left[\frac{d_m}{2(N+K)}\right]$ $W_{r9} = 364\left[\frac{8.374}{2(3.5+2.5)}\right] = 254$	$W_{r12} = W_x\left[\frac{d_m}{2(N+K)}\right]$ $W_{r12} = 364\left[\frac{8.374}{2(3.5+2.5)}\right] = 254$
Total radial load	$W_r = \sqrt{\sum (\text{loads})^2}$	$W_{rA} = \sqrt{W_{r2}^2 + (W_{r1} - W_{r3})^2}$ $W_{rA} = \sqrt{2.353^2 + (473 - 398)^2}$ $= 2364$	$W_{rB} = \sqrt{W_{r5}^2 + (W_{r4} - W_{r6})^2}$ $W_{rB} = \sqrt{545^2 + (109 - 398)^2}$ $= 617$	$W_{rC} = \sqrt{W_{r8}^2 + (W_{r7} - W_{r9})^2}$ $W_{rC} = \sqrt{758^2 + (606 - 254)^2}$ $= 836$	$W_{rD} = \sqrt{W_{r11}^2 + (W_{r10} + W_{r12})^2}$ $W_{rD} = \sqrt{(1060)^2 + (848 + 254)^2}$ $= 1536$

Note: d—pitch diameter, in.; D—gear pitch diameter; $d_m = d - F\sin\gamma$; $d_{mG} = D - F\sin\Gamma$ mean pitch diameter of gear, in.; F—gear face width; K—axial distance from center of face width to center line of bearing D; L—axial distance from center of face width to center line of bearing A; M—spacing between centerline of bearings A and B; n—speed of gear; N—axial distance from center of face width to center line of bearing C; P—power transmitted, hp; W'_r—separating component; W_t—tangential tooth load, lb; W_x—axial-thrust component; γ—pitch angle; Γ—gear pitch angle.

The tangential load on this spiral bevel gear will be

$$W_{tG} = \frac{126{,}050 P_h}{d_{mG} n} = \frac{126{,}050 \times 71}{8.374 \times 588} = 1818 \text{ lb}, \quad (14.61)$$

where $P_h = 71$ hp, $d_{mG} = 8.374$ in., and $n = \frac{16}{49} \times 1800 = 588$ r/min of gear.

The tangential load on the aforementioned spiral bevel pinion will be equal to the tangential load on the gear.

$$W_{tP} = W_{tG} = 1818 \text{ lb}. \quad (14.62)$$

The axial-thrust loads for both gear and pinion are obtained from Figure 14.22:

$$W_x = (\text{percentage from Figure 14.22}) \times W_t, \quad (14.63)$$

$$W_{xP} = +0.80 \times 1818 = 1454 \text{ lb (outward)}, \quad (14.64)$$

$$W_{xG} = +0.20 \times 1818 = 364 \text{ lb (outward)}. \quad (14.65)$$

The separating force for both gear and pinion is obtained from Figure 14.23:

$$W_r' = (\text{percentage from Figure 14.23}) \times W_t, \quad (14.66)$$

$$W_{rP}' = +0.20 \times 1818 = 364 \text{ lb (separating)}, \quad (14.67)$$

$$W_{rG}' = +0.80 \times 1818 = 1454 \text{ lb (separating)}. \quad (14.68)$$

Table 14.6 tabulates the formulas and the sample calculations for the resultant bearing loads on the aforementioned pair of gears.

Note that since an overhung mounting is used on the pinion, the formulas for bearings *A* and *B* are used. If both members had been overhung mounted (Figure 14.46), the formulas for bearings *A* and *B* would have been used for both gear and pinion, or if both members had been straddle mounted (Figure 14.47), the formulas for *C* and *D* would have been used for both gear and pinion. In any case, the dimensions and the loads for the corresponding member must be used.

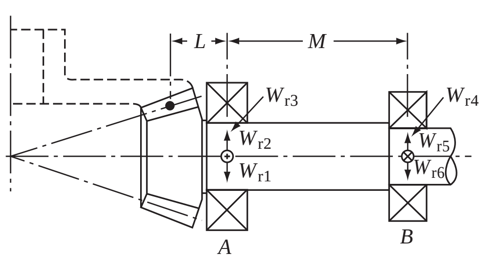


FIGURE 14.46 Overhung mounting.

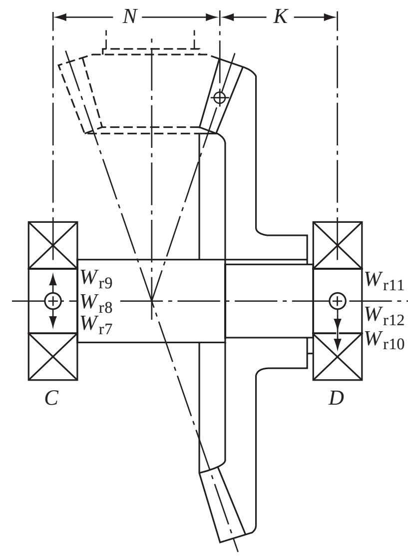


FIGURE 14.47 Straddle mounting.

PROBLEM 14.2
Example of Hypoid Gear Bearing Load Calculation

A sample problem of a 14-tooth hypoid pinion driving a 47-tooth hypoid gear is given in the following:

GIVEN

A 14/47 combination, 5.013 gear diametral pitch, 1.438 in. face width, 1.750-in. pinion offset below center. Pressure angles are 90°20′ on forward drive; 27°10′, on reverse drive. Spiral angles are 21°80′ on the gear; 45°3′, on the pinion. Pinion face angle is equal to 21°47′. Gear root angle is equal to 66°27′. (See Table 14.7 for further data relative to this problem.)

LOAD DATA

60 hp at 1500 rpm of the pinion; LH pinion driving clockwise; gear is straddle mounted; pinion, overhung.

The tangential load on the aforementioned hypoid gear will be

$$W_{tG} = \frac{126{,}050 P_H}{d_{mG} h} = \frac{126{,}050 \times 60}{8.057 \times 447} = 2100 \text{ lb}, \quad (14.69)$$

where $P_H = 60$ hp, $d_{mG} = 8057$, and $n = (14/47) \times 1500 = 447$ rpm (gear).

The tangential load on the hypoid pinion will be

$$W_{tP} = W_{tG} \frac{\cos \psi_P}{\cos \psi_G} = 2100 \times \frac{0.70649}{0.93060} = 1594 \text{ lb}, \quad (14.70)$$

$$\psi_P = 45°3', \cos \psi_P = 0.70649, \quad (14.71a)$$

$$\psi_G = 21°28', \cos \psi_G = 0.93060. \quad (14.71b)$$

TABLE 14.7
Hypoid Gear Sample Data and Bearing Arrangement (English Units)

	Pinion		Gear	
Item	**Symbol**	**English Units**	**Symbol**	**English Units**
Pitch diameter			$D = \dfrac{N}{\text{diam. pitch}}$	9.375″
Pitch angle	γ_o	21°47′	Γ_R	66°27′
	$\sin\gamma_o$	0.37110	$\sin\Gamma_R$	0.91671
Mean pitch diameter	$d_{mP} = d_{mG}\dfrac{n\cos\psi_G}{N\cos\psi_P}$	3.161″	$d_{mG} = D - F\sin\Gamma_R$	8.57′
Distance from center line of mate to center of bearing	H (from layout)	5.615″	G (from layout)	1.581″
	$J_P = \dfrac{d_{mG}}{2}\cos\varepsilon$	3.740″	$J_G = \dfrac{d_{mP}}{2}$	
Overhung Mounting (See Figures 14.44 and 14.46)				
	$L = H - J_P$	–	1.875″	–
	M	–	4.750″	–
Straddle Mounting (See Figures 14.45 and 14.47)				
	$K = G - J_G$	–	–	3.000″
	N	–	–	6.250″

The axial-thrust loads for both gear and pinion for forward drive are obtained from Figure 14.22:

$$W_x = (\text{percentage from Figure 14.22}) \times W_t, \quad (14.72)$$

$$W_{xG} = +0.11 \times 2100 = 231 \text{ lb (outward)}, \quad (14.73)$$

$$W_{xP} = +1.08 \times 1594 = 1722 \text{ lb (outward)}. \quad (14.74)$$

The separating forces for both gear and pinion are obtained from Figure 14.23:

$$W_r' = (\text{percentage from Figure 14.23}) \times W_t, \quad (14.75)$$

$$W_{rG}' = +0.48 \times 2100 = 1008 \text{ lb (separating)}, \quad (14.76)$$

$$W_{rP}' = -0.01 \times 1594 = -16 \text{ lb (attracting)}. \quad (14.77)$$

Table 14.8 tabulates the formulas and the sample calculations for the resultant bearing loads on the hypoid pair.

14.7.6 Required Data for Bearing Load Calculations

To calculate the bearing loads, the following data must be available:

Bevel gears:
- Gear ratio (number of teeth in gear and pinion)
- Gear pitch diameter or diametral pitch
- Pitch angles (both gear and pinion)
- Face width
- Pressure angle
- Spiral angle
- Horsepower or gear torque to be transmitted
- Gear revolutions per minute
- Hand of spiral
- Direction of rotation (viewed from the back)
- Sketch showing method of mounting and bearing spacing
- Shaft angle

Hypoid gears:
- Gear ratio (number of teeth in gear and pinion)
- Gear pitch diameter or diametral pitch
- Gear root angle
- Pinion face angle
- Gear face width
- Pressure angles (both sides of tooth)
- Spiral angles (both gear and pinion)
- Horsepower or gear torque to be transmitted
- Gear revolutions per minute
- Hand of spiral
- Direction of rotation (viewed from the back)
- Sketch showing method of mounting and bearing spacing
- Shaft angle

14.8 BEARING LOAD CALCULATIONS FOR WORMS

Basically, the worm-and-gear bearing load calculations are similar to helical gear reactions except for three main differences. All the members of the worm gear family run on crossed axes. Generally, the axes are crossed at 90°.

TABLE 14.8
Solution to Sample Problem on Hypoid Gear Bearing Loads (English Units)

		Bearing Loads			
		Overhung Mounting		Straddle Mounting	
Force	**Load Calculations**	**Bearing *A***	**Bearing *B***	**Bearing *C***	**Bearing *D***
Tangential, W_t	$W_t = \frac{126{,}050P}{d_m n}$	$W_{r2} = W_t\left(\frac{L+M}{M}\right)$ $W_{r2} = 1594\left(\frac{1.875+4.750}{4.750}\right)$ $= 2223$	$W_{r5} = W_t\left(\frac{L}{M}\right)$ $W_{r5} = 1594\left(\frac{1.875}{4.750}\right) = 629$	$W_{r8} = W_t\left(\frac{K}{N+K}\right)$ $W_{r8} = 2100\left(\frac{3.00}{6.25+3.00}\right)$ $= 681$	$W_{r11} = W_t\left(\frac{N}{N+K}\right)$ $W_{r11} = 2100\left(\frac{6.25}{6.25+3.00}\right)$ $= 1419$
Thrust, W_x	$W_x = W_t \times$ thrust factor See Figure 14.22	$W_x =$ thrust factor $\times W_t$ $W_x = +1.08 \times 1594 = 1722$	–	–	$W_x =$ thrust factor $\times W_t$ $W_x = +0.11 \times 2100 = 231$
Separating, W_r'	$W_r' = W_t \times$ separating factor See Figure 14.23	$W_{r1} = W_r'\left(\frac{L+M}{M}\right)$ $W_{r1} = -16\left(\frac{1.875+4.750}{4.750}\right) = -22$	$W_{r4} = W_r'\left(\frac{L}{M}\right)$ $W_{r4} = -16\left(\frac{1.875}{4.750}\right) = -6$	$W_{r7} = W_r'\left(\frac{K}{N+K}\right)$ $W_{r7} = 1008\left(\frac{3.00}{6.25+3.00}\right) = 327$	$W_{r10} = W_r'\left(\frac{N}{N+K}\right)$ $W_{r10} = 1008\left(\frac{6.25}{6.25+3.00}\right) = 681$
Thrust couple	$W_x \frac{d_m}{2}$	$W_{r3} = W_x\left(\frac{d_m}{2M}\right)$ $W_{r3} = 1722\left(\frac{3.161}{2 \times 4.750}\right) = 573$	$W_{r6} = W_x\left(\frac{d_m}{2M}\right)$ $W_{r6} = 1722\left(\frac{3.161}{2 \times 4.750}\right) = 573$	$W_{r9} = W_x\left[\frac{d_m}{2(N+K)}\right]$ $W_{r9} = 231\left[\frac{8.057}{2(6.25+8.00)}\right] = 101$	$W_{r12} = W_x\left[\frac{d_m}{2(N+K)}\right]$ $W_{r12} = 231\left[\frac{8.057}{2(6.25+3.00)}\right] = 101$
Total radial load	$W_r = \sqrt{\sum(\text{loads})^2}$	$W_{rA} = \sqrt{W_{r2}^2 + (W_{r1} - W_{r3})^2}$ $W_{rA} = \sqrt{2.223^2 + (-22-573)^2}$ $= 2301$	$W_{rB} = \sqrt{W_{r5}^2 + (W_{r4} - W_{r6})^2}$ $W_{rB} = \sqrt{629^2 + (-6-573)^2}$ $= 855$	$W_{rC} = \sqrt{W_{r8}^2 + (W_{r7} - W_{r9})^2}$ $W_{rC} = \sqrt{681^2 + (327-101)^2}$ $= 718$	$W_{rD} = \sqrt{W_{r11}^2 + (W_{r10} + W_{r12})^2}$ $W_{rD} = \sqrt{1419^2 + (681+101)^2}$ $= 1620$

Note: D—gear pitch diameter, in.; $d_m = D - F \sin\Gamma_R$ (mean pitch diameter of hypoid gear; in.) $= d_{mG}\frac{n}{N}\left(\frac{\cos\psi_G}{\cos\psi_P}\right)$ (mean pitch diameter of hypoid pinion; in.); F—gear face width, in.; K—axial distance from center of face width to center line of bearing D; L—axial distance from center of face width to center line of bearing A; M—spacing between center line of bearings A and B; n—number of teeth in hypoid pinion; n—speed, rpm (values for corresponding member); N—axial distance from center of face width to center line of bearing C; N—number of teeth in hypoid gear; P—power transmitted, hp; W_r'—separating component; W_t—tangential tooth load, lb; W_x—axial-thrust component; Γ_R—gear root angle; ψ_G—gear spiral angle; ψ_P—pinion spiral angle.

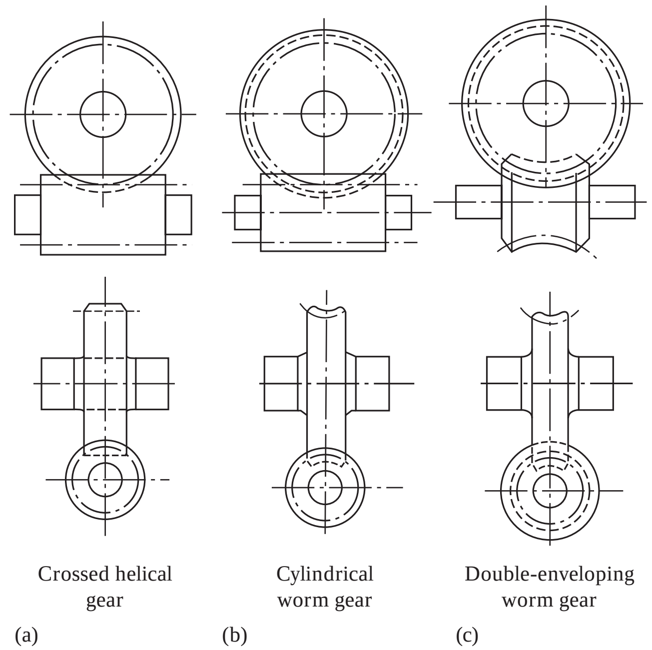


FIGURE 14.48 Types of worm gears: (a) nonthroated crossed helical (point contact), (b) single-throated cylindrical worm gear (line contact), and (c) double-throated, double-enveloping gear (line contact or area contact).

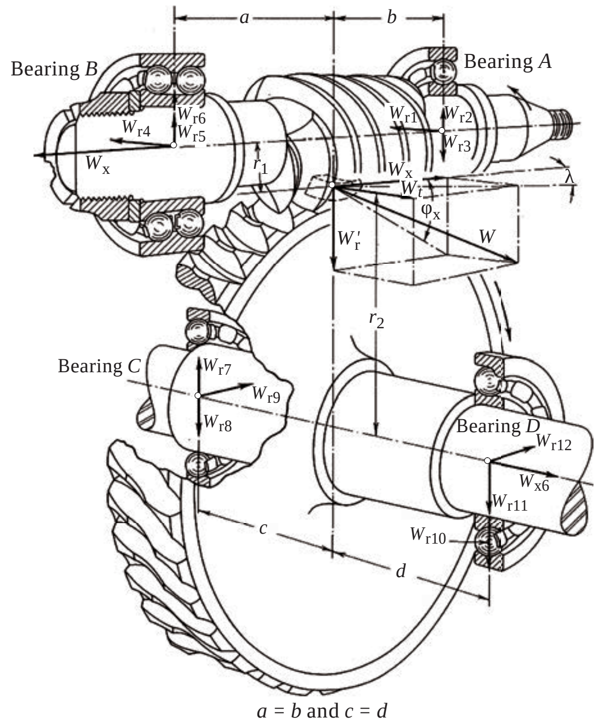


FIGURE 14.49 Worm gear bearing loads.

Besides the shaft arrangement, worm gears have overlap. There may be no overlap, overlap on one plane, or overlap on two planes. Overlap is the characteristic whereby one tries to envelop the mating part by being so curved as to tend to wrap around the mating part. Overlap gives a greater area of tooth engagement. It also makes the mounting more sensitive. See Figure 14.48 for worm types.

The third unusual feature of worm gears is that of ratio. The ratio is de ned as the number of gear teeth divided by the number of worm threads (or worm starts). A single-start worm may have several turns of thread, but if it meshes with a gear with 100 teeth, the ratio is 100:1. If the worm had two starts and the gear had 100 teeth, the ratio would be 50:1.

14.8.1 Calculation of Forces in Worm Gears

Figure 14.3 showed the basics of a force that acted on three planes. This concept is helpful in understanding how the force at the worm gear mesh acts.

The ef ciency of worm gears is quite variable. A single-thread worm may have ef ciency as low as 50%. A multiple-thread worm with at least 15° lead angle and reasonable rubbing speed will usually have ef ciency in the range of 90% to 95%. Since worm gear ef ciency is so variable, it cannot be neglected in calculating load reactions.

The worm gear overlap gives a relatively wide zone of contact. The pressure angle and the thread angle vary quite appreciably throughout this area. It is customary to calculate the worm gear load reactions if all the contact were at the theoretical pitch point. This permits the use of the theoretical pressure angle, the theoretical lead angle, etc., as variables in the equations. Although this method is not really exact, it is good enough to get load reactions to size bearings and design casing structures.

Figure 14.49 shows the case of a worm and a gear mounted on 90° axes and each straddle mounted. Load reactions at the mesh and at each of the bearings are shown.

The tangential force on the worm may be calculated by

$$W_t = \frac{19{,}098{,}600P}{n_1 d_{p1}} \text{ (metric)}, \tag{14.78}$$

$$W_t = \frac{126{,}050P_h}{n_w d} \text{ (English)}, \tag{14.79}$$

where

W_t (W_t)—tan driving force, N (metric) (lb [English])
P (P_h)—input power, kW (hp)
n_1 (n_w)—rotations per minute of worm
d_{p1} (d)—pitch diameter of worm

The separating force is

$$W_r' = \frac{W_t \tan\alpha_n}{\tan\gamma} \text{ (metric)}, \tag{14.80}$$

$$W_r' = \frac{W_t \tan\phi_n}{\tan\lambda} \text{ (English)}, \tag{14.81}$$

where

W_r' (W_r')—separating force, N (metric) (lb [English])
γ (λ)—worm lead angle
α_n (ϕ_n)—normal pressure angle

The worm thrust force is

$$W_x = \frac{W_t}{\tan(\text{lead angle})}. \tag{14.82}$$

The details for worm gear bearing load calculation are shown in Table 14.9. Figure 14.49 illustrates the bearing loads in a worm gearset.

The method of worm gear bearing load calculations just described may be used for crossed-helical gears on right-angle axes, for cylindrical worm gears, and for double-enveloping worm gears.

14.8.2 Mounting Tolerances

The worm gear types vary all the way from the nonenveloping crossed-helical to the double-enveloping type. In the crossed-helical, alignment is not critical in any direction. Axial movement merely shifts the contact area along the length of the part. Center-distance error changes the backlash and the amount of tooth contact, but neither of these things is highly critical in most applications.

The cylindrical worm gearset is critical on center distance, shaft angle, and axial position of the gear. Slight errors in any of these three things will tend to change a full face width contact pattern to only a small amount of contact at the end of the tooth. The contact area in a worm gearset may drop from 100% to 15% with an error in alignment of as little as 0.010 in. in a foot.

The double-enveloping worm gearset is critical in the three directions of the cylindrical set plus a fourth direction of the axial position of the worm.

Table 14.10 shows some typical mounting tolerances for high-capacity worm gearset. The most precise worm gears used in timing devices, radar work, etc., are mounted with about half the error shown in Table 14.10. On the other hand, worm gearsets for commercial service and relatively light load service are mounted with about twice the error shown in Table 14.10.

TABLE 14.10
Typical Mounting Tolerances for High-Capacity Worm Gearsets

Center Distance	Tolerance on Center Distance	Tolerance on Axial Position	Tolerance on Alignment
	Metric Values (mm)		
0–75	±0.013	±0.0025	±0.650/m
75–150	±0.025	±0.050	±0.400/m
150–375	±0.050	±0.075	±0.225/m
375 up	±0.075	±0.100	±0.150/m
	English Values (in.)		
0–3	±0.0005	±0.001	±0.008/ft
3–6	±0.0010	±0.002	±0.005/ft
6–15	±0.0020	±0.003	±0.003/ft
15 up	±0.0030	±0.004	±0.002/ft

14.8.3 Worm Gear Blank Considerations

The worm member is usually made of steel. Generally, this member offers no problem in design. In a few cases, the worm may be long and slender with a wide span between its bearings. In this case, there may be danger of serious bowing of the worm between supports. Calculations should be made in such a case to make sure that the worm shaft is not too highly stressed. Also, the deflection should be calculated. This deflection acts as a change in center distance and it should be judged on this basis.

The worm gear is generally made of bronze in the toothed area. Quite often, the hub is made of steel or cast iron. The

TABLE 14.9
Equations for Worm Gear Bearing Loads with Straddle Mounting

Force	Worm Bearing Loads: Bearing *A*	Worm Bearing Loads: Bearing *B*	Worm Gear Bearing Loads: Bearing *C*	Worm Gear Bearing Loads: Bearing *D*
Worm tangential force, W_t	$W_{r1} = W_t\left(\frac{a}{a+b}\right)$	$W_{r4} = W_t\left(\frac{b}{a+b}\right)$	$W_{r7} = W_t\left(\frac{r_2}{c+d}\right)$	$W_{r10} = W_t\left(\frac{r_2}{c+d}\right) = W_{r7}$
Separating, W_r'	$W_{r2} = W_r'\left(\frac{a}{a+b}\right)$	$W_{r5} = W_r'\left(\frac{b}{a+b}\right)$	$W_{r8} = W_r'\left(\frac{d}{c+d}\right)$	$W_{r11} = W_r'\left(\frac{c}{c+d}\right)$
Thrust, W_x	$W_{r3} = W_x\left(\frac{r_1}{a+b}\right)$	$W_{r6} = W_x\left(\frac{r_1}{a+b}\right) = W_{r3}$	$W_{r9} = W_x\left(\frac{d}{c+d}\right)$	$W_{r12} = W_x\left(\frac{c}{c+d}\right)$
Total radial load	$W_{rA} = \sqrt{W_{r1}^2 + (W_{r2} - W_{r3})^2}$	$W_{rB} = \sqrt{W_{r4}^2 + (W_{r5} + W_{r6})^2}$	$W_{rC} = \sqrt{W_{r9}^2 + (W_{r7} - W_{r8})^2}$	$W_{rD} = \sqrt{W_{r12}^2 + (W_{r10} + W_{r11})^2}$
Total thrust	W_x may be applied to either bearing *A* or *B*		$W_{xG} = W_t$ (may be applied to either bearing *C* or *D*)	

Note: See Figure 14.49 for definition of terms and bearing locations.

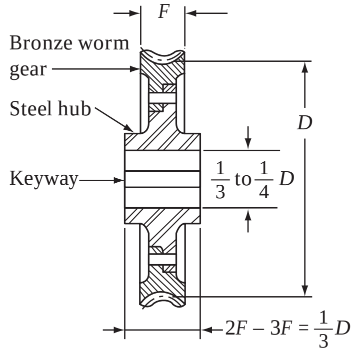


FIGURE 14.50 Worm gear blank proportions.

hub should have a large enough hole in it to assure a strong shaft. The length of the hub should be about one-third the gear pitch diameter to assure a stable mount.

Figure 14.50 shows a typical worm gear with a bronze ring bolted to a steel hub.

14.8.4 Run-In of Worm Gears

All types of worm gears bene t by a run-in. This is particularly true when the gear member is made of bronze.

The as-cut worm gear generally has generating ats on its surface, and the surface itself is somewhat imperfect because of oversize effects of the hob. As the gear is broken in, its surface wears a slight amount to conform to that of the worm. In addition, the surface of the gear becomes smooth and polished. Tests on bronze gears have shown that the amount of load that will cause seizure of the surface may go up as much as tenfold after a careful break-in!

In breaking in worm gears, the load is gradually increased and the speed is kept low. The surfaces are carefully watched. If small amounts of bronze start adhering to the worm surface, it is necessary to stop the break-in and dress down the worm threads with ne abrasive paper to remove the bronze. The break-in may then continue.

After the break-in is complete and the tooth surfaces polished, and the contact pattern has been obtained to a satisfactory level of quality, the gears are stamped as matched set. Exact axial distance settings used in the break-in are stamped on the set to aid in led installation.

14.9 BEARING LOAD CALCULATIONS FOR SPIROID GEARING

The Spiroid gear is a crossed-axes-type gear somewhat intermediate between a worm gear and a hypoid gear. Its originators, Illinois Tool Works of Chicago, Illinois, considers it to be a screw-type gear. The pinion of the Spiroid set is like a tapered worm. The pinion meshes on the side of the gear rather than the outside diameter of the gear. The gear does not wrap around the worm as a conventional worm gear does. The Spiroid gear is coned.

Another member of the Spiroid family is the Helicon gear. A Helicon pinion is cylindrical rather than coned.

The Spiroid pinion has two effective pressure angles, which are called the *high* pressure angle and the *low* pressure angle. Figure 14.51 shows the high side and the low side of a typical Spiroid pinion. The low side of the Spiroid pinion is preferred for driving because it has the largest amount of tooth contact area and it exerts the least separating force on the gear.

Spiroid and Helicon gears may be designed for a RH or a LH system. The choice of hand on the pinion determines the direction of rotation of the gear. The position of the pinion (normally in one of four quadrants) and the offset between the axes of the pinion and the gear have much to do with the direction of bearing reactions. Figure 14.52 illustrates the con gurations just described for Helicon gearsets.

Before bearing load calculations can be made for a Spiroid set, it is necessary to establish design details like ratio, cone angle, offset, and pressure angle. The pinion has a constant lead, but, because of the taper, the lead angle is variable. Figure 14.53 shows how the lead angle varies along the length of the cone.

Bearing load calculations are based on the lead angle, the pressure angles, the pitch diameters, and the center distance at the mean length of the pinion.

The exact calculation of Spiroid bearing loads is a complicated process. Generally, it is satisfactory to calculate approximate bearing loads using special constants. The error is usually less than 10%. If exact load data are needed, Illinois Tool Works is willing to furnish assistance on a speci c request.

The approximate tangential driving force may be obtained from the output horsepower or the output torque as follows:

$$W_t = \frac{19{,}098{,}600P}{n_2 d_{p2}} \text{ (metric)}, \tag{14.83}$$

$$W_t = \frac{126{,}050P_h}{n_G D} \text{ (English)}, \tag{14.84}$$

where

W_t (W_t)—tangential driving force, N (metric) (lb [English])
P (P_h)—output power, kW (hp)
n_2 (n_G)—speed of gear, rpm
d_{p2} (D)—gear pitch diameter, mm (in.)

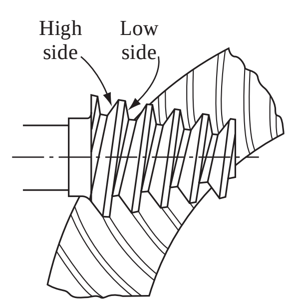


FIGURE 14.51 High and low sides of Spiroid gear.

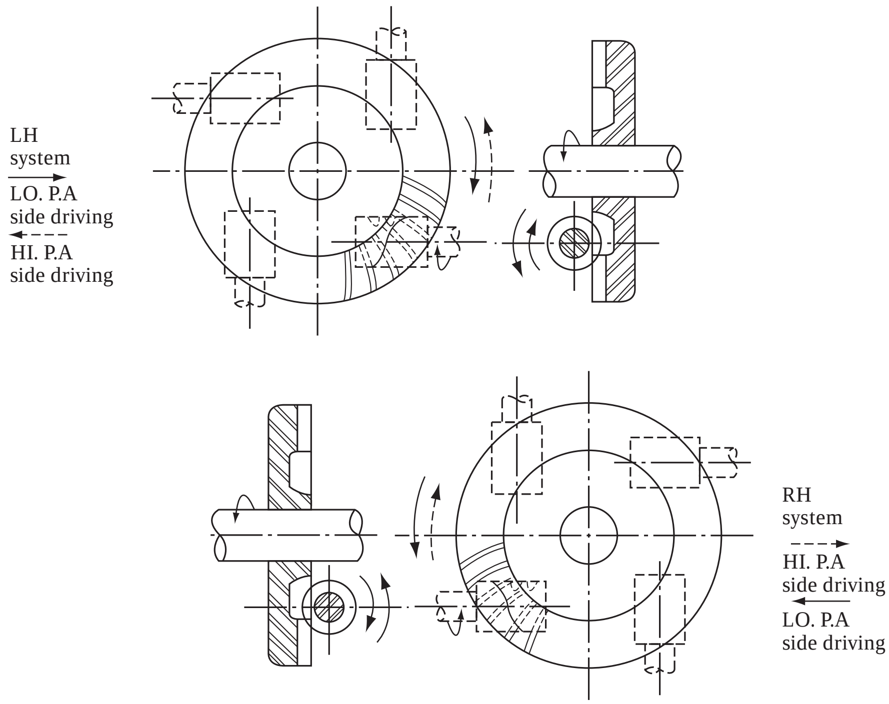


FIGURE 14.52 Examples of mounting positions for LH and RH Helicon gearsets.

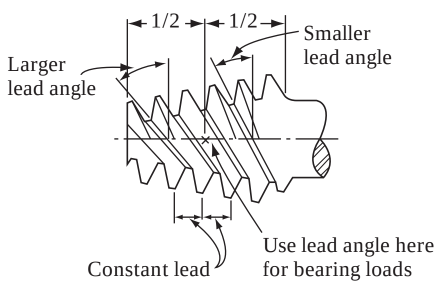


FIGURE 14.53 Spiroid pinion lead angle.

An approximate radial force on the gear is

$$W_{rG} = 0.91\ W_t. \tag{14.85}$$

The radial force on the pinion is

$$W_{rP} = \wp\, W_t, \tag{14.86}$$

where

$\wp$ = 0.60 for 35° pressure angle on high side
= 0.52 for 30° pressure angle on high side
= 0.35 for 15° pressure angle on low side
= 0.21 for 10° pressure angle on low side.

The thrust force on the pinion is equal to the radial force of the gear. Likewise, the thrust force on the gear is equal to the radial force on the pinion. Thus,

$$W_{xP} = W_{rG}, \tag{14.87}$$

$$W_{xG} = W_{rP}. \tag{14.88}$$

The point of application of these forces should be taken at the midface of the gear and the middepth of the tooth.

For straddle-mounted Spiroid set, the bearing loads may be determined by a procedure similar to that shown for worm gears in Table 14.8. Substitute W_{rP} for the worm tangential force W_t. Assume zero for the separating force. Substitute W_{xP} for the worm thrust W_x in the Spiroid gear calculation; use zero for the separating force and substitute W_{rG} for the W_x term in the worm gear bearing load part of the table. The radii r_1 and r_2 should be taken to the midpoint of the Spiroid mesh.

Figure 14.54 shows the Spiroid gear meshing point and the direction of the tangential force.

The Spiroid Division of Illinois Tool Works in Chicago, Illinois, has established computer programs to determine bearing reactions for the commonly used Spiroid and Helicon arrangements.

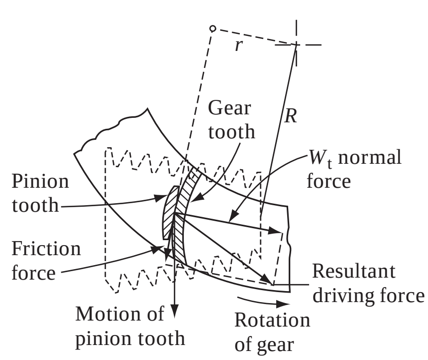


FIGURE 14.54 Spiroid gear bearing loads.

14.10 BEARING LOAD CALCULATIONS FOR OTHER GEAR TYPES

Space does not permit detailed bearing load equations for several additional types of gears. The Planoid*, Helicon, face gear, Beveloid†, and others are not covered. However, the principles used in this chapter make it relatively easy for designers to set up equations for any special types. The first step in solving the problem is to find the radial, separating, and thrust forces at some center point of the contact area. Then calculations are made to determine the final radial and axial reactions at each bearing location. If there is more than one load point on the same shaft, proper vector addition must be made to get the overall resultant.

In solving complex bearing load problems, the following rules should be kept in mind:

- When all the gearing forces have been transferred to a given mount, the final result is one resultant radial load and (depending on the bearing type) one resultant thrust (or axial) load.
- The summation of all forces must be equal to zero.
- The summation of all moments must equal to zero.

In the case of new gear types, the manufacturer of the type or the inventor of the type is usually glad to furnish potential customers with detailed calculations for the forces at the meshing point.

14.11 DESIGN OF THE BODY OF THE GEAR

To conclude this chapter, some comments should be made relative to the structure of the body that supports the gear teeth. It sometime happens that a gear designer does an excellent job of designing gear teeth and then making bearing selections and calculations to adequately support the gear, but does almost no engineering work relative to the gear body. In a gearset, the pinion is often small and essentially amounts to teeth cut on a section of shafting. The gear member of the pair, though, tends to be much larger in diameter and therefore is apt to involve a hub, a web, and a rim. Each of the three things just mentioned need to be so designed as to have adequate strength and adequate rigidity, and be manageable from the standpoint of resonance and vibration.

The prime usage of gearset may be the transmission of power, or it may be the transmission of motion. The following kinds of gear applications need to be recognized before design work is done to settle details of a rim, a web, or a hub:

- *Data gears*: Data gears are generally sized to fit an arrangement scheme and a need to bridge the distance from one shaft to another shaft. Some examples are tachometer drives, governor drives, or instrument drives. The amount of power transmitted through the gearing is usually relatively insignificant for the size of gears chosen to meet arrangement requirements.
- *Power gears*: Power gears are primarily used to transit power and change speed. They may be either speed reducing or speed increasing. The size of the gears is usually determined by making the gear large enough to carry the required power. For weight reasons and cost reasons, the tendency is to design them as small as possible but still have adequate power capacity and reliability.
- *Accessory gears*: In a power-producing engine or motor, there may be a train of power gearing that takes the main power from the prime mover to the driven device. This might be a turbine driving gear connected to a pump, a compressor, or a propeller. In addition to the main power drive, there is often a need to have a package of gear-driving accessories. These accessories can involve oil pumps, fuel pumps, pumps for hydraulic power, and so forth. The arrangement of the accessory package frequently requires gears larger in diameter and wider in face width than what is needed for the transmitted power. This means that the body structure of accessory gears can often be less rugged than that needed for main drive power gears.

A further consideration in gear body structures has to do with the kind of application. For instance, rocket engines and aircraft engines have to be very lightweight. This means the use of very high–strength materials and extra cost in manufacture to achieve the ultimate capacity in power per kilogram of metal or per pound of metal. At the other end of the spectrum, industrial gearing used in mills and in processing plants can be much heavier in weight, but there is much concern to keep the cost relatively low. Relatively heavy structures are used. The allowable stresses are held to much lower values than in aerospace work.

Table 14.11 has been put together to give some general guidance relative to rim thickness, web thickness, holes in the web, and hub thickness. Those using this table should consider the table as primarily historical with regard to what is commonly done rather than as positive design values that must be used. For instance, a well-designed aircraft gear may be acceptable with a rim thickness underneath the gear teeth that is only 1.25 times the gear tooth height. If the face width happened to be rather wide and the rim underneath the gear lacked stiffness due to a thin web or the tendency to have spokes connecting to time to the hub, it may be necessary to increase the rim thickness to two or three times the tooth height to get adequate stiffness in the gear body.

The tendency to have a spoked condition is quite common in marine applications and gearing for stationary power. Gears of this type often have a rim thickness that is as much as four times the tooth height.

At times, rather thin webs are used that tend to go beyond Table 14.11 recommendations. This helps save weight, but there may be failures of the web due to resonant vibrations.

* Registered trademark of the Spiroid Division of Illinois Tool Works, Chicago, Illinois.

† Registered trademark of Vinco Corp., Detroit, Michigan.

TABLE 14.11
Approximate Ratio of Certain Gear Body Dimensions to the Whole Depth of the Gear Tooth

Gear Drive Application	Rim Thickness	Web Thickness	Lightening Holes Recommended?	Hub Thickness
		Data Gears		
Instrument or data accessory	0.6–0.7 (0.07 in. min)	0.5–0.7	Normally if space available	0.6–1.0 (min: 0.07 in.)
		Power Gears		
Rocket engine main power	1.0 (min: 0.125 in.)	1.0 (min: 0.125 in.)	Not recommended	1.5–2.0
Accessory rocket	0.7	0.5	Recommended	1.5–2.5
Accessory aircraft	1.0–1.24 (min: 0.125 in.)	0.7–1.0 (min: 0.125 in.)	Odd number of holes	1.5–2.5
Aircraft and helicopter main drives	1.25–1.50	1.00–1.35	Normally gears only	–
Planets	1.5 avg.	1.35	Gear only	–
Accessory	1.25	1.00	Gear only	–
Commercial hypoid	2.0	2.0	Yes	8
Commercial spiroid	1.0–2.5	2.0	Yes	11
Marine, submarine	3–4	3 to 4	Holes required for welding, cleaning, etc.	–
Stationary power	4–6	Depends on design	–	–

Note: Antibacklash: Use values appropriate for data gears or for power gears. Ratio of 1.0 means value is equal to whole depth; 2.0 is twice whole depth.

This risk can be handled by using damper rings under the rim of the gear to stop vibration. If stresses induced by vibrations are not too high, the shot peening of a thin web in critical areas may give the added capacity that is needed to handle stresses from vibrations.

Another general area of concern is resonance in vibration. A webbed gear tends to have up to three or more critical speeds. If the gear is not running too fast and the web is fairly heavy, the resonant frequencies tend to be very high compared with the frequencies that the teeth are meshing. In aerospace applications, though, it often happens that a critical frequency can be rather close to the operating frequency. This is a bad situation and should be avoided. Sometimes the first critical comes at about two-thirds of the operating frequency and the second critical is well above the operating frequency. This situation may be tolerable if the gear unit tends to run at a constant speed and the gear unit comes up to full speed rather quickly so that there is almost no time that the gear is operating at the first critical.

To sum up, the design of the body structure of the gear should be handled just as thoroughly as the design of the gear teeth. It is beyond the scope of this book to present many pages of engineering data on the design of structures such as gear bodies, turbine wheel bodies, and so forth. The main point is that structure design is just as important in gearing as it is in the design of other types of machinery.

15 Gear Vibration

In this chapter we consider the general subject of gear vibration. This is an important technical subject. Gearboxes that have much vibration may be rejected by the purchaser. Gear units that have operated for a few years' time with relatively low vibration may start to have high vibration. This onset of vibration is often a timely warning that a breakdown of the gearing is only a few weeks away.

Aside from the considerations just mentioned, there are other mechanical system considerations with regard to vibration. Vibration in vehicles or aircraft is unwelcome—even if the gears are quire able to run for a long time without failure. Vibrations induced in a closely connected mechanical system may cause premature failure of somewhat fragile components such as instruments, control mechanisms, small motors, or even bolts. (Critical bolted joints may suffer from bolts coming loose.)

15.1 FUNDAMENTALS OF VIBRATION

Suppose we take a gearbox with two stages of gearing. If we measure the vibration at one of the four corners of the box with sensitive instruments, we will detect a rather tiny motion that is moving back and forth at high frequency. The character of this motion tends to be like a sine wave. Figure 15.1 depicts the sine-wave character of a vibration at some point. The mathematics of the sine wave can be thought of in terms of an imaginary circle and trigonometry.

In Figure 15.1, the radial displacement is the vector distance d. The size of this vector is determined by a point moving around a circle at a rate ω for radians per second. The vibration is both positive and negative from a no-vibration position. If d_a is the total amount of movement from the neutral position, then the diameter of the imaginary circle is $2d_a$. As a point moves around the circle, its position is speci ed by the angle θ. For a complete cycle of the vibration, θ goes through 360° (or 2π rad).

With the foregoing relations de ned, some equations can be written.

For displacement,

$$d = d_a \sin\theta. \tag{15.1}$$

The maximum displacement, peak to peak, is D. Figure 15.1 shows that D equals $2d_a$ at the time θ equals 90°. The angle θ is taken in degrees (when taking the sine of an angle by computers or tables reading sine values versus degrees for the angle).

The *velocity* of a vibration motion is an important variable. Considering that velocity is de ned as displacement divided by time, the equation for velocity becomes the rst derivative of displacement with respect to time:

$$V = \omega d_a \cos\theta. \tag{15.2}$$

The acceleration is the second derivative of displacement with respect to time. The acceleration curve is established by

$$A = -\omega^2 d_a \sin\theta. \tag{15.3}$$

Figure 15.2 shows schematically how the three curves for vibration behavior relate to each other from a *time* standpoint. Note that if the peak displacement occurring at θ equals 90°, the peak velocity occurring at θ equals 180° and the peak acceleration occurs at 90°—but acceleration is negative. However, at 270° there is another acceleration peak that is positive, *and* the displacement has now become negative.

From the standpoint of vibration measurements, the common practice is to interpret vibration at double amplitude for displacement but single amplitude for velocity. Acceleration is also normally given as a single amplitude value. The units of vibration are usually in g values—where g is the acceleration of gravity.

Table 15.1 shows simpli ed equations for these values in both metric and English units.

To illustrate the vibration relations just discussed, we take a sample problem.

PROBLEM 15.1

Suppose that the peak-to-peak displacement at a location on a gear unit measures 0.001 in. What is the peak velocity in inches per minute and the peak acceleration in g value? The frequency is 100 Hz (100 Hz is 6000 cycles/min; it is also 628.3 rad/s). The peak displacement is 0.0005 in., which is one-half the peak-to-peak displacement.

Using Equation 15.7, we nd that the peak velocity is

$$V = 0.314 \text{ in./s}. \tag{15.10}$$

From Equation 15.9, the peak acceleration is

$$A = 0.511g. \tag{15.11}$$

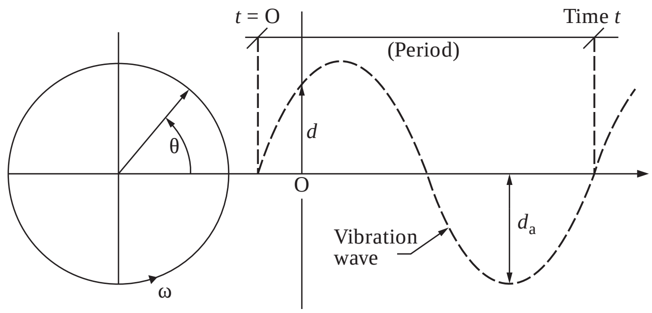


FIGURE 15.1 The vibration excursion is like a sine wave.

Figure 15.3 shows a sample of special graph paper where displacement, velocity, and acceleration can all be read—*if any one of these values has been measured*.

The Figure 15.3 graph is based on the equations just given. Note that the point in the circle gives all the answers for the given sample problem. (Incidentally, there is a small error in printing the paper. The paper reads acceleration at 0.47*g* instead of 0.511*g*.)

15.2 MEASUREMENT OF VIBRATION

The measurement of vibration involves a sensing device that detects the vibratory motion, recording equipment that records the signals from the sensing device, and data-processing equipment that converts the raw data to some kind of analytical data in the form of vibration amount at different frequencies.

The device that does the sensing is generally considered to be a transducer. The output of the transducer is an electric signal that goes by wires to the data recorder. The data recorder may record these signals on magnetic tape (to be analyzed later). It is possible to have a data recorder that will instantly process incoming signals and give a readout of vibration intensity.

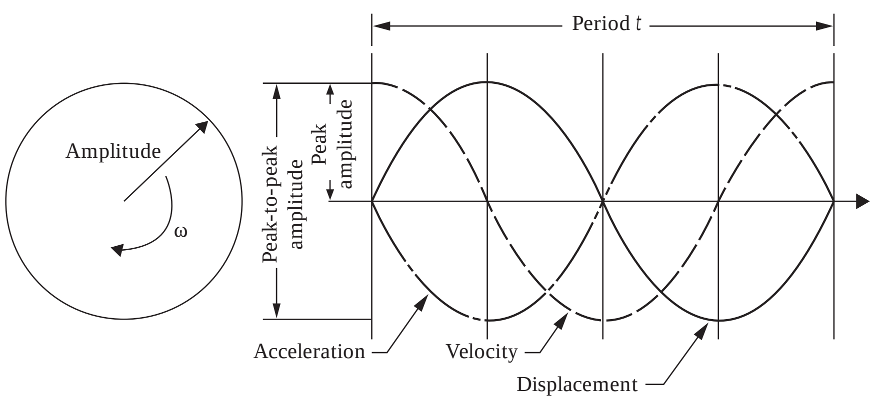


FIGURE 15.2 Waveforms for displacement, velocity, and acceleration tend to be sine waves. Note that these forms are out of phase with each other.

TABLE 15.1
Relation between Vibration Peak Values

	Equation[a]		
Item	**Metric**	**English**	
Peak-to-peak displacement	$D = 2 \times$ peak amplitude (mm)	–	(15.4)
	–	$D = 2 \times$ peak amplitude (in.)	(15.5)
Peak velocity	$V = \omega \dfrac{D}{2}$ (mm/s)	–	(15.6)
	–	$V = \omega \dfrac{D}{2}$ (in./s)	(15.7)
Peak acceleration (in *g* values)	$A = -\omega^2 \dfrac{D}{2}$ (mm/s^2)		(15.8)
	–	$A = -\dfrac{\omega^2}{386} \dfrac{D}{2}$ (in./s^2)	(15.9)

[a] ω = radians per second = 2π Hz; 1 Hz is 1 cycle per second. *D* is in millimeters for the metric equations and in inches for the English equations.

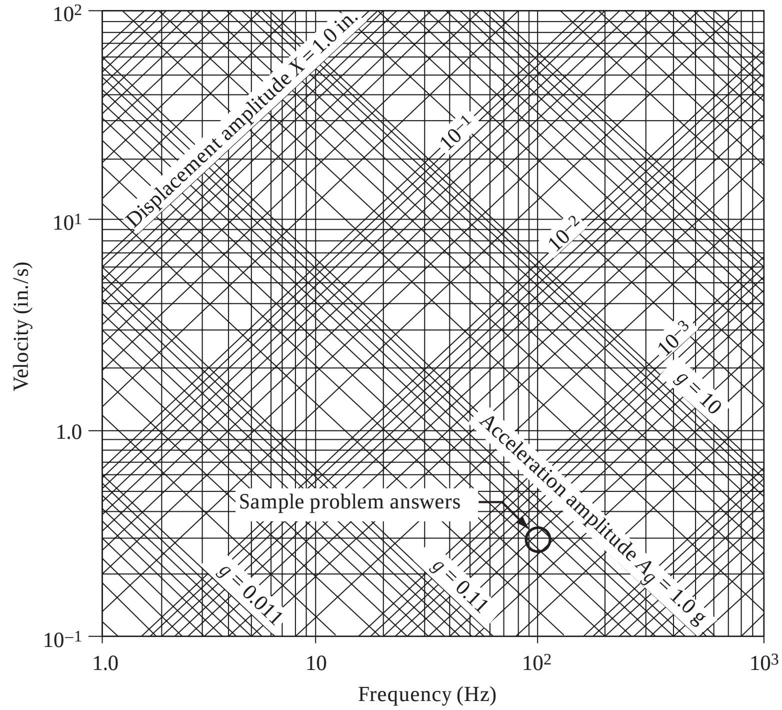


FIGURE 15.3 Special graph paper where matching values of velocity, displacement, and acceleration can all be read. Circled area shows sample problem results.

The processing equipment may filter the recorded data and give the maximum vibration for something such as each third of an octave band. This permits plots to be made of vibration intensity versus frequency.

The processing equipment will usually have the capability of taking a signal in any one of the three vibration modes (displacement, velocity, and acceleration) and converting this signal to the other two modes. When this conversion is done, it is done on the assumption that the vibration wave is a simple harmonic motion and, therefore, the relations are the same as those given in Table 15.1.

15.2.1 Examples of Sensing Devices

We discuss three commonly used sensing devices (or transducers). These are as follows:

- A proximity probe
- A velocity transducer
- An accelerometer

Figure 15.4 shows the schematic arrangement of a proximity probe. There is an air gap between the end of the probe and a rotating shaft. The probe end is positioned about 1 mm away from the shaft. A fairly high-frequency alternating current is emitted from the probe end. Variation of probe distance causes a change in the inductance, the capacitance, or the eddy current losses. All three types are nonlinear. They can be compensated for so that over a restricted distance they will measure displacement with an accuracy that is within about 1% of being linear.

For good results the shaft must be homogenous, have a good surface finish, and be very accurate for roundness. The probe needs to be calibrated for the material in the shaft. The principal use of proximity probes is in measuring vibration displacement in either shaft or bearings.

A velocity transducer can be used to sense vibration velocity in a gear casing. Figure 15.5 shows a schematic of such a device.

The coil in the transducer moves along its axis in a radial magnetic field provided by a permanent magnet. The coil is attached to a probe that is spring loaded against the vibrating surface. The main mass of the magnet (and its case) is either clamped to a vibration-free base or handheld.

The coil that is moving in a magnet field in time with the vibration has relatively good linearity of electrical output with respect to vibration velocity.

Most designs of velocity transducers will work satisfactorily up to about 2000 Hz (120,000 cycles/min). Special miniature coil designs of velocity transducers will work satisfactorily to about 10,000 Hz.

Another commonly used sensing device for vibration measurement is an accelerometer. Figure 15.6 shows the type that is piezoelectric.

A crystal of quartz or barium titanate will produce an electrical charge proportional to the stress induced in the crystal. As vibration occurs, a mass in the accelerometer produces either shear stress or compressive stress in the crystal.

Very small accelerometers are able to measure vibration frequency up to about 100,000 Hz. The accelerometer has to be quite large to measure acceleration accurately at 0.2 Hz (12 cycles/min).

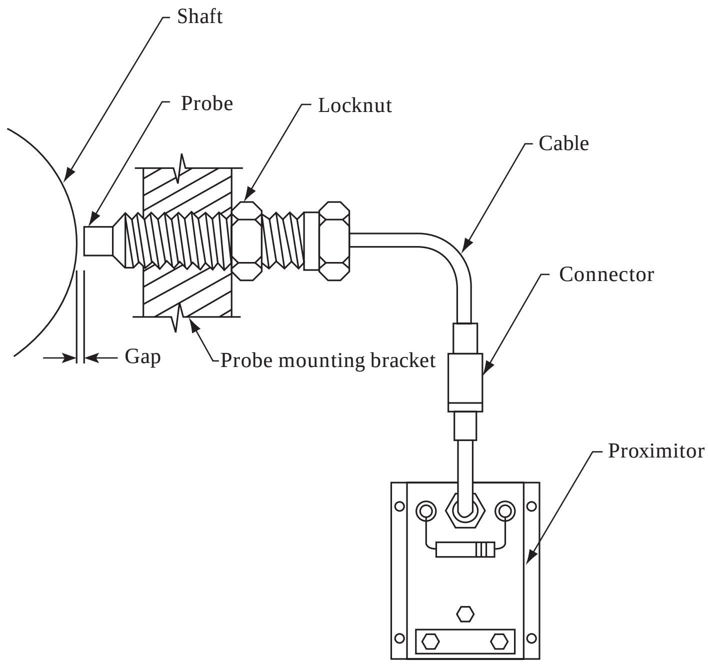


FIGURE 15.4 Schematic of a proximity probe. It is used to measure displacement.

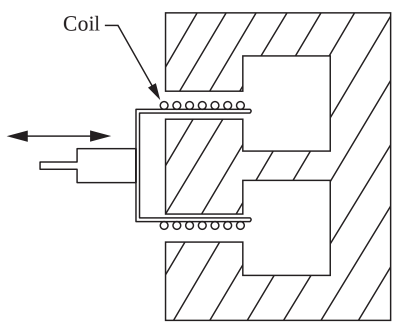


FIGURE 15.5 Schematic of a velocity transducer. It is used to measure velocity on a casing.

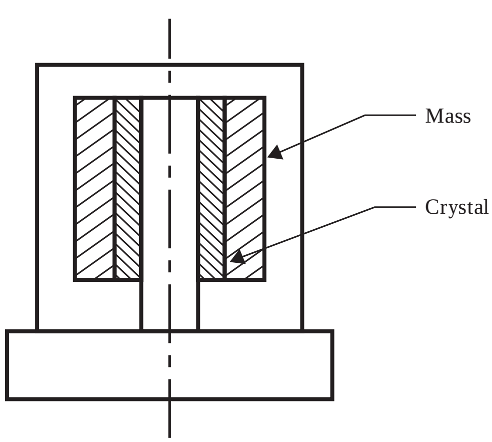


FIGURE 15.6 Schematic of a shear type of accelerometer. It is used to measure acceleration on a casing.

15.2.2 Practical Problems in Vibration Measurement

The measurement of vibration is not an easy task. With an abundance of devices on the market to sense the signal, transmit the signal, record the signal, and then process the signal, the rst problem in vibration measurement is the choice of equipment to procure. If equipment is on hand, there is the related question of whether or not the equipment on hand represents the latest technology—or is appropriate to measure the kind of vibrations likely to be present in the gears involved.

By one means or another, equipment is obtained to do vibration measurements. The next set of problems is apt to include one or more of the following requirements:

- Obtain a calibration of the transducers to be used.
- Determine how to mount each transducer so that a weakness in the mount or a vibration on the mount does not seriously affect the delity of the vibrations sensed.
- Use calibrated wires of a proper length so that the tiny signals transmitted through the wires do not lose their accuracy from signal transmission disturbances.
- Process the signal so that the results will give a valid indication of the character of the vibration present.
- Be alert to environmental things that can spoil vibration measurement accuracy. For instance, high temperatures or rather low temperatures may seriously impair the accuracy of the sensing transducer. Other things such as water vapor, oil mist, and corrosive fumes may soon cause trouble.
- Be sure that the transducers used have the right range to cover the frequency of vibration that may be present.

In looking at the foregoing list, it might seem that all could be resolved by just reading the manufacturer's speci cations for the vibration equipment on the market. Actual experience often reveals that the manufacturer's speci cations may not give all the needed details. Also, claims of accuracy limits may be under ideal conditions and not realizable in the actual environment of the job at hand.

15.3 SOME EXAMPLES OF VIBRATION IN GEARED UNITS

When vibrations are measured on a gear unit, several locations are chosen to take the readings. In the beginning, it is common practice to take readings in three directions. These are normally horizontal, vertical, and axial. Later on, when a unit has developed a vibration history, it may be possible to take readings at only one or two locations and only one or two directions and still know whether or not the vibration characteristics of the unit are satisfactory or unsatisfactory.

Figure 15.7 shows the full number of locations that might be used to take vibration readings.

A set of vibration readings taken by an accelerometer in the horizontal direction of position A on a high-speed gear unit is shown as an example. Note that there are peaks at gear mesh frequency and at twice gear mesh frequency. There is also a peak at nine times the pinion frequency. (The pinion teeth of this unit were somewhat worn and a pattern of nine bumps was developing in the tooth spacing pattern.) See Figure 15.8. Note that the g values for acceleration are not high, but there are three pronounced peaks. The peak at nine times the pinion frequency is quite ominous. There is no reason to have any peak at all this frequency unless something is going wrong.

Figure 15.9 shows an example of vibration readings in velocity taken for a large, fast-running epicyclic gear unit. These readings were taken in three directions shown for positions A, B, and C. Note that the highest value of vibration in the survey was 0.20 in./s at tooth mesh frequency in the horizontal direction.

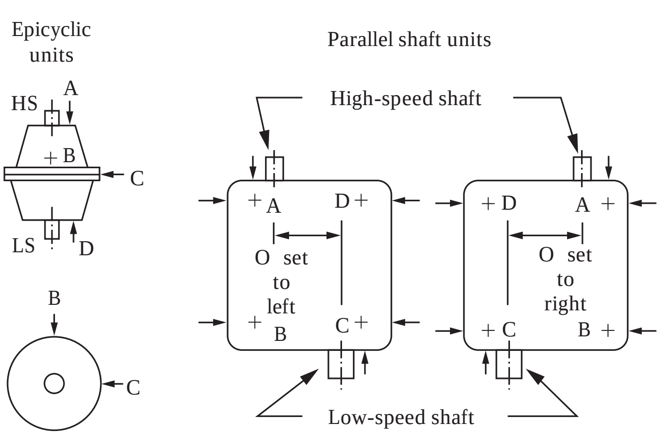


FIGURE 15.7 Typical locations for taking vibration readings on gear casings. HS: high side; LS: low side.

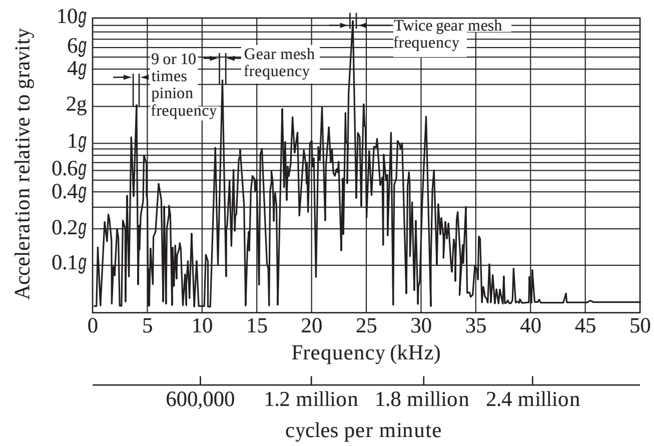


FIGURE 15.8 Spectrum analysis of vibration for a high-speed gear unit. This unit failed due to tooth wear, but the vibration amounts were not high at the time this analysis was made.

When proximity probes are used on shafts, it is possible to use two probes 90° apart. The data from these two can be processed such that the actual shaft motion can be seen on an oscilloscope.

Figure 15.10 shows the oscilloscope pattern for a " oating" sun pinion in an epicyclic gear unit running at about 60% of full rating. Note that a fast camera exposure can show what happens in 2½ turns of the shaft. A longer exposure shows 6 turns of the shaft. The oscilloscope data show that the orbit pattern is constantly changing due to vibrations at different frequencies.

Vibration data are particularly useful in monitoring units in service that may be prone to failure. A serious defect in geometric quality or metallurgical quality may get through even extensive and well-disciplined quality control inspections. Vibration checking of equipment in service is the last chance to catch a defect before a serious failure occurs.

Figure 15.11 shows an example of vibration behavior related to an impending failure. Shortly after start-up, a set of vibration readings was taken. The reading in the horizontal direction at one location was higher than the other readings. A month later this reading was rechecked and found to be still higher. In another month the reading had become substantially higher. Then, 2 days later, the reading was even higher. At this time the unit was taken out of service. When the unit was taken apart, serious bearing damage and gear tooth damage were found. After replacing the damaged parts, the rebuilt gear unit functioned normally for many years after.

15.4 APPROXIMATE VIBRATION LIMITS

When we consider a wide range of gear sizes, such as 100 kW to over 20,000 kW, and all types of prime movers such as electric motors, turbines, and internal combustion engines, it is obvious that the vibration criteria for a well-built, good-running gearbox is bound to vary over some range of values. Some other things that make vibration vary are the kind of

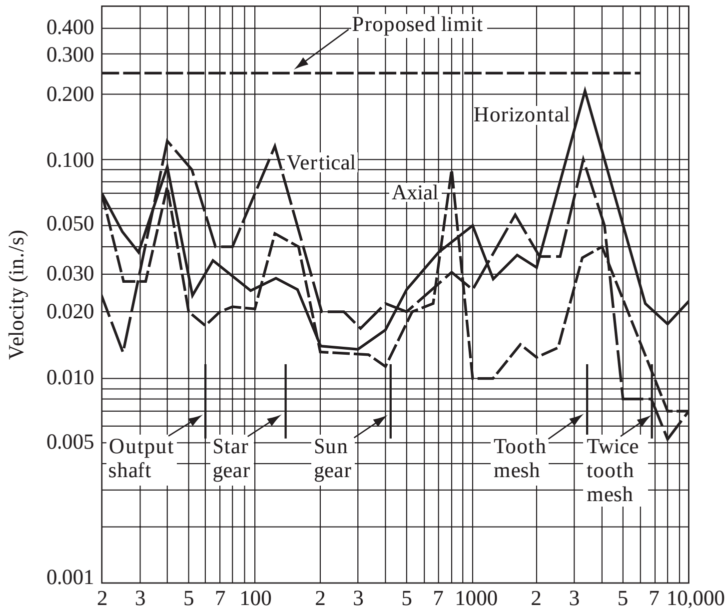
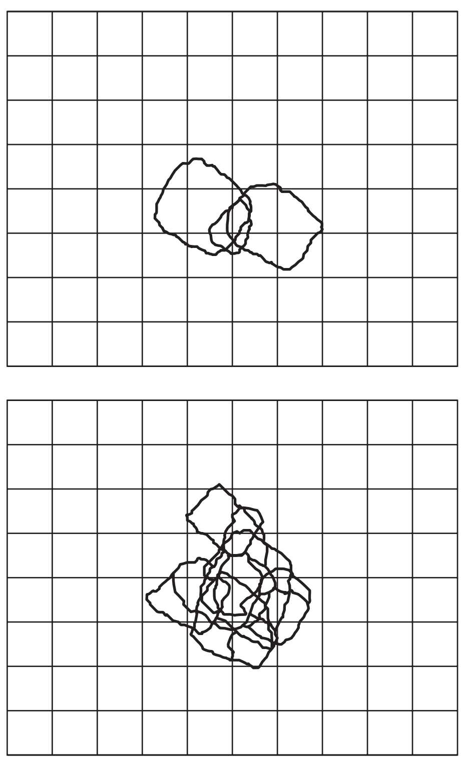


FIGURE 15.9 Examples of vibration readings taken on an epicyclic gear unit.



View A: Exposure 1/100 s

Shows about 2½ loops of orbit due to 240 Hz vibration

View B: Exposure 1/30 s

Shows about six loops of 245 Hz vibration, three loops of small 92 Hz vibration and pattern for 30 Hz

FIGURE 15.10 Examples of orbiting patterns for a oating sun pinion in an epicyclic gear unit. Note how vibrations at different frequencies can be seen on the oscilloscope.

teeth (spur or helical), the style of manufacture (low-hardness cut teeth or hard-hardness ground teeth), and the casing type (cast iron with heavy wall sections, or steel weldments with somewhat thin walls made of steel plate).

Figure 15.12 shows vibration limits based on the author's experience. Instead of showing one curve as a proposed limit, three curves are shown:

- *Upper curve with long dashes:* This curve represents values that are on the high side. The statement is that the unit is *probably in trouble* if these values are exceeded.
- *Middle curve with solid line:* This curve represents a middle-of-the-road value. It can be considered a *nominal design value.*

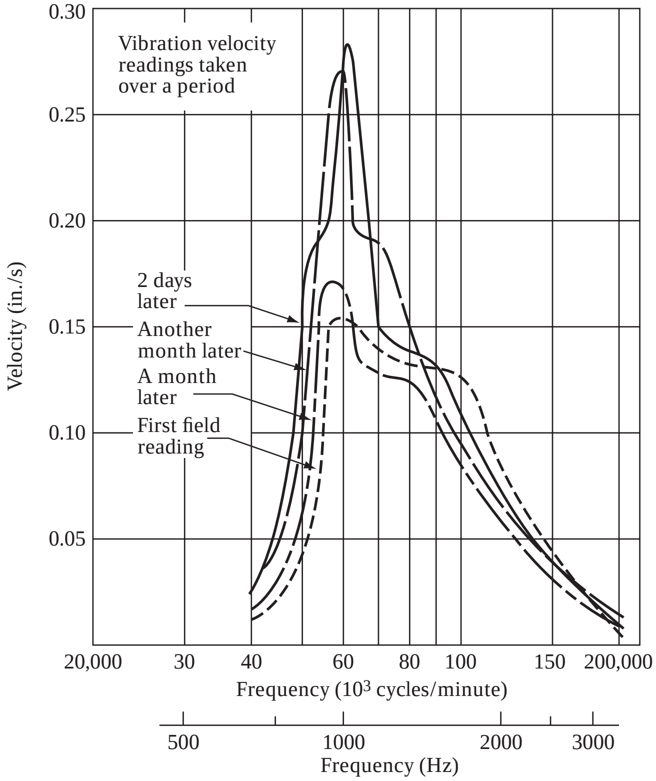


FIGURE 15.11 A turbine-driven gear unit is found to be in a failure mode and shut down before breakage of parts.

- *Lower curve with short dashes:* This curve represents close control of vibration. Even units that are quite sensitive to mechanical failure are *probably OK* if the curve of vibration intensity versus frequency is below these values.

15.4.1 Velocity Limits

Part 1 of Figure 15.12 shows the three curves for casing velocity limits. Note that these curves run from 20 to 1000 Hz. This is the frequency range where most velocity transducers can be expected to work well.

It is rather consoling to the gear engineer that the appropriate velocity limit tends to stay xed over a rather wide frequency range. This means, in many cases, that a measurement of vibration intensity tells the experienced engineer whether or not there is a vibration problem—without consideration of the frequency and how allowed vibration changes with frequency.

15.4.2 Acceleration Limits

Part 2 of Figure 15.12 shows three curves for acceleration in *g* levels. These curves are plotted from 60 to 20,000 Hz. This a range where it should be possible to use the accelerometer as the transducer to pick up casing vibration and get relatively accurate readings without undue effort. At vibration readings

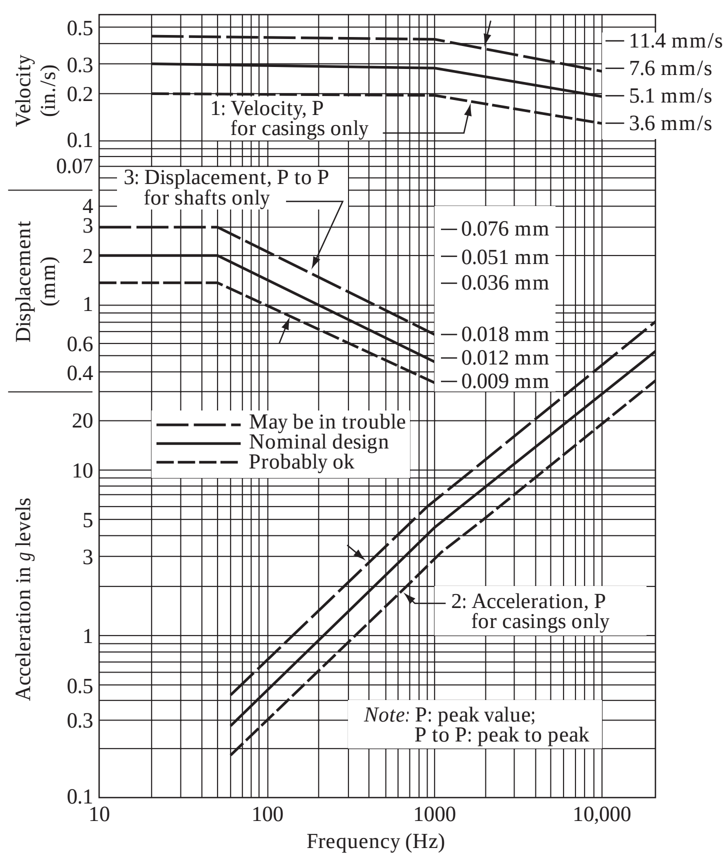


FIGURE 15.12 Range of vibration values recommended for consideration by gear designers or users.

of more than 20,000 Hz, the problem of avoiding vibration in the accelerometer mounting device becomes quite difficult to handle.

When the vibration readings go below 60 Hz, the allowable *g* values become very low. This means a very low electrical current in the pickup wires and difficulty in transmitting this signal accurately back to the recording and analyzing equipment.

Quite often accelerometers are used to measure vibrations in the range of 60 to 10,000 Hz, and then the readout is given in velocity. (See Table 15.1 for equations relating displacement, velocity, and acceleration.) This works fairly well. Of course, if the vibration is not close to being simple harmonic (sine wave) in pattern, then velocity converted from acceleration will not be the same as velocity read directly.

Below 60 Hz, there is apt to be trouble in getting reliable velocity readings from conversion of accelerometer readings. The recommendation is to use velocity transducers or other means when measuring rather low-frequency vibrations.

15.4.3 Proximity Probes

These devices are generally used on rotating shafts rather than on casings. It is possible, though, to use them on casings.

The proximity probe has no particular problem working down to very low speeds. Even a speed as low as 0.01 Hz (0.6 cycles/min) is practical.

At high speeds, the proximity probe is usually not practical when the limit reading is very small. For instance, a displacement reading less than 0.0075 mm (0.0003 in.) tends to be a trouble due to possible surface irregularities of the shaft and small variations in composition of the shaft. If the proximity probes give a reading of 0.0003 in., it is hard to decide whether this reading is really vibration or whether it represents shaft imperfections.

15.4.4 Displacement Limits

At frequencies below 50 Hz, one might question whether the limit values shown in part 3 of Figure 15.12 really represent a vibration hazard. (The matching values of either velocity or acceleration get very low.)

A displacement reading of 0.05 mm (0.002 in.) at 20 Hz does not shake a gearbox very much. Higher readings, though, tend to indicate misfit in bearings. This can spell trouble, for instance, to a low-speed output shaft in a gearbox that is turning at only 1 rpm. For good gear machinery, there is a concern to keep shaft run-outs relatively low even at slow speed and low vibration frequencies.

For epicyclic gears, there is often one member freely floating. This may be the sun, the planet carrier, or the annulus gear. Displacement readings by proximity probes are quite helpful to determine whether or not the orbit pattern of the free-floating member is acceptable. The displacement limits in part 3 of Figure 15.12 do not apply to free-floating epicyclic gear elements.

15.4.5 General Vibration Tendencies

A gear designer may be working on a low-cost, short-life gear unit where weight is somewhat important. The design may use features that are bound to make the vibration levels some somewhat high. If this unit is used in an environment where noise and vibration can be tolerated, then it is quite reasonable to use somewhat high vibration limits.

In this case, the key to what vibration limits should be used comes from building a number of units that meet all the design objectives—and taking enough vibration data to understand what vibration values are *normal* for this application. After the normal values are established, the gear builder and the gear user can use these values as a quality control check to help determining if additional units are really as good as the first units built and found to be satisfactory.

In a similar vein, a gear designer working on a long-life, expensive gear unit to be used in critical environment will find that the vibration values for units meeting all project objectives are rather low. This means that rather low limits must be set on vibration to maintain the desired quality in a long production run.

To help designers and users understand the many factors that enter into vibration values, Table 15.2 is based primarily on overall design considerations rather than just vibration logic. For instance, a turbine-driven, epicyclic gear unit made rather lightweight may show a *g* level of more than 40 at a frequency over 10,000 Hz. Such units at more than 1000 kW have a very large amount of kinetic energy for the mass of the parts. With good materials, good accuracy, and appropriate lubrication systems, a given design may demonstrate a service life of more than 30,000 h, even with a 40*g* vibration level.

TABLE 15.2
Factors That Raise or Lower Allowable Vibration

Factor	What Raises Vibration Limit	What Reduces Vibration Limit
Required life at or near full rating	Less than 2000 hours	Over 20,000 hours
Material hardness	38 HRC or lower (through-hardened)	50 to 65 HRC (case-hardened)
Kind of gear	Straight bevel gear/Spiroid gear	Spiral bevel gear/helical gear
Accuracy of teeth	Medium precision (cut gears)	High precision (ground gears)
Pitch-line velocity	Less than 25.4 m/s (5000 fpm)	Over 25.4 m/s (5000 fpm)
Rated power	Less than 400 kW (500 hp)	Over 1500 kW (2000 hp)
Weight	Light weight	Heavy weight

15.4.6 Trade Standard

AGMA has a standard for linear vibration on gear units. This standard is ANSI/AGMA 6000-B96.

Much useful data are given in this standard in regard to de nitions, instrumentation, measurement, and proposed test conditions when acceptance tests are being run. The standard aims to help the gear buyer and the gear maker negotiate a reasonable contract for the vibration part of gear acceptance tests.

Those designing or purchasing gear units should study ANSI/AGMA 6000-B96 carefully.

Figures 15.13 through 15.15 are extracted from AGMA standard. They show the class A and class B limits for displacement, velocity, and acceleration. These AGMA limits are not binding unless speci cally agreed to by the manufacturer and the purchaser.

15.5 CONTROL OF VIBRATION IN MANUFACTURING GEARS AND IN THE FIELD

The obvious starting point for vibration control is the mechanical design of the gears, shaft, bearings, and castings.

The gear teeth need to have suitable geometric accuracy. This involves tooth pro le, tooth spacing, helix angle, and concentricity. It also involves tooth surface nish, pro le

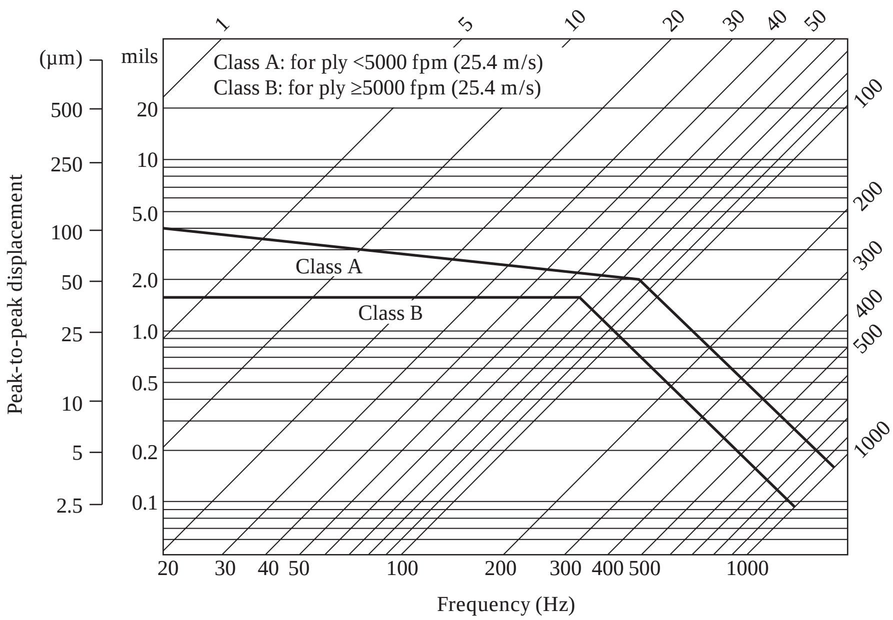


FIGURE 15.13 Displacement limits.

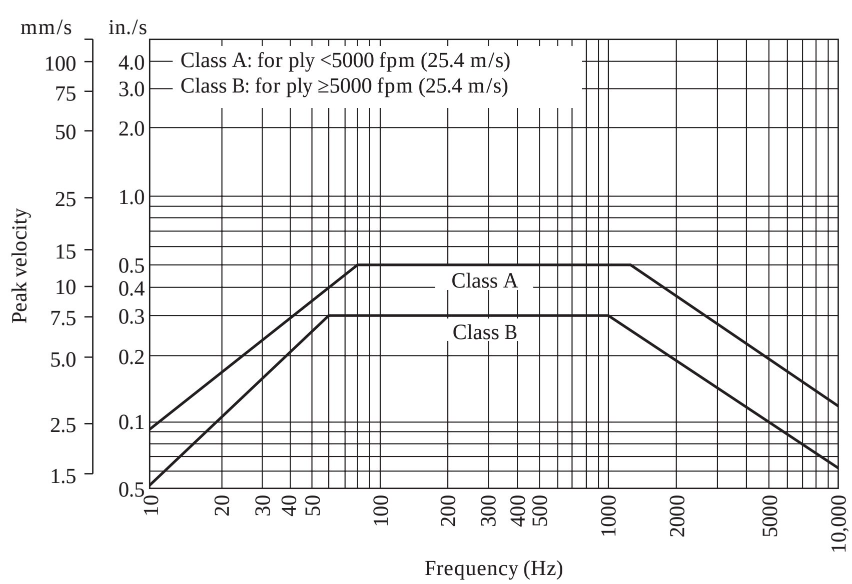


FIGURE 15.14 Velocity limits.

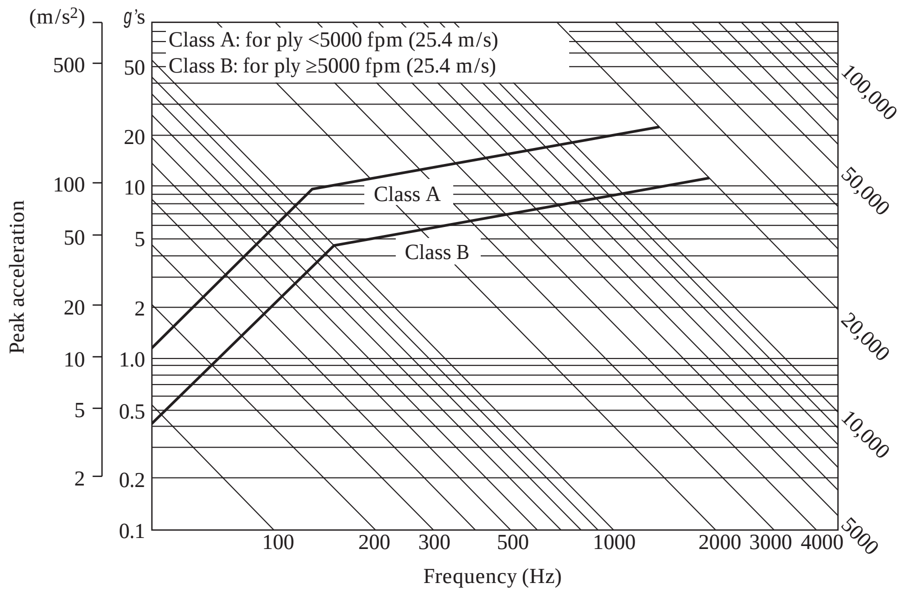


FIGURE 15.15 Acceleration limits.

modi cation, and helix modi cation. The teeth need to roll through mesh very smoothly with no undue shock as tips of teeth and ends of the teeth enter and leave mesh.

Chapter 7 gives some general advice on gear accuracy limits that are used in a variety of gear applications. The accuracy needs for gear teeth become quite critical when one or more of the following apply:

- The power transmitted across a single gear mesh is over 1000 kW, or over 1340 hp.
- The pitch-line speed is over 50 m/s, or over 10,000 fpm.
- The required life of the gearing at a high level of loading is over 10,000 h.

The shafts in a gear system need to support the meshing gears as to provide good alignment even when sizable changes occur in operating speed, percentage of load being carried, or operating temperature. If the shafts are rather long, the shaft windup may result in a torsional critical speed that is within the operating range of the gear unit. Large gear drives in ships or mills are often subject to torsional criticals, and, therefore, critical speeds must be calculated for the whole power system. If a critical speed comes too close to an operating speed, it is usually possible to shift the critical speed away from the operating speed by changing shaft diameters and masses of the rotating gears (or masses of other rotating elements).

The bearings in a gear system may be subject to half-frequency whirl, ball path excursions, or axial runout effects. Both sliding-element bearings and rolling-element bearings have their problems. The design of a critical gear unit must involve a consideration of possible vibration hazards coming from the bearings. It is usually possible to pick a kind of bearing and a mechanical design for the bearing so that the bearing does have a bad effect on the vibration behavior or the gear unit.

Large gear casings may have side panels that have critical vibration frequencies that come at some mesh frequency present in the gearbox. The attachment of the gearbox to a support structure may be such as to result in serious vibrations of the whole structure under certain conditions of operation (speed and load).

Both the mounts for a gear unit and the casing walls may need special devices to suppress vibration. To sum up, the control of vibration starts with the mechanical design of gears, shafts, bearings, and casing structures.

15.5.1 Testing of Gear Units at the Gear Factory

Gear units for turbine drive equipment in the oil and gas industry are often given a full-speed, no-load type of test before shipment. Typically, a unit is operated for about 1 hour at half speed, three-quarter speed, and full speed. Vibration readings are taken. Oil ow, oil temperature rise, and critical bearing temperatures are also measured.

The full-speed, no-load test is useful to make sure that the rotating components are all balanced well enough to run without undue vibration. The oil system is also checked out. A bearing that is too tight may run hot. If an oil passage is not drilled, the oil ow will not be right.

In very critical units, the purchase contract may call for a factory test at full speed and full power. The full-power tests may be made with a dynamometer or water brake to absorb the rated power of the gear unit. Sometimes the full-power test is made with one gear unit loaded against another gear unit.

The full-power test is very useful in catching defects, such as the following:

- Too much spacing error is accumulated in the gear teeth. (Vibration is high when full torque is applied.)
- Bearing fit is not right. Bearings show distress after running 20 or more hours at full torque.
- The tooth fit (under load) is not right. Early signs of tooth distress show up.
- Vibration shows up under torque loading at some speed in the operational range.
- A torsional or lateral critical may be present. Another possibility is that some bearing may not be loaded enough to seat itself in its proper position. (Gear weight or coupling reaction causes abnormal conditions in the bearing when running at partial torque.)

15.5.2 Tests of the Assembled Power Package

After the gear unit has been tested, it is often shipped to another factory where the whole power package is put together and tested. The power package involves a prime mover, a gear unit, and a driven unit. Some examples are the following:

- Turbine, gear unit, electric generator
- Turbine, gear unit, compressor
- Diesel engine, gear unit, pump
- Electric motor, gear unit, fan

The test of the whole power package involves starting, stopping, running at different power settings, overspeed test, high- or low-temperature tests, and endurance test. During tests such as these, vibration is measured, along with many other things such as oil flow, bearing temperature, fuel rate, and output energy.

Supposed, the gear unit vibration will be satisfactory—since the gear unit has already had a gear factory test. Quite often, though, the vibration behavior of the gear may be a problem. Here are some possibilities:

- The gear unit vibrates due to misalignment or unbalance in couplings between the gear unit and the prime mover or between the gear unit and the driven equipment.
- The heavy gear unit is within vibration limits but unbalance in the gear unit causes lightweight accessory equipment on the turbine to vibrate excessively.
- The diesel engine is not timed to fire just right on all cylinders. This causes vibration to be high on the gearbox.
- A centrifugal pump has a whirl mode. The pump is within limits on vibration, but it causes the gear unit to be over limits.

At the time of the power package test, it is very strategic to get a *vibration signature* of the gear unit (as well as a vibration signature of the prime mover and the driven machine).

When the power unit is installed in the field, the vibration signature can be compared with the vibration pattern at start-up. If the installation is made properly, the vibration pattern in the field should match the factory vibration signature quite closely.

15.5.3 Vibration Tests in the Field

After a power package is installed and operating in the field, vibration testing becomes most important. An increase in vibration very often means an impending failure. If it is an aircraft, it could mean that an engine would have to be removed before another flight was made. On a large ship, high gear vibration is likely to mean that gear repair work must be done before the ship can be insured to carry another load of cargo across the ocean.

In the oil and gas industry, there is always the danger of explosions or fire in an oil refinery or on an oil platform. The equipment that is running constantly is generally checked each day with some simple vibration equipment. If some gear unit or other piece of power equipment reads significantly higher one day than it did the day before, it is a matter of immediate concern. Even when vibration values are low, a change upward is an immediate warning.

When simple vibration checks show an upward shift, it is likely that more complex vibration equipment will be brought to the site. A vibration expert will make a complete analysis of the situation, and may make one of the following recommendations:

- Shut down the unit immediately.
- Observe the unit on a daily basis for some time to see if the vibration continues to increase.
- The simple vibration check (perhaps using a handheld instrument) was faulty—there is nothing to worry about.

In a situation such as this, a vibration signature is most helpful to the person making the decision. With the vibration signature, the whole pattern of change from a new unit running under factory conditions to a used unit running under field conditions is revealed.

In many situations, the risk of failures has led equipment operators to want to purchase gear units with installed vibration monitors. One or two strategic points are picked for a gear unit, and the vibration monitor reads continuously as the unit runs. The usual range is as follows:

- Normal
- Near maximum
- Shutdown

The continuous monitoring equipment has the obvious advantage that there can be automatic shutdown any time the vibration is dangerous.

TABLE 15.3
Analysis Methods for Gear Vibration

Method	Advantages	Disadvantages
Vibration amplitude plotted against frequency (may be called "spectrum analysis")	A standard means of presenting vibration data; specified in ANSI/AGMA 6000-A8; good to determine average vibration at each shaft frequency and each mesh frequency	If there are one or more bad gear teeth in the gear circumference, these are not located
Cepstra	Can find individual bad teeth here; instrumentation is available	Can confuse tooth defects with other shaft-related problems
Time domain averaging	Improved vibration spectrum; improved defect detection; instrumentation available	Requires shaft encoder signal; interpretation can be complex
Root-mean-square vibration level	Easy to implement; simple instrumentation	Cannot isolate gear tooth vibration from other vibrations; high false alarm rate
Tooth by tooth	The most effective method to find defects in all the teeth of a gear	Instrumentation developed but not generally available in gear making shops; requires shaft encoder; will become better known to industry through the 1990s

However, there are a couple of problem areas. At start-up a unit is cold and may be out of alignment if there is a need to allow for a change in alignment when normal steady-state running conditions are reached. This can mean that a unit keeps going over the shutdown vibration limit when someone is trying to start the unit.

Another problem is that operating people assume that all is well when the vibration is low. Quite often a vibration at some location may measure no more than 0.05 in./s. If the shutdown was 0.30 in./s and normal went up to 0.25, then a change of 0.05 to 0.10 in one day and a change from 0.10 to 0.15 a day later could almost pass unnoticed. If there is a quick change of 0.10 in vibration, something is going wrong and needs to be considered—even if the vibration is still in a "normal" range. From practical standpoint, *change in field readings is more important than the measured value.*

15.6 VIBRATION ANALYSIS TECHNIQUE

The techniques of analyzing and presenting gear unit vibration data have become somewhat numerous. Vibration specialists keep trying to develop better ways to determine what is wrong—or not wrong—with a gear unit from the vibration signals picked up on the gear unit when it is running at some output speed and at some output torque.

The concern over vibration becomes most critical when a gear unit may have a gear tooth with a developing fatigue crack—and the breakage of this tooth might lead to costly damage to the gear unit or (even worse) be a safety hazard in an oil refinery or on a passenger aircraft.

Even in a new gear unit, the risk of one or more gear teeth being accidentally damaged in installation or being malformed due to a local final tooth grinding mistake can be a matter of serious concern.

Table 15.3 shows some of the analysis methods in current use. Note that the commonly used method of plotting vibration intensity versus discrete frequencies does not have the capability to locate a single bad tooth. Of course, if there is just one bad tooth, there may be a high vibration "spike" coming in a shaft frequency—rather than gear mesh frequency.

The method that can locate any or all bad teeth in a gear is the tooth-by-tooth measuring system using encoders. (See the last item in Table 15.3.)

For most gear units used in factories, aboard ships, or in oil refineries and pumping stations, the spectrum analysis, shown as the first entry in Table 15.3, is quite adequate.

From a practical standpoint, may very important gear units are built and shipped without any vibration testing being done either in the gear factory or on site in the field. Although improvements and new developments in vibration analysis will be helpful, the greatest need (from the author's viewpoint) is for more gear production with a vibration control being exercised along the lines discussed earlier in this chapter.

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Appendix A: The Evolution of the Gear Art*

FOREWORD

The administrative family and members of the American Gear Manufacturers Association are happy and proud to held in bringing this authoritative work by Mr. Darle W. Dudley to your attention.

Even a casual reading discloses that this is a genuine labor of love, coupled with a remarkable knowledge of our industry. A more thorough study will enrich the student, engineer, gear manufacturer, or gear user by giving each an insight into the manner in which gears have been and are being utilized to soften man's bumpy traverse of land, sea, and space.

No one but Mr. Dudley could have authored this book, because we know of no one so uniquely experienced in the rami cation of the gear of the gear eld. Substantial evidence of this conclusion was shown in 1958 when the AGMA Awards Committee and Board of Directors named Mr. Dudley as recipient of the "Edward P. Connell Award," AGMA's highest annual honor that can be tendered to a gear oriented individual.

John C. Sears
Executive Director
American Gear Manufacturers Association
Washington, D.C.

PREFACE

Anyone who has worked in the gear eld for many years becomes impressed with the long, continued use of gears in all manner of machinery. Geared machines always seem to go right back to the beginning. The earliest automobiles, for instance, used gears. The early water mills, a few centuries ago, used gears. The Romans, two millenniums ago, had geared devices. When, then, did the art of gearing start?

In recent years, I have tried to nd out just how the gear art evolved. Many friends like Earle Buckingham, Gilbert Dannehower, Charles Staub, Russell Ball, Jr., Granger Davenport, and Ted Miller have told me things they know about gear history and have suggested where I might search to nd more information. The Smithsonian Institution in Washington, D.C., (and Edwin Battison of the Smithsonian) was very helpful. Books by Robert Woodbury, Sprague De Camp, Abbott Payson Usher and Ralph Flanders have provided excellent source material. Out of these sources and personal discussions with a great number of AGMA members, I have pieced together a story about how gears and the theory of gear engineering has developed.

The story that is told in this book is rather surprising. Gears have been around for at least 5000 years! The gear art, born in the earliest times of antiquity, is growing (like so many other things) at an astonishing rate as we approach the end of the 20th century.

I am very pleased that the American Gear Manufacturers Association (AGMA) has been interested enough in this subject to sponsor the publication of *The Evolution of the Gear Art*.

My secretary, Mrs. M. Irene Galarneau, worked late at night to organize, edit, and type my manuscript material for this book. I greatly appreciate her very capable assistance.

Darle W. Dudley
San Diego, California

A.1 INTRODUCTION TO GEARING AND ITS PLACE IN THE ARTS AND CRAFTS OF MAN

The gearing industry of today is concerned not only with tooth gear wheels but with gear transmission packages involving casings, gears, shafts, bearings, clutches, couplings, lubrication system and even the structure to support the gear plus its driving or driven device. All in all, gears and the things that go to make up gear packages are a very important part of the mechanical products manufactured in the United States. John Sears, the executive director of AGMA, has told me that their studies at AGMA Headquarters show an annual gear business of over one billion dollars in the United States. My own studies indicate that that the total value of the gear transmissions in every device built—vehicle, machine, appliance, instrument, power system, tool, clock, implement, military weapon, electronic device, toy, gadget, etc.—may be closer to two billion dollars per year.

Gears are one of man's oldest mechanical devices. The toothed wheel takes its place with the lever, the inclined plane, the screw and the pulley as one of man's earliest devices to increase the force that could be applied to an object. The gear has been a basic element of machinery throughout all time from the earliest beginnings of machinery.

As a matter of fact, the candidates to be considered as the earliest machines of man would have to include the primitive gear train along with the primitive form of the potter's wheel, the primitive lathe and the ancient water-lifting devices.

Sentimentally, gears have always been associated in the popular mind with mechanical machinery. A Rube Goldberg cartoon of an invention will invariably depict gears in the device. Abbott Payson Usher's excellent book, *A History of Mechanical Inventions*,* chose some gear types as its cover illustration (see Figure A.1). Figure A.2 shows the familiar gear wheel insignia

* This slightly modi ed version of the book by Darle W. Dudley entitled *The Evolution of the Gear Art* is represented in this section of the book. Originally published by AGMA as early as in 1969 (93 pages in total), this publication is not widely known to the gear society.

* A selected bibliography is given at the end of this section of the book.

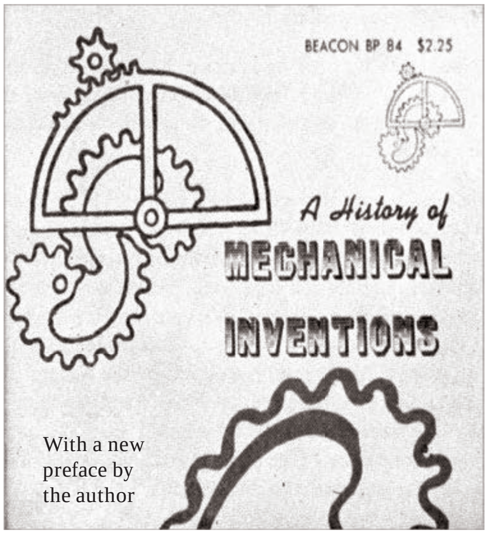

FIGURE A.1 Gears are used to illustrate the cover of Abbott Payson Usher's book. (Courtesy of Beacon Press, Boston, Massachusetts.)

FIGURE A.3 The symbol of interlocked gears indicates positive, coordinated action. (Courtesy of National Petroleum Council, Washington, D.C.)

of Rotary International that is a road-sign all over the United States. Figure A.3 shows an example of the gear symbol used by the National Petroleum Institute on a brochure published in 1966. Numerous manufacturing companies—like National Cash Register—have chosen a gear to be part of the symbol of their company. The term *geared for progress* has a symbolic meaning to anyone concerned with the mechanical arts.

In the public mind, gears are one of the most well recognized kinds of machinery. They create the impression of positive action, coordinated—interlocked—precise application of effort to secure a desired result.

The gear trade can be very proud of the fact that their craft is as old as civilization itself. For 5000 years gears have been used by man—and their use is ever increasing. Our newest supersonic airplanes will have hinged wings with gears used in the actuation system to move the wings. The supermarket of the future will deliver goods to a customer like a gigantic coin machine. Geared mechanisms will mechanically transfer the purchased articles from storage to the customer's car. Doctors will soon be implanting extremely tiny electro-mechanical devices in the human body to regulate body functions that a diseased organ can no longer properly control. Miniature gears of very high quality and miniature electronic components make this very humane work possible.

The gear industry has bene ted greatly from the early establishment and forward looking development of a national trade association called "American Gear Manufacturer's Association" (AGMA). This history of the gear art was rst presented at the AGMA Annual Convention at Hot Springs, Virginia, June 1966, as part of the program to commemorate the 50th anniversary of the founding of AGMA. The widespread interest in gears and gear history has led to the publication of this revised and enlarged work on the evolution of the gear art.

A.2 GEARING IN PRE-CHRISTIAN TIMES (3000 BC–100 BC)

Most writers trace gears back to the writings of Aristotle (around 330 BC). Aristotle writes of gears as if they were commonplace so the beginning must go back much further.

FIGURE A.2 Rotary International uses the gear to indicate the sterling quality of the organization.

The earliest known relic of gearing from ancient times is the South Pointing Chariot, circa 2600 BC. This chariot was not only geared but it contained a very complex differential gear train that is hard for even a modern gear engineer to analyze! A miniature replica of this chariot is on exhibit at the Smithsonian at Washington, D.C.

The ancient Chinese apparently used this chariot to keep from getting lost while traveling through the Gobi Desert. It can be set so that the figure *points south and continues to point south regardless of which direction* the chariot is going.

The South Pointing Chariot used gearing made with wooden pins. These pins are clearly visible in Figure A.4. Figure A.5 shows, in cartoon fashion, how this type of gear can be used on either a parallel axis or a right-angle axis drive.

In view of the intricacy of the South Pointing Chariot, it seems obvious that there must have been earlier use of gearing going back to at least 3000 BC. The earliest written records about gearing are dated from about 330 BC. This leaves a blank space of almost 3000 years during which gear devices must have been in use. Since there are no written records covering this interval, we must await the spade of the archeologist to fill in our knowledge.

Writings of Philio of Byzantium and Hero of Alexandria indicate that prayer wheel devices in Egyptian temples were developed to the point where trains of gears were used. Apparently the worshipper could get blessed more thoroughly with the help of speed-increasing gears. De Camp [1] remarks that gears led to "more salvation per revolution."

From many writings it seems probable that both the Egyptians and Babylonians were using gear devices as far back as 1000 BC. The most probable uses were in clocks, temple devices and water-lifting equipment.

The earlier writings about gears did not write of gear devices as if they were new inventions; rather, the writings described gear devices with the implication that they were items of machinery or devices that were in common everyday use. It can thus be inferred that Greeks, Egyptians, and Babylonians were using gear devices for many centuries prior to 300 BC. The discovery of the South Pointing Chariot proves that the Chinese had a knowledge and use of gearing back as far as about 3000 BC.

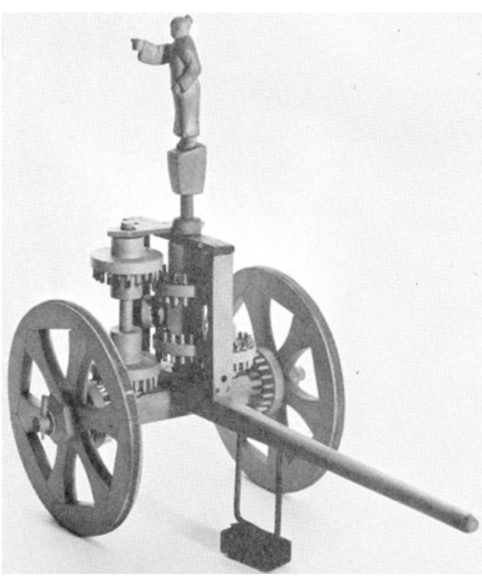

FIGURE A.4 South-pointing chariot, circa 2600 BC. (Courtesy of Smithsonian Institution, Washington, D.C.)

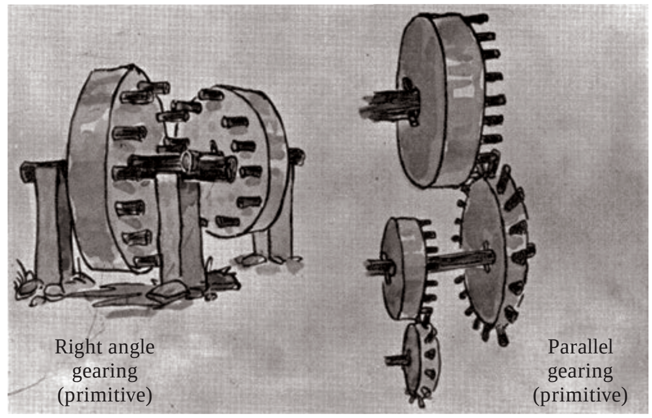

FIGURE A.5 Primitive gear types. (Courtesy of L. O. Witzenburg, Cleveland Worm & Gear Division, Eaton Yale & Town Inc., Cleveland, Ohio.)

After Aristotle, several ancient writers mentioned gears and left sketches (in some instances). Table A.1 shows some of these.

By 100 BC, the gear art included both metal and wooden gears. Triangular teeth, buttressed teeth, and pin-teeth were all in use. Spur gears, racks and pinions and worm gears had all arrived on the scene. Right-angle pin-tooth drives were in use and perhaps the first primitive bevel gear.

A.3 THE USE OF GEARING IN THE ROMAN EMPIRE (100 BC–400 AD)

As the Roman legions marched to conquer and develop the world's greatest empire, all the arts and the crafts went through an extensive development. Gears were used for the first time to supply continuous power. Marcus Vitruvius, a colleague of Agrippa (writing about 16 BC), describes a flour mill driven by water poser through right-angle gears (pin-gear type). See Figure A.6.

The Romans and Greeks made wide use of gearing in clocks and astronomical devices. Gears were also used to measure distances or speed. For instance, a small paddle wheel on the side of a ship was geared to a speed-indicating device. The ship was powered by oar or sail.

Hero of Alexandria wrote books describing the various hydraulic and mechanical devices used in the first* century AD. In his book, *Mechanics*, he describes the five simple machines used to apply force to move a weight. These were:

- Wheel and axle
- Lever

* The approximate date of his writings is established by an eclipse that was seen at 62 AD in Alexandria and noted by Hero.

TABLE A.1
Early Gear Information

Name	Approximate Date	Item
Aristotle	330 BC	Explains gear wheel drives in windlasses. Points out that the direction of rotation is reversed when one gear wheel drives another gear wheel.
Otesibius (Greek)	250 BC	Made water clocks and water organs using gears.
Philio of Byzantium	230 BC	Made rack and pinion device to raise water. Wrote of elasticity of metals. Tells how to test Spanish and Celtic swords.
Archimedes	220 BC	Made devices to multiply force or torque many times—studied spirals.

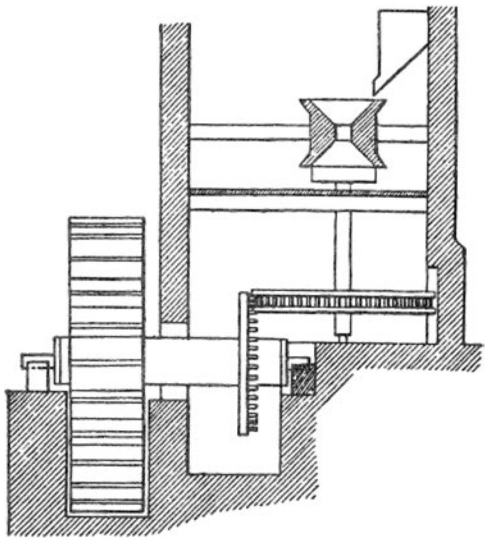

FIGURE A.6 Schematic of Roman grist mill (as depicted by Marcus Vitruvius), circa 16 BC.

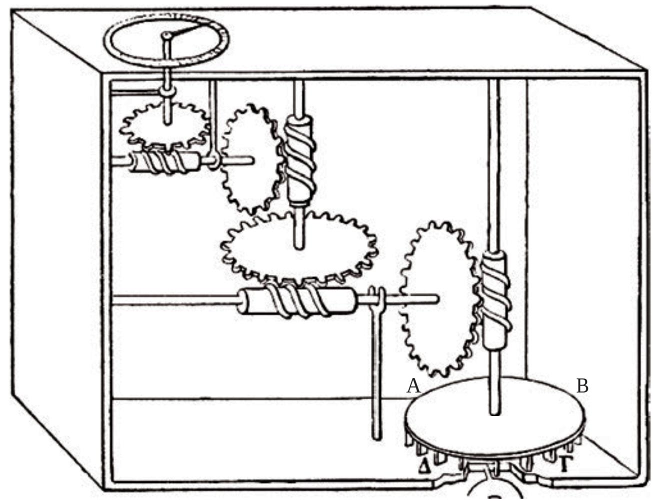

FIGURE A.7 Cyclometer (odometer) described by Hero, circa 62 AD.

- Pulley
- Wedge
- Screw

Hero wrote of gears but he apparently did not recognize them as a separate machine. For example, he shows a very practical arrangement of gear wheels for lifting a weight. Hero no doubt considered this as an advanced development of the wheel and axle machine.

A cyclometer device (odometer) is shown by Hero. He says that the rst wheel should be made of brass but neglects to specify material for the other parts of the gear train. This device (see Figure A.7) is believed to have been used as an attachment to a wagon wheel to measure a day's journey.

Hero worked to develop an advanced surveying instrument. Figure A.8 shows his *dioptra* [2]. Note the gear trains used for azimuth and elevation settings. Today's designers of radar pedestals should understand that they have a real hero as the pioneer of their art!

Besides using gears to drive our mills, the Romans (in Gaul) used gears to saw marble. Iron came into frequent use as a gear material. It is possible that the rst case carburized gears were made by Romans.

An interesting device using a rack and a pinion is shown in Figure A.9. Hero describes this lamp and speci es that the rack should be made of *iron*.

One of the most interesting relics of this era is the Antikythera machine discovered by sponge divers off the Greek island of Antikythera. Recent studies by Derek Price, a British historian, reveal that this machine is an astronomical computer. The device was built about 82 BC (when Julius Caesar was a youth). The ship carrying it sank about 65 BC.

The Antikythera machine had many gear trains in it—some of which were planetary [3]. There was evidence of tooth breakage and repair of parts. Apparently, the Romans had some gear troubles (just as we do today).

Figure A.10 shows a schematic of the gear trains of the Antikythera machine as reconstructed by Rear Admiral Jean Theophanidis. Figure A.11 shows some fragments of gearing of this machine as photographed by the National Museum, Athens. Note the triangular tooth shape. Some teeth are obviously worn or damaged. Others reveal the original tooth shape when new.

A very puzzling aspect of the Antikythera machine is that al-Birûna, an Iranian savant traveling in India wrote of a similar machine about 1000 years later. Did the technical know-how of this machine go from Rome to India and last for over a thousand years?

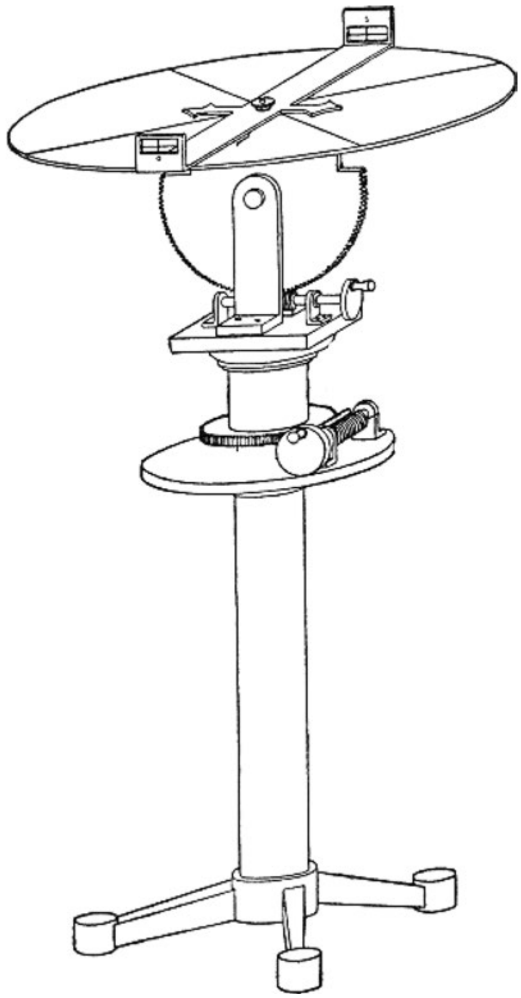

FIGURE A.8 Hero's dioptra, an early surveying instrument with azimuth and elevation gearing, circa 62 AD. (From Usher, A. P., *A History of Mechanical Inventions*, Beacon Press, Boston, Massachusetts, 1929. With permission.)

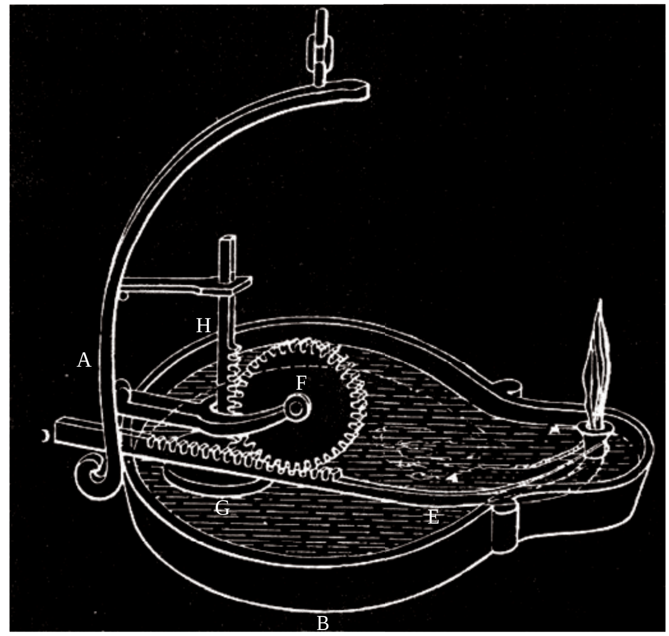

FIGURE A.9 Ingenious Roman lamp with self adjusting wick; racks are iron.

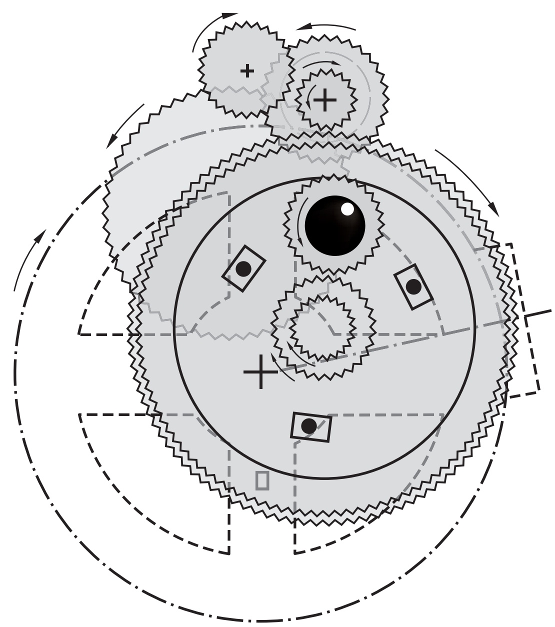

FIGURE A.10 Schematic of Antikythera machine gearing by Theophanidis.

The Antikythera machine as well as the one in India had approximately 30-degree pressure angle teeth.

Figure A.12 shows a segment of a Roman water clock gear. Note the interesting artwork on the gear blank.

In the Roman era, gears were first extensively used for power.* Handcut gear teeth made in metal were frequently used. Pin wheel gears were still used extensively. A very wide variety of gear arrangements was used. Some of the gear arrangements invented in recent times were undoubtedly first used in Rome!

A.4 GEAR USES AND DEVELOPMENTS THROUGH THE DARK AGES AND MEDIEVAL TIMES (400–1600 AD)

After the fall of Rome, there was a dark age in Western Europe. The center of learning moved to the Eastern Empire of Constantinople. (This empire lasted another thousand years!)

Not much is known about gearing in China but the gear art was apparently not lost there. Figure A.13 shows a geared water clock from about 1090 AD. This complex clock measured time by the flow of water. Every 15 minutes, the weight of water unlatched an escape wheel. This mechanism has been traced back to 725 AD. The Chinese developed this device without any known contact with the Western world.

* There is evidence that the Chinese made some use of power gearing in the 4th century BC.

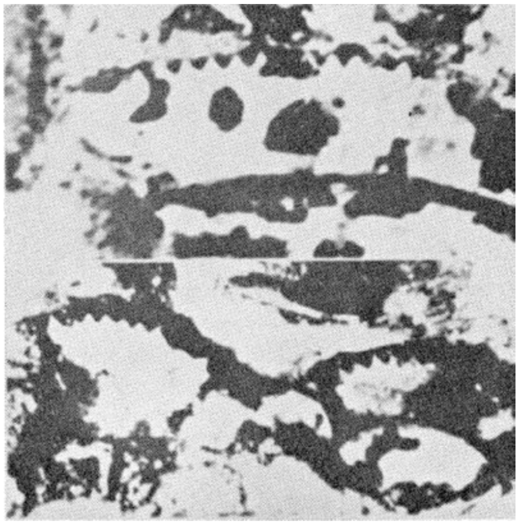

FIGURE A.11 Fragments of Antikythera machine gears. (Courtesy of National Museum, Athens and Smithsonian Institution, Washington, D.C.)

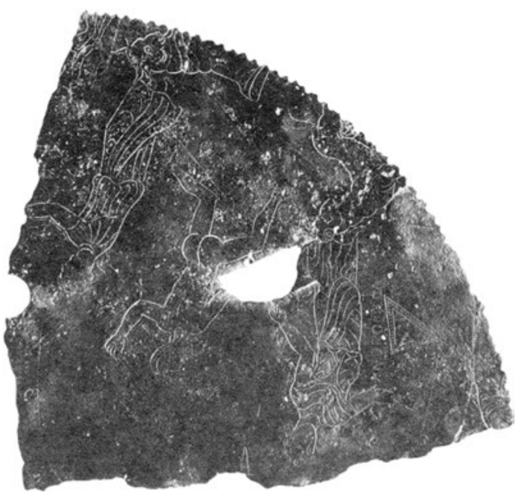

FIGURE A.12 Segment of Roman water clock gear. (Courtesy of Smithsonian Institution, Washington, D.C.)

At the time of the Crusades, we find the Arabic world of Islam leading in mathematics and mechanical arts. Figure A.14 shows a replica of a geared calendar assembly by Muhammed B. Avi Bakr of Isfahan, 1221–1222 AD. The *original* gear assembly is in the Science Museum, London. This is believed to be the oldest workable set of gearing in existence. Furthermore, this drive is based on al-Birûna's design, circa 1000 AD. The al-Birûna design, as was noted earlier, undoubtedly goes back to the Antikythera machine of Roman times. Figure A.15 shows the al-Birûna design.*

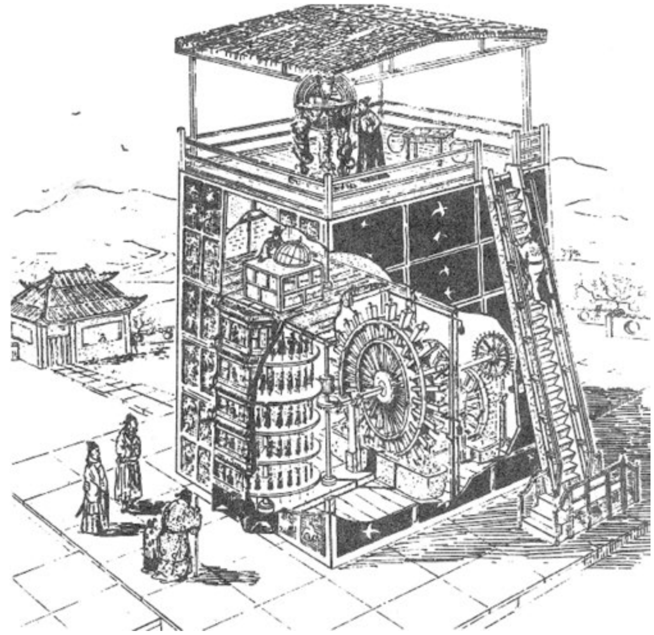

FIGURE A.13 A pictorial reconstruction of the astronomical clock-tower built by Su Sung and his collaborators at K'ai-fêng in Honan province, then the capital of the empire, in AD 1090. The clockwork, driven by a water-wheel, and fully enclosed within the tower, rotated an armillary sphere on the top platform and a celestial globe in the upper storey; puppet figures giving notice meanwhile of the passing hours and quarters by signals of sight and sound. (Original drawings by John Christiansen. Courtesy of Smithsonian Institution, Washington, D.C.)

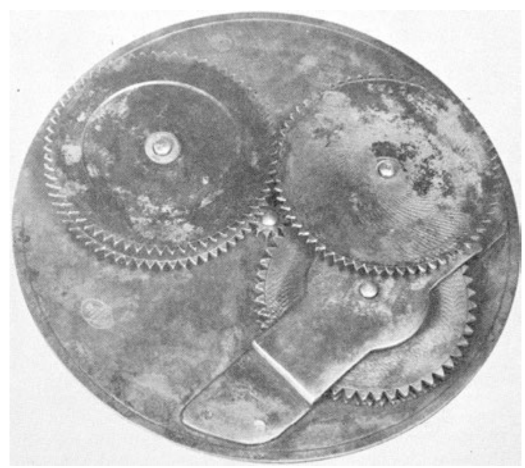

FIGURE A.14 Geared astrolabe of 1221–1222 AD by Muhammed B. Abi Bark of Isfahan. (Courtesy of Smithsonian Institution, Washington, D.C.)

Most writers assume that the mechanical arts of Greece and Rome were lost to the world and then rediscovered. Several pieces of evidence indicate that they were never really lost. Many items of evidence, like the one above, indicate that technical knowledge was preserved in the Near East, India, and Egypt and then reintroduced into Western Europe. The Moors, for instance, brought many technical ideas into Spain. Table A.2 shows, in chronological order, the many

* Figures A.10, A.11, A.13, A.15, A.16, A.17, A.18, and Table A.2, reproduced from the article "On the Origin of Clockwork, Perpetual Motion Devices and the Compass" by Derek J. DeSolla Price in the United States National Museum Bulletin 218, *Contribution from the Museum of History and Technology*, published by the Smithsonian Institution, 1959. Figure A.15 data came originally from E. Weidemann's 1913 paper in Der Islam, 1913, Vol. 4, p. 5.

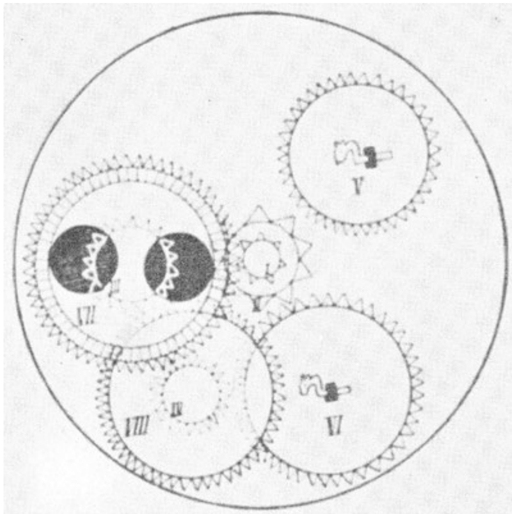

FIGURE A.15 Calendrical gearing as designed by al-Birûna, circa 1000 AD.

developments in clock gearing, astronomical devices using gears, and related gear devices.

Figure A.16 shows an Islamic gear device used for pumping water. (The teeth shown look like they had better involute profiles than would normally be expected of a medieval Arab—probably the Smithsonian Institution redrew the old drawings with some improvements in details.)

Figures A.17 and A.18 show a French geared astrolabe and the details of the gear teeth. The tooth design is of a lower pressure angle (about 12°) and the teeth are slightly rounded (like an involute). This is a difference—and an improvement—over the straight sided, triangular teeth shown in Figure A.14. The teeth in clocks and watches, to this day, tend to have deep teeth with a low pressure angle. Clock gearing of this design is less apt to stick and bind due to tooth errors.

The French development of the astrolabe and English developments of the astrolabe and equatorial (Chaucer wrote a treatise on the astrolabe) can be traced to Arabic learning in the mechanical arts that flooded into Europe from about 1100 to 1300 AD.

The earliest mechanical clock for which the design has survived is the clock of Giovanni de Dondi of Padua, constructed from 1348 to 1364. This imposing clock has been built again and put on display at the Smithsonian Institution in Washington, D.C., at their new Science and Technology building. Figure A.19 shows the dial of Jupiter. The clock has four dials and an intricate gear and weight drive system. Figure A.20 shows the weight and the drive gearing for Dondi's clock.

Leonardo da Vinci, 1452–1519, designed all manner of things—including gear drives. This early-day genius designed battle cars, guns, cannons, a clockwork automobile, and differential gears—among many other things. His notes have survived and provided an interesting insight into the mechanical knowledge and thinking of his day. Figure A.21 shows a model of a da Vinci designed water pumping mill. Note pin-tooth gears.

One of the most notable developments of the early Middle Ages was the lantern pinion. This spur pinion had pins fixed

TABLE A.2
Chronological Chart

Classical Europe

3rd C., B.C., Archimedes planetarium
2nd C., B.C., Hipparchus Stereographic Projection
1st C., B.C., Vitruvius hodometer and water clocks
65, B.C. (ca.) Antikythera machine
1st C., A.D., Salzburg and Vosges anaphoric clocks

Islam

807 Harun-al-Rashid
850 (ca.) Earliest extamp astrolabes
1000 Geared astrolabe of al-Birûni
1025 Equatorium text
1050 Salsdin clock
1200 (ca.) Ridwān water clocks, perpetual motion, and weight drive
1206 as-Jazari clocks, etc.
1221 Geared astrolabe
1232 Charlemagne clock
1243 al-Konpas (compass)
1272 Alfonsine corpus clock with mercure drum, equatoria

Europe

1000 Gerbert astronomical model
1187 Neckham on compass
1198 Jocelin on water clock
1245 Villard clock water, "escapement," perpetual motion
1267 Villers Abbey clock
1269 Peregrinus, compass and perpetual motion
1271 Robertus Anglicus, animated models and "perpetual motion" clock
1285 Drover's water clock with wheel and weight drive
1300 (ca.) French geared astrolabe
1320 Richard of Wallingford astronomical clock and equatorium
1364 de Dondi's astronomical clock with mechanical escapement
Later 14th C. Tradition of escapement clocks continues and degenerates into simple timekeepers

China

4th C., B.C. Power gearing
2nd C., A.D. Chang Hêng animated globe hodometer. Continuing tradition of animated astronomical models
725 Invention of Chinese escapement by I-Hsing and Liang Ling-tsan
1074 Shen Kua, clocks and magnetic compass
1080 Su Sung clock built
1101 Su Sung clock destroyer

India

1100 (ca.) Sūrya Siddhānta animated astronomical models and perpetual motion
1150 (ca.) Siddhānta Siromani animated models and perpetual motion

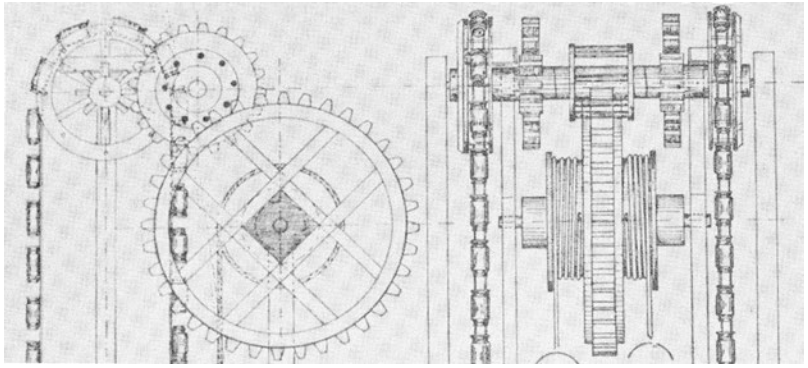

FIGURE A.16 Islamic water pump driven by a weight drive, circa 1200 AD.

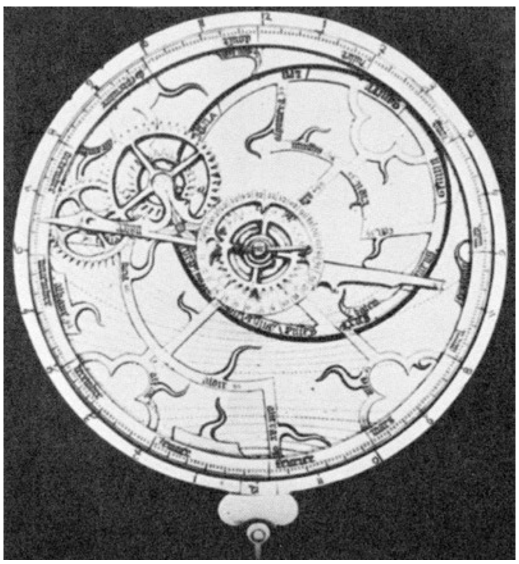

FIGURE A.17 French Geared Astrolabe, circa 1300 AD.

at both ends. Thus, it had much more load-carrying capacity. Figure A.22 shows a drawing of a hand-driven gristmill using lantern pinions.

Figure A.23 shows another example of a hand-driven gristmill. This example shows two sets of *speed reducing* gears and a set of *speed increasing* gears. In many cases, the ancient engineers made mistakes in their logic and designed things wrong. It is illogical to reduce speed and then increase speed in the same drive—unless one is just fascinated by gears!

Figure A.24 shows some interesting gears including a worm gear drive and some sort of pounding machine to pulverize material. Obviously this machine wouldn't work. It is a *perpetual motion* machine. Water drives the big wheel but then is lifted by buckets to the upper through so it can work again. Medieval engineers did not understand energy and friction loss principles. They kept designing many kinds of perpetual motion machines even though the machines would never work.

A practical machine is shown in Figure A.25. This machine uses water power to drive a boring tool. This was probably a wooden pipe making machine.

A gristmill driven by two persons walking in the large tread wheel is shown in Figure A.26. From medieval times down to the colonial days of the United States, tread mills were important power machines. People, horses, donkeys, and goats were used in different kinds of tread mills. Troy, New York, was prominent at one time in the manufacture of tread mills.

A gristmill driven by a medieval water turbine is shown in Figure A.27.

The rst known gear cutting by machine was that developed by Juanelo Torriano (1501–1575) in constructing a great

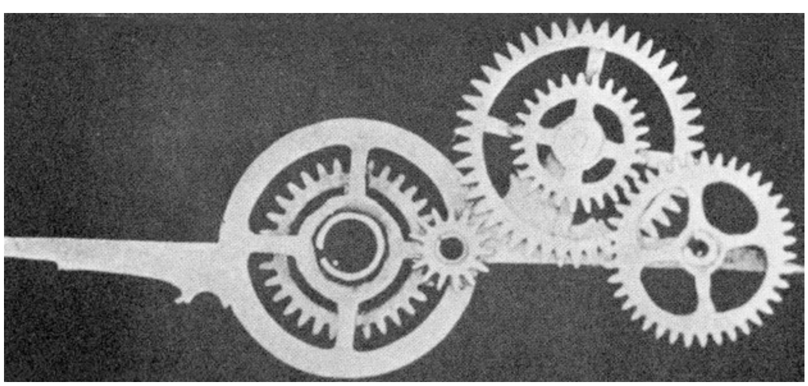

FIGURE A.18 Details of gear train in Figure A.17.

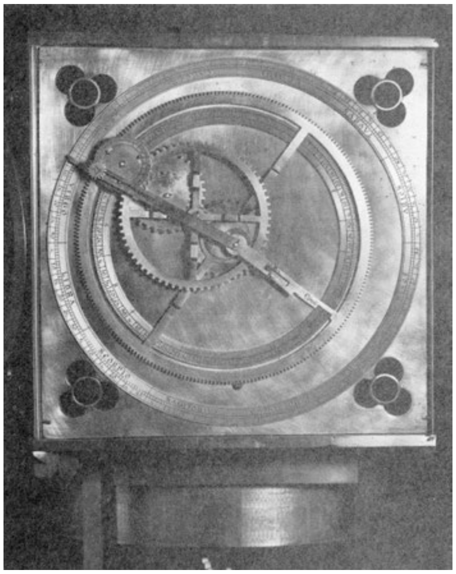

FIGURE A.19 Dial of Jupiter, Dondi clock. (Courtesy of Smithsonian Institution, Washington, D.C.)

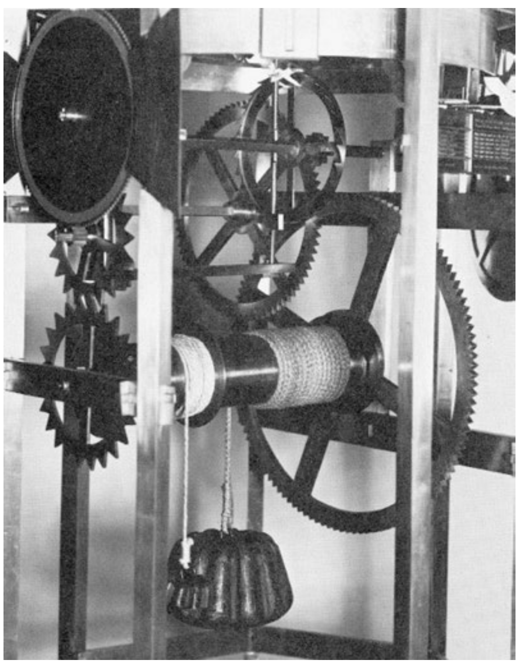

FIGURE A.20 Weight and drive gears for Dondi clock; note triangular teeth. (Courtesy of Museum of Science, Boston, Massachusetts.)

clockwork for Charles V of Spain. The device has about 1800 gear wheels. Torriano got up to a production rate of three wheels per day using a homemade, hand-powered gear-cutting machine. The cutter was in the form of a rotary le. Morales claims that Torriano's gear cutting device came into common use in Spain within 25 years' time. Morales' account, translated by Woodbury (1958a), is very interesting.

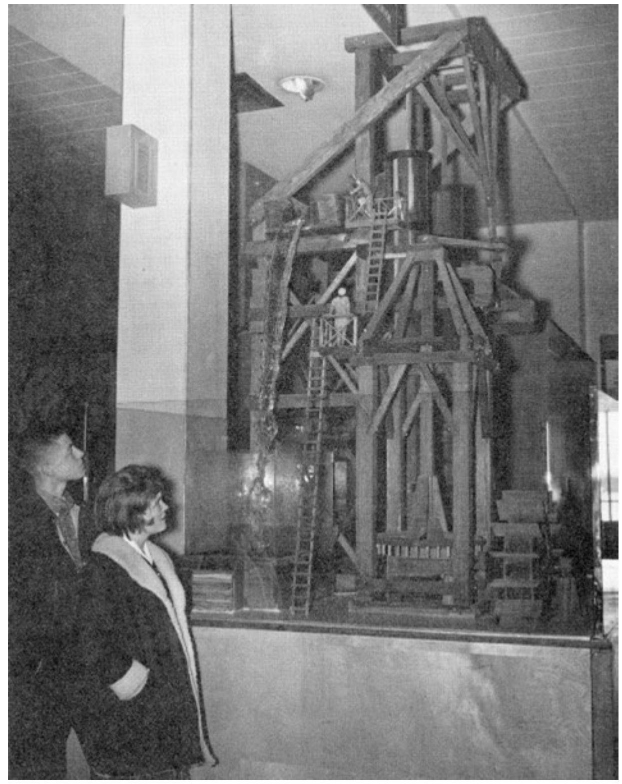

FIGURE A.21 Model of water pumping mill designed by Leonardo da Vinci. (Courtesy of Museum of Science, Boston, Massachusetts.)

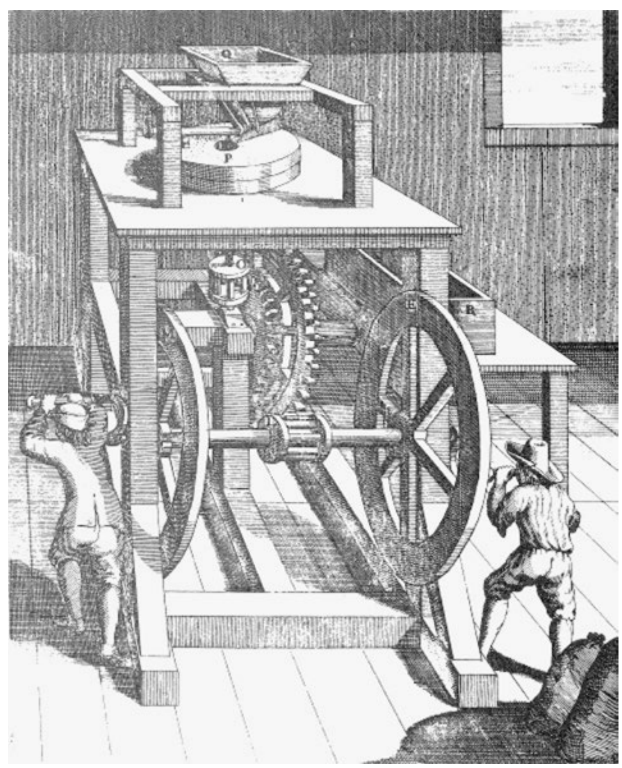

FIGURE A.22 Lantern pinion used in grist mill drive, circa 1500 AD. (Courtesy of Smithsonian Institution, Washington, D.C.)

FIGURE A.23 Small grist mill drive, circa 1500 AD. (Courtesy of Smithsonian Institution, Washington, D.C.)

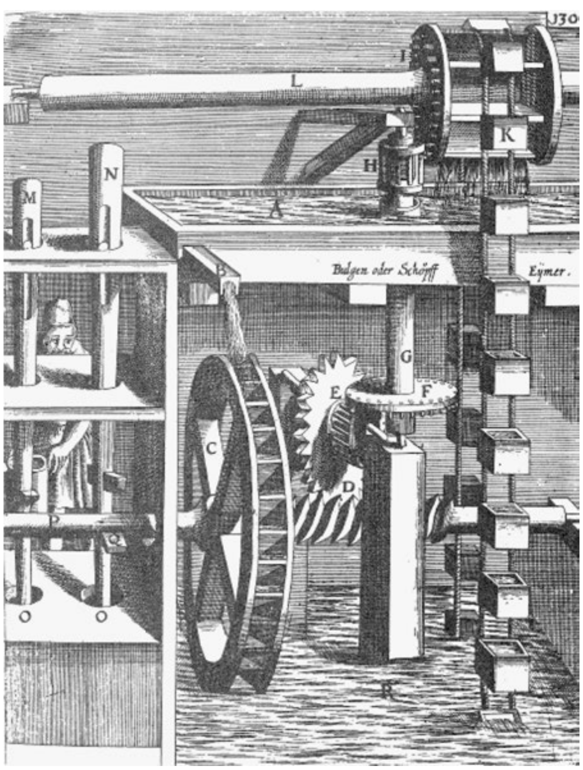

FIGURE A.24 Medieval perpetual motion machine—not practical! (Courtesy of Smithsonian Institution, Washington, D.C.)

> It took him as he told me, all of twenty years to conceive and work out the plan; and because of the great amount of energy and concentration in thinking about it, he got sick twice during that period and almost died. Then having spent so much time in conceiving it, it did not take him more than three and a half years to make it by hand. This was quite a feat because the whole clockwork (relox) had more than 1800 wheels not counting many other parts of iron and brass that are involved. So every day (not counting holidays), he had to make . . . more than three wheels that were different in size, number and shape of teeth, and in the way in which they are placed and engaged. But in spite of the fact that this speed is miraculous, even more astounding is a most ingenious lathe (torno) that he invented (and we see then today) to carve out with a file iron wheels (*labrar ruedas de hierro con la lima*) to the required dimensions and degree of uniformity of the teeth. And in spite of all this and knowing that he did it all by hand, it is not surprising that Januelo says, as he does, that no wheel was made twice because it always came out right the first time. And if all he said were not better in actual fact, it would be very surprising.*

Gears in the medieval times were still rather crude. The learning of Rome slowly found its way into Western Europe. Crude mills were built. Clockwork made some notable progress. All in all, the medieval mechanic did not go much beyond the Roman of over 1000 years earlier. It is in the next interval that gearing began to develop rapidly.

A.5 THE BEGINNINGS OF MODERN GEARING (1600–1800 AD)

In this period, the theory of gear tooth action was first worked out. Theorists claimed—even then—that the involute tooth form should be used. Table A.3 shows in capsule form some of the early work done on gear theory.

Most of the gears made in this period did not benefit much from the theories that a few people were writing about. Gear making was certainly a craft and an art. The skilled craftsman could make gears that would fit and run smoothly without having even heard of an involute or a cycloid. I have personally examined worm gear drives with the gear *driving the worm*. The tooth action was amazingly good considering the fact that gear cutting machines and gear checking machines would not show up for another century or more! (It should be added, of course, that some mighty crude cogwheels were being made in this period, too.)

Figure A.28 shows epicyclic gear tooth drives of the kind designated by Camus. Note that two pairs of teeth are in contact even with only 5 teeth on the pinion. This means a contact ratio greater than 1.0. With *involute* teeth, it is not practical to get a contact ratio over 1.0 with a 5 tooth pinion.

Woodbury (1958) makes the statement that Willis used a gear addendum equal to 1.0 divided by the diametral pitch. This is still the standard of the gear trade right down to

* Reprinted from *History of the Gear-Cutting Machine* by Robert S. Woodbury, pp. 45–46 (see Ref. [4]), by permission of MIT Press, Cambridge, MA.

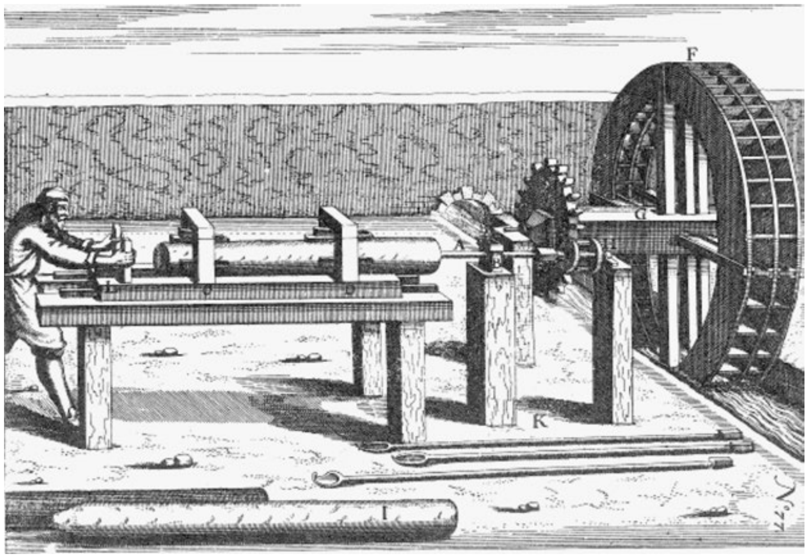

FIGURE A.25 Medieval machine to bore wooden pipe. (Courtesy of Smithsonian Institution, Washington, D.C.)

present times. The origin of 14.5° pressure angle is explained by Woodbury as follows:

> Willis comes to advocate the involute form from a study of the path of the point of contact and of the smallest number of teeth possible for spur gears, both external and internal, and for racks. This led him to consider the ideal working depth and addendum, as well as the thickness of the tooth and the breadth of space. He also introduced the constant 14.5° because it had a sine of very close to ¼. Later this value was retained because it also coincident closely with the pressure angle usual in epicycloidal teeth. It is also the angle used for worm threads, too, making the straight-sided rack of the involute system correspond in angle, as well as in other

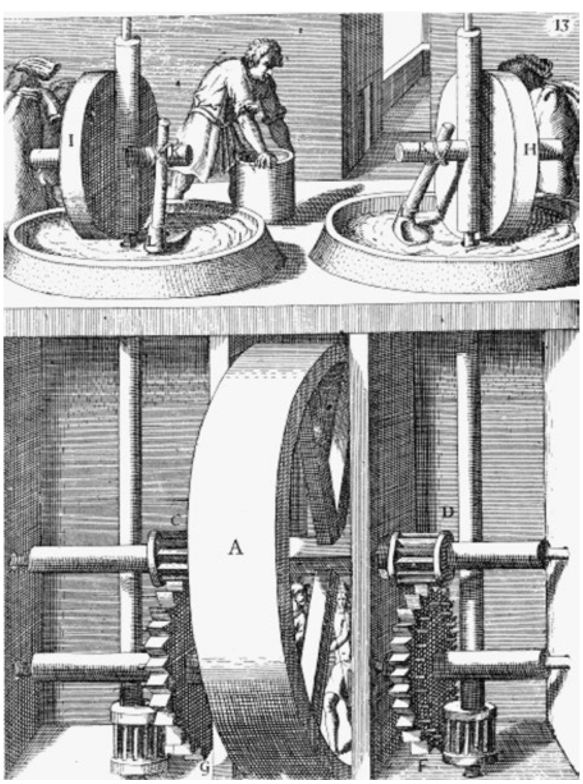

FIGURE A.26 Tread wheel used to drive a grist mill. (Courtesy of Smithsonian Institution, Washington, D.C.)

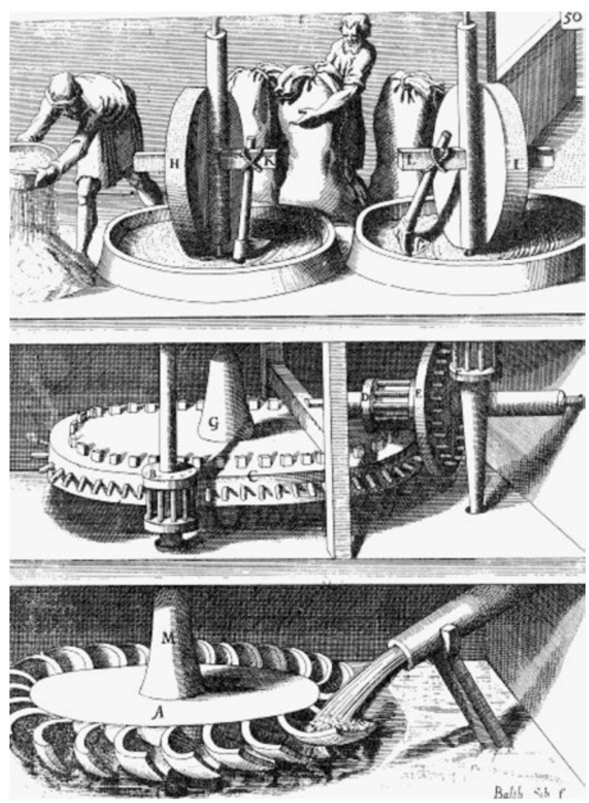

FIGURE A.27 Water turbine used to drive a grist mill. (Courtesy of Smithsonian Institution, Washington, D.C.)

TABLE A.3
Gear Theorists

Name	Approximate Date	Nationality	Contribution
Nicolas of Cusa	1451	French	Studied cycloidal curve.
Albrecht Dürer	1525	German	Discovered epicycloids.
Girolamo Cardano	1557	Swiss	First mathematics of gears in print.
Philip de la Hire	1694	French	Full mathematical analysis of epicycloids. Recommended involute curve for gearing. (Involute not used in practice until about 150 years later.)
Charles Camus	1733	French	Expanded on work of la Hire, developed theories of mechanisms, studied lantern pinion and gear, crown gears and beveled gears.
Leonard Euler	1754	Swiss	Euler worked out design principles, worked out rules for conjugate action. Some consider him "the father of involute gearing."
Abraham Kaestner	1781	German	Wrote up practical methods for computing tooth shapes of epicycloid and involute gear teeth. Considered 15° to be a minimum pressure angle.
Robert Willis	1832	English	Wrote and taught extensively in gear eld. A pioneer in gear engineering.
Edward Sang	1852	Scotch	General theory of gear teeth. Provided theoretical basis on which all gear tooth generating machines are based.

proportions, with the worm thread. All this work was based upon pure mechanism.

The result is a clear indication of the complexity resulting from epicycloidal teeth, especially for the cast teeth common at that time—separate molds would be required for each gear if they were to t each other. Willis recognized the limitations of the epicycloid for an interchangeable system of gearing. The advantages of the involute form stand out in the greater strength of this form, especially as compared to the epicycloidal with radial anks. See Figure A.29.

An 18th century style of clock movement is shown in Figure A.30. This movement was made for a belfry of German Reformed Church of Frederick, Maryland, where it was used until 1931. Its maker, Frederick Heisely, apparently learned his trade from George Hoff, his father-in-law, a European trained clock maker. This particular clock had a 14-foot pendulum.

During this period, water power came into general use to drive relatively heavy duty mills. Figure A.31 shows an example of a paper mill from about 1750 driven by a water wheel. This mill was about 65 miles south of Paris in l'Anglée. Mills like this made France an early leader in the production of paper.

Another water driven mill is shown in Figure A.32. Note the heavy duty gear wheels. These were made by inserting wooden teeth in cast iron hubs. The wooden teeth were generally made of maple.

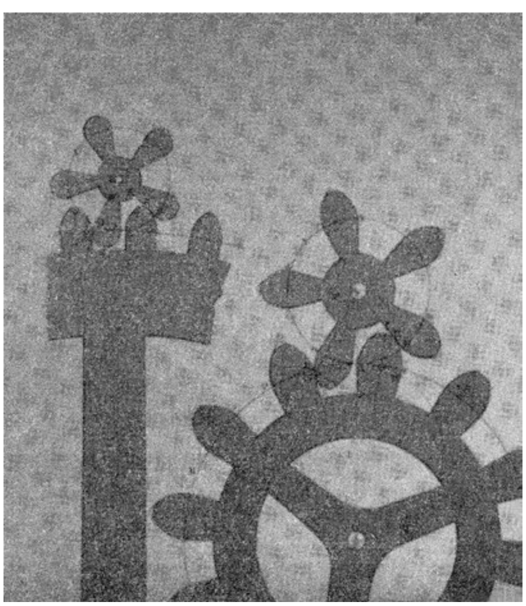

FIGURE A.28 Examples of epicyclic gear drives using a 5-tooth pinion. (Courtesy of MIT Press, Cambridge, Massachusetts.)

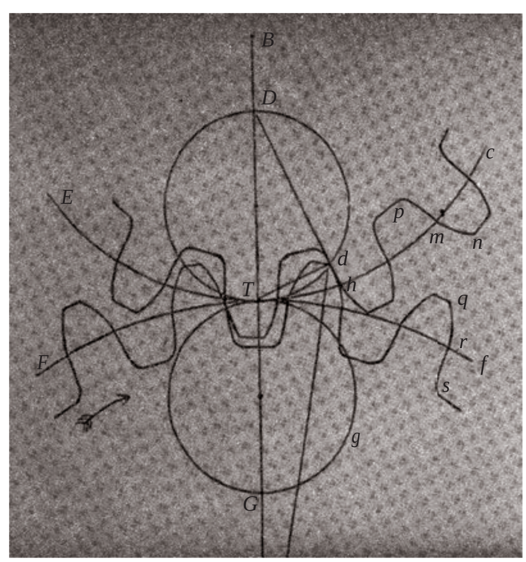

FIGURE A.29 Old style involute teeth. (Courtesy of MIT Press, Cambridge, Massachusetts.)

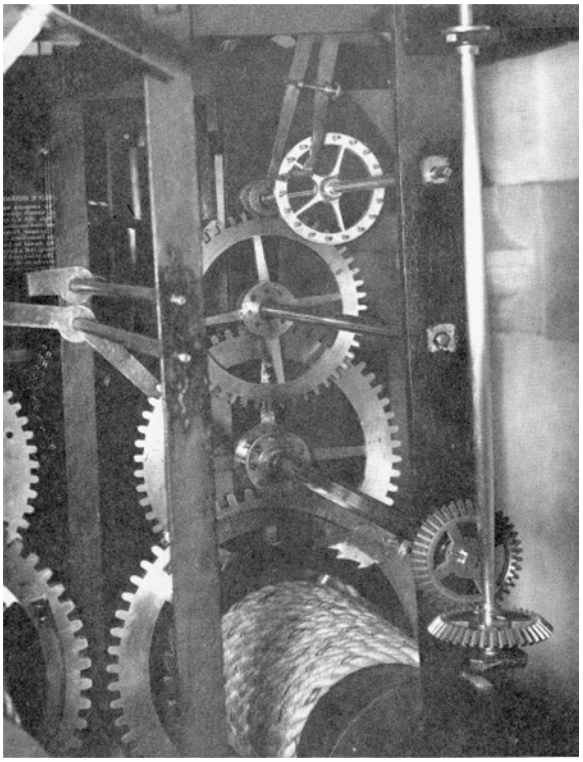

FIGURE A.30 An 18th century clock movement from Frederick, MD. (Courtesy of Smithsonian Institution, Washington, D.C.)

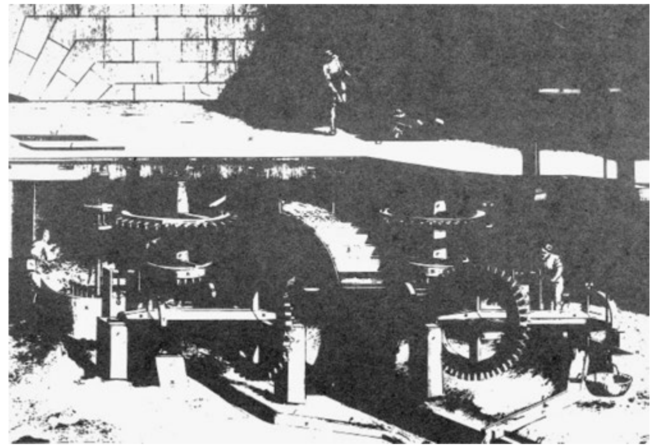

FIGURE A.32 Water driven mill, circa 1775 AD. (Courtesy of *Fortune* magazine, New York, New York.)

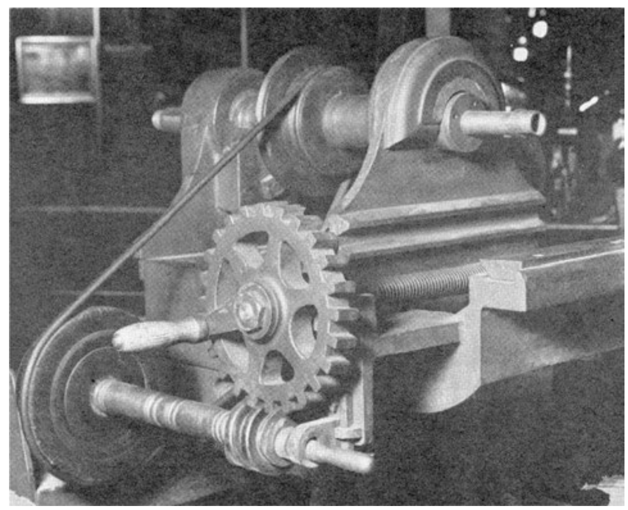

FIGURE A.33 Feed gearing in Eli Whitney's milling machine, circa 1800 AD. (Courtesy of Smithsonian Institution Washington, D.C.)

The art of making metal gears for power drives really got going in this period. (Previously, metal gears had been used primarily in clocks, instruments, and just plain gadgets.) Figure A.33 shows Eli Whitney's feed gearing drive for a milling machine. Whitney is most remembered for his invention of the cotton gin. After some years in the south (1792–1798). Whitney began the manufacture of rearms near New Haven, Connecticut. Whitney is noted for promoting the concept of interchangeable parts. This skilled craftsman and inventor made many important contributions to the mechanical arts.

John Stevens developed a multi-tubular boiler and a high pressure steam engine. He was concerned about safe and speedy transportation across the Hudson River (Stevens owned a New York to New Jersey ferry). Stevens' concern for his inventions led him to petition Congress for patent protection. Congress responded and passed the 1790 Patent law, which is the foundation of our present patent system.

Stevens built a steamboat in 1802 with a *twin-screw* drive. Figure A.34 shows the gearing for Stevens' boat. Note that both Stevens and Whitney were using iron gears with about 5° pressure angle. (The improved 14.5° teeth of Willis had not yet come into general use.)

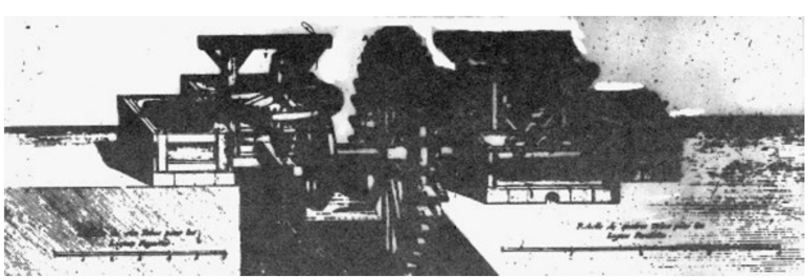

FIGURE A.31 French paper mill drive, circa 1750 AD.

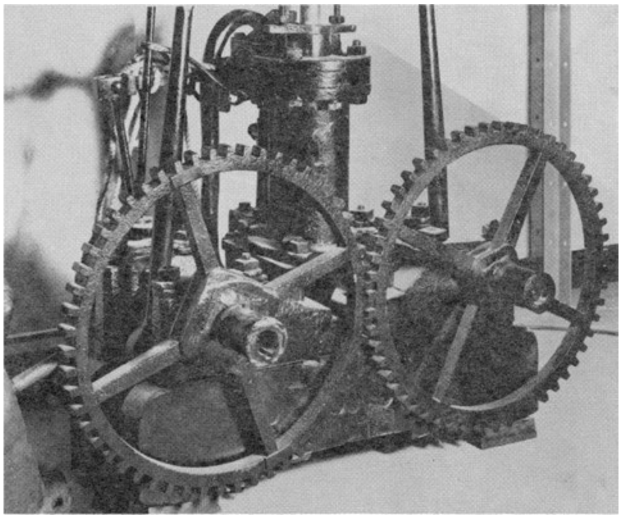

FIGURE A.34 Cast iron gears from twin screw drive in Stevens' steamboat, 1804 AD. (Courtesy of Smithsonian Institution, Washington, D.C.)

A.6 THE MANY DEVELOPMENTS OF THE RECENT PAST (1800–1916 AD)

Although gear cutting machines had gotten started in an earlier period, this period was the one in which most of the types which we know today got started. Table A.4 shows in chronological form some of the more important machine developments.

A gear cutting machine—believed to be Robert Hook's—is shown in Figure A.35. The cutter, index mechanism, and hand crank can all be seen. Note that even in this early machine, gears are used to make gears! Christopher Polhem's very nicely built machine for production gear cutting is shown in Figure A.36.

Figure A.37 shows the oldest gear cutter that is still in existence. (Torriano's cutters lost!) The French were very active in making horological machines (clocks) in the 18th century.

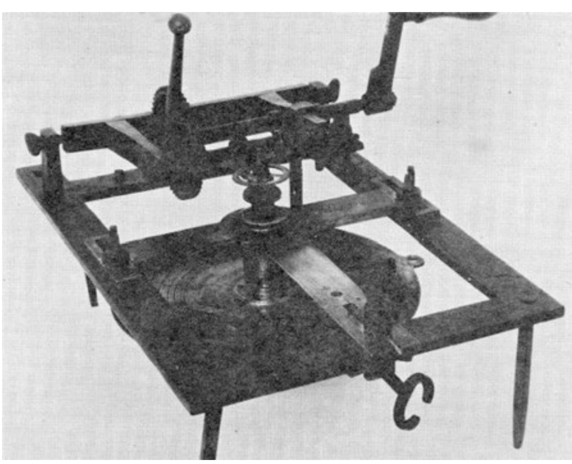

FIGURE A.35 Robert Hook's wheel cutting engine, circa 1672 AD. (Courtesy of the Science Museum, London.)

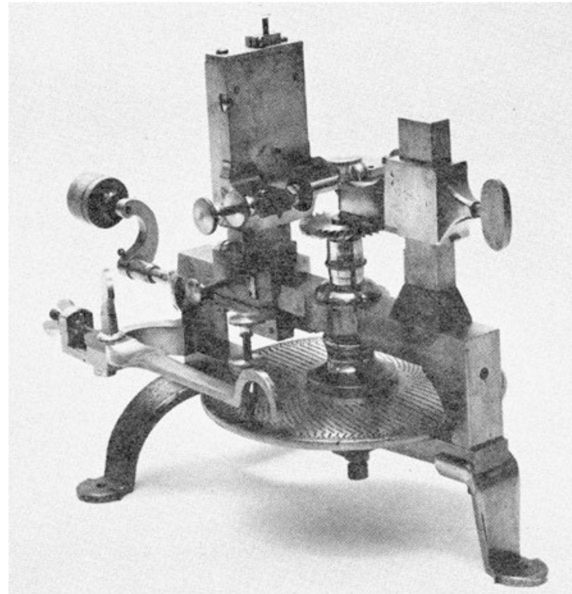

FIGURE A.36 Christopher Polhem's machine to cut clock-work gears, circa 1729 AD. (Courtesy of Smithsonian Institution, Washington, D.C.)

FIGURE A.37 The "oldest" gear cutter. (Courtesy of Brown & Sharpe, North Kingstown, Rhode Island.)

Nicolas Bion (1652–1733) wrote of French gear cutting machines using formed rotary cutters.

The Brown & Sharpe Company of Providence, Rhode Island, was an early pioneer in gear work in the United States. Figure A.38 shows their rst gear cutting machine. The well known Brown & Sharpe form cutter is shown in Figure A.39.

The great pioneer in the bevel gear eld was William Gleason and the Gleason Works. Figure A.40 shows the rst bevel gear planer. The principle of the machine is depicted in Figure A.41.

Another bevel gear machine developer was Hugo Bilgram. His machine is shown in Figure A.42.

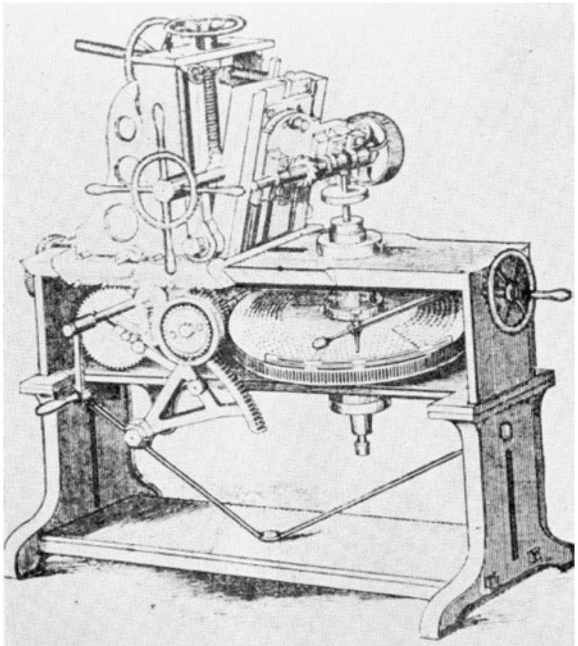

FIGURE A.38 First Brown & Sharpe gear cutting machine, 1855 AD.

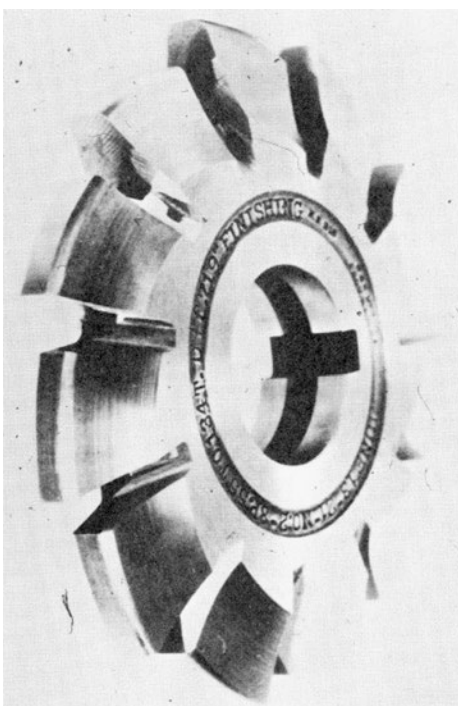

FIGURE A.39 Brown & Sharpe form relied gear cutter.

The bevel gear generator was developed by James E. Gleason, a son of William Gleason. The principle of his machine is shown in Figure A.43.

The gear cutting machines developed by Gould & Eberhardt achieved early fame. As early as 1889, these machines were being shipped all over the world. Figure A.44 shows the spur gear cutting machine. By 1900, this type of machine had been developed to form cut helical gears. It also had (by then) an automatic return stroke. Eberhardt's pioneering work has had a lasting effect on the gear cutting industry that is still evident in many gear shops.

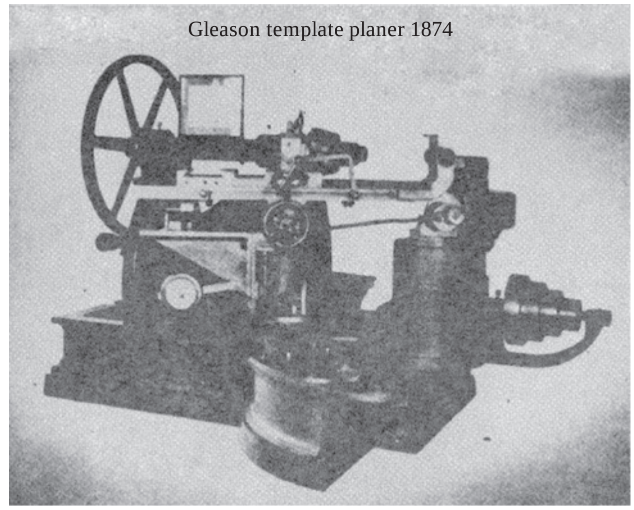

FIGURE A.40 Gleason bevel gear planing machine. (Courtesy of Gleason Works, Rochester, New York.)

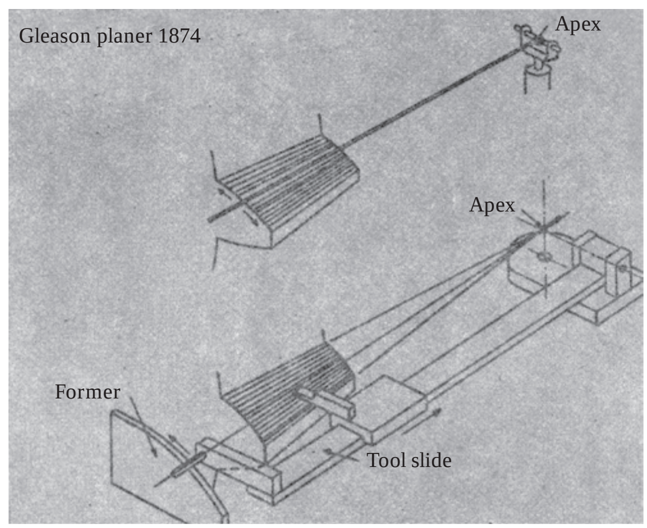

FIGURE A.41 Principle of Gleason bevel gear planing machine. (Courtesy of Gleason Works, Rochester, New York.)

Although Table A.4 does not show it, the rst attempts had been made to generate grinding gears with disk wheels and with treaded wheels. Woodbury (1958a) states in the *History of the Gear-Cutting Machine* that by 1909, all the basic types of gear cutting machines had been built to use the grinding method.

In the period 1800–1916, the rst elements of what we might call gear engineering or gear technology appeared.

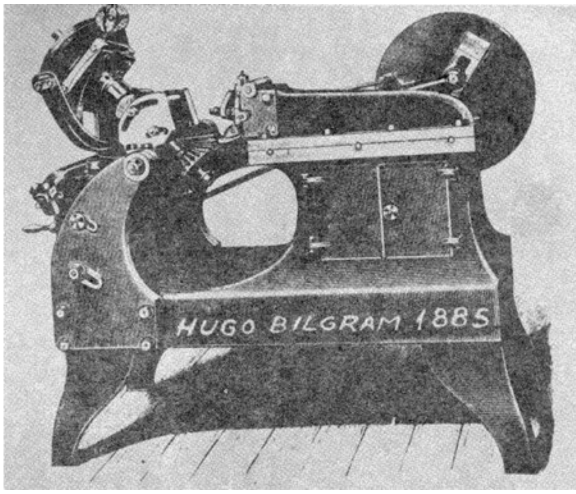

FIGURE A.42 Bilgram gear generator, 1885 AD. (Courtesy of Gleason Works, Rochester, New York.)

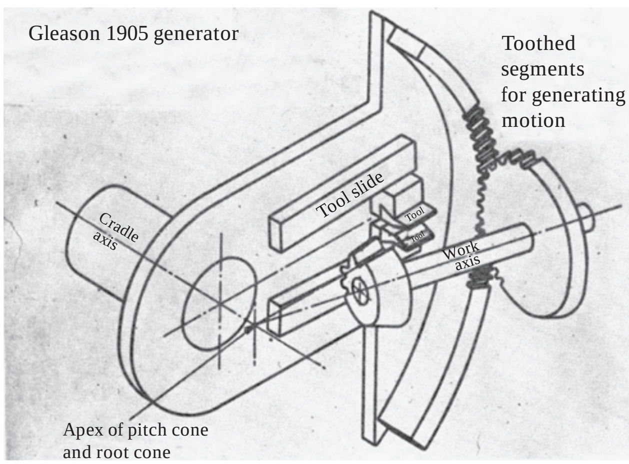

FIGURE A.43 Gleason bevel gear generator. (Courtesy of Gleason Works, Rochester, New York.)

A little booklet written by Sereno Newton and published in 1832 by G. & C.& H. Carvill, New York City, gives an interesting insight into the thinking of that period. The rule of Mr. James Carmichael is quoted for load carrying capacity:

> Rule—Multiply the breadth of the teeth by the square of the thickness, and divided the product by the length; the quotient will be the proportionate strength in horses power, with a velocity of 2.27 feet per second.

The results of Mr. Carmichael's rule are shown in the following table in Sereno Newton's book [see Table A.5].

Some very interesting—and very practical to this day—principles of gear design are given by Sereno Newton. The following is a good example:

On the Teeth of Wheels

The proper method of forming the teeth of wheels so as to communicate equal motion, with as little friction as possible, is a matter of very great importance, and has given rise to much study among mechanics; the result of which shows that the face of the teeth of wheels which are to act on one another, should be epicycloids. But as this is a curve not easily applied, part of circles are frequently employed as a substitute. A straight line tending to the centre of the wheel appears to be the most advantageous form for the ank of the tooth, or that part which is within the pitch line as it causes least pressure on the axis. The proper form for the curved part of the tooth of a wheel or pinion to act on a rack is an involute of a circle, or, which is the same, an epicycloid, whose generating circle is in nitely great; and the form of the teeth of a rack acting on a wheel or pinion is a cycloid. But as in the case of wheels acting on one another, parts of circles are generally used instead of these compound curves.

A very common method of setting out the teeth of a wheel is to describe the teeth by a circular arc of which the radius is equal to the pitch added to half the thickness of the tooth, and of which the centre is the pitch line. Another method is to describe the teeth by an arc of which the radius is only equal to the pitch: but, in either case, the form of the teeth is better adapted to wheels that act as drivers, than to those that act as the driven, in consequence of the action of the teeth before they reach the line of centers.

The friction of teeth approaching the line of centers is much greater than in receding* from it; and this is not the only disadvantage; for when a machine becomes worn it causes irregularity I nits movements in consequence of the action in approaching the line of centers, tending to spread the axis of the wheels, while the action, in receding from that line, tends to draw these axis together, and consequently occasions more irregular action, friction, and wear, in the machine; therefore the teeth ought, if possible, never to begin to act before they

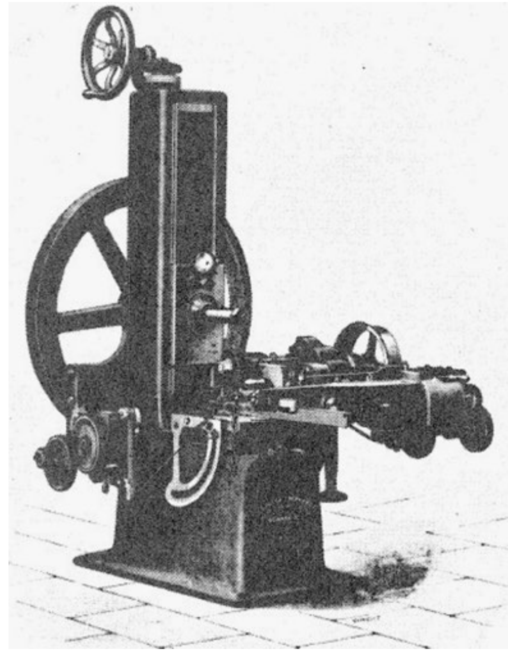

FIGURE A.44 Eberhardt's automatic gear cutting machine, circa 1889 AD. (From *Practical Mechanics*, Worcester, MA, II, 10, April 1889.)

* The advantage of recess action tooth design were appreciated even in 1832. There has been renewed interest in recess action gear designs in recent years.

TABLE A.4
Some Early Landmarks in the Development of Machines to Make Gear Teeth

Kind of Machine	Name of Inventor or Developer	Approximate Date	Description
Gear cutter	Juanelo Torriano	1540	Cut spur gears for a clock at a rate of three per day. Used rotary file instead of milling cutter. Hand powered.
Gear cutter	Robert Hook of England	1672	Cut spur gears with a formed wheel. Hand powered.
Gear broaching Gear cutting	Christopher Polhem of Sweeden	1729	Machine to cut teeth with a reciprocating broach. Later developed machine to cut teeth with rotary cutters. Hand powered.
Gear cutter	Henry Hindley	1741	Differential indexing. Rugged construction. Could cut pinions and racks. Hand powered.
Gear cutter	Samuel Rehé	1783	Used true milling cutters rather than rotary file cutters. Cutters ground to approximate form for conjugate action. Hand powered.
Gear cutter	J. G. Bodmer of Switzerland	1825	Made spurt wheel cutting machines.
Gear cutter	Joseph Whitworth of England	1834	Involute cutters. Power driven by belt and pulley. Could cut spur and bevel gears.
Hobbing machine	Joseph Whitworth of England	1835	Hobbed spiral gears. Whitworth took out first patents on the hob.
Gear generator	Joseph Sexton of England	1842	Straight-sided milling cutter used to generate an involute tooth.
Gear cutter	Joseph R. Brown of United States	1854	A “precision” machine for making index plates and master gears. Used “backed-off” form relieved cutter of modern type.
Bevel gear planing machine	William Gleason of United States	1874	First practical bevel gear making machine built.
Bevel gear shaper	Huge Bilgram of United States	1884	Used a generating type of process. Made bevel gears for bicycles.
Gear cutter	H. E. Eberhardt (Gould & Eberhardt, USA)	1884	Patent on automatic indexing.
Hobbing machine	Reinecker of Germany	1894	Heavy-duty machine.
Gear shaper	E. R. Fellows (of Fellows Gear Shaper, USA)	1895	Gears cut by a “pinion” cutter with hardened and ground involute profile teeth.
Bevel gear generator	James E. Gleason (Gleason Works, USA)	1898	Rotary cutters used to generate bevel teeth. Automatic type of machine.
Hobbing machine	Pfauter of Germany	1900	Hob axis could swivel. Differential gearing could be used.
Form grinding	Ward and Taylor (Gear Grinding Machine Co., USA)	1908	Automatic, diamond point dressing of formed wheel. Spur gear making machine.

TABLE A.5
Proportional Strength of the Teeth of Wheels, in Horses’ Power (Calculated by Mr. Carmichael’s Rule)

Pitch, in inches	Thickness of Teeth, in inches	Breadth of Teeth, in inches	Length of Teeth, in inches	Horses’ Power at 2.27 feet per second	Horses’ Power at 3 feet per second	Horses’ Power at 4 feet per second	Horses’ Power at 6 feet per second	Horses’ Power at 8 feet per second	Horses’ Power at 11 feet per second
4.02	2.0	8.0	2.40	13.33	17.61	23.48	35.23	46.97	64.60
3.99	1.9	7.6	2.28	13.03	15.90	20.20	31.80	40.40	58.30
3.78	1.8	7.2	2.16	10.80	14.27	19.02	28.54	38.05	52.32
3.57	1.7	6.8	2.04	9.63	12.72	17.02	25.54	34.05	46.68
3.36	1.6	6.4	1.92	8.53	11.27	15.02	22.54	30.05	41.32
3.15	1.5	6.0	1.80	7.50	9.91	13.21	19.82	26.43	36.33
2.94	1.4	5.6	1.68	6.53	8.63	11.50	17.26	23.00	31.64
2.73	1.3	5.2	1.56	5.63	7.44	9.92	14.88	19.84	27.28
2.52	1.2	4.8	1.44	4.80	6.34	8.45	12.68	16.91	23.24
2.31	1.1	4.4	1.32	4.03	5.32	7.09	10.64	14.19	19.54
2.10	1.0	4.0	1.20	3.33	4.40	5.87	8.81	11.75	16.15
1.89	.9	3.6	1.08	2.70	3.57	4.76	7.14	9.52	13.09
1.68	.8	3.2	.96	2.13	2.81	3.74	5.62	7.49	10.33
1.47	.7	2.8	.84	1.63	2.15	2.86	4.30	5.73	7.88
1.26	.6	2.4	.72	1.20	1.59	2.12	3.18	4.24	5.83
1.05	.5	2.0	.60	.83	1.10	1.46	2.20	2.93	4.03

Note: The strength being directly as the breadth, and the stress inversely as the velocity, it is easy to find the horses’ power equal to any given breadth of these pitches; or the horses’ power equal to any of these pitches and breadth, for any other velocity.

arrive at the line of centers. But, in cases where the pinion is small, the action in approaching the line of centers cannot be altogether prevented. For it can be demonstrated that a pinion with less than ten teeth cannot be uniformly driven by a wheel of any number of teeth whatever, unless they act partly before they arrive at the line of centers: but a pinion of 12 teeth by a wheel having not less than 15 teeth; so that the whole of the action may be after the teeth arrive at the line of centers.

When the pinion drives the wheel, the number of teeth in a wheel should be prime to the number of teeth in its pinion; that is, the number of teeth in the wheel should not be divisible by the number of teeth in the pinion. For example: Suppose there are 20 teeth in a pinion which is required to make about ve revolutions to one of the wheel. If this were the exact ratio, there would be just 100 teeth in the wheel; and after each revolution of the wheel, the same teeth would be continually brought in contact. But, if the diameter of the wheel by made somewhat greater—so as to admit of 101 teeth—then after ve revolutions of the pinion each tooth will come in contact with the one on the wheel immediately before that on which it had worked at the commencement, and after ve more revolutions will be engaged with the second tooth form those on which they acted in the rst instance; therefore it is evident that the wheel must revolve 101 times, and the pinion 5 × 101 or 505 times before the same teeth will again come in contact. By this means the inequalities of wear, form, and material, will compensate each other.

One of the most noteworthy of the early pioneers of the gear industry was George B. Grant. This dynamic businessman founded no less than ve gear companies in the United States. Three of these companies, Philadelphia Gear, Boston Gear Works, and Grant Gear Works, are still going strong today. George Grant was an inventor, a writer of gear books, and an outstanding gear engineer. Grant is described [5] as having a owing red beard and a commanding appearance somewhat like General U. S. Grant.

Some excerpts from Grant's book [6] called *Gearing* give a good insight into the historical development of gears and gear engineering. The following is article 47 from the tenth edition of *Gearing* published by Grant Gear Works in 1907.

The Mortise Wheel

Another venerable relic of the last century is the "mortise" gear, having wooden teeth set in a cored rim, in which they are driven and keyed.

Where a gear I subjected to sudden strains and great shocks, the mortise wheel is better, and works with less noise than a poor cast gear, and will carry as much as or more power at a high speed with a greater durability. But in no case is it equal of a properly cut gear, while its cost is about as great.

In times where large gears could not be cut, and when the cast tooth was not even approximately of the proper shape, the mortise wheel had it place, but now that the large cut gear can be obtained the mortise gear should be dropped and forgotten.

Figures A.31 and A.32 show examples of mortise wheel gears. It is interesting to note that perhaps a hundred mills using mortise gears are still in operation in the United States right now in 1968.

The business of making replacement teeth for these mills is mostly handled by the Thompson Manufacturing Company of Lancaster, New Hampshire. In 1967, President Robert D. Hillard showed the writer through the plant and explained that there was still a very active business in supplying maple gear teeth (literally tens of thousands per year) for use primarily in paper mills, roo ng mills, and grist mills for customers in nearly every state east of the Rocky Mountains. Many old mills are scattered throughout New England, upstate New York, and Pennsylvania. Most of these mills are still using water power and are carrying on enterprises that go back over 100 years. The Thompson Manufacturing Company (incidentally) was established in 1858.

At the time of my visit, several hundred nished maple gear teeth were in the shipping room ready for shipment and a whole stack of un lled orders were in the plant.

In article 51 of his book, Grant gives an interesting story about how to gure horse-power for cast gears.

Horse-Power of Cast Gears

The horse-power of a gear is the amount of power it may be depended upon to carry in continual service.

It is very well settled that continual strains and impact will change the mature of the metal, rendering it more brittle, so that a tooth that is perfectly reliable when new may be worthless when it has seen some years of service. This cause of deterioration is particularly potent in the case of rough cast teeth, for they can only approximate to the true shape required to transmit a uniform speed, and the continual impact form shocks and rapid variations in the power carried must and does destroy the strength of the metal.

There are about as many rules for computing the power of a gear as there are manufacturers of gears, each foundryman having a rule, the only good one, which he has found in some book, and with which he ll gure the power down to so many horses and hundredths of a horse as con dently as he will count the teeth or weigh the casting.

Even among the standard writers on engineering subjects, the agreement is no better, as shown by Cooper's collection of twenty-four rules from many different writers, applied to the single case of a ve-foot gear. See the *Journal of the Franklin Institute* for July, 1879. For the single case over twenty different results were obtained, ranging from forty-six to three hundred horse-power, and proving conclusively that the exact object sought is not to be obtained by calculations.

This variety is very convenient, for it is always possible to t a desired power to a given gear, and if a badly designed gear should break, it is a simple matter to nd a rule to prove that it was just right, and must have met with some accident.

Although no rule can be called reliable, the one that appears to be the best is that given by Box, in his Treatise on Mill Gearing. Box's rule, which is based on many actual cases, and which gives among the lowest, and therefore the safest results, is by the formula

$$\text{Horse-power of a cast gear} = \frac{12 \cdot c^2 \cdot f \cdot \sqrt{d \cdot n}}{1000}$$

in which c is the circular pitch, f is the face, d is the diameter, all in inches, and n is the number of revolutions per minute.

> Example: A gear of two feet diameter, four inches face, two inches pitch, running at one hundred revolutions per minute, will transmit
>
> $$\frac{12 \cdot 2 \cdot 2 \cdot 4 \cdot \sqrt{24 \cdot 100}}{1000} = 9.4 \text{ h.p.}$$
>
> For bevel gears, take the diameter and pitch at the middle of the face.
>
> It is perfectly allowable, although it is not good practice, to depend upon the gear for from three to six times the calculated power, if it is new, well made, and runs without being subjected to sudden shocks and variation of load.
>
> The influence of impact and continued service will be appreciated when it is considered that the gear in the example, which will carry 9.4 horse-power, will carry seventy horse-power if impact is ignored, and the ultimate strength of the metal is the only dependence.
>
> A mortise gear, with wooden cogs, will carry as much as, or more than a rough cast-iron gear will carry, although its strength is much inferior. The elasticity of the wood allows it to spring and stand a shock that would break a more brittle tooth of much greater strength. And, for the same reason, a gear will last longer in a yielding wooden frame that it will in a rigid iron frame.

In the example shown, the 24″ gear running at 100 RPM with 4″ face width and 2″ circular pitch would have had a unit load *index* of 194 psi at the 9.4 HP allowed. AGMA 225.01 shows 5000 psi as design limit for s_{at} for low grade cast iron. For a slow speed cast tooth of this size a stress limit of 5000 psi would make the allowable unit load come out to be about 400 for 14.5° pressure angle teeth of the old style. This shows that Grant was advocating design loads that would look reasonable by modern methods.

The data of 1832 given by Sereno Newton calculates to have an allowable unit load for cast iron of 300 and an allowable unit load for wood of 75. These values are not too different than those used by Grant almost 100 years later. The big difference to be noted was that Grant was worried about the effects of impact and the fatigue (and wear) due to long time operation. Grant's recommended formula has the effect (due to the square root sign) of *severely de-rating* the higher speed gears.

The period 1800–1916 showed a great expansion in the use of gears. Gears were used for the first time in ship propulsion, bicycles, textile mills, machine tools, automobiles, and even in the early airplane. Older uses in guns, clocks, process machinery, etc., were greatly expanded.

The gears cut in the early part of the nineteenth century showed a marked improvement in both design and craftsmanship. Figure A.45 shows the gear drives used to turn the 10-foot lens at the historic Navesink Light on the New Jersey coast. With 9,000,000 candle power, Navesink (erected in 1828) was the most powerful lighthouse in the United States in its day. Note the rather good looking spur and bevel gears—made before modern gear cutting machines were invented! Note also that the *lantern* pinions were still used.

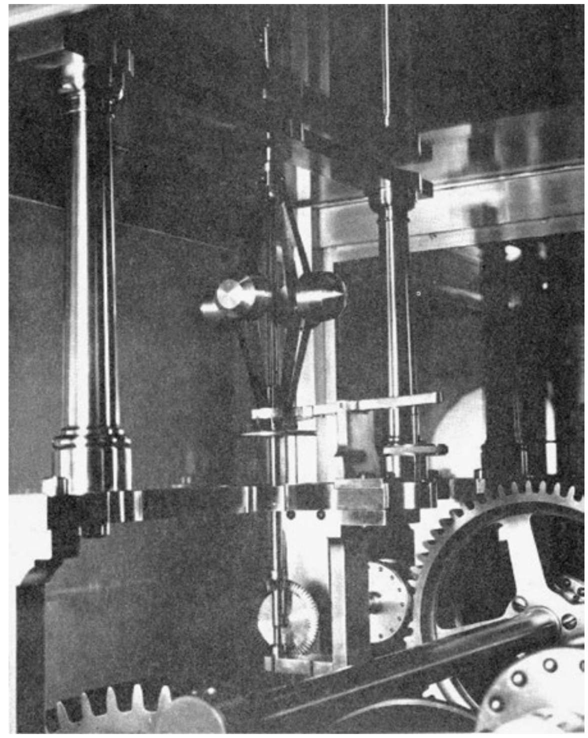

FIGURE A.45 Gear drives used at Navesink Lighthouse, circa 1828. (Courtesy of Museum of Science, Boston, Massachusetts.)

A.7 THE GREAT UPSURGE OF THE GEAR ART IN THE HALF CENTURY OF AGMA (1916–1968 AD)

So much progress has occurred in the gear art in the last fifty years that it is difficult to describe it in a few words.

The first models of most of the machine tools used in the gear trade had appeared by 1916. The 1915 model, though, compared with the 1968 model is like comparing a 1915 Model T Ford with the Oldsmobile Tornado of today!

New additions to the family of gear machine tools in the last fifty years include gear shaving machines, Shear-speed* machines, gear honing machines, bevel gear grinding machines, worm thread grinders, helical gear grinders and machines to broach splines or gears.

Besides the vast improvements made in the machine tools, to make gear teeth, a whole family of machine tools has appeared to measure gear teeth. These include involute checkers, lead checkers, composite error checkers and spacing checkers. Very recently the Fellows Gear Shaper Company and Hofler (of Germany) have each made breakthroughs on machines to rapidly measure index error (accumulated tooth spacing).

The man running a modern gear shop will often comment rather ruefully that it costs as much to tool up and measure gears as it does to make then in the first place!

Both gear making and gear checking have become highly automated. In the large production plants for automotive gearing,

* Trademark of Michigan Tool Company (Division of Ex-Cell-O Corp.), Detroit, Michigan.

TABLE A.6
Comparison of Typical Maximum Unit Load Values Used in Design

Approximate Date	Kind of Application	Approximate Unit Load in psi	Material
1832	Water-powered mill	75	Hard wood
	Water-powered mill	300	Cast iron, as cast
1900	Steam engine	1100	Cast iron, cut teeth
1968	Electric motor	5000	High strength cast iron, precision machined
	Steam turbine	6000	Steel gears, precision machined
	Steam turbine	9000	Hardened steel gears, precision machined
	Aircraft gas turbines	15,000	Case hardened and precision ground
	Rocket engine	30,000	Case hardened, super quality steel, ground to extreme accuracy

it can be said truthfully that the gear is made, measured, and then accepted or rejected *untouched by human hands.*

Although basic involute theory for parallel axis gears was worked out long ago, much work remained to be done to settle the mathematics of *any tooth shape* meshing on *any skew, right-angle* or *nonintersecting axis.* The difficult mathematics of gears and gear cutting for *any tooth form* on *any axis* has been almost completely solved. Some of the better known contributors to this work of modern gear theory are Ernest Wildhaber, Earle Buckingham, H. E. Merritt (England), Allen Candee, Werner Vogel, Hillel Poritsky, G. Niemann (Germany), Darle Dudley, Oliver Saari, Charles B. King, and Meriwether Baxter.

Seventy-five year ago, the load rating of gears was handled by very simple rules. That great pioneer in American gear work, George Grant, had tried to make a little order of chaos and had set down rules for wooden gear teeth, cast iron teeth and steel teeth. Wilfred Lewis presented a technique which is still the basis of most gear strength calculations. (This was about 1900 AD.)

The early builders of power gears were concerned with the fact that errors in gear teeth produced dynamic loads and made a fast running gear unable to carry as much torque as a slow running gear. The historical turning point in gear engineering relative to dynamic load came in 1931 when a research committee of the American Society of Mechanical Engineers—under the chairmanship of Earle Buckingham—published the

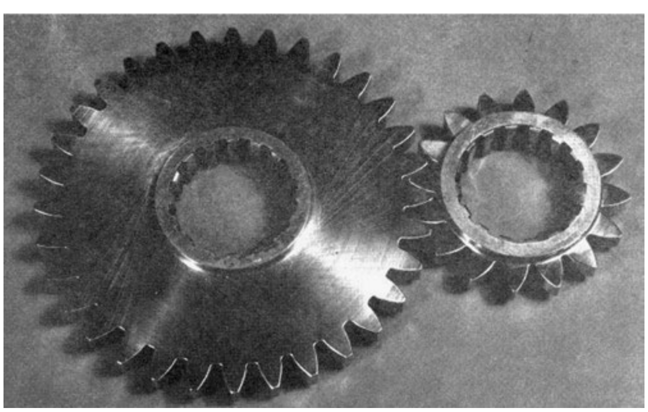

FIGURE A.46 Involute teeth of modern design.

results of an extensive gear test program. The results of this work gave test data on dynamic load and led to the establishment of formulas to calculate dynamic loads. These methods have been widely published* and used.

In recent years investigators have made much more sophisticated dynamic load tests. The fear of dynamic loads has led gear designers and gear manufacturers to become greatly concerned about gear accuracy. It soon became clear to the whole gear trade that gears had to be made to great precision if they were to run fast and last a long time. In modern turbine drives the first reduction generally has a pitch line speed of more than 10,000 billion feet per minute. Typical gears in a 5-year period will run for as much as 10 billion contact cycles without serious wear! The old cast iron gears of a hundred years earlier often had trouble running a year at 500 feet per minute.

Modern research and development testing plus a large amount of field experience has led to a whole stable of sophisticated rating formulas [11] for spur, helical, bevel and worm gears. Gears are rated for strength, surface durability—and if necessary—given a thermal rating and a scoring rating. Some of the better known men in gear rating work of the last fifty years are Aubrey Ross, Walter Schmitter, John Almen, Earle Buckingham, Darle Dudley, Edward Wellauer, G. Niemann, John Seabrook, Bruce Kelley, Martin Hartman, H. E. Merritt, Eugene Shipley, Paul Gustavson, S. L. Crawshaw, and Wells Coleman.

The dramatic progress in gear load carrying capacity can be seen if we look at the Table A.6 list of unit *load*† values. (The unit load index of tooth strength is the only criterion of rating that goes back to 1800. Other index criteria such as *K-Factor*, *scoring factor*, *etc.* were not used until the early 1900's.)

If we compare a good wood tooth of 1800 with a good rocket gear tooth of 1968, there is a gain in ability to carry tooth loads of about 400 to 1! Even in ferrous metal the gain from cast teeth of 1800 to steel rocket gear teeth of 1968 is a good 100 to 1!

The gains in tooth load carrying capacity have resulted from improvements in gear metallurgy [9,10] and lubricants

* See Ref. [8] in the Bibliography for dynamic load calculation methods.

† $\text{Unit load} = \dfrac{\text{tan driving force} \times \text{diam. pitch}}{\text{face width} \times \text{cosine of helix angle}} = \dfrac{W_t \cdot P_d}{F \cos\psi}$ (psi)

as much as from design improvements and accuracy improvements. Highly alloyed steels and cast irons came into wide use about 1920. Nitriding and improved case hardening developed rapidly from about 1930 onwards. Induction hardening of gear teeth came into use about 1950. Vacuum melt steels of unusual purity came into general use about 1960.

"Extreme pressure" lubricants for hypoid (and other gears) were first used around 1930. Several very sophisticated oil additives came into use in the 1955–1960 period. Synthetic lubricants for hot running gas turbine engines and gears have developed rapidly since 1950. A wide array of special lubricants permit gears to operate under load, temperature and speed conditions that would have otherwise been impossible.

During the 1916–1966 period, the 20° pressure angle involute teeth with extra depth to provide a full-fillet, root radius became the general standard of the gear industry. For extra heavy load carrying capacity, 25° pressure angle teeth became popular. Tooth designer frequently made the pinion *long addendum* and the gear *short addendum* to improve load carrying capacity. Figure A.46 shows a modern design with very high load carrying capacity.

The old 14½° involute remained in use but was not the most popular design. Epicyclic teeth passed out of use except for a few special cases.

Some new tooth forms and arrangements have appeared in the last fifty years. These include the Formate* gear tooth, Wildhaber-Novikov tooth form,[†] and Spiroid[‡] kinds of gears, the Harmonic Drive[§] and Cone-Drive[¶] gears. The hypoid gear was known before 1916 but it took machine tool development and automotive needs of the 1920–1930 era to push it into high production. Ernest Wildhaber and the Gleason Works did the pioneering in the gear machinery development and gear geometry to make the modern hypoid gear practical.

The newcomers just mentioned are either already important in the gear market place, or they have a potential which makes them important to consider. It seems probable that there will be even more tooth forms and tooth kinds to consider in the near future.

The work of AGMA has led to standards, information sheets, and technical investigations covering the whole gamut of gear technology. This work has become the unquestioned authority for responsible design and application of gearing in the United States. Outside the United States, AGMA work is studied (and often followed) by those concerned with the design and application of gearing. No other country has come even close to the work done by AGMA in the United States in advancing and codifying gear technology.

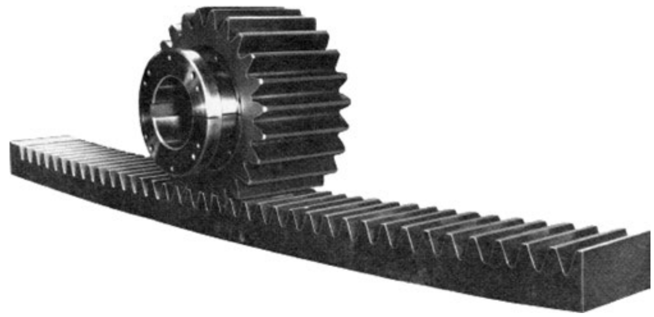

FIGURE A.47 Segment of an internal gear of slightly over 100 feet pitch diameter. Pinion is 24 teeth, 1½ pitch, 10″ face width and 20° pressure angle. (Courtesy of Sewall Gear Manufacturing Company, St. Paul, Minnesota.)

A.8 THE PIONEERING OF TODAY PORTENDS THE TRENDS OF TOMORROW

In laboratories and shops all over the United States (and the world) new things are being tried or built for the first time. This is true for the field of gearing just as it is true in so many other fields. Out of this pioneering work will come the new things of tomorrow.

In many areas, significant new developments are taking place that will be very important in the future. These are:

- Giant gears
- Huge gear teeth
- Ultra-tiny gear teeth
- Hard gears in very large sizes
- Gears to run at high load without oil
- Noiseless gears (almost noiseless)
- Gears to run at 1000°F
- Gears to run in non-lubricating fluids
- New tooth forms beyond involute, cycloid, Wildhaber-Novikov**
- New knowledge of stress pattern in odd-shaped gear teeth
- New knowledge of scoring phenomena
- Gears and splines formed by rolling
- Toothed gears formed by high energy forging
- Electron beam welding of *finished* parts of a gear assembly
- Statistical reliability data as a full-fledged tool of gear design

* Formate is a trademark of the Gleason Works, Rochester, New York.

† It is a huge mistake to combine the Wildhaber "Helical gearing" (U.S. Pat. No. 1.601.750, 1926), and Novikov gearing (USSR Pat. No. 109113, 1956) into a so-called "Wildhaber-Novikov gearing" as the gearings of these two types are based on completely different concepts of operating. The loosely introduced term "Wildhaber-Novikov gearing" has no engineering meaning, and it cannot be defined. Therefore, it is wrong way to use the term "Wildhaber-Novikov gearing." The two abovementioned gear systems, namely, (a) the Wildhaber "helical gearing" (U.S. Pat. No. 1.601.750, 1926), and (b) Novikov gearing (USSR Pat. No. 109113, 1956), should be considered separately and there are no way to combine them into a strange "Wildhaber-Novikov gearing." The last can be done only by those who has no understanding of the difference between the Wildhaber gear system and between the Novikov gear system (S. P. Radzevich [2012, 2015]).

‡ Spiroid is a trademark of the Illinois Tool Works Inc., Chicago, Illinois.

§ Harmonic Drive is a trademark of the United Shoe Machinery Co., Beverly, Massachusetts.

¶ Cone-Drive is a trademark of the Michigan Tool Company, Detroit, Michigan.

** The incorrectness of the term "Wildhaber-Novikov gearing" is explained above (see footnote †).

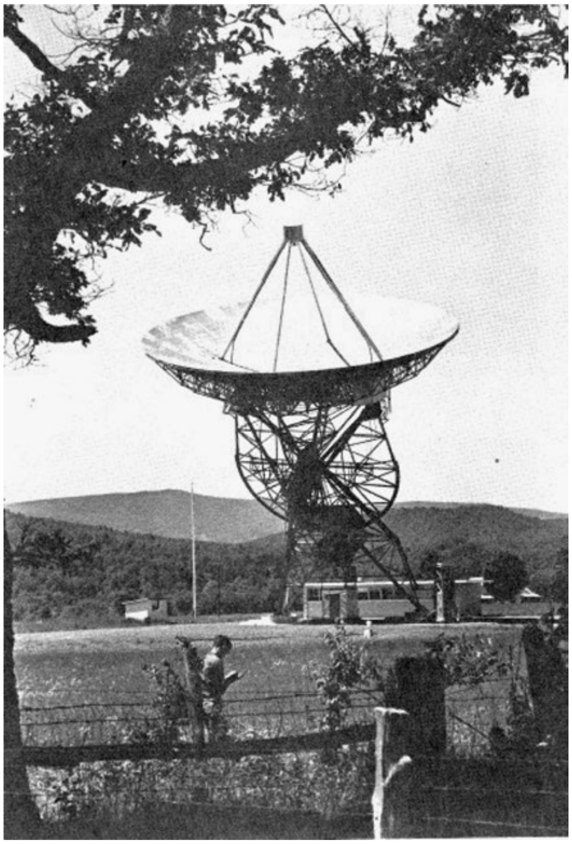

FIGURE A.48 Radio telescope. Note very large gear segment. (Courtesy of Sewall Gear Manufacturing Company and Blaw-Knox Company.)

A large gear used to be one almost 200 inches in diameter (16.7 feet). Now some gears are being made in the giant range of 70 to 100 feet in diameter. Since no machine tools are this large, the gear wheels are being made is sectors and assembled into wheels. Earle Gear, Philadelphia Gear, Brad Foote, Sewall Gear (and possibly others) have been active in this field. The colossal 500-foot gear is probably just around the corner! Figure A.47 shows a pinion and a segment of a gear having over 100 foot pitch diameter. Figure A.48 shows a large radio telescope. Note the giant segment gear used for elevation drive. A segment of a radar for a deep space probe unit is shown in Figure A.49.

It is still very difficult to cut a gear tooth coarser than ¾ pitch. Lufkin Foundry and Machine Company have recently made some very well-formed gear teeth twice this big at ⅜ pitch. See Figure A.50.

The finest pitch used to be around 128 pitch. Gears are now being made in the 200 to 400 pitch range. Fellows Gear Shaper is one of the pioneers in this field. In contrast to the giant gears and gear teeth just mentioned, note the very tiny gears in Figures A.51 and A.52.

Most companies still consider a gear around 30-inch diameter as the upper limit for case carburizing without too much distortion. A few have handled 50-inch gears via case carburizing. Recently, Brad Foote successfully carburized a 72-inch gear. See Figure A.53 for a 54-inch carburized gear.

Large, wide face helical pinions are perhaps the most difficult part to harden without excessive distortion. Unusual skill and very special heat treat facilities have been developed by Tool Steel Gear and Pinion in making extra heavy duty gearing for still mills. Figure A.54 shows a pair of pinions that were carburized and hardened with a case depth of over 150″. These pinions—for a steel mill pinion stand—are so accurate after hardening that satisfactory contact can be obtained over a tooth height of about 2 inches and face width of over 5 feet without grinding. The ability to make hard gears this large without significant distortion is a remarkable new development in the gear trade.

Gears in large sizes are being nitrided successfully. One Swiss company claims to be making good 100-inch gears by

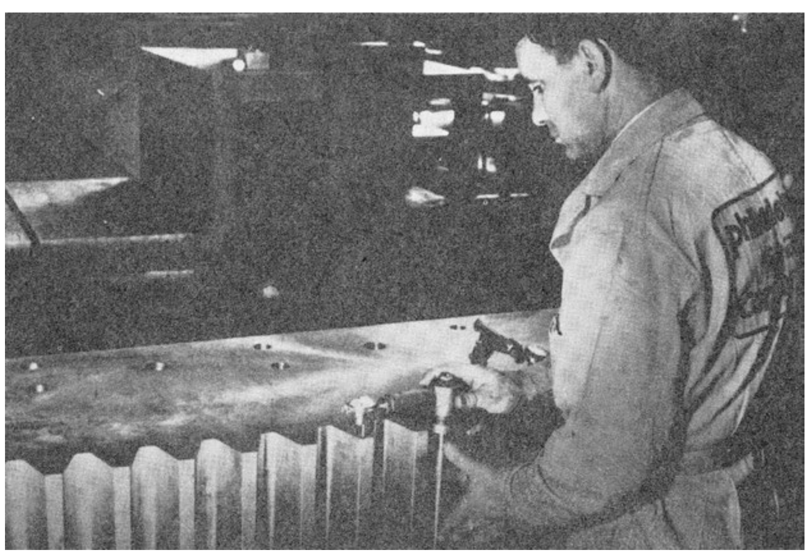

FIGURE A.49 Checking a segment of an 80 foot gear for a deep space radar unit. (Courtesy of Philadelphia Gear Corporation, King of Prussia, Pennsylvania.)

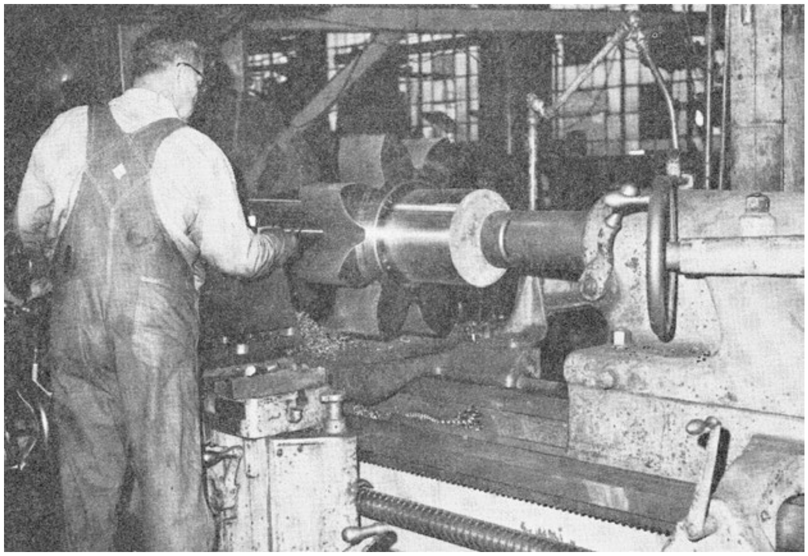

FIGURE A.50 Finishing an 8-tooth pinion, ⅜ pitch, 21.333″ pitch diameter, 6″ face width. This pinion was made by torch cutting and is used to drive a giant rack on an offshore drilling platform. (Courtesy of Lufkin Foundry & Machine Co., Lufkin, Texas.)

nitriding. In the United States 60 inches is still about the limit for nitriding facilities.

Spline forming and worm tread forming by rolling have become well established production process. Gear teeth are harder to make because of the deeper tooth depth but gear tooth rolling is beginning to become a new production process. Figures A.55 and A.56 show the start and near finish of a gear rolling operation.

High energy forging can make hot steel flow into a die almost like molten metal. Very powerful new forging machines will now form gear teeth and other parts with an accuracy and complexity that was only previously possible in precision die forgings of nonferrous metals (zinc, for instance). The forged gear can use very high strength steel and make a part that as good or better than a high strength machined part.

Figure A.57 shows an example of a gear with teeth finish forged. The spacing and lead accuracy are around AGMA quality level 8. With further development, it is probable that many small gears can be finish forged to as high as AGMA quality level 10. (Incidentally, forged gears have to have the "flash" trimmed off the ends of the teeth and the bores or journals finish ground.)

For very high accuracy gears of the future, the *high energy* forging of gear teeth could readily be teamed up with a finishing operation only of shaving or grinding to get accuracy in *the AGMA* quality level 12 to 14.

The pioneering work of Dynamic Forge Corporation has shown that high energy forging can be used to advantage in several ways in the gear trade. For instance, a complex

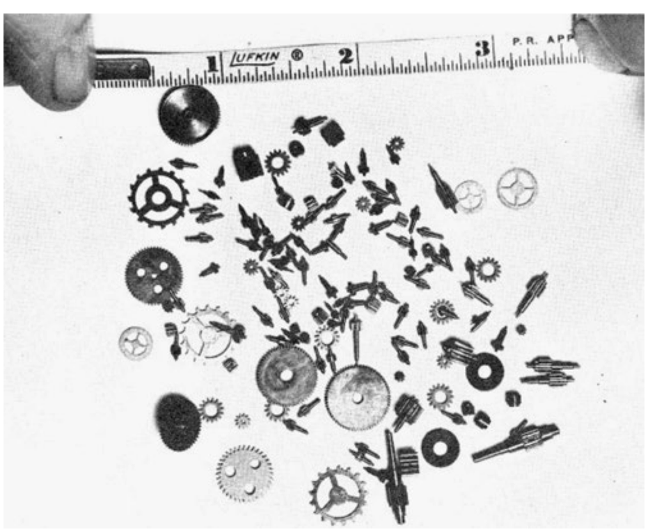

FIGURE A.51 An assortment of very fine pitch pinions and gears. (Courtesy of Fellows Gear Shaper Co., Springfield, Vermont.)

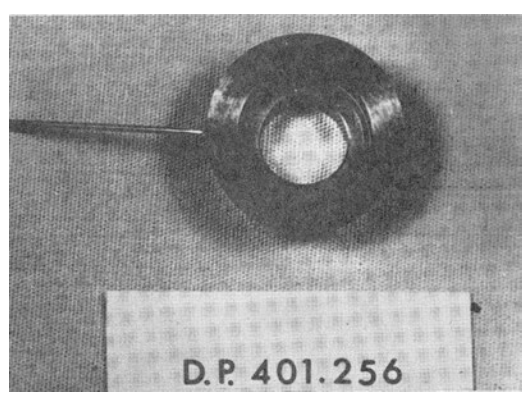

FIGURE A.52 A gear with teeth so tiny that they cannot be seen without a magnifying glass. Note weave of cloth and tiny brad for comparison—401.256 pitch. (Courtesy of Fellows Gear Shaper Company, Springfield, Vermont.)

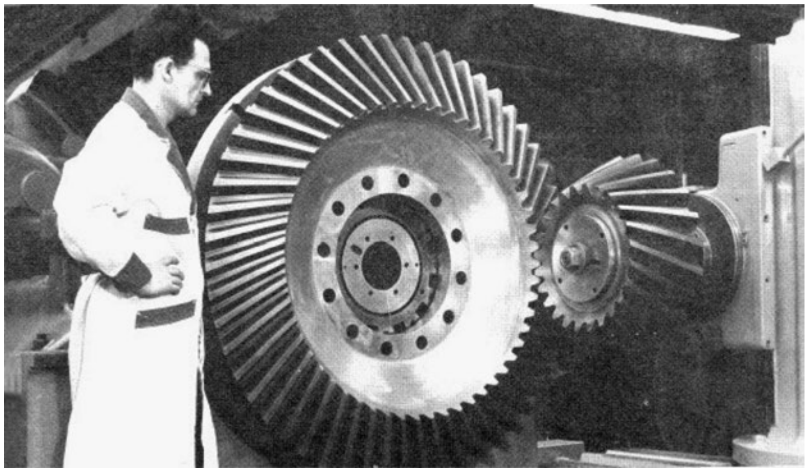

FIGURE A.53 A 54″ gear carburized and nished without grinding passes contact test with its pinion. (Courtesy of Brad Foote Gear Works, Inc., Chicago, Illinois.)

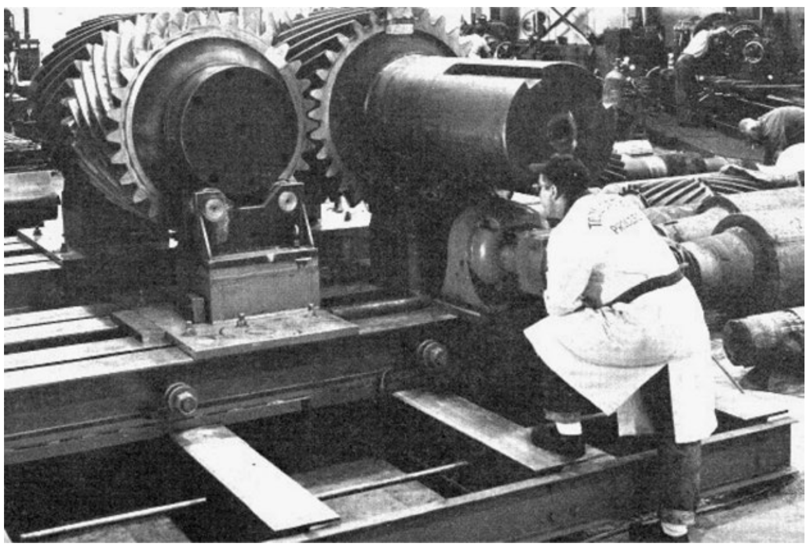

FIGURE A.54 Giant carburized pinions, ¾″ pitch—68″ face width pass contact test without grinding. (Courtesy of Tool Steel Gear and Pinion Company, Cincinnati, Ohio.)

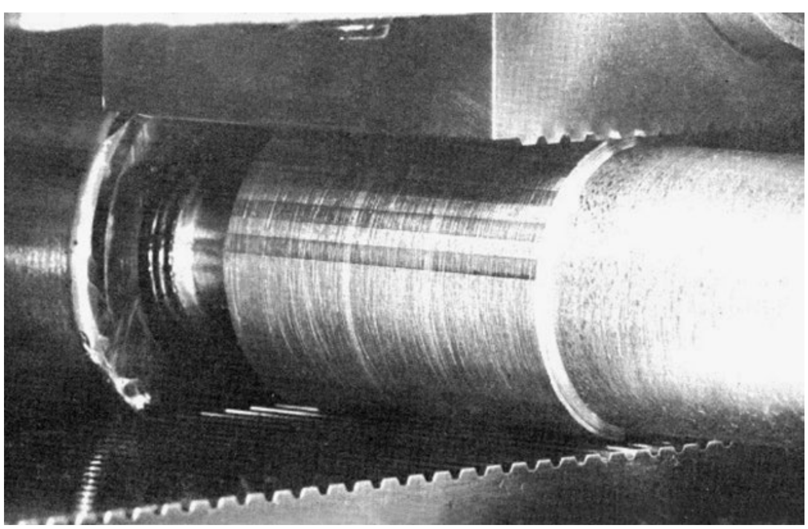

FIGURE A.55 Start of a rolled gear. (Courtesy of Michigan Tool Company, Detroit, Michigan.)

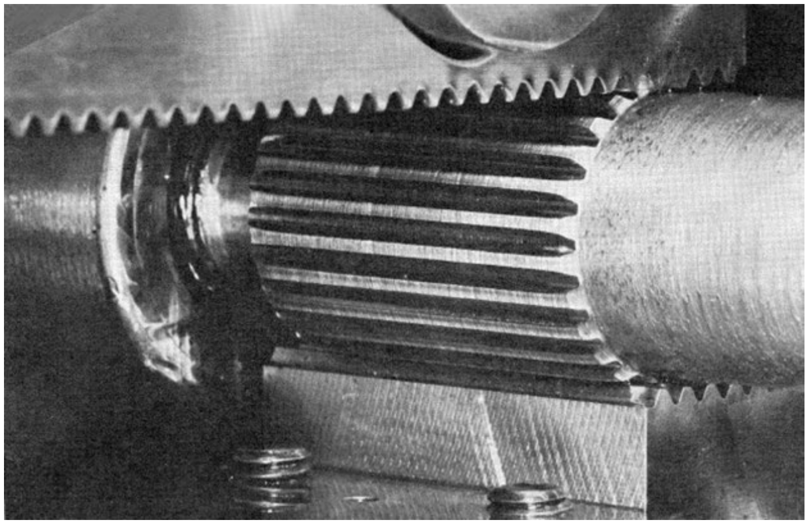

FIGURE A.56 Gear in Figure A.55 almost finish rolled.

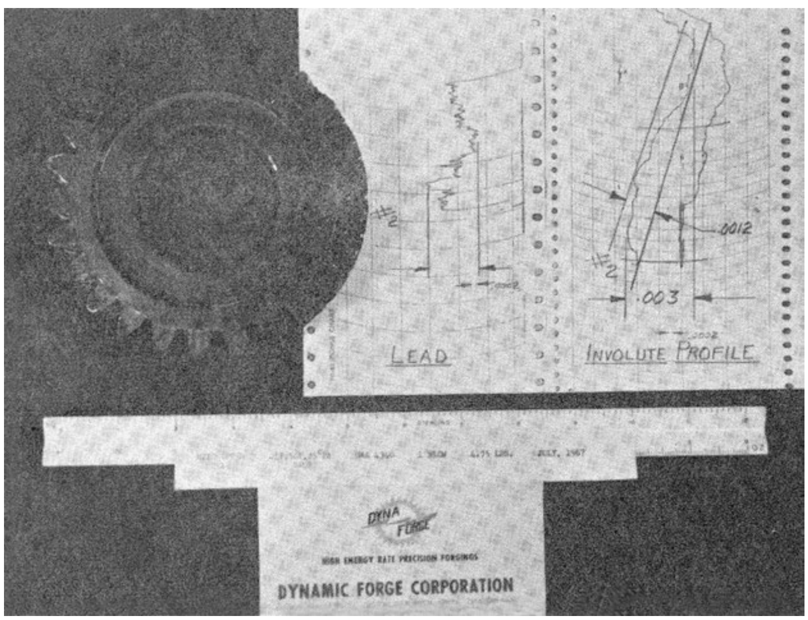

FIGURE A.57 Experimental helicopter gear finish forged in one blow. Weight is 4.75 lbs. (Courtesy of Dynamic Forge Corporation, Royal Oak, Michigan.)

jet engine gear can be forged to shape in one blow. (See Figure A.58.) A minimum amount of premium steel is used and an unusually good structure is obtained.

Figure A.59 shows a blank shape, complicated to machine, can be readily produced by forging. At present the internal teeth and splines are cut. With more development of the process and die, it will soon become possible to make complex parts, teeth and all, by this remarkable new process.

Gears (of any size) are being induction hardened a tooth at a time. Philadelphia Gear and National Automatic Tool are both very active in this field. See Figures A.60 and A.61.

Military people need capability to run their helicopters (or other equipment) for a few hours after they have been shot up. They want to get home! Advanced work on how to run gearing at high load without oil is being done at Mechanical Technology Inc. in Latham, New York; Bell Helicopter, Ft. Worth, Texas; and by the Vertol Division of the Boeing Company in Morton, Pennsylvania. Peculiar steel compositions and special tooth surface treatments make it possible to run at high load without oil.

In many situations, powerful gears do everything that is desired of them but they make enough noise to impair human hearing. Pioneering work in noise reduction is being

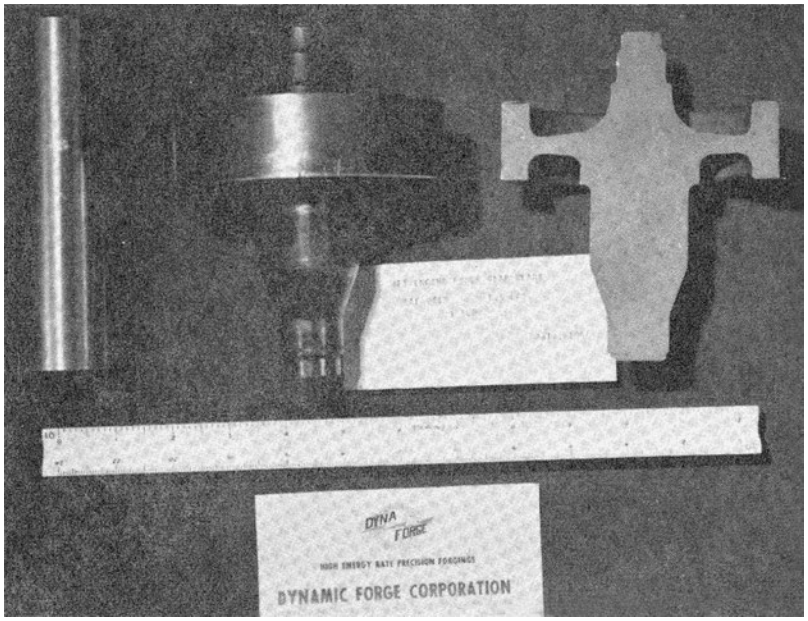

FIGURE A.58 Blank of jet engine gear made in one blow. Bar of stock is shown at left and cross section of this very intricate gear is shown at right. (Courtesy of Dynamic Forge Corporation, Royal Oak, Michigan.)

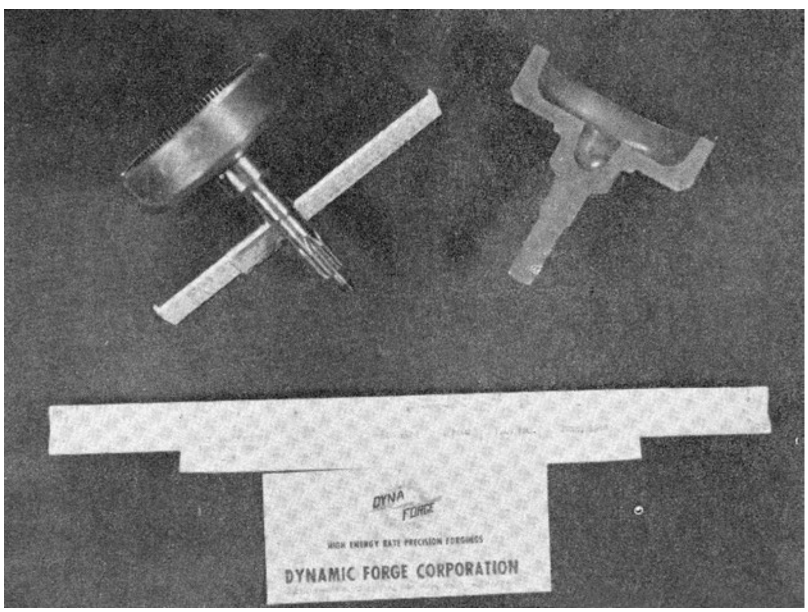

FIGURE A.59 Actuator gear blank for jet transport is made in one blow. Part weighs 1.40 lbs. Forged blank is shown on the right. Finish gear is shown on left. (Courtesy of Dynamic Forge Corporation, Royal Oak, Michigan.)

done by General Electric, De Laval, Mechanical Technology and others. The gear of the future must be a gentleman who speaks with a soft voice. It is becoming uncouth to aggravate people and impair their hearing with noisy, ill-mannered gears!

We can look to a steady advancement in all the areas of gear development mentioned above (plus many other areas). The next fifty years will certainly turn out to be even more surprising than the last fifty.

Those of us in the gear field can be most proud that our product has been around for about five thousand years and is *here to stay*. We have one of the finest trade associations in the world, “AGMA.” Gear manufacturers and gear engineers under AGMA leadership will continue to match progress in

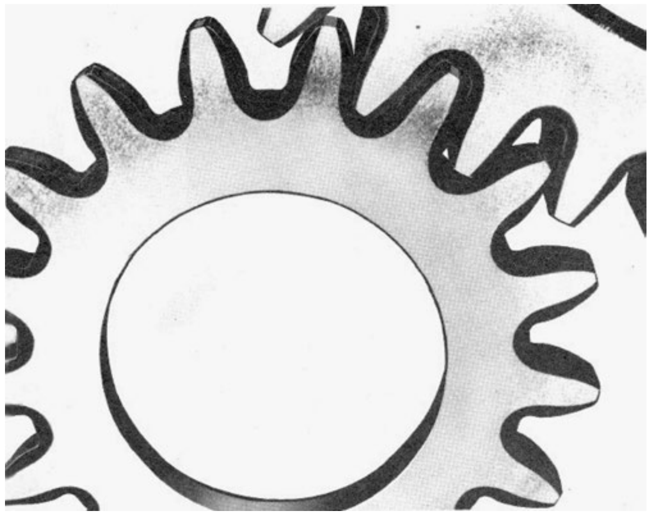

FIGURE A.60 Large gear teeth induction hardened by scanning a tooth at a time with a moving induction coil. (Courtesy of Philadelphia Gear Corporation, King of Prussia, Pennsylvania.)

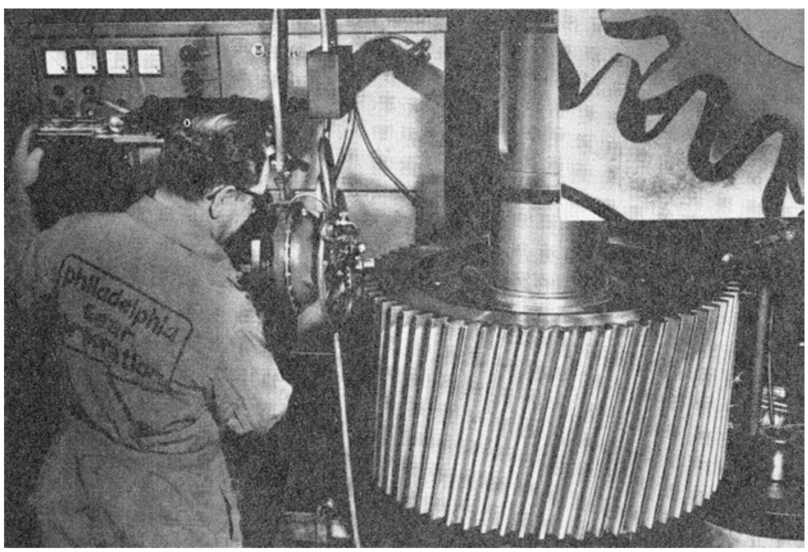

FIGURE A.61 Induction hardening a large pinion. (Courtesy of Philadelphia Gear Corporation, King of Prussia, Pennsylvania.)

other elds with outstanding progress in the very important eld of gearing.

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Appendix B: Complementary Material

The previous chapters of this book covered the prime text material: the selection, the design, the rating, and the manufacture of gears. The analysis of gear failures and the handling of certain special problems in gear design are discussed as well. This appendix essentially contains all appendix-type material, with additional explanations and calculations to complement the material in the earlier chapters.

B.1 INTRODUCTION TO GEARS (SUPPLEMENT TO CHAPTER 1)

Gears are toothed wheels which have been used for about 5000 years to transmit circular motion or rotation force (torque) from one part of a machine to another. Today gears are used in most machinery, and they range in size from the tiny gears in watch mechanisms to the giant 100 ft diameter gears in radar antennas. Gears are used in pairs, and each gear is usually attached to a rotating shaft. When the teeth of two gears are meshed, the rotation of one shaft and its gear causes the other gear and its shaft to rotate also. If the two gears have different diameters, the smaller one (called the *pinion*) will turn faster and with less rotational force than the larger (called the *gear*) one. Thus, gears can be designed to regulate rotational velocity and torque.

B.1.1 Types of Gears

There are a number of different types of gears, each especially suitable for a particular job. The spur gearset is the simplest to design and manufacture (except for friction gears, which have no teeth at all). The spur gear and pinion both have teeth which run straight across the width of the gear face.* The spur gearset transmits rotary motion and torque from one shaft to a parallel shaft; it is widely used where rotation velocities and torques are relatively low, and cost is an important consideration. The rack and pinion (see Figure 1.18) is a variation of the spur gear in which the gear (rack) is at rather than round. The rack and its round pinion convert rotation to linear motion and vice versa.

In the usual spur gearset, the gear and the pinion will always rotate in opposite directions. If the parallel shafts must rotate in the same direction, there are two solutions: (1) The internal gearset (see Figure 1.20), consisting of one internal gear and one pinion, is used when the two axes are close together and the pinion diameter is not too close to the gear diameter; and (2) if there is room between the shafts, an idler gear can be mounted on a bearing between the gear and the pinion. The idler gear will freely rotate on its shaft, whereas most gears are locked to their shafts by splines or by small pieces of metal called *keys*.

The bevel gear (see the upper right-hand corner of Figures 1.7 and 1.21) is used to transmit rotation from one shaft to a nonparallel shaft. (Perpendicular shafts are most common.) The basic shape of bevel gears is conical rather than cylindrical. Bevel gears are very common in household appliances and power tools. They are also used to a considerable extent in vehicles, helicopters, and mills.

The straight teeth of spur gears and straight bevel gears tend to make them noisy in operation, since the mating teeth mesh along their whole width all at once. To solve this problem and to obtain gears that can carry a higher load, some spur and bevel gears are made with teeth that twist across the face width. Spur gears with twisted teeth are called *helical gears* (see Figure 1.19), because the teeth are made in a helix shape. Bevel gears with twisted teeth are commonly called *spiral bevel gears* (Figure 1.23). As helical gear teeth mesh, the contact between the two gears begins at one end of a tooth and gradually extends in a diagonal line across the width of the tooth. If the face width of the gears is long enough, two or more teeth on each gear will be engaged at all times, and this overlapping enables the gears to carry a higher load. Helical gears are smoother and quieter in operation than spur gears, and so they are favored for high-speed applications.

Unfortunately, helical gears convert a portion of the rotational force into a thrust along the gear shaft, which can have an undesirable effect on the shaft bearings. This sideways component of the force, called the *axial thrust load*, occurs in helical gears because the teeth mesh along a diagonal line. When this axial thrust would be a serious problem, it can be eliminated by mounting two gearsets side by side with teeth sloping in opposite directions. This arrangement is referred to as a double helical gear (see Figure 1.11). When the RH and LH gears are both cut onto a single cylinder, the gear is sometimes called a *herringbone gear*.

Sometimes gears are needed to connect shafts which are neither parallel nor intersecting. For this purpose, a variation of the spiral bevel gear, called a *hypoid gear* (see Figures 1.24 and 1.25), has been developed. The unusual geometry of the hypoid gear allows the pinion to be large and strong even though it only has a few teeth.

Cylindrical helical gears can also be used for nonparallel, nonintersecting shafts, and such gears are called *crossed-helical gears* (see Figure 1.27). When crossed-helical gears mesh, their teeth meet in only one point at a time, and load-carrying capacity is limited. For this reason, variations have been designed in which the gear is throated or curved outward on the edges of its face so that it wraps part way around the pinion like a nut envelopes a screw. The small cylindrical pinion is called *worm*, and the gearset is called a *cylindrical worm gear* or a single-enveloping worm gear (see Figure 1.28). When the worm is not cylindrical, but is throated into an hourglass shape so that it also curves part way around the gear, the result

* See illustrations of gears in Chapter 1. For spur gears, see Figure 1.7 and 1.18.

is the double-enveloping worm gear (see Figure 1.30). Crossed-helical gears are sometimes called *nonenveloping worm gears*.

A worm can be designed with only one tooth, or thread, which wraps around and around the worm like the thread on a screw. Usually, however, a worm will have several threads. Worm gears provide the easiest way of greatly decreasing rotation speeds between two shafts, but friction losses can be high as the worm threads slide sideways along the gear teeth.

In practice, gears are used not only in pairs, but also in more complex interactions called *gear trains*. Automatic transmissions in automobiles use epicyclic or planetary gear systems (see Figure 4.19). The sun or central gear meshes with three planet gears. All the planet gears also mesh with an internal ring gear. Three different gear ratios can be obtained in an epicyclic gear train by a clutch system which prevents the rotation of either the sun gear, the internal gear, or the arm which is attached to the three planet gears through bearings.

B.1.2 Gear Design

Designing and manufacturing complex gears which quietly and efficiently mesh at high speeds and under high loads for millions of cycles require intricate planning, manufacturing, and testing. If the designs of simple spur gears look like an easy task, try it! There is no better way to learn about gears.

Pretend that you have been asked to design a spur gearset which will reduce the rotational speed of an engine shaft from 40 to 30 rpm. This requires that your gear-to-pinion tooth ratio be 4 to 3. Design your gear and pinion accordingly, and cut them out of hard paper. Check to see whether the pinion will effectively turn the gear when you pin them through their centers onto a piece of cardboard. This attempt will acquaint you with some of the angle and the distance that are familiar tools of the gear designer. For example, the distance between your pins is called, logically enough, the center distance.

In your design, you might start by drawing a root circle, with the idea of drawing your gear teeth sticking out from the root circle. Around this root circle, you will probably draw a large outside circle so that all the gear teeth will have the same whole depth. Then you will decide how many teeth your gear will have, and you will divide 360° by the number of teeth to determine the angle allotted for each tooth. Finally you must design the shape of the tooth, or the tooth profile.

After the preliminary design of your gear is finished, a critical question arises: How big should the pinion be?

For two spur gears to properly mesh, it is essential that they have the same tooth spacing. But tooth spacing is an ambiguous term on round gears. It sounds logical to define tooth spacing as the distance from the center of one tooth to the center of the next tooth. But do we measure this distance along the outside circle, along the root circle, or somewhere in between? (See Figure B.1). Unfortunately, *somewhere in between* is the only meaningful choice, since that is where the two gears mesh. The circle that is used is called the pitch circle, and the tooth-spacing term *circular pitch* is defined as the circumference of the pitch circle divided by the number of teeth. The pitch-circle circumference cannot be directly measured on a gear, but we can calculate the circumference for a meshed pair of gears.

The pitch circles of two mounted gears must be tangent (just touch each other at a single point, called the pitch point), so

$$\text{Center distance} = \text{gear's pitch radius} + \text{pinion's pitch radius}. \tag{B.1}$$

We also know that these gears will have the same circular pitch only if the ratio of these two radii is the same as the tooth ratio, so

$$\frac{\text{Radius of gear's pitch circle}}{\text{Radius of pinion's pitch circle}} = \frac{\text{no. of gear teeth}}{\text{no. of pinion teeth}}. \tag{B.2}$$

By counting the numbers of teeth and measuring the center distance, we can use Equations B.1 and B.2 to calculate the pitch-circle radii for gear and pinion. The circumference of any circle can be obtained by multiplying its radius by 2π. Dividing the pitch-circle circumference by the number of teeth will then give the circular pitch.

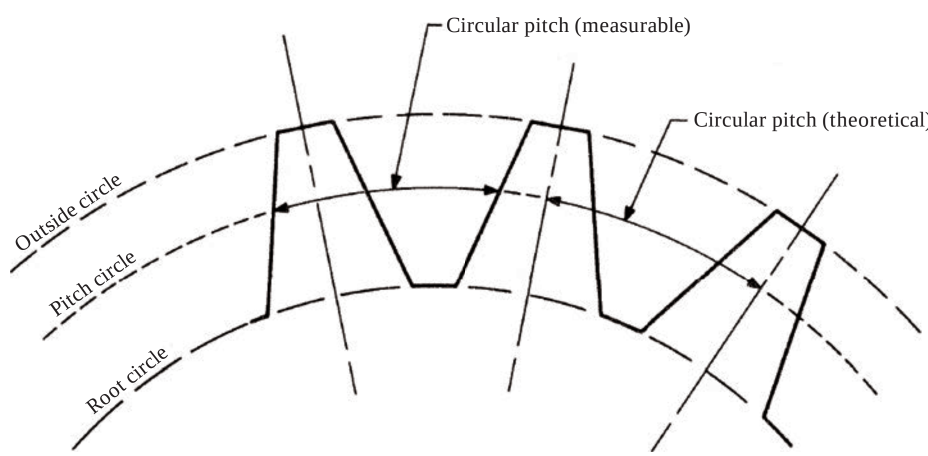


FIGURE B.1 Circular pitch.

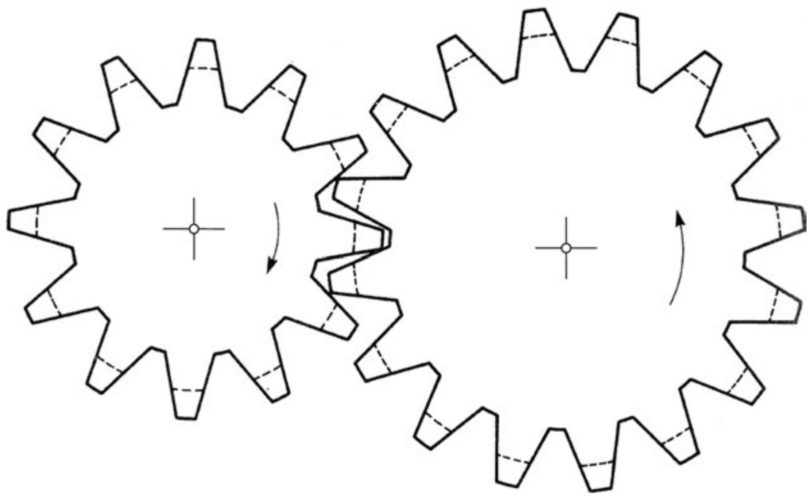


FIGURE B.2 Straight cardboard teeth.

For a gearset you are designing, the gear tooth ratio is 4/3. Thus, the ratio of pitch radii must also be 4/3, i.e., the radius of your pinion pitch circle must be 3/4 the size of your gear's pitch-circle radius.

After many trials with paper gears, you may evolve a design that actually works and that looks something like the gearset in Figure B.2. Congratulations! You have learned a lot, and you will have a much easier time absorbing gear design terminology and equations. However, there are problems with straight-sided tooth pro les like those shown.

First, notice that much of the contact between the two gears is on the leading corners of the driving gear. As the gear wears in, these corners will tend to round. Such wear is not necessarily detrimental—it may actually improve the running characteristics of the gearset if the break-in period is gentle.

The major dif culty with this design is that the angular velocity of the driven gear will not be uniform, even when the angular velocity of the driving gear is uniform. A gear of this design will turn with a slight jerkiness, and such nonuniformity is most unwelcome in precision machinery. For this reason, gear teeth must usually have a curved tooth pro le. If the tooth curvatures are just right to transmit uniform angular velocity from one shaft to another, the tooth pro les are said to be conjugate.

Of the various curves which maintain uniform angular velocity, the easiest for gear manufacturers to produce is called the *involute curve.* You can generate an involute curve by wrapping a string around a circle. The circle you chose will be called the *base circle* of the involute curve. Put a pencil point inside a loop at the end of the string, and slowly unwrap the string while pulling it taut without letting the circle rotate and without letting the string slip. See Figure B.3. The curve $SP_1P_2P\ldots$ which your pencil draws as it is pulled away from the base circle is an involute curve. Notice that at every point, the unwound string lies in a straight line which is tangent to the base circle and also perpendicular to the involute curve at each point. Since the string did not change length during the unwinding, the length of string PQ which has been unwound is always equal to the arc length SQ on the base circle. These

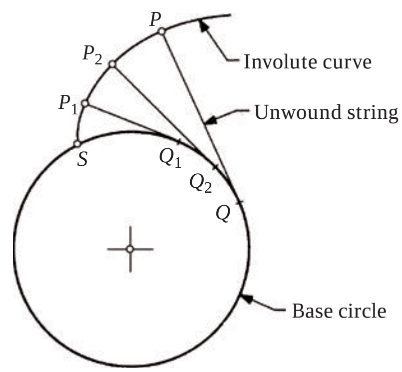


FIGURE B.3 Drawing an involute of a circle by unwrapping string.

facts directly lead to the mathematical involute relations of Figure 4.10 and Section 5.1.8.

Any line perpendicular to the involute tooth pro le surface is called a *line of pressure* because it corresponds to the direction of the force whenever the gear driving its mate is in contact at that point. All the lines of pressure for each involute gear tooth are tangent to the base circle (remember the unwound string!). When an involute gear and a pinion roll together in mesh, their two lines of pressure through the point of contact always lie along the same line. This very special line, called the *line of action*, is tangent to both base circles and contains the pitch point. The point of contact between the pinion and gear is always on this line, moving back and forth as the involute gear teeth roll through the mesh.

Before we add an involute curve to our paper gear pro les, let us consider how big we should make the base circle. Figure B.4 shows two different base circles and the involute pro les that result from each. Base circle c_1 has a radius r_{b1} which is only a little smaller than the pitch-circle radius r. Base circle c_2 is much smaller. The involute curve 2 determined by c_2 is much more slanted at the pitch-line level than involute curve 1. This difference is also re ected in the size of the pressure angle, which is de ned as the angle at a pitch point between the line perpendicular to the involute tooth surface and the line tangent to the pitch circle.

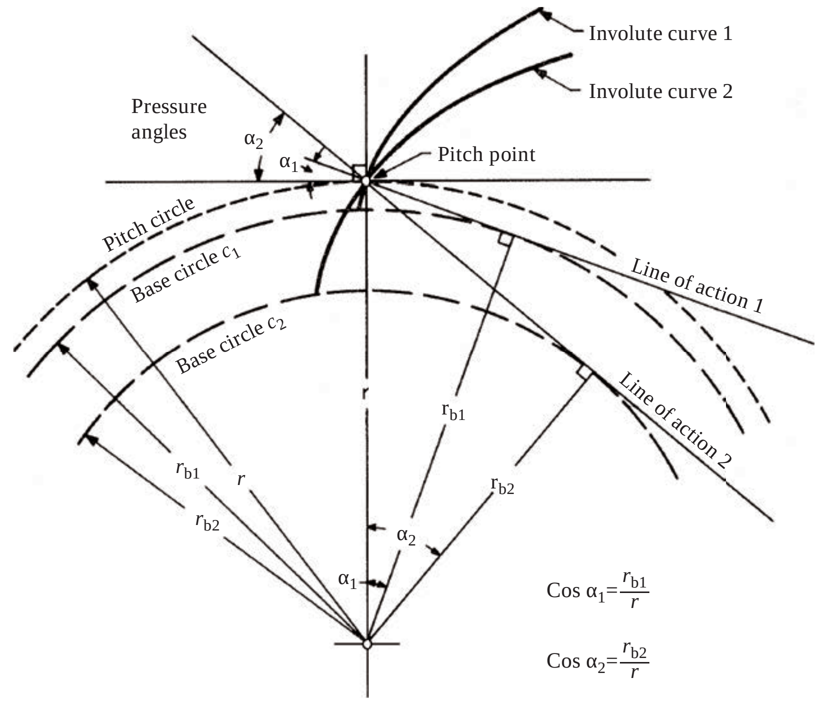


FIGURE B.4 The pressure angle α of an involute gear tooth is determined by the size ratio between the base circle and the pitch circle.

Some simple geometric logic will con rm that the pressure angle α is equal to the pro le angle between the tangent to the involute pro le and the radial line through the pitch point. This pro le angle is also equal to the angle labeled α at the center of the circle. It can be seen from the right triangle formed by the unwound string that the cosine of α is equal to the ratio of the base-circle radius to the pitch-circle radius. For Figure B.4, the base circle c_1 was chosen to be about 94% the size of the pitch circle, so the pressure angle α_1 is about 20° (cos 20° = 0.9397). A pressure angle of 20° is very commonly used in modern gears. The base circle c_2 is only about three-fourths the size of the pitch circle, so the pressure angle α_2 is about 40° (cos 40° = 0.766). If you imagine the other side of the 40° pressure angle tooth, you will see why such a large pressure angle is never used for actual gear teeth. The tooth would be extremely wide at the base, but the two sides would come together at a point before the tooth could reach a workable whole depth. Also, it would be impossible for more than one tooth of the gear to be in contact with a mating gear tooth at the same time, so the gears would not run nearly as quietly or as smoothly as 20° pressure angle gears. However, a pressure angle of 25° is sometimes used for fairly low-speed gears which are to be loaded heavily enough to bene t from the extra strength of a wide base.

If we now redesign the tooth pro le of our paper gears shown in Figure B.2 to incorporate a 22.5° pressure angle involute curve on the mating surfaces and a smooth curve in the root area between the teeth, our gears will look like those shown in Figure B.5. Now our gears will smoothly mesh, and they are close to the design of some current commercial spur gears, although most gears have more teeth than those shown in these illustrations.

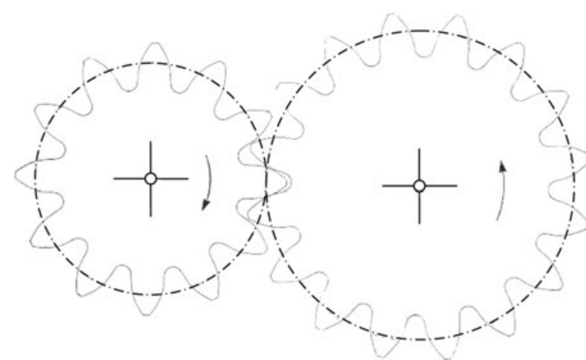


FIGURE B.5 Involute added to Figure B.2 teeth.

B.2 DYNAMIC LOAD THEORY (SUPPLEMENT TO CHAPTER 4)

At the International Federation for the Promotion of Mechanism and Machine Science (IFToMM) Gear Committee meeting in Eindhoven, Holland, in June 1983, Dr. Aizoh Kubo* gave an informal presentation covering dynamic load research. Dr. Kubo pointed out that there were really three kinds of dynamic load:

1. Fluctuations of input and output torque caused by more or less rough running of the engine making the power and the machine absorbing the power. (A piston engine driving a stone crusher would certainly be a good example of a uctuating torque situation.)

* Associate professors, University of Kyoto, Kyoto, Japan. Dr. Kubo is an IFToMM Gear Committee member.

2. Fluctuations of torque resulting from torsional vibration of the power transmission system. (A system like a propeller drive on a large ship can have a natural frequency that is excited by the frequency of propeller blades passing the rudder post.)
3. Fluctuation in torque as teeth roll through mesh. This is caused by periodic changes in tooth mesh stiffness and by errors in tooth profile, tooth spacing, and tooth alignment across the face width. (In some badly worn large spur gear drives, it is possible to feel and hear the disturbance as each tooth goes through mesh.)

Errors of these three kinds are handled in gear design work in the following manner:

- Misbehavior of driving and driven equipment is handled by an application factor. (Table 5.37 and Section 5.2.4 give general information on application factors.)
- It is necessary to calculate critical torsional frequencies in a power transmission system and then design and operate the equipment so as to avoid any appreciable operation at or near a critical speed. Fast-running equipment that operates above the first critical speed should be brought through the first critical speed quickly and at light load. (See Section 12.3.1 for more discussion of torsional vibration.)
- The dynamic load resulting from tooth stiffness effects and tooth errors is handled by a factor called *dynamic factor*. (See the part of Section 4.1.3 relating to dynamic load. Also see Section 5.2.4 and Figure 5.34).

In a private communication in December 31, 1982, Dr. Kubo sent me (i.e., Darle W. Dudley) a short statement of dynamic load for possible inclusion in this book. In Dr. Kubo's words:

> Although a torque applied to a pair of gears is kept constant, momentary overloads act on gear teeth. With progress of meshing, the number of tooth pairs in contact alternates, which results in the periodical change of the rotation of driven gear to the driving gear due to tooth deflections, even if the gears have no tooth errors. When a tooth error goes through mesh, it makes a delay or advance or rotational motion of driven gear to that of the driving gear. These forced changes of rotational movement of gear motion owing to tooth error and tooth stiffness changes prevent the masses of the driven and driving gears and apparatus from rotating at uniform velocities. The change in velocity causes momentary overloads.
>
> Since most of spur gear pairs have contact ratios between 1.3 and 2.0, one tooth pair meshing and two tooth pair meshing alternate cyclically with the progress of meshing. Because the difference of tooth stiffness under one tooth pair meshing and under two tooth pair meshing is large, non-uniform rotational movement of gears, i.e., vibrational excitation owing to tooth deflections, is also large, particularly if no adequate tooth profile correction to obtain uniform rotational movement of gears is made on spur gears. Consequently, the dynamic loads of spur gears without adequate tooth profile correction receive strong mutual influences of tooth stiffness change and tooth errors.
>
> On the other hand, for helical gears with considerably large overlap ratio, the total length of the sum of contact lines on the simultaneously meshing tooth pairs does not change so strongly as for the case of spur gears, and non-uniform rotational movement of gears owing to tooth deflection is in most cases rather small. The vibrational excitation of helical gears is therefore caused mainly by the non-uniform rotational movement of gears owing to tooth errors. In the case of helical gears with large overlap ratio, if tooth error is not the form of undulation which changes in the direction perpendicular to the contact lines on the plane of action, the tooth profile errors are averaged on the contact lines of the simultaneously meshing tooth pairs. And such helical gears can rotate almost uniformly, although they have considerably large profile errors. In this case, dynamic load increment is not large under wide driving speed range, but the tooth suffers from an overload increment caused by the local deflection of tooth pair corresponding to the tooth profile error whose magnitude is different at each part of contact line. In gear rating calculations this statical load increment due to tooth errors of helical gears is often not considered: In many gear rating calculation methods, e.g., AGMA procedure, the sum of both static and dynamic load increment due to tooth errors is usually taken in account by the "dynamic factor" for helical gears. But in some gear rating calculation methods, e.g., ISO/DIS 6336, DIN 3990 procedure, pure dynamic load increment is treated for both spur and helical gears. Consequently the dynamic factors from AGMA and ISO procedures for helical gears of large overlap ratio may show a very large difference between their values (depending on speed and accuracy). By the way, the statical load increment due to tooth errors for helical gears is not well considered in every gear rating method until now, and the use of dynamic factor which considers pure dynamic load increment for the rating calculation of larger helical gears without considering the statical load increment due to tooth errors results in an underestimation of real tooth loading in operation.
>
> When the vibration of gears becomes large enough or transmitting load is small enough, driving and driven tooth flanks lose their contact momentarily. Then the tooth flanks collide in the next moment with each other and a large momentarily overload acts on the tooth, with a magnitude proportional to the inertia masses of the driving and driven apparatus and to the collision velocity of tooth flanks. The backlash between driving and driven teeth has a very large influence on dynamic overload, when teeth lose their contact.
>
> When the gears are run at high enough speeds, the kinetic energy becomes so great that tooth errors and tooth stiffness change cannot change their velocity appreciably. Under such super critical driving speed, the overload is simply whatever load is required to bend the tooth out of the way (that is, the static load which would cause the deflection corresponding to the tooth error).

Figures B.6 and B.7 show two test rigs used by Dr. Kubo and his associates at Kyoto University in Kyoto, Japan, to get the dynamic load data. The first illustration shows a back-to-back gear test, while the second shows a power-absorbing type of gear test.

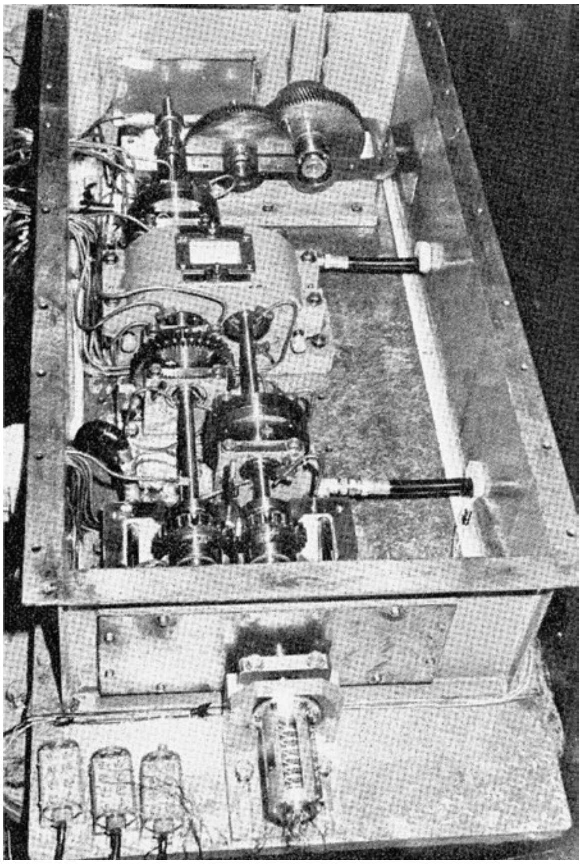


FIGURE B.6 Circulating torque test rig for dynamic load study. (Courtesy of Aizoh Kubo, Kyoto University, Kyoto, Japan.)

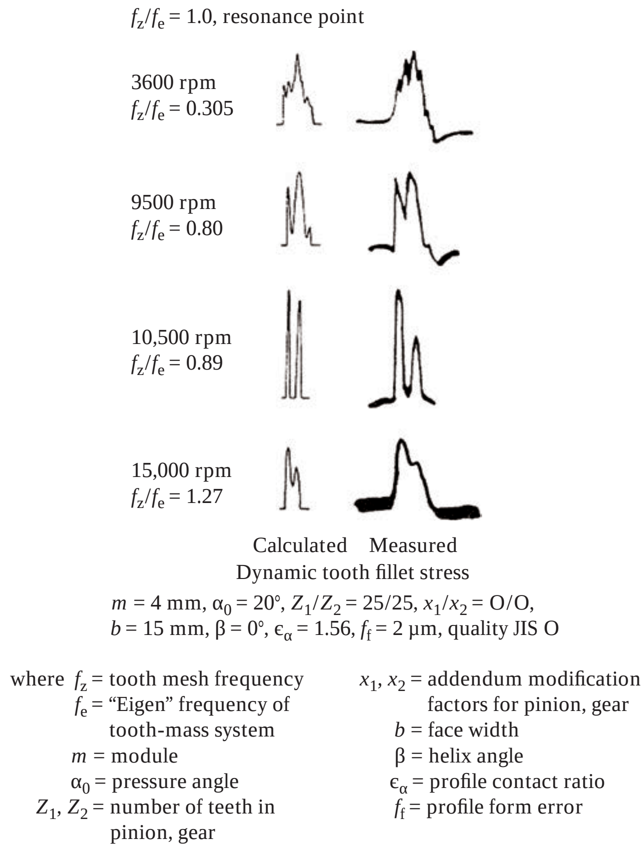


FIGURE B.8 Changing of dynamic tooth llet stress of high precision spur gears at different driving speeds.

The data obtained from the tests made at Kyoto University are most revealing and interesting. With good instrumentation and considerable number of tests, it was possible to thoroughly explore the dynamic load behavior of typical small gearsets. Figures B.8 through B.12 were supplied by Dr. Kubo to show some of his results.

Figure B.8 shows the general characteristic of the actual data taken. Note how the dynamic load changes as the speed changes.

Figure B.9 shows a summary of many tests for high-precision spur gears. Note how the dynamic load reaches a peak when there is a vibration resonance between the masses of the two gears in mesh. In Figure B.9, this resonance point comes just below 10,000 rpm. Also note the superimposed calculate values for an AGMA dynamic load and an ISO dynamic load.

Figure B.10 shows a eld of data for spur gears of medium quality. The test points are not all shown, but the area bounded

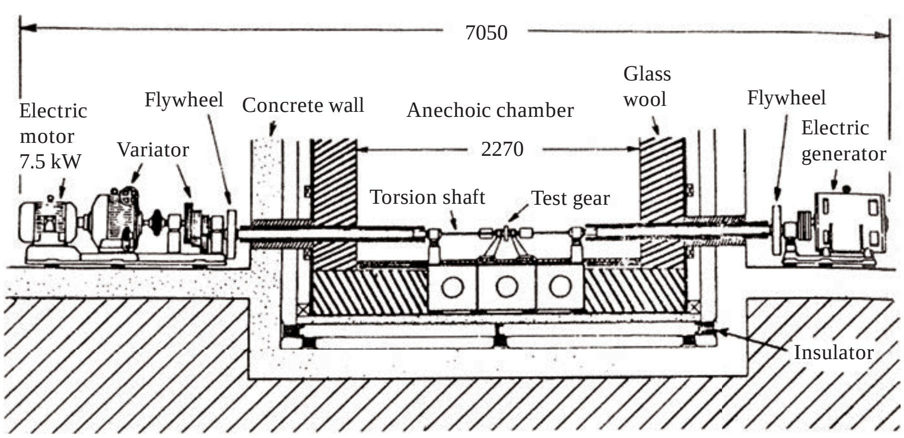


FIGURE B.7 Power-absorbing test rig for dynamic load study. (Courtesy of Aizoh Kubo, Kyoto University, Kyoto, Japan.)

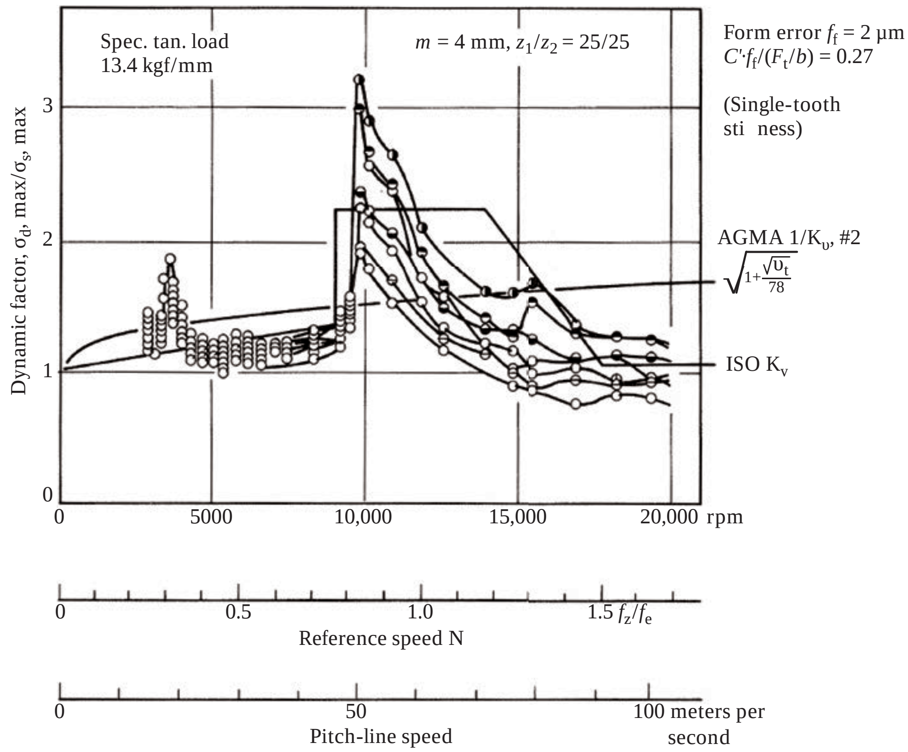


FIGURE B.9 Comparison of ISO and AGMA dynamic factors with measured dynamic factor for llet stress of high-precision spur gears.

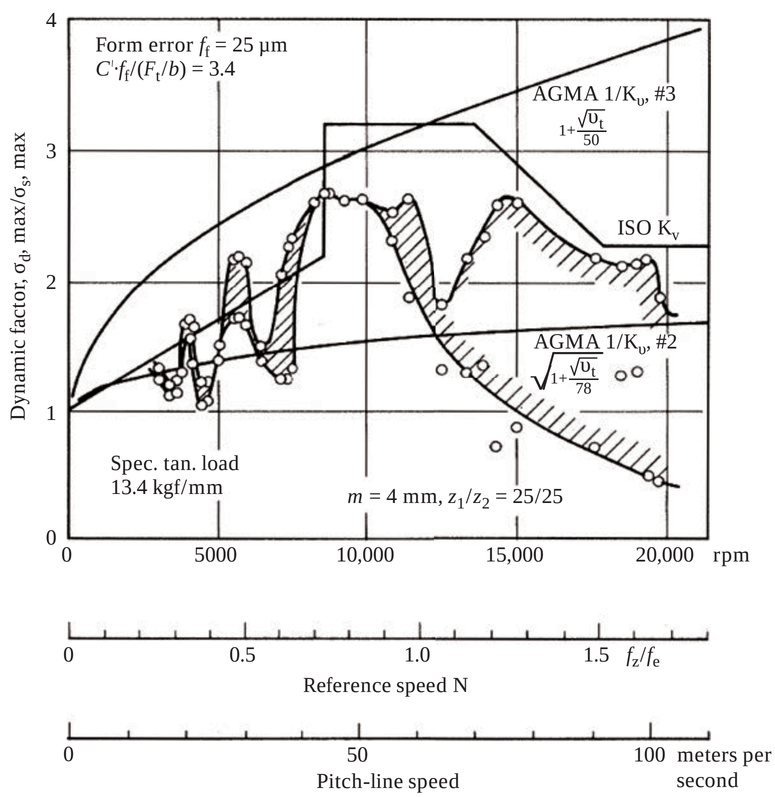


FIGURE B.10 Comparison of ISO and AGMA dynamic factors with measured dynamic factor for llet stress of spur gears of medium quality.

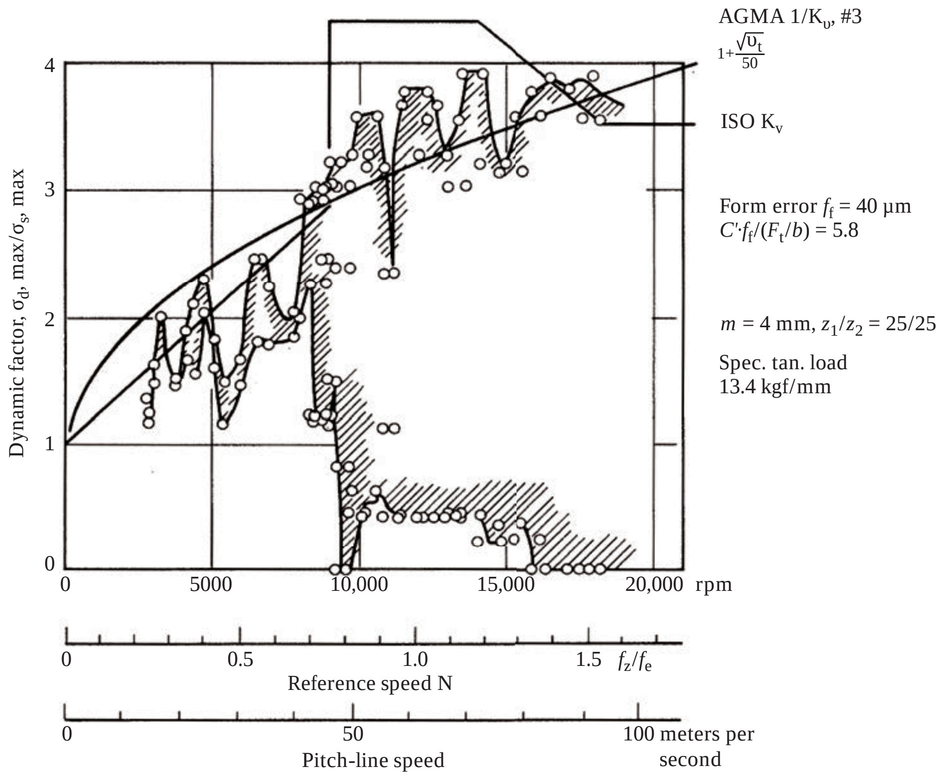


FIGURE B.11 Comparison of ISO and AGMA dynamic factors with measured dynamic factor for llet stress of spur gears of poor quality.

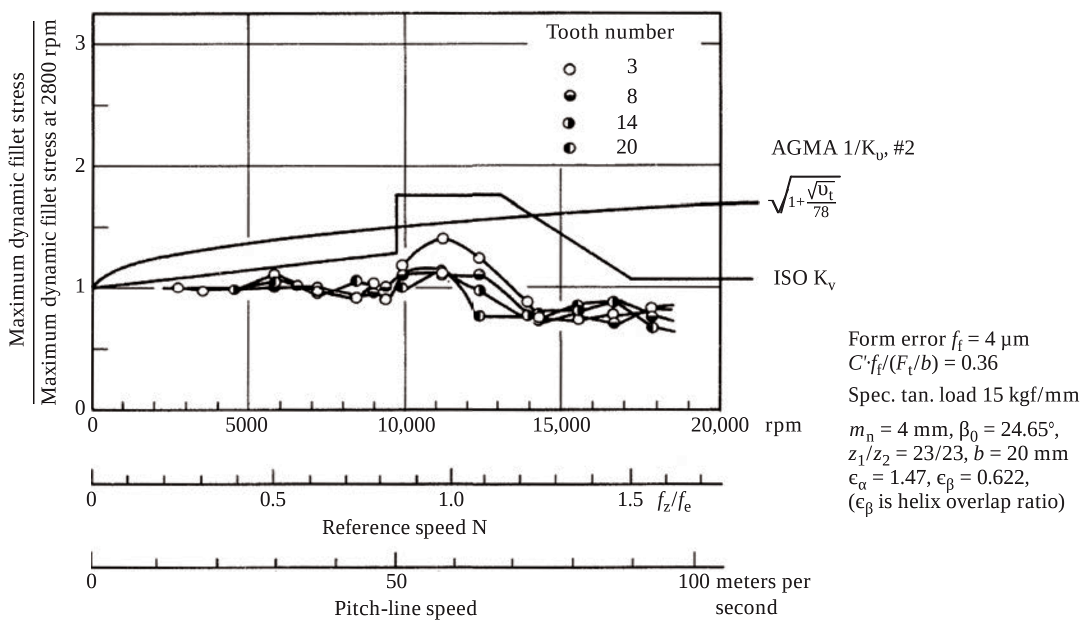


FIGURE B.12 Comparison of ISO and AGMA dynamic factors with measured dynamic factor for llet stress of high-precision helical gears.

by the test points is shown. In this case, the relative AGMA and ISO values that would be calculated are shown.

Figure B.11 shows more data for spur gears. In this case, the quality was really poor. Note the very high values of dynamic load. Data such as these teach the gear designer that gear quality is most important. Inaccurate gears are just not capable of running at high speed. They generate so much dynamic overload that the overload will quickly wear out the gears—even when the transmitted overload is relatively small.

Figure B.12 shows the situation for helical gears with relatively high accuracy. Note the rather low values of the dynamic overload. Of course, it should be kept in mind that these tests

by Dr. Kubo determined dynamic overload by strain gages mounted in the root fillets of gear teeth. Because the form of tooth fillet stress with progress of gear rotation for helical gears is somewhat different at each individual position of fillet along tooth width, the dynamic overload is difficult to determine by one measurement as shown in Figure B.12. But many measurements indicate that the situation of dynamic loading does not differ much from that of Figure B.12.

The general conclusion to be drawn from test work on dynamic load is that an exact calculation is nearly impossible. The designer can handle dynamic load by making a reasonable allowance for the effective dynamic load, then specifying close control on gear tooth accuracy so that relatively small amounts of dynamic load are developed. In addition, it is necessary to operate the gears away from the resonance point, particularly if they are spur gears.

B.3 HIGHEST AND LOWEST POINTS OF SINGLE-TOOTH CONTACT (SUPPLEMENT TO CHAPTERS 4 AND 5)

Spur gears with 20° pressure angle have contact ratios that are usually in the range of 1.6 to 1.7. Those with 25° pressure angle have contact ratio that are usually in the range of 1.4 to 1.5.

Figure 5.13 shows curves of contact ratio for different numbers of meshing teeth; 20°, 22.5°, and 25° pressure angles; and both standard addendum design and design where the pinion is 25% long addendum and the gear is 25% short addendum. Figure 5.13 is drawn for standard-depth teeth. (Working depth is 2.0 × module for metric dimensions or 2.0 ÷ transverse diametral pitch for English dimensions). The range of Figure 5.13 goes from a contact ratio of 1.34 for a 10/35 tooth ratio at 25° pressure angle, 25% long addendum to 1.78 for a 40/100 tooth ratio, 20° pressure angle, and standard addendum.

Given this, it is evident that in most of the spur gear designs used in the gear trade, two pairs of teeth are in contact when a tooth enters the mesh and two pairs will be in contact when a tooth leaves mesh. In the middle of the tooth contact area, though, only one pair of teeth will be in mesh. It is only when the calculated contact ratio exceeds 2.0 that two pairs of teeth can be meshed in the middle of the tooth contact area. The general theory of involute spur gear contact can be expressed as follows:

- Contact ratio equals to 1, only one pair of teeth in contact.
- Contact ratio over 1.0 but less than 2.0, two pairs of teeth in contact at start and finish of meshing, but one pair of teeth in contact for part of meshing interval in the middle of the tooth contact areas.
- Contact ratio over 2.0 but less than 3.0, three pairs of teeth in contact at start and finish of meshing, but two pairs of teeth in contact for part of meshing interval in the middle of the tooth contact areas.

Since normal spur gear designs have a contact ratio over 1.0 but less than 2.0, the locations where contact shifts from one pair of teeth to two pairs of teeth have been of much interest to those rating spur gears. It can be theorized that the most critical bending stress should come at the highest point on the tooth at which the tooth has to carry the full transmitted load. It can also be theorized that the most critical contact stress should come at the lowest point on the tooth at which one tooth has to carry the full transmitted load.

In the 1940s and the 1950s, much bench-test work was done on gear teeth in an attempt to find out whether or not these concepts in rating spur gears were valid. The general answer was that they are valid. (The ASME paper by Seabrook and Dudley [1963] gives an insight into one body of test work that probed the issues just mentioned.)

In recent years, it has become generally accepted that the changeover points from one pair to two pairs of teeth are critical. (It is only in the cases where there is a serious profile error or an unusual modification that these points are not critical from the standpoint of load rating calculations.)

Figure B.13 shows the basic definitions of where the highest and lowest points of single-tooth (single-pair) contact occur. If the pinion drives, the highest point of single-tooth contact (HPSTC) is one base pitch from the first point of contact. (If the pinion is driven, the first point of contact becomes the last point of contact, and the highest point of single-tooth contact is still at the same place—now one base pitch away from the last point of contact.)

The lowest point of single-tooth contact (LPSTC) is one base pitch away from the last point of contact, with the pinion driving.

In Figure B.13, point 1 is the highest point on the pinion at which single-tooth contact is carried. This position is often referred to in gear writings as HPSTC.

The lowest point of single-tooth contact in Figure B.13 is at point 2. This point is often referred to as LPSTC.

B.4 LAYOUT OF LARGE CIRCLES BY CALCULATION (SUPPLEMENT TO CHAPTERS 4 AND 5)

In making gear tooth layouts, it is frequently necessary to draw accurate arcs of circles that are quite large. For instance, a 50-tooth gear of 3 module may be laid out 10 times the size. This would make the pitch diameter used in the layout 1500 mm (59.055 in.).

With an ordinary beam compass, it is usually not possible to draw circles, or arcs of circles, greater than about 500 mm (20 in.). How can the gear designer draw accurate arcs of large circles?

The solution to this problem is to calculate the deviation from a straight line for a distance of about 100 mm (4 in.). With the tooth centerline in the center of a sheet of standard paper, it is possible to draw the circular arc for a distance of about 200 mm, with only four guide points on either side of the centerline.

Figure B.14 shows how four points may be precisely calculated (with a small pocket calculator). The example shown is

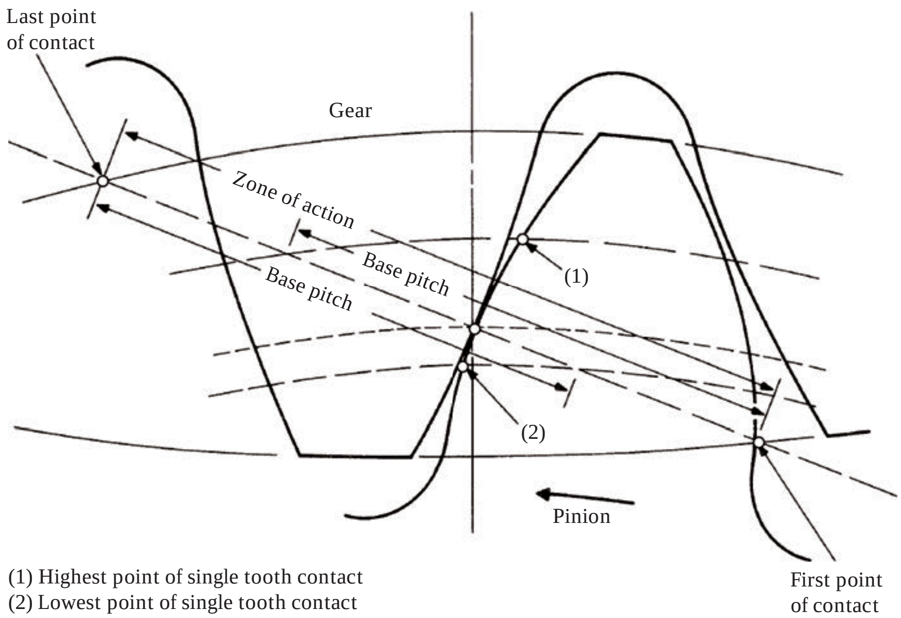
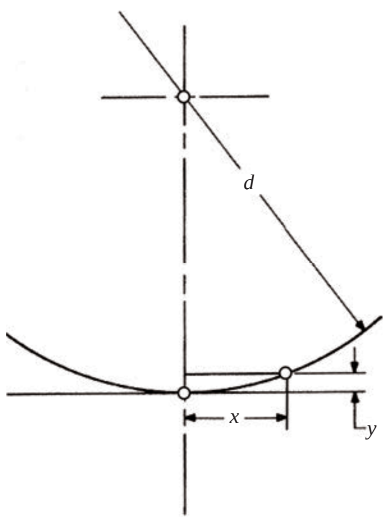


FIGURE B.13 De nition of the highest and lowest points of single-tooth contact for involute gears.



Calculation steps

1. Distance *x*, mm	25	50	75	100
2. Distance *d*, mm	1500			
3. (2) ÷ 2.000	750			
4. (3) × (3)	562,500			
5. (1) × (1)	625	2500	5625	10,000
6. $[(4) - (5)]^{0.50}$	749.583	748.331	746.241	743.303
7. $y = (3) - (6)$	0.417	1.669	3.759	6.697

FIGURE B.14 Method of calculating points for the layout of large circles.

for a 1500 mm diameter. Note that the fourth point calculates a deviation in the *y* direction of 6.697 mm. The deviations at the other four *x* values are

First point, 25 mm	*y* is 0.417 mm or 6.2%
Second point, 50 mm	*y* is 1.669 mm or 25%
Third point, 75 mm	*y* is 3.759 mm or 56%
Fourth point, 100 mm	*y* is 6.697 mm or 100%

The percentage values for four evenly spaced points are essentially constant for all large circles. Thus, calculating the value of the fourth point makes it easy to get other three points good enough for a layout.

The following list of *y* deviations at $x = 100$ mm can be used to check calculations or to quickly estimate the deviation of any large-diameter circle from a straight line:

Circle Diameter (mm)	*y* Deviation (mm) at $x = 100$ mm	Circle Diameter (mm)	*y* Deviation (mm) at $x = 100$ mm
550	18.826	1200	8.392
600	17.157	1500	6.697
650	15.767	1800	5.573
700	14.590	2250	4.453
750	13.579	2800	3.576
850	11.932	3375	2.966
1000	10.102	4000	2.502

The given values, for x = 100 mm, can be approximated closely enough for layout purposes by the relation

$$y = \frac{10,300}{d}. \tag{B.3}$$

B.5 SPECIAL CALCULATIONS FOR SPUR GEARS (SUPPLEMENT TO CHAPTER 5)

In this section, several special calculation procedures that are needed for spur gears will be covered. Other calculation procedures, of course, are covered in Chapter 5.

The supplementary material that will now be discussed covers tooth thickness, involute geometry dimensions, involute pro le dimensions, and root llet trochoid dimensions. Each of these will be covered in a separate subsection.

B.5.1 Tooth Thickness Measurement by Diameter over Pins

It is common practice in the gear trade to check spur gears by taking a measurement over pins. If the part has an even number of teeth, the pins are placed 180° apart. If the part has an odd number of teeth, the pins are placed as near to 180° apart as is possible.

In some countries (particularly in the United States), standard pin sizes have been established. Gear makers buy standard-sized pins to go with the commonly used pitches of teeth. Then there are published tables which make determining the measurement over pins for a standard set of pins and a standard pitch very simple.

Appendix C gives tables for the measurement of standard external and internal gears. In many cases, though, the pitch may not be standard or the pin size may not be standard. The gear designer then needs to make a special calculation.

Table B.1 shows the sizes of pins commonly used to check spur gears. These sizes are satisfactory for most gear designs. They are also the sizes that have been standardized in the United States.

The pin size used should touch the involute pro le of the gear tooth somewhat above the middle of the tooth height. The top of the pin should stick out beyond the outside diameter of the gear at least a small amount. The bottom of the pin should not go into the gear tooth space so deeply that it touches the root diameter. These limits on pin size, of course, make it possible to check a gear with several different sizes of pins. Given some choices, the designer should either pick a pin in accordance with Table B.1 or pick a pin close to the standard size given in Table B.1.

Table B.2 shows an operation sheet for calculating the diameter over pins for spur gears. A 25-tooth pinion of 3 module is shown as an example.

TABLE B.1
Pin Sizes Used to Check the Tooth Thickness of Spur Gears

Type of Tooth	Pressure Angle	Pin Diameter Constant
External, standard or near standard proportions	14½° to 25°	1.728
		1.920
External, long-addendum pinion design	14½° to 25°	1.680
		1.920
Internal, standard design	14½° to 25°	1.680
		1.440

Note: The pin diameters change as the tooth size changes. For metric design, multiply the pin size above by the module. For English design, divide the pin size by the diametral pitch. As an example, a 5-module spur tooth would normally use an 8.64 mm pin. A 5-pitch spur tooth would use a 0.3456 in. pin. If the teeth were 5 module and the only standard pin available was the 5-pitch pin, it could be used, since its size is 8.77826 mm, and that is close to the desired size. Calculations in Table B.2 could be made using the 8.77826 mm pin size for item (6).

Table B.3 shows the calculations that need to be made to determine the form diameters, the roll angles at the form diameters, and the roll angle at the start of pro le modi cations. In addition, this sheet has a calculation for the contact ratio.

The general theory behind Table B.3 is given in Sections 5.1.8 and 5.1.9. These sections give equations that could be used in direct calculation. Once the equations are understood, though, it is much simpler in day-to-day work either to use an operation sheet like Table B.3 or to make a computer routine to carry out the steps in Table B.3.

In designing spur gears, there is general concern about the relative sizes of the angle of approach and the angle of recess. Figure B.15 de nes these angles. In the angle of approach, the tooth sliding velocity is directed toward the root diameter of the pinion, but the tooth rolling velocity is toward the outside diameter of the pinion. In the angle of recess, the rolling velocity continues to be toward the outside diameter of the pinion and the sliding velocity is also toward the outside diameter of the pinion.

When the sliding and rolling velocities are opposite to each other, the tooth surface is much more apt to pit than when they are in the same direction. As a further consideration, the tooth friction makes the pinion run more roughly when the sliding and rolling velocities are opposed.

The designer should tend to favor making the arc of recess longer than the arc of approach, wherever possible. The small pinion will make several revolutions per turn of the gear, so it is more sensitive to wear than the gear. Also, the small pinion will be rotating several times faster than the large, so the pinion is more subject to vibration trouble.

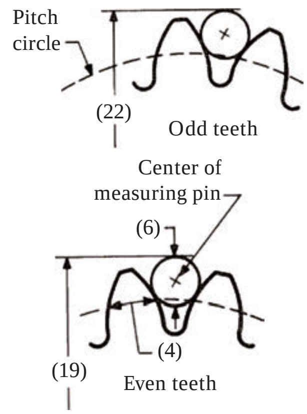
B.5.2 Calculation of Involute Geometry Values

Table 5.12 shows dimensions for a spur gear design, giving numerical values for a 25/26 tooth ratio with 3-module teeth.

B.5.3 Arc Tooth Thickness Values

In new design work, it is often desirable to plot the complete pro le of the gear tooth. This will show the designer what

TABLE B.2
Diameter over Pins for External Spur Gears



	Given Data				
1	Pitch diameter	75.00			
2	Number of teeth	25			
3	Pressure angle (PA) at (1)	20°			
4	Arc tooth thickness (TT)	4.991	4.915		
5	Base circle diameter, (1) × cos (3)	70.47695			
6	Diameter of measuring pin	5.334			
7	3.14159265 ÷ 2	0.1256637			
8	inv(3) = [tan(3) − (3)]/57.29578	0.0149044			
9	(6) ÷ (5)	0.0756843			
10	(4) ÷ (1)	0.0665467	0.0655333		
11	(9) + (10) + (8) – (7)	0.0314717	0.0304583		
12	Inv. angle[a] for (4), arc inv. (11)	25.3874	25.12653		
Steps					
13	Pressure angle, pin center	25.3874	25.12653	254500	25.0600
14	inv(13)	0.0314717	0.0304583	0.0317185	0.0302037
15	(14) − (8)	0.0165673	0.0155539	0.0168141	0.0152993
16	(15) + (7) − (9)	0.0665467	0.0655333	0.0667935	0.0652787
17	Arc thickness, (1) × (16)	4.991000	4.915000	5.009511	4.895900
18	Cos(13)	0.9034296	0.9053723	0.9029606	0.9058647
19	Diameter over pins, (5) ÷ (18) + (6) for odd number of teeth	–	–	–	–
20	cos[90° ÷ (2)]	0.998027	0.998027	0.998027	0.998027
21	[(5) ÷ (18)] × (20)	77.85651	77.68945	77.89695	77.64722
22	Diameter over pins, (21) + (6)	83.19051	83.02345	83.23095	82.98122

Note: Normally, (12) is used for (13) in the rst column, and then (17) in the rst column should be equal to (4). Other values of (13) are assumed and entered in the remaining columns. Four values are then obtained for tooth thickness versus diameter over pins. Item (19) is for even tooth numbers. Calculate (22) if tooth number is odd. Inv.: involute.

[a] See Table B.10.

the tooth will look like when it is machined in metal. If the resulting tooth needs changing, it is much better to learn this fact early in the design process than to be surprised at how the teeth look when they are rst cut in the shop.

Table B.4 shows how to calculate the arc tooth thickness of the tooth at any diameter from the base circle to the outside diameter. By calculating a few arc tooth thickness values, it is possible to make a layout of the involute part of the tooth pro le.

Table B.4 shows arc tooth thickness values for a 25-tooth pinion, 3 module, and 20° pressure angle. This is the same pinion shown in Table 5.12.

B.5.4 Root Fillet Trochoid Calculations

After the involute part of a tooth has been calculated, it may be necessary to calculate the root llet region. If this region is generated by a hob, there will be a trochoidal curve in the root llet. Table B.5 shows how to calculate a root llet trochoid. The same 25-tooth pinion, 3 module and 20° pressure angle was used for this calculation.

After both the involute part of the tooth and the root llet part of the tooth have been calculated, it is possible to plot the complete tooth pro le. Figure 5.14 shows the 25-tooth pro le, which was plotted at an enlarged size from the data in Tables B.4 and B.5.

The resulting tooth form in Figure 5.14 looks good. The width of the tooth at the outside diameter (top land) is generously wide. There would certainly be no dif culty carburizing a tooth with this width of top land.

The root region of the tooth also looks good. There are no sharp curvatures. The relatively gentle curvature should result in a low stress-concentration factor. It should be easy to design pregrind or preshave hobs with a protuberance tip and then have an easy manufacturing operation to blend the ground or the shaved involute pro le with the unground root llet.

B.6 SPECIAL CALCULATIONS FOR INTERNAL GEARS (SUPPLEMENT TO CHAPTER 5)

An internal gearset does not involve two internal gears running with each other. This is geometric impossibility. An

TABLE B.3
Calculation of Form Diameters, Roll Angles, Contact Ratio, and Start Profile Modification for External Gearsets

		Metric		English	
		1	2	1	Pinion
No.	Data Item or Operation	Pinion	Gear	Pinion	Gear
1	Number of teeth	25	96	25	96
2	Pitch diameter (PD)	75.00	288.00	2.9627	11.3386
3	Addendum	3.54	2.46	0.1394	0.0968
4	Outside diameter (OD), (2) + 2.0 × (3)	82.08	292.92	3.2315	11.5322
5	Pressure angle, transverse	20°	20°	20°	20°
6	Base diameter (BD), cos(5) × (2)	70.47695	270.6315	2.77463	10.65480
7	Gear ratio, $(1)_2 \div (1)_1$	3.84	3.84	3.84	3.84
8	Inverse ratio, 1.0 ÷ (7)	0.26042	0.26042	0.26042	0.26042
9	$\cos^{-1}[(6) \div (4)]$	30.83608	22.49556	30.83802	22.49435
10	OD roll, tan(9) × 57.29578	34.204	23.727	34.207	23.726
11	PD roll, tan(5) × 57.29578	20.854	20.854	20.854	20.854
12	Addendum roll, (10) − (11)	13.35	2.87	13.35	2.87
13	Pinion roll, $(12)_1 + [(12)_2 \times (7)]$	24.38	–	24.38	–
14	Gear roll, $(12)_2 + [(12)_1 \times (8)]$	–	6.35	–	6.35
15	Limit diameter (LD) roll, pinion, (10) − (13)	9.8196	–	9.8251	–
16	LD roll, gear, (10) − (14)	–	17.377	–	17.377
17	$\tan^{-1}[(15) \div 57.29578]$	9.72512	–	9.73042	–
18	LD, pinion, (6) ÷ cos(17)	71.50450	–	2.81513	–
19	$\tan^{-1}[(16) \div 57.29578]$	–	16.87215	–	16.87152
20	LD, gear, (6) ÷ cos(19)	–	282.8049	–	11.13403
21	Circular pitch, π × (2) ÷ (1)	9.42478	9.42478	0.37105	0.37105
22	Extra involute, 0.0160 × (21)	0.1508	0.1508	0.00594	0.00594
23	Form diameter (FD), pinion, (18) − (22)	71.3537	–	2.8092	–
24	$\cos^{-1}[(6) \div (23)]$	8.99114	–	8.99690	–
25	FD roll, pinion = tan(24) × 57.29578	9.07	–	9.07	–
26	FD, gear, (20) − (22)	–	282.6541	–	11.1281
27	$\cos^{-1}[(6) \div (26)]$	–	16.77107	–	16.77044
28	FD roll, gear = tan(27) × 57.29578	–	12.77	–	17.27
29	Roll per tooth, 360° ÷ (1)	14.4	–	14.4	–
30	Contact ratio, $(13) \div (29)_1$	1.693	–	1.693	–
31	Start modi cation diameter (SD) (see Table 5.54)	80.880	291.570	3.18426	11.47905
32	$\cos^{-1}[(6) \div (31)]$	29.38098	21.84605	29.3832	21.84478
33	SD roll, tan(32) × 57.29578	32.26	22.97	32.26	22.97

Note: For metric calculations, use millimeter dimensions, and for English calculations, use inches. All angles are in degrees. See Section 5.1.9 for cases when (22) may be too much extra involute. Subscript numerals are for column numbers.

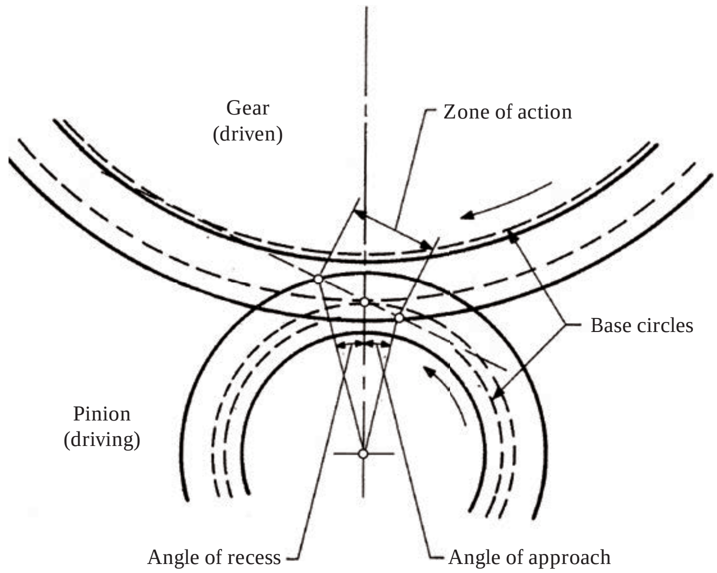
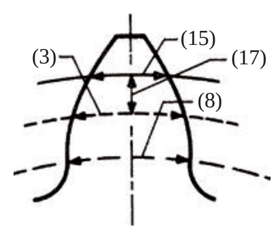


FIGURE B.15 Example of arc of approach and arc of recess.

TABLE B.4
Arc Tooth Thickness at Any Diameter, External Spur Gears



	Given Data	
1	Pitch diameter	75.00 mm
2	Pressure angle at (1)	20°
3	Arc tooth thickness at (1)	5.030 mm
Step A: Solve for the tooth thickness at base diameter.		
4	Involute[a] (2)	0.014904
5	cos(2)	0.9396926
6	(4) × (1)	1.117829
7	(3) + (6)	6.147829
8	TT at BD, (5) × (7)	5.7770689
9	Base diameter, (5) × (1)	70.47695

Step B: Solve for a series of tooth thicknesses to study the tooth pro le. For any desired diameter, the cosine of the pressure angle at the diameter is

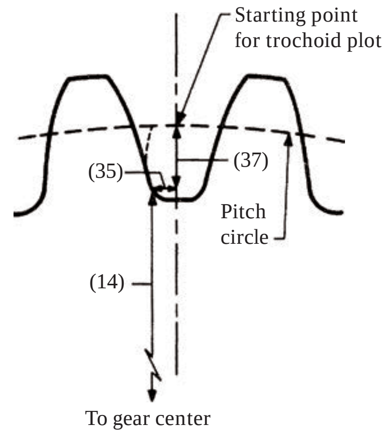
$$\cos \text{PA at any diameter} = \text{BD} \div \text{any diameter}.$$

A series of pressure angles is assumed based on diameters of interest. Then calculations are made to get the arc thicknesses (line 15) that go with each diameter.

10	Assumed pressure angles	30.83608	28.24139	24.58019	16.48985	11.80586
11	cos(10)	0.858637	0.880962	0.90938	0.95887	0.97885
12	Involute (10)	0.058782	0.04422	0.02841	0.00822	0.00297
13	(12) × (9)	4.142800	3.11653	2.00242	0.57922	0.20907
14	(8) − (13)	1.634269	2.66054	3.77465	5.19785	5.56800
15	Arc TT, (14) ÷ (11)	1.903329	3.02004	4.15080	5.42080	5.68833
16	Any diameter, (9) ÷ (11)	82.08000	80.00000	77.50000	73.50000	72.00000
17	Δ rad., [(16) − (1)] ÷ 2	3.5400	2.5000	1.2500	−0.7500	−1.5000

[a] The involute of an angle α is de ned as inv α = tan α − (α in radians). If α is in degrees, then inv α = tan α − (α × π − 180). Involutes can also be read from the involute function table (see Table C.12).

TABLE B.5
Calculation of Spur Gear Root-Fillet Trochoid Produced by a Round Corner Hob



	Given Data					
1	Pitch diameter (as hobbed)	75.00 mm				
2	Pressure angle (as hobbed)	20°				
3	Dedendum of gear (as hobbed)	3.51 mm				
4	Circular pitch (as hobbed)	9.42478 mm				
5	Edge radius of hob	1.05 mm				
6	Tooth thickness of gear (at pitch)	5.030 mm				
Step A: Solve for some intermediate distances and angles.						
7	Distance to edge radius center, (3) − (5)	2.46				
8	Hob tooth thickness, (4) − (6)	4.39478				
9	(7) × tan(2)	0.895367				
10	(5) ÷ cos(2)	1.117387				
11	0.5 × (8) − (9) − (10)	0.184637				
12	[2.0 × (11)] ÷ (1)	0.004924				
13	(12) × 180 ÷ π	0.282104				
Step B: Solve for points (x, y) or trochoidal path by assuming a series of radius values from middedendum to root radius.						
14	Assumed radius values	36.200	36.000	35.600	35.200	35.100
15	(1) ÷ 2.0	37.5				
16	(15) − (7)	35.04				
17	$[(14)^2 - (16)^2]^{0.5}$	9.09057	8.25823	6.28955	3.35237	2.05144
18	$\tan^{-1}$[(17) ÷ (16)] (°)	14.54383	13.26148	10.17601	5.46501	3.35059
19	(18) × π ÷ 180 (radians)	0.25384	0.23146	0.17760	0.09538	0.05848
20	(17) ÷ (15)	0.24242	0.22022	0.16772	0.08940	0.05471
21	(19) − (20)	0.01142	0.01124	0.00988	0.00599	0.00377
22	$(15) \times (16) - (14)^2$	3.56	18.0	46.64	74.96	81.99
23	(15) × (17)	340.8962	309.6837	235.8580	125.7140	76.92894
24	$\tan^{-1}$[(22) ÷ (23)] (°)	0.59832	3.32651	11.18571	30.80654	46.82407
25	$(14)^2 + (5)^2$	1311.543	1297.103	1268.463	1240.143	1233.113
26	(5) × sin(24)	0.01096	0.06093	0.20369	0.53775	0.76572
27	2.0 × (26) × (14)	0.79384	4.38676	14.50266	37.85745	53.75347
28	$[(25) - (27)]^{0.5}$	36.20426	35.95436	35.41130	34.67398	34.34180
29	(14) − (26)	36.18904	35.93907	35.39631	34.66225	34.33428
30	$\cos^{-1}$[(29) ÷ (28)] (°)	1.66184	1.67067	1.66687	1.49039	1.19875
31	(30) × π ÷ 180 (radians)	0.02900	0.02916	0.02909	0.02601	0.02092
32	(31) + (21)	0.04043	0.04040	0.03898	0.03200	0.02470
33	(32) + (12)	0.04535	0.04532	0.04390	0.03692	0.02962
34	(33) × 180 ÷ π (°)	2.59842	2.59660	2.51527	2.11546	1.69708
35	x = (28) × sin(34)	1.64133	1.62886	1.55405	1.27993	1.01704
36	(28) × cos(34)	36.16704	39.91744	35.37718	34.65035	34.32673
37	y = (15) − (36)	1.33296	1.58256	2.12282	2.84965	3.17327
38	Point on path = (35), (37)	(1.64, 1.33)	(1.63, 1.58)	()1.55, 2.12	(1.28, 2.85)	(1.02, 3.17)

internal gearset consists of a pinion with external teeth running with a gear with internal teeth.

The internal gear is generally checked for tooth thickness with measuring pins, like the external gear. However, the measurement is made *between* pins instead of over pins. Generally, the measurement is taken as a diameter under two pins placed as near to 180° apart as possible.

Table B.6 shows the calculation procedure. The numerical example shown is for a 28-tooth internal gear of 3 module and 25° pressure angle. This is the internal gear shown in Table 5.15.

Table B.7 is a calculation sheet for form diameters, roll angles, and contact ratio. The numerical example shown is for an 18-tooth spur pinion meshing with a 28-tooth internal gear. This is the gearset shown in Table 5.15.

Table B.8 is a calculation sheet for the arc tooth thickness at any diameter on an internal gear. The numerical data shown are for 28 teeth, 3 module, and 25° pressure angle.

Figure B.16 shows a layout of the internal gear teeth using the involute pro le data from Table B.8. The root llet trochoid would be formed by the tip of an internal shaper-cutter rather than by the tip of a hob tooth. (Internal teeth are usually cut by shapers.)

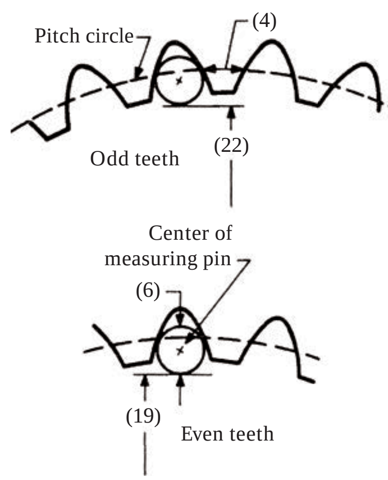
The example shown in Figure B.16 would require a shaper-cutter with rather narrow tips on the teeth. The generating action that would produce a trochoid would have to be very limited. For this reason, the llet region of the internal tooth was approximated by an arc of a circle.

Figure B.16 shows that the internal tooth is not really a good design. The root llet region is so small that there would

TABLE B.6
Diameter between Pins for Internal Spur Gear

Given Data

1	Pitch diameter	84.00	
2	Number of teeth	28	
3	Pressure angle at (1)	25	
4	Arc tooth thickness at (1)	4.240	4.160
5	Base-circle diameter, (1) × cos(3)	76.12985	
6	Diameter of measuring pin	5.33401	
7	3.14159265 ÷ (2)	0.22220	
8	inv(3) = [tan(3) − (3)]/57.29578	0.02998	
9	(6) ÷ (5)	0.07006	
10	(4) ÷ (1)	0.05048	0.04952
11	(7) + (8) − (9) − (10)	0.02164	0.02259
12	inv. angle for (4), arc inv. (11)	22.54	28.85



Steps:

13	Pressure angle, pin center	22.54	22.23	22.70	22.85
14	inv (13)	0.02164	0.02072	0.02212	0.02258
15	(7) + (8)	0.14218			
16	(15) − (14) − (9)	0.05048	0.05139	0.04999	0.04953
17	Arc thickness, (1) × (16)	4.2403	4.3170	4.1992	4.16045
18	cos(13)	0.92361	0.92567	0.92254	0.92152
19	Diameter under pins, (5) ÷ (18) − (6)	77.092	76.909	77.188	77.279
For odd number of teeth:					
20	cos[90° ÷ (2)]	–	–	–	–
21	[(5) ÷ (18)] × (20)	–	–	–	–
22	Diameter under pins, (21) − (6)	–	–	–	–

Note: Normally, (12) is used for (13) in the rst column, and then (17) in the rst column should be equal to (4). Other values of (13) are assumed and entered in the remaining columns. Four values are then obtained for tooth thickness versus diameter between pins. Item (19) is for even tooth numbers. Calculate (22) if tooth number is odd. This sample calculation is in the metric system (using millimeters and degrees).

TABLE B.7
Calculation of Form Diameter, Roll Angles, and Contact Ratio of Internal Spur Gears

		Metric		English	
Data Item or Operation		**Pinion**	**Gear**	**Pinion**	**Gear**
1	Number of teeth	18	28	18	28
2	Pitch diameter	54.00	84.00	2.12599	3.30709
3	Addendum	3.39	2.12	0.133465	0.083465
4	Outside diameter, pinion (2) + [2.0 × (3)]	60.78	–	2.39291	–
5	Inside diameter, gear, (2) − [2.0 × (3)]	–	79.76	–	3.14016
6	Pressure angle, transverse	25	25	25	25
7	Base diameter, cos(6) × (2)	48.94062	76.12985	1.92680	2.99724
8	Gear ratio, $(1)_{gear} \div (1)_{pinion}$	1.55556	1.55556	1.55556	1.55556
9	Inverse ratio, 1.0 ÷ (8)	0.64286	0.64286	0.64286	0.64286
10	$\cos^{-1}[(7) \div (4)]$	36.36953	–	36.36953	–
11	$\cos^{-1}[(7) \div (5)]$	–	17.35275	–	17.35275
12	OD roll, pinion, tan(10) × 57.29578	42.19507	–	42.19507	–
13	OD roll, gear, tan(11) × 57.29578	–	17.90353	–	17.90353
14	PD roll, tan(6) × 57.29578	26.71746	26.71746	26.71746	26.71746
15	Addendum roll, pinion, (12) − (14)	15.47761	–	15.47761	–
16	Addendum roll, gear, (14) − (13)	–	8.81393	–	8.81393
17	Pinion roll, (15) + $[(16)_{gear} \times (8)]$	29.18818	–	29.18818	–
18	Gear roll, (16) + $[(15)_{gear} \times (9)]$	–	18.76383	–	18.76383
19	LD roll, pinion, (12) − (17)	13.00690	–	13.00690	–
20	LD roll, gear, (13) + (18)	–	36.66736	–	36.66736
21	$\tan^{-1}[(19) \div 57.29578]$	12.79012	–	12.79012	–
22	LD, pinion, (7) ÷ cos(21)	50.18586	–	1.97582	–
23	$\tan^{-1}[(20) \div 57.29578]$	–	32.61786	–	32.61786
24	LD, gear, (7) ÷ cos(23)	–	90.38498	–	3.55846
25	Circular pitch, π × (2) ÷ (1)	9.42478	9.42478	0.37105	0.37105
26	Extra involute, 0.016 × (25)	0.15080	0.15080	0.00594	0.00594
27	Form diameter, pinion, (22) − (26)	50.03506	–	1.96988	–
28	$\cos^{-1}[(7) \div (27)]$	12.00580	–	12.00580	–
29	FD roll, pinion, tan(28) × 57.29578	12.18465	–	12.18465	–
30	FD, gear, (24) + (26)	–	90.53577	–	3.56440
31	$\cos^{-1}[(7) \div (3)]$	–	32.76668	–	32.76668
32	FD roll, gear, tan(31) × 57.29578	–	36.87747	–	36.87747
33	Roll per pinion tooth, 360° ÷ (1)	20	–	20	–
34	Contact ratio, (17) ÷ (33)	1.45941	–	1.45941	–

Note: Metric dimensions are in millimeters and degrees. English dimensions are in inches and degrees. See Section 5.1.9 for cases where (26) may be too much extra involute. FD: form diameter; LD: limit diameter; OD: outside diameter; PD: pitch diameter.

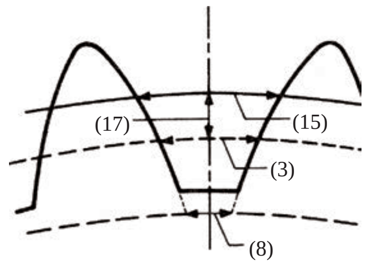
be danger of sludge and foreign material collecting and jamming against the outside diameter of the mating pinion. The design could be improved by using a large number of internal teeth or by using a pressure angle 1° or 2° smaller than 25°.

Figure B.17 shows a layout of the 18-tooth spur pinion intended to mate with the 28-tooth internal gear. The pinion layout looks good. A change in proportions, though, to improve the 28-tooth internal gear would require a change in the 18-tooth pinion also.

If the designer still wants to use 18 teeth meshing with 28 teeth, the first step is to change 28-tooth gear to get more room in the root region. The changed gear would, of course, require a changed pinion. The new pinion design should then be laid out to see how it looks.

TABLE B.8
Arc Tooth Thickness at Any Diameter, Internal Spur Gearsets

	Given Data	
1	Pitch diameter	84.00 mm
2	Pressure angle at (1)	25°
3	Arc tooth thickness at (1)	4.240 mm
Step A: Solve for the tooth thickness at base diameter.		
4	Involute (2)	0.029975
5	cos(2)	0.906308
6	(4) × (1)	2.51793
7	(3) − (6)	1.72207
8	TT at BD, (5) × (7)	1.56073
9	Base diameter, (5) × (1)	76.12985



Step B: Solve for a series of tooth thicknesses to study the tooth pro le. For any desired diameter, the cosine of the pressure angle at that diameter is

cos PA at any diameter = base diameter ÷ any diameter.

A series of pressure angles is assumed, based on diameters of interest. Then calculations are made to get the arc thicknesses that go with each diameter.

10	Assumed pressure angles	17.35276	21.8	28.9	32.2	34.4128
11	cos(10)	0.95449	0.9285	0.8755	0.8462	0.8250
12	inv (10)	0.00961	0.0195	0.0476	0.0677	0.0844
13	(12) × (9)	0.73184	1.4838	3.6261	5.1570	6.4274
14	(8) + (13)	2.29914	3.0445	5.1868	6.7177	7.9881
15	Arc TT, (14) ÷ (11)	2.4088	3.2790	5.9247	7.9388	9.6827
16	Any diameter, (9) ÷ (11)	79.760	81.993	86.960	89.968	92.280
17	Δ radius, [(16) − (1)] ÷ 2	−2.120	−1.004	1.480	2.984	4.140

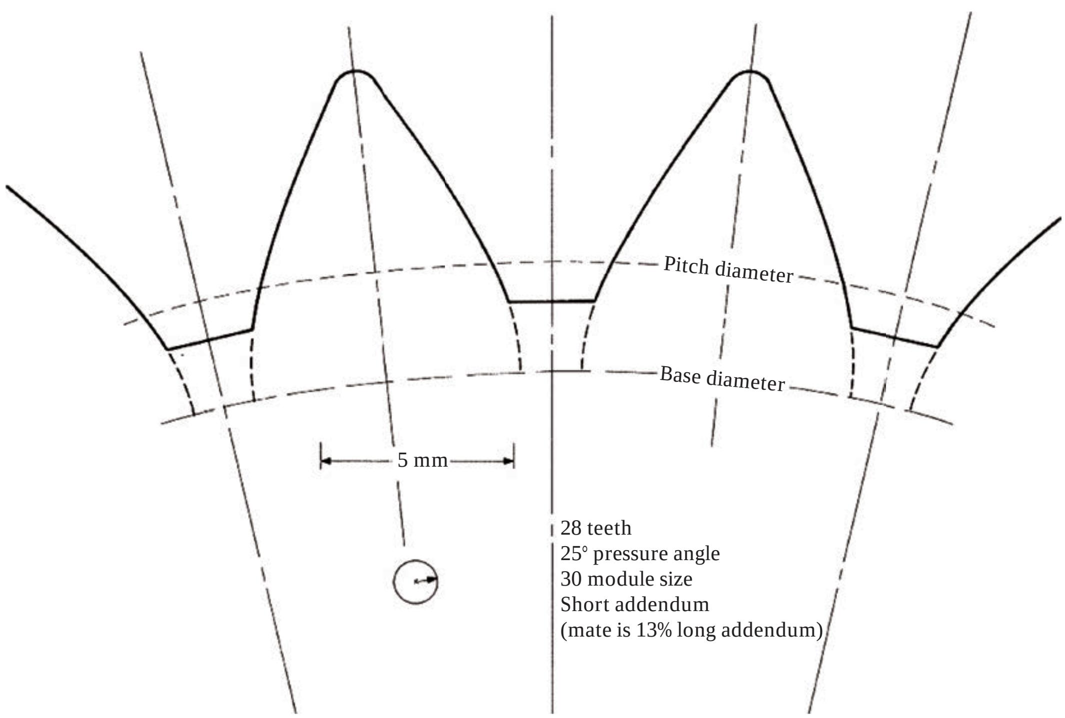


FIGURE B.16 Layout of 28-tooth internal gear.

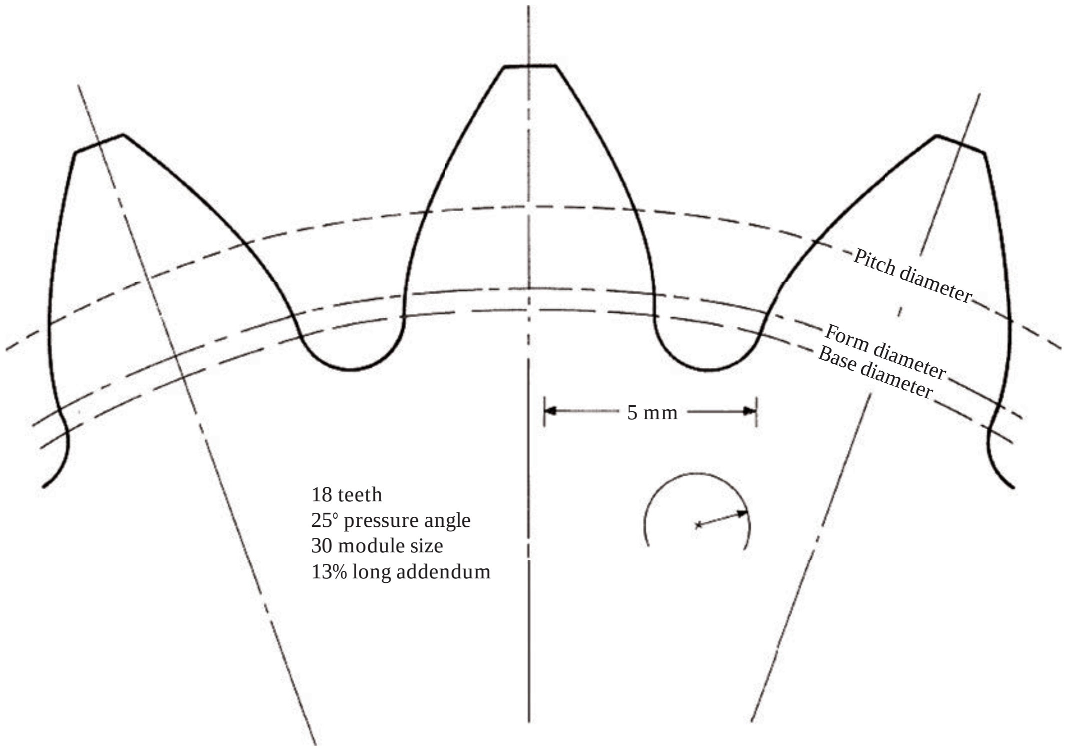


FIGURE B.17 Layout of 18-tooth external pinion.

The ratio of 18 to 28 is 1.556. Another way of solving the problem would be to use something like a 23-tooth pinion meshing with 36-tooth internal gear. This would give a ratio of 1.565, which is almost the same as 1.556. With larger numbers of teeth, the dif culty in the rst design would tend to disappear.

B.7 SPECIAL CALCULATIONS FOR HELICAL GEARS (SUPPLEMENT TO CHAPTER 5)

The tooth thicknesses of helical gears are often checked by measurement over balls or pins. Theoretically, the measurement over balls for helical gears can be calculated in a manner similar to that for the measurement over pins for spur gears. Table B.9 shows a calculation sheet for diameter over balls for a 25-tooth helical pinion and a 96-tooth helical gear. These are the same parts that are shown in Table 5.17.

When the number of teeth is odd, it is not possible to get the balls 180° apart and in the same transverse plane. However, if three balls are used, it is possible to have two balls each 180° apart from the third ball. This is done by putting two balls on one side, one axial pitch apart.

The possibilities just mentioned are all sketched in Table B.9, and the calculation procedure will handle the two different situations that are possible with an odd number of teeth.

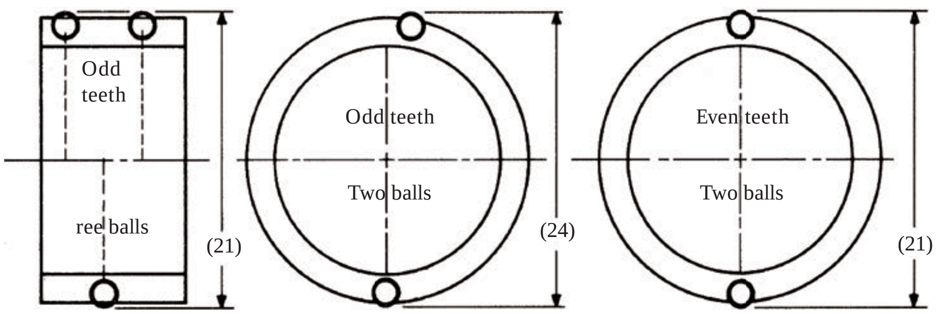
It is often practical to use pins in place of balls on helical teeth. There is a slight geometric error in calculating a diameter over balls and then using this dimension for a measurement of pins. In most helical designs, this discrepancy is so slight as to be of no consequence.

Unfortunately, a precise calculation for the measurement of a diameter over pins on helical gears is too complicated to be included in this book. The usual practice in the gear trade has been to make the calculations for balls, then start a product job with both balls and pins of the same diameter available. A few carefully taken measurements will reveal the difference (if any) between the diameter over pins and the diameter over balls for the same part. Once this difference is known, the part can, of course, be checked by diameter over pins, with an approximate allowance being made for the known difference in reading between pins and balls.

The helical gear can be considered as a gear with two sets of tooth data. One set of data goes with a normal section through the gear teeth. Things like pressure angle, circular pitch, and tooth thickness change in going from the normal section to the transverse section. The addendum and the whole depth are the same in either section.

TABLE B.9
Diameter over Balls for Helical Gears

		Pinion	Gear
1	Pitch diameter	77.6457	298.1595
2	Number of teeth	25	96
3	Pressure angle, transverse	20.64689	20.64689
4	Arc tooth thickness, transverse	4.7799	4.7799
5	Base helix angle	14.07610	14.07610
6	Base circle diameter, (1) × cos(3)	72.6586	279.0091
7	Diameter of ball	5.33401	5.33401
8	3.14159265 ÷ (2)	0.1256637	0.0327249
9	inv (3), [tan(3) − (3)] × 57.29578	0.0164534	0.0164534
10	(7) ÷ (6)	0.0734120	0.0191177
11	10 ÷ cos(5)	0.0756845	0.0197095
12	(4) ÷ (1)	0.0615604	0.0160314
13	(11) + (12) + (9) − (8)	0.0280346	0.0194694
14	Involute angle for (4), arc involute[a] (13)	24.47616	21.79269



		Pinion	Gear
15	Pressure angle, ball center	24.47616	21.79269
16	Involute (15)	0.0280343	0.0194693
17	(16) − (9)	0.0115809	0.0030159
18	(17) + (8) − (11)	0.0615601	0.0160313
19	Arc thickness, (1) × (18)	4.7799	4.7799
20	cos(15)	0.910134	0.928533
21	Diameter of balls, (6) ÷ (20) + (7)	–	305.8177
For odd teeth:			
22	cos[90° ÷ (2)]	0.9980267	–
23	[(6) ÷ (20)] × (22)	79.67533	–
24	Diameter over balls, (23) + (7)	85.0093	–

Note: The arc thickness in the transverse plane (4) is equal to the arc thickness in the normal plane divided by the cosine of the helix angle (helix angle at the pitch circle). The base helix angle (5) is determined from the helix of the pitch circle: tan(basic helix) = tan(pitch helix) × cos(pressure angle, transverse). Normally, (14) is used for (15) in the rst column; then (19) should be equal to (4). Somewhat different values of the pressure angle are entered into (15) for the remaining columns. This will give four values of tooth thickness versus diameter over balls. Item (2) is for even teeth. Item (24) can be used for odd teeth, but only if two balls are used in a transverse plane. If balls are used in an axial plane, then the even calculation must be used for odd teeth as well as even teeth.

[a] See Table C.12 for involute functions.

The calculations for form diameter, roll angle, and contact ratio are made using transverse section data, with a helical gear being treated as if it were a spur gear with an in nitesimally small face width (a wafer-thin spur gear). This means that the calculations just mentioned can be made for a helical gear in the same manner that they are made for a spur gear, provided transverse section data are used.

The pro le of a helical tooth is best judged in the normal section. It is conventional practice in the gear trade to make layouts of helical gear teeth in the normal section, using the technique of a virtual spur gear.

The virtual spur gear will have almost exactly the same tooth shape as the helical gear has in its normal section. Table B.10 shows how to get the needed dimensions of a virtual spur gear which will match the normal section of a helical gear.

The calculation procedure involves these steps:

1. Convert the normal section of the helical gear to a set of tooth data describing the virtual spur gear. Use the rules of Table B.10.
2. Make calculations for the involute part of the virtual spur gear just as if it were a regular spur gear.
3. Make calculations for the trochoid of the virtual spur gear just as if it were a regular spur gear. (The hob tooth data for a helical gear will match the normal section of the helical gear.)
4. For what happens when the helical pinion meshes with the helical gear, *do not* use the virtual-spur-gear concept. The involute helical gear is handled by calculations for the real helical gears and their transverse sections.

TABLE B.10
Virtual Spur Gear to Match Normal Section of a Helical Gear

Item	Helical Gear	Virtual Spur Gear
Number of teeth	Actual number of teeth	$\dfrac{\text{Helical no. teeth}}{\cos^3(\text{helix angle})}$
Pitch diameter	Actual pitch diameter[a]	$\dfrac{\text{Helical pitch diameter}}{\cos^2(\text{helix angle})}$
Tooth thickness	Actual normal-section tooth thickness[a]	Helical normal section tooth thickness
Addendum	Actual value[a]	Helical value
Whole depth	Actual value[a]	Helical value

[a] When there are both cutting and operating data on the drawing, use the operating data values for these items. (See text for a discussion of how operating data are used for virtual-helical-gear layouts.)

When a helical gear is cut (or ground) at a pitch diameter different from the operating pitch diameter, the layout of the virtual spur gears is made to match the normal section for operating conditions, not the normal section for cutting conditions. This can be rationalized by the fact that the addendum that meshes is the addendum base on the operating pitch diameter, not an addendum that is valid in cutting only.

Section B.13 has an example that shows how data are obtained for the normal section for operating conditions, as well as for the normal section for cutting.

B.8 SUMMARY SHEETS FOR BEVEL GEARS (SUPPLEMENT TO CHAPTER 5)

When bevel gears are to be manufactured on machines built by Gleason Works in Rochester, New York, United States, it is common practice to get a summary sheet of tooth data computed by Gleason Works. These sheets can be obtained at a small charge.

Table B.11 shows an example of a summary sheet for a straight bevel gearset of 16/49 teeth, 5 module. This is the same set shown in the straight bevel dimension sheet, Table 5.22.

The Gleason summary sheet is quite useful to the gear manufacturer. From a design standpoint, it gives all the tooth geometric data. These data will agree, of course, with the latest design practice recommended by Gleason. (Presumably, the designer will want to use the latest design practice.)

The summary sheet will be based on the kind of machine and the model of machine that is expected to be used by the gear manufacturer. (A builder who owns many kinds and models of Gleason machines will need to inform Gleason which machine or machines are to be used in the manufacture of the particular gearset.)

The summary sheet gives tool data and machine settings. This information is very helpful for the shop engineers who make the bevel gear processing plans.

The summary sheets will give some values useful in gear rating equations.

The values given may or may not t standard rating practice of the various bodies that issue rules on ratings. The designer should consider the rating values on the summary sheet as representing Gleason practice. (The later part of Chapter 5 covers gear rating practices and explains how a given gear contract may have speci c details that specify how the gear rating is to be completed.)

TABLE B.11
Dudley Engineering Company Summary Sheet for 16/49 Teeth, Straight Bevel Gearset

Straight Bevel Gear Dimensions No. M W004474

	Pinion	Gear
Number of teeth	16	49
Part number		
Module	5.000	
Fact width	40.00	40.00
Pressure angle	20°0′	
Shaft angle	90°0′	
Transverse contact ratio	1.517	
Outer cone distance	128.87	
Circular pitch	15.71	
Working depth	10.00	
Whole depth	10.99	10.99
Clearance	0.99	0.99
Pitch diameter	80.00	245.00
Addendum	7.05	2.95
Dedendum	3.94	8.05
Limit point width		
Limit point width—large end	0.130″	0.143″
Limit point width—small end	0.091″	0.100″
Stock allowance	0.026″	0.030″
Max. radius—cutter blades	0.081″	0.089″
Max. radius—mutilation	0.062″	0.069″
Max. radius—interference	0.048″	0.121″
Tool edge radius	0.025″	0.025″
Two Tool Generator		
Machine		
Cutter diameter		
Blade pressure angle		
Cutter depth designation		
Blade point width	0.065″	0.070″
Blade edge radius	0.025″	0.025″
Axial factor	Out 0.085	Out 0.085
Separating factor	Sep 0.260	Sep 0.028
Control Data		
Pattern length factor		
Pro le mismatch factor		

Note: All dimensions are in metric unless denoted otherwise. Angles are in degrees (°) and minutes (′). Coni ex is a trademark of Gleason Works.

Form C Date 8/3/81 Time 12:1

Data Sheet for Coniflex[a] Bevel Gears

	Pinion	Gear
Outside diameter	93.41	246.83
Pitch apex to crown	120.31	37.20
Circular thickness	9.514	6.194
Mean normal top land	2.27	3.43
Pitch angle	18°5′	71°55′
Face angle of blank	21°38′	73°39′
Root angle	16°21′	68°22′

(*Continued*)

TABLE B.11 (CONTINUED)
Dudley Engineering Company Summary Sheet for 16/49 Teeth, Straight Bevel Gearset

Form C Date 8/3/81 Time 12:1

Data Sheet for Coniflex[a] Bevel Gears

		Pinion		**Gear**
Dedendum angle		1°44′		3°33′
Chordal addendum		7.32		2.96
Chordal thickness		9.43		6.13
Backlash		Min. 0.13		Max. 0.18
Tooth proportions	Std			
Face in percent of cone dist				31.040
Undercut	No			
Geometry factor-strength—J		0.2373		0.1920
Strength factor—Q		5.74969		2.32077
Size factor—K_s		0.666		
K_i factor		1.3188		
Strength balance desired	Strs			
Strength balance obtained	Tpld			0.189
Geometry factor-durability—I		0.0756		
Durability factor—Z		3651.26		2086.44
Root line face width		40.00		40.00
Position load application	Hpt1			
Machine Setting—Two Tool Generator				
Sliding base		ADV 0.00		ADV 0.00
Top tool height		RISE 0.00		RISE 0.00
Bottom tool height		LOWR 0.00		LOWR 0.00
Tooth angle		2°45′		2°40′
Tool advance		0.05		0.05
Cradle test roll		20°0′		30°0′
Work test roll		64°26′		31°34′
Decimal ratio		0.6873		0.6873
Nc/75 ratio gears		47/45 × 50/76		47/45 × 50/76
Depth Checking Data—No. 15 Black Checker				
Checking diameter		72.52		240.01
Backing	Md−	123.72	Md−	47.65

[a] Factor given on this data sheet applies to Coniflex straight bevel gears with localized tooth bearing. For gears cut with nonlocalized tooth bearing the stresses will be increased by 20%–50%.

Table B.12 shows a summary sheet for spiral bevel gears. This sheet patches the spiral gear dimension sheet in Table 5.24.

Table B.13 shows a summary sheet for Zerol bevel gears. It matches the Table 5.25 dimension sheet for a set of Zerol bevel gears.

B.9 COMPLETE AGMA AND ISO FORMULAS FOR BENDING STRENGTH AND PITTING RESISTANCE (SUPPLEMENT TO CHAPTER 5)

During the 1970s, an international technical committee worked on gear rating formulas and developed a draft of standards for rating the strength and the pitting resistance of gear teeth running on parallel axes. The two standards first developed were intended to be mother standards for all types of applications. In the late 1970s and the early 1980s, this work was extended to develop product standards. (Some of the product areas were marine gears, aircraft gears, vehicle gears, and industrial gears.)

The committee doing this work was designated ISO/TC 60/WG 6. This designation means that the work was sponsored by the ISO. *WG 6* means "working group 6 of the general technical committee group 60."

The president (chairman) of WG 6 was Dr.-Ing. Hans Winter. The handling of processing of committee was assigned by ISO to DNA (Deutscher Normenausschuss). H. Schwartz has been the secretary of the WG 6 committee. His office has provided drafts of the standard and the minutes of meetings in several languages.

TABLE B.12
Dudley Engineering Company Summary Sheet for 16/49 Teeth, Spiral Bevel Gearset

Spiral Bevel Gear Dimension No. M S006651

	Pinion	Gear
Number of teeth	16	49
Part number		
Module		5.000
Face width	38.00	38.00
Pressure angle	20° 0′	
Shaft angle	90° 0′	
Transverse contact ratio		
Face contact ratio		1.192
Face contact ratio		2.006
Modified contact ratio		2.333
Outer cone distance		128.87
Mean cone distance		109.87
Pitch diameter	80.00	245.00
Circular pitch	15.71	
Working depth	8.28	
Whole depth	9.22	9.22
Clearance	0.94	0.94
Addendum	5.91	2.38
Dedendum	3.32	6.85
Outside diameter	91.23	246.48
Face angle junction diameter		
Theoretical cutter radius	3.751″	
Cutter radius	3.750″	
Calc. gear finish. pt. width		0.100″
Gear finishing point width		0.100″
Roughing point width	0.045″	0.080″
Outer slot width	0.071″	0.100″
Mean slot width	0.083″	0.100″
Inner slot width	0.073″	0.100″
Finishing cutter blade point	0.045″	0.100″
Stock allowance	0.026″	0.020″
Max. radius—cutter blades	0.043″	0.074″
Max. radius—mutilation	0.061″	0.076″
Max. radius—interference	0.045″	0.096″
Cutter edge radius	0.025″	0.025″
Calc. cutter number	3	9
Max. no. blades in cutter		11.295
Cutter blades required	Std depth	Std depth
Gear angular face—concave		26°47′
Gear angular face—convex		29°30′
Gear angular face—total		31°52′

Note: All dimensions are in metric unless denoted otherwise. Angles are in degrees (°) and minutes (′). Std, standard.

Form M Date 10/15/82 Time 16:41

	Pinion	Gear
Pitch apex to crown	120.67	37.74
Face ang junct to pitch apex		

(*Continued*)

TABLE B.12 (CONTINUED)
Dudley Engineering Company Summary Sheet for 16/49 Teeth, Spiral Bevel Gearset

Form M Date 10/15/82 Time 16:41

		Pinion	Gear
Mean circular thickness		8.13	5.04
Outer normal top land		2.18	2.28
Mean normal top land		2.25	2.66
Inner normal top land		2.41	2.36
Pitch angle		18°5′	71°55′
Face angle of blank		20°55′	73°3′
Inner face angle of blank			
Root angle		16°57′	69°5′
Dedendum angle		1°8′	2°50′
Outer spiral angle			42°22′
Mean spiral angle			35°0′
Inner spiral angle			28°14′
Hand of spiral		LH	RH
Driving member	Pin		
Direction of rotation driver	Rev		
Outer normal backlash		Min 0.13	Max 0.18
Depthwise tooth taper		Generated	
Gear type			29.488
Face in percent of cone dist			
Depth factor—K			
Addendum factor—C_1			
Geometry factor-strength—J		0.2483	0.2639
Strength factor—Q		5.78453	1.77692
Edge radius used in strength	v	0.025″	0.025″
Cutter radius factor—K_x		1.038	
Factor	Min	0.7856	
Strength balance desired	Givn		
Strength balance obtained	Givn		−0.055
Geometry factor-durability—I		0.1254	
Durability factor—Z		2932.37	1675.64
Geometry factor-scoring—G		0.004934	
Scoring factor—X		0.3052	
Root line face width		38.00	38.00
Profile sliding factor		0.00354	0.00537
Ratio of involute/mean cone		1.228	
Axial factor—driver cw		Out 0.598	Out 0.050
Axial factor—driver ccw		In 0.393	Out 0.156
Separating factor—driver cw		Sep 0.153	Sep 0.195
Separating factor—driver ccw		Sep 0.476	Att 0.128
Duplex sum of dedendum ang		3°58′	
Roughing radial		3.764″	
Input data	Cutm		

The gear rating standards developed by ISO/TC 60/WG 6 are generally referred to as *ISO Standards* in the gear trade. Although draft copies of the proposed ISO Standards have been available for study for several years, these standards are in limited use for international gear work. Some countries (outside the United States) have decided to use ISO Standards in their domestic work.

TABLE B.13
Dudley Engineering Company Summary Sheet for 16/49 Teeth, Zerol Bevel Gearset

Zerol Bevel Gear Dimensions No. M Z006652

	Pinion	Gear
Number of teeth	32	98
Part number		
Module		2.500
Face width	32.00	32.00
Pressure angle	20°0′	
Shaft angle	90°0′	
Transverse contact ratio		1.463
Face contact ratio		
Modi ed contact ratio		1.463
Outer cone distance		128.87
Mean cone distance		112.87
Pitch diameter	80.00	245.00
Circular pitch	7.85	
Working depth	5.00	
Whole depth	5.47	5.47
Clearance	0.47	0.47
Addendum	3.53	1.47
Dedendum	1.94	4.00
Outside diameter	86.71	245.91
Face angle junction diameter		
Theoretical cutter radius		
Cutter radius	3.000″	
Calc. gear nish. pt. width		0.070″
Gear nishing point width		0.070″
Roughing point width	0.035″	0.050″
Outer slot width	0.065″	0.070″
Mean slot width	0.070″	0.070″
Inner slot width	0.062″	0.070″
Finishing cutter blade point	0.035″	0.040″
Stock allowance	0.027″	0.020″
Max. radius—cutter blades	0.037″	0.046″
Max. radius—mutilation	0.063″	0.068″
Max. radius—interference	0.022″	0.035″
Cutter edge radius	0.015″	0.025″
Calc. cutter number	0	0
Max. no. blades in cutter		14.781
Cutter blades required	Std depth	Std depth
Gear angular face—concave		23°29′
Gear angular face—convex		25°2′
Gear angular face—total		24°21′

Note: All dimensions are in metric unless denoted otherwise. Angles are in degrees (°) and minutes (′). Zerol is a trademark of Gleason Works. STD, Standard.

Form M Date 10/15/82 Time 16:46

	Pinion	Gear
Pitch apex to crown	121.41	38.60
Face ang junct to pitch apex		

(*Continued*)

TABLE B.13 (CONTINUED)
Dudley Engineering Company Summary Sheet for 16/49 Teeth, Zerol Bevel Gearset

Form M Date 10/15/82 Time 16:46

		Pinion	Gear
Mean circular thickness		3.92	2.86
Outer normal top land		1.50	1.88
Mean normal top land		1.72	2.07
Inner normal top land		1.93	1.85
Pitch angle		18°5′	71°55′
Face angle of blank		21°22′	73°17′
Inner face angle of blank			
Root angle		16°43′	68°38′
Dedendum angle		1°22′	3°17′
Outer spiral angle			11°21′
Mean spiral angle			0°0′
Inner spiral angle			−13°8′
Hand of spiral		LH	RH
Driving member	Pin		
Direction of rotation-driver	Rev		
Outer normal backlash		Min 0.05	Max 0.10
Depthwise tooth taper	Dplx		
Gear type			Generated
Face in percent of cone dist			24.832
Depth factor—K			
Addendum factor—C_1			
Geometry factor strength—J		0.2363	0.2330
Strength factor—Q		12.138	4.01942
Edge radius used in strength		0.015″	0.025″
Cutter radius factor—K_x			
Factor	K_i	1.3666	
Strength balance desired	Givn		
Strength balance obtained	Givn		0.013
Geometry factor-durability—I		0.0785	
Durability factor—Z		4037.51	2307.15
Geometry factor-scoring—G		0.002801	
Scoring factor—X		0.2639	
Root line face width		32.00	32.00
Pro le sliding factor		0.00228	0.00348
Ratio of involute/mean cone		1.383	
Axial factor—driver cw		Out 0.082	Out 0.082
Axial factor—driver ccw		Out 0.082	Out 0.082
Separating factor—driver cw		Sep 0.251	Sep 0.027
Separating factor—driver ccw		Sep 0.251	Sep 0.027
Duplex sum of dedendum ang			
Roughing radial		5.361″	
Input data	Cutm		1

In 1983, the situation was as follows:

AGMA has just formally approved a new general standard for the rating of gears on parallel axes for both tooth strength and tooth surface durability. Its designation is AGMA 2001-D04 and AGMA 908-B89. The older general rating standards, AGMA 2001-D04 and 908-B89, and 6010-F97, are now considered obsolete (and replaced by 2001-D04 and 908-B89).

ISO rating work by WG 6 is continuing. Several product standards are being developed. The general standard, ISO/DIS 6336 I-IV, is considered to be nished, but it has not been accepted as the world standard.

The general AGMA and ISO gear rating equations may be compared as follows:

Total rating stress in bending

$$s_t = \frac{W_t K_a}{K_v} \times \frac{P_n}{F} \times K_m \times \frac{\cos\psi}{J} \quad \text{(AGMA)} \tag{B.4}$$

$$\sigma_F = \frac{F_t}{bm_n} Y_F Y_s Y_\beta K_A K_v K_{F\beta} K_{F\alpha} \quad \text{for load at HPSTC (ISO)} \tag{B.5}$$

$$\sigma_F = \frac{F_t}{bm_n} Y_{Fa} Y_{sa} Y_\varepsilon Y_\beta K_A K_v K_{F\beta} K_{F\alpha} \tag{B.6}$$

for load at tip of tooth (ISO).

Total rating stress for surface durability

$$s_c = C_p \sqrt{\frac{W_t}{dF} C_a \frac{1}{I} \frac{C_m}{C_v}} \quad \text{(AGMA)} \tag{B.7}$$

$$\sigma_H = Z_E \sqrt{F_t K_A \frac{1}{d_1 b} \frac{u+1}{u} K_v K_{H\alpha} K_{H\beta}} (Z_H Z_\beta Z_\varepsilon). \quad \text{(ISO)} \tag{B.8}$$

Word de nitions of all terms in these equations are given in Table B.14. The AGMA numerical de nitions of the terms used are given (for the most part) in Chapter 5. The ISO numerical de nitions are too lengthy to be included in this book. The reader who wishes to study ISO-proposed standards should obtain original copies from DIN. (They are available in English, German, and French.)

The equations just given do not show size factors like K_s or C_s. Since the size factor is often handled by using a value of 1.0 (for small to medium gears), it is possible to compare the general equations without getting into the uncertainty of what size factor should be used.

TABLE B.14
AGMA and ISO Nomenclature Used in Rating Calculations

AGMA		ISO	
	Bending Stress Calculation		
s_t	Calculated tensile stress at root of tooth (psi)	σ_F	Calculated tensile stress at root of tooth (N/mm^2)
W_t	Transmitted tangential load at operating pitch diameter (lb)	F_t	Transmitted tangential load at operating pitch diameter (N)
K_a	Application factor	K_A	Application factor
F	Contacting face width (in.)	b	Contacting face width (mm)
P_n	Normal diametral pitch	m_n	Normal module (mm)
J	Bending geometry factor	Y_F	Tooth form factor (HPSTC)
K_v	Dynamic factor (derates by being less than 1.0)	K_v	Dynamic factor (derates by being over than 1.0)
ψ	Helix angle	Y_β	Helix angle factor
K_m	Load distribution factor	Y_S	Stress concentration factor (HPSTC)
		Y_{Sa}	Stress concentration factor (tip loading)
		Y_{Fa}	Tooth form factor (tip loading)
		Y_ε	Contact ratio factor
		$K_{F\beta}$	Longitudinal load distribution factor for bending stress
		$K_{F\varepsilon}$	Transverse load distribution factor for bending stress
	Contact Stress Calculation		
s_c	Calculated contact stress (psi)	σ_H	Calculated contact stress (N/mm^2)
C_p	Coef cient for elastic properties of materials used	Z_E	Coef cient for elastic properties of materials used (N/mm^2)$^{0.5}$
W_t	Transmitted tangential load at operating pitch diameter (lb)	F_t	Tangential load (N)
C_a	Application factor	K_A	Application factor
F	Contacting face width (in.)	b	Contacting face width (mm)
d	Pinion operating pitch diameter (in.)	d_1	Pinion pitch diameter (mm)
I	Durability geometry factor	$Z_H Z_\beta Z_\varepsilon$	Total durability geometry factor
C_v	Dynamic factor for durability (derates by being less than 1.0)	K_v	Dynamic factor (derates by being over than 1.0)
C_m	Durability load distribution factor	$K_{F\beta}$	Longitudinal load distribution factor, for pitting
		$K_{H\alpha}$	Transverse load distribution factor, for pitting
		u	(No. of gear teeth)/(No. of pinion teeth)

The AGMA and ISO systems do not calculate the same stress values for like gears under the same loading. Also, the allowable stress values for a de ned material in each system are not the same. This makes it somewhat dif cult to compare rating results by the two systems. The engineer has to consider

- Which system more correctly calculates the true total stress, and
- Which system best knows the real ability of a given steel (or other material) to withstand tensile stress or compressive stress in a gear tooth.

One of the best ways to compare the two systems is to look at the load intensity for gear units in service. True comparisons can be made by calculating the unit load and the *K*-factor. These indices of tooth loading are calculated in exactly the same way by AGMA and ISO. The calculation is simple, and all variables are directly measurable dimensions or forces.

An excellent comparative study of 54 gear designs was made by Imwalle et al. (1980) of Cincinnati Gear Co., Cincinnati, Ohio, United States. Their results were published in a 1980 ASME paper given at the International Power Transmission & Gearing Conference in San Francisco, California, United States. This paper shows in considerable detail how to make comparative calculations by AGMA and ISO methods. Calculation results in terms of stress, derating factors, and hardness of steel are compared.

At the same ASME meeting, an excellent analytical study of AGMA and ISO methods for calculating tooth strength was given by Castellani and Castelli (1980). (Castellani is a gear consultant in Modena, Italy, and Castelli is a professor at the University of Bologna, Italy.)

The papers just mentioned, plus other studies made since 1980, show that there are unresolved differences in rating by the two methods. The most serious problems have to do with gear strength rating. More worldwide gear research and more follow-up evaluation of gears in service are needed to establish better formulas and data on gear material capabilities. Both AGMA and ISO are continuing their work on gear rating. New or revised rating standards can be expected—at least through the 1980s.

Figure B.18 shows a comparison of unit load allowable for a substantial number of gear designs made by Imwalle and Labath (1981). These data were presented and discussed at the IFToMM Gear Committee meeting in Eindhoven, Holland, in June 1982.

The allowable unit load is obtained by rst calculating a power rating using AGMA or ISO calculating methods. Then the allowed power rating is used to calculate the unit load and *K*-factor indices of tooth loading intensity.

Figure B.18 shows these comparisons for unit load:

- For fully hard helical gears, ISO would allow more load than AGMA 2001-D04 and 908-B89. (AGMA 2001-D04 and 908-B89 would allow more than the obsolete AGMA 225.)
- For medium-hard helical gears, ISO would again allow more load than AGMA 2001-D04 and 908-B89.
- For fully hard spur gears, ISO would allow more load that AGMA 2001-D04 and 908-B89.
- Spur and helical gears are somewhat closer to the same rating in ISO than in AGMA.
- The allowable ratings of Figure B.18 are too high for long-life gears of high reliability. (See Chapter 5 for effects of grades of material, levels of reliability, and numbers of cycles like 10^9 or more.)

In a similar fashion, Figure B.19 shows allowable *K*-factors for the same group of designs studied by Imwalle and Labath (1981).

Figure B.19 shows these comparisons:

- For fully hard helical gears, ISO and AGMA 2001-D04 and 908-B89 are close to the same level of loading. (AGMA 2001-D04 and 908-B89 will allow more load than the obsolete 225.)

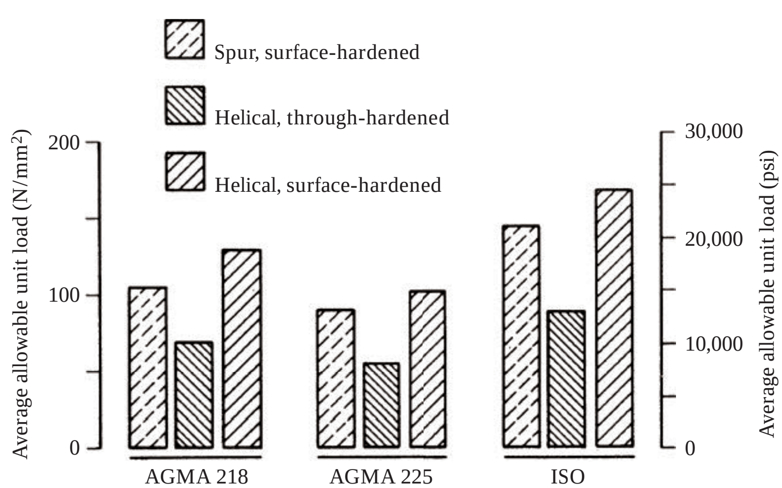


FIGURE B.18 Average allowable unit load, AGMA and ISO. (Courtesy of the Cincinnati Gear Co., Cincinnati, Ohio.)

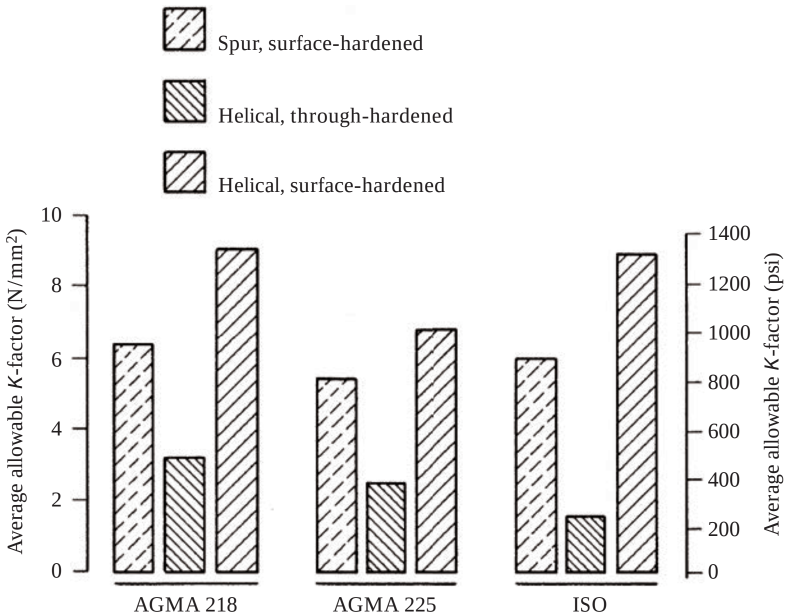


FIGURE B.19 Average allowable *K*-factor, AGMA and ISO. (Courtesy of the Cincinnati Gear Co., Cincinnati, Ohio.)

- For medium-hard helical gears, ISO would allow less load than AGMA 2001-D04 and 908-B89—or even less than the old AGMA 908-B89.
- For fully hard spur gears, the allowed ratings are close to the same value. ISO is slightly less than AGMA 2001-D04 and 908-B89.
- The allowable ratings of Figure B.19 are too high for long-life gears of high reliability. (See Chapter 5 for effects of grades of material, regimes of lubrication, levels of reliability, and numbers of cycles like 10^9 or more.)

The comparisons of allowable unit loads and *K*-factors for AGMA and ISO should be considered as generalizations developed by Imwalle and Labath in their published study. Dr. Hans Winter, chairman of the ISO/TC 60/WC 6 committee, has reminded that recommended safety factors vary between AGMA and ISO, and the hardness of a through-hardened gear may be somewhat different. Exact comparisons can only be made by studying all the details in individual cases.

B.10 PROFILE MODIFICATION CALCULATION PROCEDURE (SUPPLEMENT TO CHAPTER 5)

Pro le modi cations are generally made to reduce the risk of tooth scoring. Section 5.2.8 gives general information on how to design gears that may have a scoring hazard. Table 5.54 shows a general guide for depth of the pro le modi cation.

The best procedure is to put pro le modi cation at the tip of the pinion ant at the tip of the gear. The involute has less curvature here, and it has more length for an increment of roll angle.

In some cases, a gear builder will own a pinion-grinding machine capable of making modi ed involute pro les, but the larger machine that will grind the gear is not equipped to grind a modi ed involute. What can be done?

The pinion can be made with a modi cation at the tip and a second modi cation near the root llet. This is reasonably practical for pinions with large enough numbers of teeth to keep the form diameter a few degrees of roll away from the base circle. (The base circle is at 0° roll.)

The calculation procedure for this kind of design involves locating a diameter on the pinion that *matches* the diameter at which the involute pro le modi cation might have started on the gear.

Table B.15 is a calculation sheet that can be used to get a diameter on the pinion matching a diameter on the gear.

The calculation procedure is as follows:

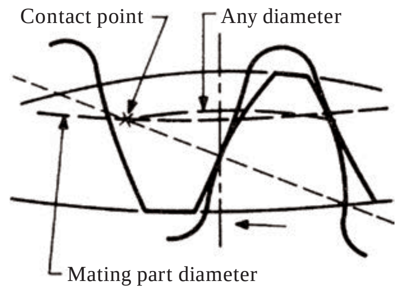
- Determine the diameter at which the pro le modi cation might have started on the gear. Enter these values as any diameter in (5).
- Fill in the column for the gear number of teeth and the gear pitch diameter.
- Show the number of pinion teeth in (2) for the mating part.
- Item (19) is the pinion diameter for start of modi cation—in the dedendum.

The numerical values in column 1 of Table B.15 are for the sample problem on scoring in Section 10.2. If the gear modi cation had been put on the lower ank of the 26-tooth pinion, the theoretical pinion pro le would have these values in the metric system (see Table B.16).

TABLE B.15
Assume Any Diameter on a Gear and Calculate the Matching Diameter on the Mating Gear

Basic Data (for Information Only)

Pinion addendum, 22 mm
Gear addendum, 18 mm
The part chosen for calculation may be either the pinion or the gear. In metric system use millimeters; in English system use inches.



	Location	At Start of Modification		At Outside Diameter	
	Column	**1**	**2**	**3**	**4**
1	Number of teeth	51		26	51
2	Number of teeth in mate	26		51	26
3	Pressure angle, transverse	20.410311		20.410311	20.410311
4	Pitch diameter	1042.787		531.617	1042.787
5	Any diameter	1060.808		575.620	1078.790
6	cos(3) × (4)	977.3200		498.2417	977.3200
7	(6) ÷ (5)	0.921298		0.865574	0.905941
8	$\cos^{-1}$(7)	22.88345		30.05169	25.04969
9	tan(8) × 57.29578	24.18318		33.14869	26.77798
10	tan(3) × 57.29578	21.31985		21.31985	21.31985
11	(9) − (10)	2.86333		11.82884	5.45813
12	(1) ÷ (2)	1.961538		0.509804	1.961538
13	(11) × (12)	5.61654		6.03039	10.70633
14	(10) − (13)	15.70331		15.28946	10.61352
15	(14) ÷ 57.29578	0.274074		0.266851	0.185241
16	$\tan^{-1}$(15)	15.32694		14.941299	10.49456
17	cos(16)	0.964433		0.966190	0.983272
18	(6) ÷ (12)	498.2417		977.3201	498.2417
19	(18) ÷ (17)	516.61606		1011.5196	506.71797

Note: Item (19) is the diameter on the mate that contacts with the diameter on the part chosen for item (5). If item (5) is the outside diameter of a part, item (19) is the limit diameter of the mating part. (See columns 3 and 4.) Item (9) is the roll angle of the part to the contact point. Item (14) is the roll angle of the mating part to the contact point. Column 1 is an example showing how to find a start modification diameter on a pinion to match the diameter that might have been used for a modification on the gear.

TABLE B.16
Modification in the Lower Flank of a 26-Tooth Gear (in Metric System)

Location			
Name	**Diameter**	**Degrees Roll**	**Involute Basic Value**
Limit diameter	506.72	10.49	−0.101
End modification	516.62	15.70	0.00
Pitch diameter	531.62	26.778	0.00
Start modification	559.61	29.30	0.00
Outside diameter	575.62	33.15	−0.063

With both modifications on the pinion, the gear profile would be a theoretical true involute from the limit diameter to the tip radius at the outside diameter.

B.11 THE BASICS OF GEAR-TOOTH MEASUREMENT FOR ACCURACY AND SIZE (SUPPLEMENT TO CHAPTER 10)

In cartoon style, Figure B.20 shows the principal kinds of checks used to determine the geometric accuracy and the size of gear teeth. Section 10.4.1 has already discussed the accuracy of gears needed for different kinds of applications.

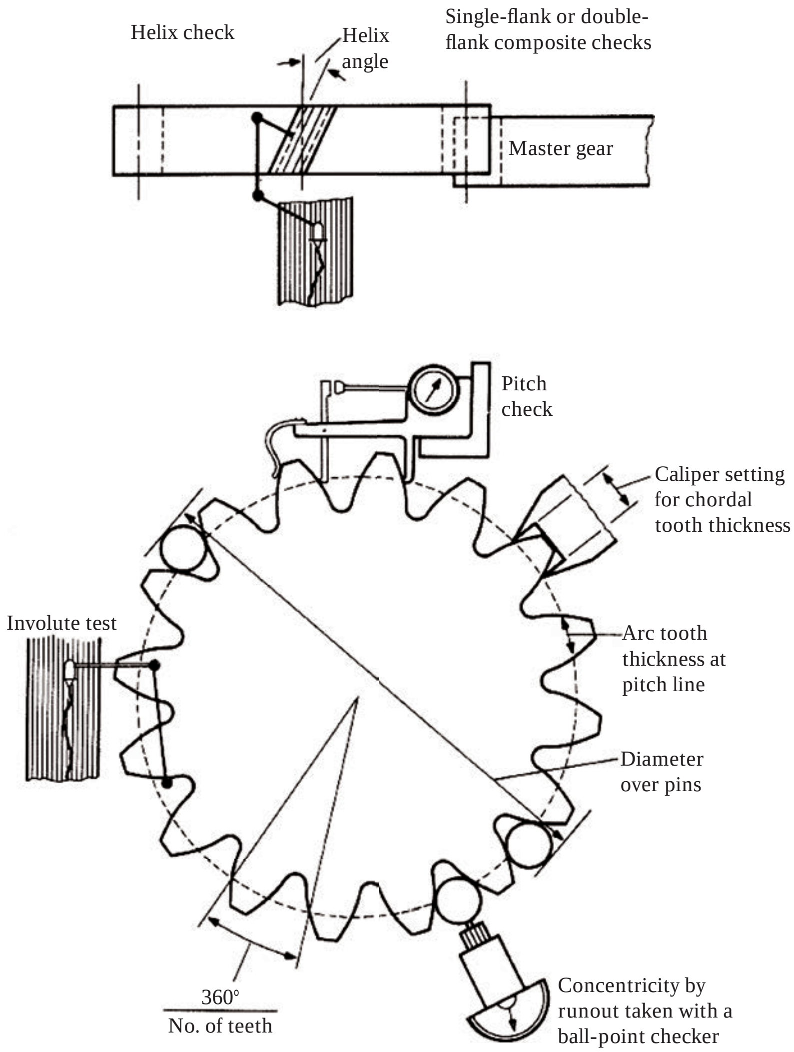


FIGURE B.20 Schematic of different gear tooth checks.

Figure 10.50 de ned gear tooth tolerances. Some discussion is needed to clarify how the checks are taken.

In Figure B.20, the upper view on the schematic gear shows a tooth-to-tooth basic checking method. The checking device is set to read zero at the approximate circular pitch of the gear being checked. The rst reading might be from the working side of tooth 1 to the working side of tooth 2. The second reading would then be taken from tooth 2 to tooth 3 and the third reading from tooth 3 to tooth 4. In a similar manner, readings are taken all the way around the gear.

The value that is needed is the difference in readings, not the readings themselves. The difference is obtained by subtracting the 1–2 reading from 2–3 reading. The second difference is obtained by subtracting the 2–3 reading from the 3–4 reading.

In the manner just described, as many differences are obtained as there are teeth on the gear. The drawing limit is the maximum difference. If any of the differences in the gear circumference exceeds the drawing maximum, then the gear does not meet the drawing speci cation.

The cumulative error is directly obtained by setting an angle equal to 360° divided by the number of teeth. This method is shown in the lower part of Figure B.20.

At the rst tooth, zero is set on the tooth and a reading is taken on the second tooth with an angular setting of 360° divided by the number of teeth. For the third tooth, the angular setting is doubled, and for the fourth tooth, the angular setting is tripled. In this manner, the out-of-position reading for all teeth is determined with respect to a starting point on one tooth.

In the gear trade, cumulative error is often called *accumulated* error, *index* error, or *out-of-position* error. This error does not compare adjacent teeth. The drawing limit is the greatest amount any one tooth is out of position with respect to any other tooth.

The two teeth that determine the cumulative error are often close to 180° apart. In an extreme case, two adjacent teeth could be so much in error that they set the maximum cumulative error. (The reader should carefully note that it takes three adjacent teeth to set a maximum tooth-to-tooth spacing error, while the distance setting a maximum cumulative error can

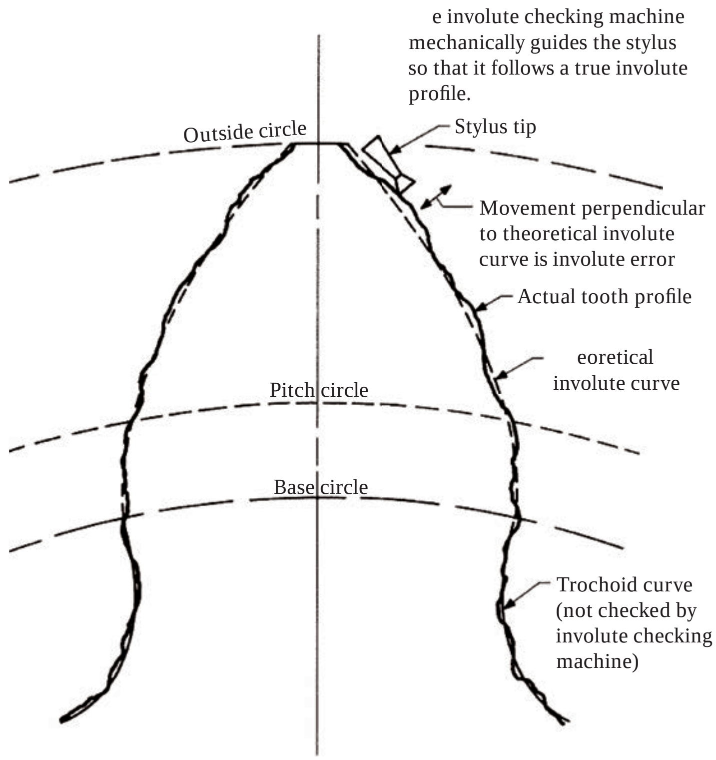


FIGURE B.21 Details of involute check.

be from two teeth to one-half the total number of teeth in the part.)

The involute check is taken by a stylus that traverses the involute pro le. Note the Figure B.20 schematic, and the details of Figure B.21. (Involute variations in Figure B.21 are exaggerated.) Involute checks measure slope, irregularity, and waviness, and determine whether or not a speci ed pro le modi cation was obtained. Note in particular how involute pro les are speci ed by a K-chart in Figure 7.50.

The surface nish may be evaluated in an involute check if the machine will read deviations at 2000× magni cation. For involute accuracy, ×500 or ×1000 magni cation is normally used for high-accuracy gears. For medium accuracy gears, ×250 magni cation may be suitable.

The helix is checked by a stylus moving lengthwise across the tooth. The stylus moves in a helical spiral. If the tooth is spur (0° helix), the stylus moves in a straight line. See upper view of Figure B.20.

The concentricity of a gear may be determined using a ball-point checker or by a double- ank check with a master gear; see upper and lower views of Figure B.20.

Production gears are generally checked by rolling with a master gear. In a double- ank check, the master gear is spring loaded to stay in the tight mesh with the gear being checked. In a single- ank check, the master gear and the production gear roll in the mesh, with only one side of the teeth contacting. A timing mechanism determines the transmission error. (See Section 10.4.2 for more information on composite checking machines.)

The tooth thickness of a gear may be directly measured with calipers, or it may be indirectly determined by diameter pins. The sizing of gears may be controlled by double- ank composite checks and center-distance settings corresponding to maximum and minimum tooth thickness speci cations.

Figure B.22 shows locations for involute and helix checks. Normally, four teeth 90° apart are checked. This method nds problems with the teeth on a gear not being alike and any serious effect on involute and helix due to a part being eccentric.

In some cases, the ends of teeth may have extra involute errors, or the helix at the tooth tip or root may be worse than at the pitch line. Figure B.22 shows how a grid of checks may be used to more fully explore a gear tooth. (Composite checks with a master gear often nd local bad spots by contacting the whole tooth surface.)

Figure B.23 shows an example of a double- ank composite check using a master gear. Note that the maximum tooth-to-tooth composite reading is found and compared with the design speci cation. Also note that a runout reading is not usually as large as the total composite variation.

Figure B.24 shows a single- ank composite check. This check shows a transmission error in a circumferential direction, not runout-type errors in a radial direction. The cumulative error is usually less than the total composite variation.

Special sources of information give a considerable amount of additional information on gear checking and types of equipment being used for these purposes. The reader who needs more information than this book can provide should refer to special sources on gear checking.

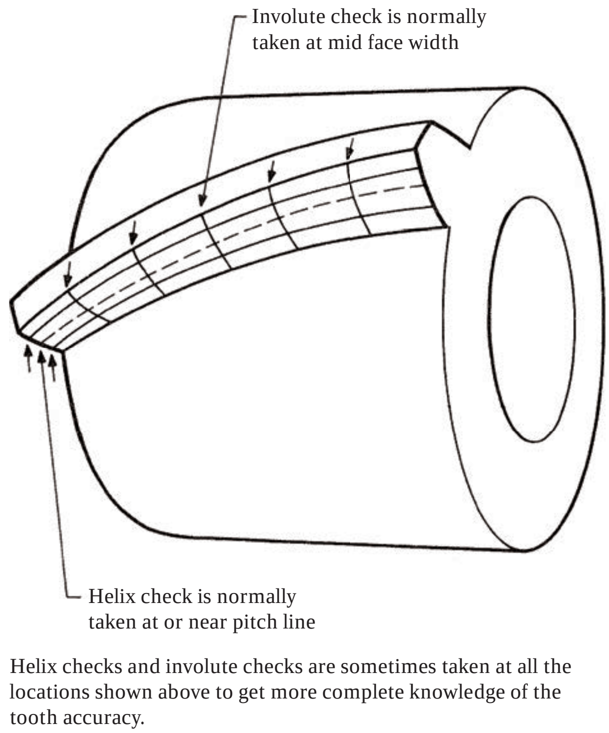


FIGURE B.22 Location for involute and helix checks.

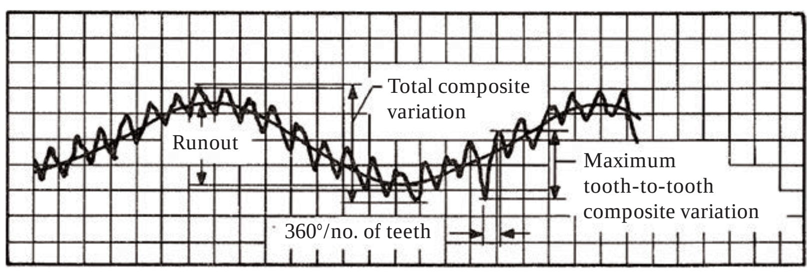


FIGURE B.23 Double- ank composite check.

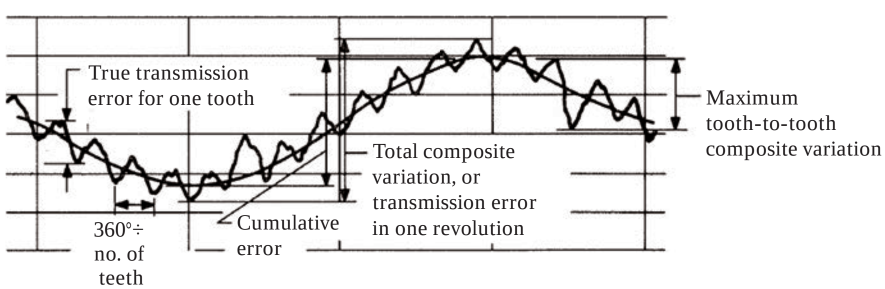
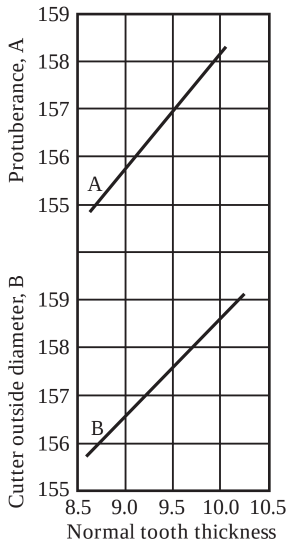


FIGURE B.24 Single- ank composite check.

B.12 SHAPER-CUTTER TOOTH THICKNESS (SUPPLEMENT TO CHAPTER 11)

Figure B.25 shows an example of a critical shaper-cutter design. This is a preshave cutter designed to produce the deepest point of undercut at a 360.8324 mm diameter on the gear. The cutter is designed to hold a constant root diameter of 362.255 mm. Curves *A* and *B* show how the cutter Hi-point of protuberance and the cutter outside diameter must change as the tooth thickness of the cutter is reduced because of sharpening. When the locations of the pro le changes are measured by a conventional involute checker, the diameters in the chart are converted to degrees of the gear. (Table B.3 shows how to calculate roll angles.)



Item		Cutter	Sample
Number of teeth		23	55
Module		6.35	6.35
Pitch diameter		146.05	349.25
Addendum		6.5024	5.080
Whole depth		–	11.5824
Normal pressure angle		25°	25°
Normal circular pitch		19.949	19.949
Depth B and S calipers		6.68	5.03
Chordal	Tooth	10.109	–
thickness	space	–	10.1854–10.287
Helix angle		0°	0°
Pressure angle		25°	25°

FIGURE B.25 Shaper-cutter design to hold xed major diameter on internal part.

Table B.17 shows the calculation sheet that was used to design the shaper-cutter in Figure B.25. In this case, the gear designer wanted the deepest point of undercut to be about midway between the end of active involute (form diameter) and the root diameter.

The calculations were made by assuming a series of operating pressure angles. A wide enough range of angles must be used to get curves that will cover the change in tooth thickness during the life of the cutter. Roughly, the change in cutter thickness is 0.060 mm/mm of cutter width.

At some time during the life of the cutter, the operating pressure angle is usually the same as the design pressure angle of the gear. When this happens, the cutter is rolling with the gear on the design pitch line of the gear. The cutter designer may choose to design the cutter so that the operating pressure angle runs either lower or higher than theoretical. If the cutter has a small number of teeth, it may be necessary to keep the operating pressure angle high to avoid undercut on the cutter. The smallest llet on the gear is produced when the operating pressure angle of the cutter is close to the pressure angle at the root of the gear tooth.

The equations for the relation of cutter tooth thickness to outside diameter are as follows: Equations B.10 and B.11 were used in Table B.17.

External gear cutters

$$s_0 = (p - s) - 2a\,(\text{inv}\,\alpha_t - \text{inv}\,\alpha_0) \quad \text{(metric)} \qquad \text{(B.9)}$$

$$t_c = (p - T) - 2C\,(\text{inv}\,\phi_t - \text{inv}\,\phi_c) \quad \text{(English)} \qquad \text{(B.10)}$$

$$d_{a0} = 2a\left(\frac{\cos\alpha_t}{\cos\alpha_0}\right) - d_r \quad \text{(metric)} \qquad \text{(B.11)}$$

$$d_{oc} = 2C\left(\frac{\cos\phi_t}{\cos\phi_0}\right) - D_R \quad \text{(English).} \qquad \text{(B.12)}$$

Internal gears

$$s_0 = (p - s) + 2a(\text{inv}\,\alpha_t - \text{inv}\,\alpha_0) \quad \text{(metric)} \qquad \text{(B.13)}$$

$$t_c = (p - T) + 2C(\text{inv}\,\phi_t - \text{inv}\,\phi_c) \quad \text{(English)} \qquad \text{(B.14)}$$

$$d_{a0} = d_r - 2a\left(\frac{\cos\alpha_t}{\cos\alpha_0}\right) \quad \text{(metric)} \qquad \text{(B.15)}$$

$$d_{oc} = D_R - 2C\left(\frac{\cos\phi_t}{\cos\phi_0}\right) \quad \text{(English)} \qquad \text{(B.16)}$$

where

s_0 or t_c—cutter tooth thickness at pitch line (mm or in.)
p—circular pitch of gear (mm or in.)
s or T—tooth thickness of gear at design pitch line (mm or in.)
a or C—design center distance (mm or in.)

$$\frac{d_{p0} + d_p}{2} \quad \text{(metric), or} \quad \frac{D + d_c}{2} \quad \text{(English, external gear)}$$

$$\frac{d_p - d_{p0}}{2} \quad \text{(metric), or} \quad \frac{d_c - D}{2} \quad \text{(English, internal gear)}$$

d_{a0} or d_{oc}—outside diameter of cutter (mm or in.)
α_t or ϕ_t—transverse pressure angle of gear (°)
α_0 or ϕ_c—operating pressure angle of cutter (°)
d_p or D—pitch diameter of gear (mm or in.)
d_{p0} or d_c—design pitch diameter of cutter (mm or in.)
d_r or D_R—root diameter of gear (mm or in.)

When some special features—such as a protuberance or a chamfering bevel—are needed on a tool, it is desired to calculate the location of the point on the cutter that will produce this feature on the gear. This permits the cutter pro le to be checked any time during its life to see if the cutter is properly

TABLE B.17
Calculation of Design Data for Internal Gear Shaper-Cutter

	Cutter Data				
1	Number of teeth	23			
2	Pitch diameter	146.05			
3	Transverse pressure angle	25°			
	Gear Data				
4	Number of teeth	55			
5	Pitch diameter	349.25			
6	Inside diameter	339.09			
7	Root diameter	362.25			
8	Form diameter	359.41			
9	Undercut diameter	360.83			
10	Helix angle	0°			
11	Arc tooth thickness	9.7384			
	Calculations				
12	(5) × π ÷ (4)	19.949			
13	(5) − (2)	203.20			
14	(5) × cos(3)	316.528			
Calculation of cutter outside diameter (22) versus cutter tooth thickness at pitch line (20)					
15	Assume operating pressure angle	25.00°	25.50°	26.00°	26.50°
16	cos(3) ÷ cos(15)	1.000	1.004124	1.008360	1.012709
17	inv[a] (3) − inv (15)	0	−0.001942	−0.003972	−0.006094
18	(12) − (11)	10.2107			
19	(13) × (17)	0	−0.39446	−80704	−1.23830
20	(18) + (19)	10.2107	9.81626	9.40368	8.97241
21	(13) × (16)	203.20	204.04	204.90	205.78
22	(7) − (21)	159.05	158.21	157.35	156.47
Calculation of diameter on cutter (3) to produce maximum undercut on gear at (9)					
23	(14) ÷ (9)	0.87722			
24	$\cos^{-1}$(23)	28.69096			
25	(24) − (15)	3.69096	3.19096	2.69096	2.19096
26	sin(25)	0.064375	0.055664	0.046949	0.038230
27	cos(25)	0.997926	0.998450	0.998897	0.999269
28	x = (9) × (26)	23.22836	20.08521	16.94053	13.79457
29	y = (9) × (27) − (21)	156.882	156.233	155.533	154.784
30	$(28)^2 + (29)^2$	25,151.4	24,812.0	24,477.6	24,148.3
31	$(30)^{0.5}$	158.5919	157.5183	156.4533	155.3973

Note: For calculation of diameter on cutter to correspond to a particular diameter on the gear, substitute the particular diameter for (9), and repeat steps (23) through (31). Linear dimensions shown earlier are all in millimeters, but the calculation procedure is the same when all dimensions are in inches.

[a] See Table C.12 for de nitions of involute function.

formed to produce the gear. This system may even be used to check and see how close a cutter will come to producing a gear it was not designed to cut.

If a point is taken on the cutter instead of on the gear, the calculation may be worked in reverse to see what the cutter does to the gear. For instance, one might take the cutter outside diameter as a starting point and solve for the diameter of a point on the gear that was cut by this point on the cutter. Such a calculation would give the exact location of the end of the involute pro le on the gear (which is also the point at which the root llet starts).

The equations to calculate from a point on one member to a point on the other member are as follows:

External gear cutter (or any two external gears with axes parallel)

$$\cos\alpha' = \frac{d_p}{d'_p}\cos\alpha \quad \text{(metric)} \tag{B.17}$$

$$\cos\phi' = \frac{D}{D'}\cos\phi_t \quad \text{(English)} \tag{B.18}$$

$$x = d'_p \sin(\alpha' - \alpha_0) \quad \text{(metric)} \qquad \text{(B.19)}$$

$$x = D' \sin(\phi' - \phi_c) \quad \text{(English)} \qquad \text{(B.20)}$$

$$y = 2a\left(\frac{\cos\alpha_t}{\cos\alpha_0}\right) - d'_p \cos(\alpha' - \alpha_0) \quad \text{(metric)} \qquad \text{(B.21)}$$

$$y = 2C\left(\frac{\cos\phi_t}{\cos\phi_c}\right) - D' \cos(\phi' - \phi_c) \quad \text{(English)} \qquad \text{(B.22)}$$

$$d' = \sqrt{x^2 + y^2} \quad \text{(metric or English)} \qquad \text{(B.23)}$$

Internal gear cutter (or internal gearset)

$$y = d'_p \cos(\alpha' - \alpha_0) - 2a\left(\frac{\cos\alpha_t}{\cos\alpha_0}\right) \quad \text{(metric)} \qquad \text{(B.24)}$$

$$y = D' \cos(\phi' - \phi_c) - 2C\left(\frac{\cos\phi_t}{\cos\phi_c}\right) \quad \text{(English)} \qquad \text{(B.25)}$$

$$d' = \sqrt{x^2 + y^2} \quad \text{(metric or English)} \qquad \text{(B.26)}$$

where

d'_p or D'—diameter to be cut on gear (or point on either member at which calculation starts)

α' or ϕ'—pressure angle at d'_p or D'

α_0 or ϕ_c—operating pressure angle of cutter

x, y—rectangular coordinates of point on cutter

d'—diameter through point on tool which cuts point on gear (or point on second member corresponding to starting point on first member)

Note that calculations on shaper-cutters are carried out just as if the shaper-cutter were an involute spur pinion. No account is taken of the fact that the cutter may have a top face angle of 5° or 10°. Since finishing cutters usually have 5° angle, this feature cannot be ignored. The actual cutting is done by a tooth profile which is equivalent to a projection of the cutter tooth. This projection is just the same as a section through an equivalent spur pinion. For this reason, calculations can be made on the basis of an equivalent pinion.

If an involute shaper-cutter is checked back of the cutting edge, in a plane perpendicular to the cutter axis, it is necessary to take into account of the top face angle. This can be done by making a small reduction in the base-circle diameter setting that is used to check the cutter. Usually, cutter drawings allow the tool manufacturer to stamp the cutter with a base-circle diameter that allows for the top face angle.

The calculations just described may be used for helical as well as spur gears. In calculating a helical job, it is only necessary to keep all dimensions in the transverse plane. The tooth action in this plane is the same for either spur or helical shaper-cutters. The transverse tooth thickness is equal to the normal tooth thickness divided by the cosine of the helix angle. Usually, design drawings will show the transverse pressure angle, but sometimes it is necessary to convert normal tooth thickness to transverse tooth thickness (which has not been given).

B.13 GENERAL METHOD FOR DETERMINING TOOTH THICKNESS WHEN HELICAL GEARS ARE OPERATED ON SPREAD CENTERS (SUPPLEMENT TO CHAPTER 13)

Problem 13.2 in Chapter 13 showed how to design spur gears to be cut with standard cutting tools and operate on spread centers. The more general problem is that of helical gears designed to run on spread centers. (A spur gear is a helical gear with 0° helix angle.) In this section, we will show the general procedure for setting the tooth thickness values of a set of helical gears on spread centers.

The method will be shown by going through a step-by-step procedure for an 18/95 teeth gearset cut at 3 normal module, 20° normal pressure angle, and 15° helix angle. This set is to run on a center distance of 180 mm so as to give an operating pressure angle of about 24°.

The first step is to calculate the basic tooth data for both cutting and operating conditions. The several equations, those given in Chapters 4, 5, and 13, are used. (Presumably, the reader is familiar with these equations by now.)

The values given in Table B.18 are the basic data for the design.

The next calculation step is to establish the tooth height dimensions. A 3-normal-module tooth has a standard working depth of 6 mm. A whole depth of at least 7 mm and a

TABLE B.18
Basic Data for the Design

Item	Pinion	Gear
Number of teeth	18	95
Pitch diameter, cutting	55.90490	295.05366
Normal circular pitch, cutting	9.42478	9.42478
Normal pressure angle, cutting	20°	20°
Transverse pressure angle, cutting	20.64690	20.64690
Helix angle, cutting	15° RH	15° LH
Lead	655.4619	3459.3824
Base diameter	52.31420	276.10272
Center distance, operating	180	180
Pitch diameter, operating	57.34513	302.65486
Transverse circular pitch, operating	10.00861	10.00861
Transverse pressure angle, operating	24.17914	24.17914
Helix angle, operating	15.368364	15.368364
Normal circular pitch, operating	9.65072	9.65072
Normal pressure angle, operating	23.40902	23.40902

TABLE B.19
Working Table Dimensioning the Tooth Height

Item	Pinion	Gear
Addendum, operating	3.75	2.25
Pitch diameter, operating	57.3451	302.6549
Outside diameter	64.8451	307.1459
Whole depth	7.1	7.1
Root diameter	50.6451	292.9549

good-sized root llet radius are needed for shaving or grinding. With a small number of pinion teeth, the pinion should be made about 25% long addendum and the gear about 25% short addendum. We will divide the working depth into a pinion addendum of 3.75 mm and a gear addendum of 2.25 mm.

With these decisions made, we can now make a working table dimensioning the tooth height (see Table B.19).

The operating circular pitch was 10.0086 mm. Our gears are to be used at a relatively slow pitch-line speed (10 m/s), and there will be frequent torque reversals. This makes it desirable to have a small amount of backlash. We will design for 0.10 to 0.15 mm backlash in the transverse plane. This means that the sum of our design thicknesses should be no greater than 9.9086 mm (10.0086 − 0.10).

With standard-addendum gears, this tooth thickness sum would be divided approximately evenly between pinion and gear. However, in this case, the pinion addendum will be 0.75 mm longer than standard addendum (3.75 − 3.00). The pinion transverse arc tooth thickness should be about 0.61 larger than standard, and the gear, being 0.75 short, should be about 0.61 thinner than standard. The 0.61 is 0.75 × 2 tan 22°. (Twenty-two degrees is about midway between the operating pressure angle of 24.1791° and the cutting pressure angle of 20.6469°.)

The next step is to calculate the tooth thickness for cutting (and shaving) operations. The arc tooth thickness at the cutting pitch diameter is determined by the method in Table B.20, using the desired operating tooth thickness as a starting point. The normal section arc tooth thickness is obtained by multiplying the cosine of the helix angle (at the cutting pitch diameter). The results are summarized in Table B.21.

A preshave hob is available to cut these parts. The hob has an addendum of 4.20 mm and a tooth thickness of 4.56 mm. It is 20° normal pressure angle. Its standard cutting depth is 7.20 mm (2.40 × module = 7.20 mm).

TABLE B.20
Determination of Arc Tooth Thickness at Cutting Pitch Diameter from Design Value at Operating Pitch Diameter

Given Data

Item		Pinion	Gear
1	Pitch diameter	57.34513	302.65486
2	Pressure angle at (1)	24.17914	24.17914
3	Arc tooth thickness at (1)	5.5643	4.3443

Step A: Solve for the tooth thickness at base diameter.

Item		Pinion	Gear
4	Involute (2)	0.0269747	0.0269747
5	cos(2)	0.912269	0.912269
6	(4) × (1)	1.54687	8.16402
7	(3) + (6)	7.11117	12.50832
8	TT at BD, (5) × (7)	6.48730	11.41095
9	Base diameter, (5) × (1)	52.31420	276.1026

Step B: Solve for a series of tooth thicknesses to study the tooth pro le. For any desired diameter, the cosine of the pressure angle at that pitch diameter is

cos PA at any diameter = base diameter ÷ any diameter.

A series of pressure angles is assumed based on diameters of interest. Then calculations are made to get the arc thicknesses that to with each diameter.

Item		Pinion	Gear
10	Assumed pressure angles	20.64690	20.64690
11	cos(10)	0.935771	0.935771
12	Involute (10)	0.0164534	0.0164534
13	(12) × (9)	0.860746	4.54282
14	(8) − (13)	5.62655	6.86813
15	Arc TT, (14) ÷ (11)	6.01274	7.33954
16	Any diameter, (9) ÷ (11)	55.90492	295.05359
17	Δ rad., [(16) − (1)] ÷ 2		

TABLE B.21
The Results of Computation of the Tooth Thicknesses for Cutting and Shaving Operations

Item	Pinion	Gear
Arc tooth thickness, operating	5.5643	4.3443
Arc tooth thickness, cutting	6.0127	7.3395
Helix angle, cutting	15°	15°
Normal arc tooth thickness, cutting	5.8079	7.0895
Pitch diameter, cutting	55.90490	295.05366

The dedendum of the pinion during hobbing is

$$\begin{aligned}\text{Pinion dedendum} &= 0.5\times(\text{pitch diameter} - \text{root diameter})\\ &= 0.5\times(55.90490 - 50.6451)\\ &= 2.6299\ \text{mm}.\end{aligned} \tag{B.27}$$

When cutting the long-addendum pinion, the hob would use an addendum close to the dedendum of the pinion, which is 2.6299 mm, instead of 4.20 mm; the hob tooth thickness would be

$$\begin{aligned}\text{Hob tooth thickness} &= 4.56 - 2.0\tan 20°(4.200 - 2.6299)\\ &= 3.41706\ \text{mm}.\end{aligned} \tag{B.28}$$

The hob cuts its own thickness at the hobbing pitch diameter. We can now subtract the hob tooth thickness from the normal circular pitch to get the normal arc tooth thickness that the chosen hob will produce. This is 9.4248 mm − 3.4171 mm = 6.0077 mm.

Our earlier calculations gave a design value of 5.8079 mm for the pinion normal tooth thickness at the cutting pitch diameter. Since the pinion is to be shaved, we need to add 0.05 mm for shaving stock. We should produce a tooth thickness of 5.8579 mm at the hobbing operation. The calculations just nished show that the hob will produce a tooth thickness of 6.0077 mm. We need to hob the pinion teeth to about 0.1498 mm thinner (6.0077 − 5.8579 = 0.1498 mm). What do we do now?

The answer is easy. If we sink the hob in 0.2058 mm, it will cut the teeth 0.1498 mm thinner (0.1498 ÷ 2 tan 20° = 0.2058 mm). Our design was set at a whole depth of 7.100 mm. Sinking the hob in for tooth thickness adjustment will give a whole depth of about 7.3 mm (7.100 + 0.206 = 7.306 mm). This is OK. The original aim in this job was to get a whole depth of at least 7.0 mm. The depth of 7.3 mm is not too much.

When hobbing the short-addendum gear, the gear dedendum is

$$\begin{aligned}\text{Gear dedendum} &= 0.5\times(\text{pitch diameter} - \text{root diameter})\\ &= 0.5\times(295.05366 - 292.9549) \qquad \text{(B.29)}\\ &= 1.0494\ \text{mm}.\end{aligned}$$

When cutting the short-addendum gear, the hob will use an addendum of 1.0494 mm (to match the gear dedendum). At this addendum, the hob tooth thickness will be

$$\begin{aligned}\text{Hob tooth thickness} &= 4.56 - 2.0\tan 20°\ (4.200 - 1.0494)\\ &= 2.2665\ \text{mm}.\end{aligned} \tag{B.30}$$

We subtract 2.2665 mm from the gear normal circular pitch (9.4248 mm) to get a gear tooth thickness of 7.1583 mm. We need 0.05 mm shaving stock, and so the gear tooth thickness that we need to hob is 7.1395 mm (7.0895 + 0.05 = 7.1395 mm).

Our hob will produce 7.1583 mm and we need only 7.1395 mm. The gear teeth will be too thick by 0.0188 mm. (7.1583 − 7.1395). This problem is quite easy to solve. We will hob the gear slightly deeper than 7.10 mm. An extra depth of 0.026 mm will make the gear tooth thickness come out just right (0.0188 ÷ 2 tan 20° = 0.0258 mm). Our approximate whole depth in hobbing will be 7.126 mm. This slight difference in whole depth is of no consequence at all. It is between the 7.100 mm design whole depth and the 7.200 mm whole depth normally used for the hob.

The calculation procedure to determine tooth thickness for a spread-center helical gearset is nished.

The reader should note that this general procedure will work for spur gears, since they are really just helical gears with 0° helix angle. Also, the procedure shows how to check out a preshave hob. If a pregrind hob is used, the procedure is the same, but the stock allowance for grinding will be much greater than the stock allowance for shaving.

B.14 CALCULATION OF GEOMETRY FACTOR FOR SCORING (SUPPLEMENT TO CHAPTER 5)

In Section 5.2.8, a general equation was given for the calculation of ash temperature. This value shows the hazard of hot scoring. A term in the formula is the *scoring geometry factor*. This factor is de ned in Equation 4.46. The historical background of this equation (and others relating to scoring) is given in Section 6.1.4.

In Section 13.2, Problem 13.5 nds the diameters at which pro le modi cations should start for gears of moderately critical scoring hazard. The problem is for 26/51 tooth mesh of 20 module. We will now show how to calculate the scoring geometry factors at the start of pro le modi cation.

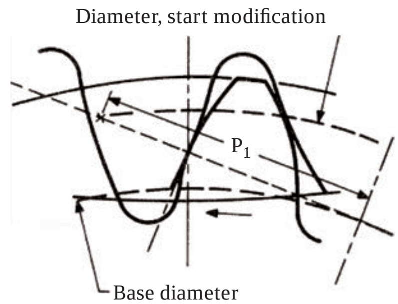
The 26-tooth pinion has a start of pro le modi cation at 559.620 mm (diameter). The pro le modi cation of the 51-tooth gear starts at 1060.808 mm (diameter).

Table B.22 shows the calculations for the scoring geometry factor. Column 1 determines Z_t at the start of modi cation near the pinion tip. The answer is 0.00789.

Column 2 determines the scoring geometry factor near the gear tip. The calculation sheet only works, though, for diameters on the pinion. This means that it is necessary to nd the

TABLE B.22
Calculation of Geometry Factor for Scoring

	Basic Data	
1	Circular pitch, transverse	64.2355
2	Pressure angle, transverse	20.410311
3	cos(2)	0.937219
4	sin(2)	0.348741
5	Center distance	787.20 mm



	Calculations	**Column 1**	**Column 2**
6	Number of teeth	26	26
7	Pitch diameter	531.6156	531.6156
8	Diameter, start modification	559.620	516.616
9	Base diameter, (7) × (3)	498.2417	498.2417
10	(9) ÷ (8)	0.890321	0.964433
11	$\cos^{-1}$(10)	27.08639	15.32692
12	tan(11)	0.511426	0.274074
13	ϱ_1 = 0.50 × (9) × (12)	127.4069	68.2775
14	(5) × (4)	274.5289	274.5289
15	ϱ_2 = (14) − (13)	147.1220	206.2514
16	$(13)^{0.5}$	11.2875	8.2630
17	(6) ÷ (no. of teeth)[a]	0.509804	0.509804
18	$[(17) \times (15)]^{0.5}$	8.66045	10.25416
19	(16) − (18)	2.62705	−1.99116
20	0.0175 × (19)	0.045973	−0.034845
21	$(3)^{0.75}$	0.952535	0.952535
22	(13) × (15)	18744.36	14082.33
23	(13) + (15)	274.5289	274.5289
24	$[(22) \div (23)]^{0.25}$	2.87455	2.67622
25	(21) × (24)	2.73811	2.55026
26	(20) ÷ (25)	0.016790	0.013663
27	m_t = (1) ÷ 3.1415926	20.4468	20.4468
28	$(27)^{0.25}$	2.1265	2.1265
29	Z_t = (26) ÷ (28)	0.00789	−0.006425
For English calculation:			
30	P_t = 3.1415926 ÷ (1)		
31	$(30)^{0.25}$		
32	Z_t = (26) ÷ (31)		

Note: The diameter at the start of modification is usually critical from a scoring standpoint. If there is no modification of involute profile, the outside diameter of the pinion and the limit diameter of the pinion should be used in item (8) for the scoring geometry factor calculation. This calculation sheet can be used for either metric or English units. Item (29) is the metric answer. Item (32) is the English answer. The same answer should be obtained either way, since Z_t is dimensionless. This calculation works only for diameters on the pinion. When the gear is modified, find the matching diameter on the pinion. (See Table B.16, Column 1, for an example.)

[a] The number of teeth in the mating gear is 51.

diameter on the pinion that matches the start of modification on the gear. Table B.22 does this operation and is all worked out in column 1 for our problem. The diameter of the pinion (near root fillet) is 516.616 mm. This diameter matches 1060.808 mm on the gear.

Column 2 of Table B.22 is general in nature. If a study of scoring at locations other than the start of profile modification is needed, the Z_t value can be calculated for a series of diameters going from the limit diameter to the outside diameter.

Appendix C: Numerical Data Tables

Certain tables are used very frequently in gear tooth calculations. These tables and the directions for using them are given in this appendix.

C.1 GEAR MEASUREMENT TABLES

The gear measurement tables in this appendix are used extensively by gear manufacturers and users and provide a quick means of checking the tooth thickness of gears at the pitch diameter. A measurement over wires is made and compared with the measurement for a perfect gear, which is obtainable from the tables. The values in the tables are accurate to 0.0001 in. for diametral pitch gears. (This is equivalent to 0.0025 mm for metric measurements.) After the value is selected in the proper table for the speci ed pressure angle, number of teeth, and wire size to be used, this value is divided by the diametral pitch of the gear being measured. If the actual measurement is greater than the theoretical value, the teeth are thick; if it is less, the teeth are thin. The change factors K_m, which permit easy conversion of oversize or undersize readings into the approximate amount the teeth are thick or thin, are tabulated alongside the measurement values.

When doing a metric calculation, the value of the *M* in Tables C.3 and C.4, etc., is the *right value* in millimeters for a 1 module tooth size provided that the measuring wire size is the right value in millimeters for the module. (For instance, the 1.728 wire for 1 module is 1.728 mm. See Tables C.1 and C.2.)

In this appendix, the measurement tables (except for Table C.7, which covers 25° gears) are presented so that, for each pressure angle measurement, values are available based on two or more wire diameters. In general, it is recommended that wires from the 1.728 in. series be used for measuring external gears and that wires from the 1.44 in. series be used for internal gears. The tabulated values for the other wire diameters, however, are available for checking involute pro les and for permitting greater freedom in wire selection. Table C.1 serves as a guide to the selection of a wire diameter for a particular application.

When spur gears are made to even pitches in the metric system, the pitch is given in terms of *module* values rather than *diametral pitch* values. The tables given in this appendix can be used for checking the tooth thickness of even-module metric gears provided that wires made to appropriate metric sizes in millimeters are on hand. (If appropriate wires are not on hand, it is not dif cult to precision grinding pairs of wires to the metric sizes needed.)

Table C.2 shows standard wire sizes for commonly used pitches in both the English system of measurement and the metric system.

C.1.1 Numerical Examples

Three illustrative examples immediately follow.

Example C.1

An even-tooth external gear of 26 teeth, 10 diametral pitch, 14½° pressure angle, is measured with 0.1728 in. diameter wires. From Table C.3, second column, the measurement for $1P_d$ is 28.4314 in. For $10P_d$ the measurement over wires is 28.4314 ÷ 10 = 2.8431 in. If the teeth are to be cut 0.003 in. thin, the reduced measurement over wires can be determined by using the change factor K_m. From the third column of Table C.3, the change factor is K_m = 2.90. Thus, the reduction in measurement for teeth 0.003 in. thin would be 0.003 in. × 2.90 or 0.0087 in. The measurement over wires with teeth 0.003 in. thin would be 2.8431 − 0.0087, or 2.8344 in.

As an example of metric dimensions, we will take a 2.500-module gear. We will assume an even-tooth external gear of 26 teeth, 14½° pressure angle measured with two wires, measured with basic 1.728 mm wires (for one module). At 2.500 module the wire size is 4.320 mm. From Table C.3 for 26 teeth, we read a measurement value of 28.4314 in. This value is for 1 module. We need to interpret this value in millimeters and then change the millimeter value from 1 module to 2.500 module. The calculation procedure is

$$\frac{M}{0.0393701\,\text{in/mm}} \times \frac{2.500\ \text{module}}{25.400} = M\ \text{at 2.500 module} \tag{C.1}$$

$$\frac{M \times 2.500\ \text{module}}{1.000000} = 28.4314 \times 2.500 = 71.0785\ \text{mm}. \tag{C.2}$$

If the teeth are to be cut thin by 0.075 mm, the change in *M* is

$$\begin{aligned} M\ \text{in millimeters} &= 0.075\ \text{mm} \times K_m \\ &= 0.075 \times 2.90 = 0.2175\ \text{mm}. \end{aligned} \tag{C.3}$$

The measure over two pins, 4.320 mm in diameter, is then 71.0785 − 0.2175, or 70.8610 mm.

Example C.2

An even-tooth external gear of 20 teeth, 20 diametral pitch, 20° pressure angle, with teeth to be cut 0.004 in.,

TABLE C.1
Guide to Wire Selection

No.	Series: English (in.)	Series: Metric (mm)	Uses
1	$\frac{1.92''}{P_d}$	$1.92 \times m$	• Enlarged pinions; wires project above OD • Alternate for 14½° and 20° pressure angle, standard-addendum external gears
2	$\frac{1.728''}{P_d}$	$1.728 \times m$	• External standard-addendum spur and helical gears; wires project above OD
3	$\frac{1.68''}{P_d}$	$1.68 \times m$	• Suitable for 14½° internal gears above 31 teeth and 20° internal gears above 29 teeth; wires project below ID • Alternate for standard-addendum external gears
4	$\frac{1.44''}{P_d}$	$1.44 \times m$	• Internal standard-addendum gears; wires make good contact on gear teeth but do not project below ID

Note: ID: inside diameter; m: the module of a spur gear; mm: millimeters; OD: outside diameter; P_d: the diametral pitch of a spur gear.

TABLE C.2
Gear Wire Sizes

English					Metric				
P_d	$\frac{1.92\text{ in.}}{P_d}$	$\frac{1.728\text{ in.}}{P_d}$	$\frac{1.68\text{ in.}}{P_d}$	$\frac{1.44\text{ in.}}{P_d}$	m	1.92 m	1.728 m	1.68 m	1.44 m
1	1.92	1.728	1.68	1.44	1	1.920	1.728	1.680	1.440
2	0.960	0.864	0.840	0.720	2	3.840	3.456	3.360	2.880
2.5	0.768	0.6912	0.672	0.576	2.5	4.800	4.320	4.200	3.600
3	0.640	0.576	0.560	0.480	3	5.760	5.184	5.040	4.320
4	0.480	0.432	0.420	0.360	4	7.680	6.912	6.720	5.760
5	0.384	0.3456	0.336	0.288	5	9.600	8.640	8.400	7.200
6	0.320	0.288	0.280	0.240	6	11.520	10.368	10.080	8.640
7	0.27428	0.24686	0.240	0.20571	7	13.440	12.096	11.760	10.080
8	0.240	0.216	0.210	0.180	8	15.360	13.824	13.440	11.520
9	0.21333	0.192	0.18666	0.160	9	17.280	15.552	15.120	12.960
10	0.192	0.1728	0.168	0.144	10	19.200	17.280	16.800	14.400
11	0.17454	0.15709	0.15273	0.13091	11	21.120	19.008	18.480	15.840
12	0.160	0.144	0.140	0.120	12	23.040	20.736	20.160	17.280
14	0.13714	0.12343	0.120	0.10286	14	26.880	24.192	23.520	20.160
16	0.120	0.108	0.105	0.090	16	30.720	27.648	26.880	23.040
18	0.10667	0.096	0.09333	0.080	18	34.560	31.104	30.240	25.920
20	0.096	0.0864	0.084	0.072	20	38.400	34.560	33.600	28.800
22	0.08727	0.07855	0.07636	0.06545	22	42.240	38.016	36.960	31.680
24	0.080	0.072	0.070	0.060	24	46.080	41.472	40.320	34.560
28	0.06857	0.06171	0.060	0.05143	28	53.760	48.384	47.040	40.320
32	0.060	0.054	0.0525	0.045	32	61.440	55.296	53.760	46.080
36	0.05333	0.048	0.04667	0.040	36	64.120	62.208	60.480	51.840
40	0.048	0.0432	0.042	0.036	40	76.800	69.120	67.200	57.600
48	0.040	0.036	0.035	0.030	48	92.160	82.944	80.640	69.120
64	0.030	0.027	0.02625	0.0225	64	122.880	110.592	107.520	92.160

TABLE C.3
External Gears: 14½° Pressure Angle

No. of Teeth	1.728 Wire Diameter M	1.728 Wire Diameter K_m	Alternate 1.92 Wire Diameter M	Alternate 1.92 Wire Diameter K_m	Alternate 1.68 Wire Diameter M	Alternate 1.68 Wire Diameter K_m
5	6.9936	1.90	7.5296	1.61	6.8485	2.03
6	8.2846	2.09	8.8551	1.77	8.1298	2.23
7	9.1116	2.17	9.6871	1.84	8.9555	2.31
8	10.3160	2.24	10.9147	1.90	10.1535	2.39
9	11.1829	2.31	11.7872	1.96	11.0189	2.45
10	12.3399	2.36	12.9617	2.01	12.1712	2.51
11	13.2317	2.42	13.8590	2.06	13.0615	2.57
12	14.3590	2.47	15.0002	2.11	14.1851	2.62
13	15.2677	2.51	15.9143	2.15	15.0925	2.66
14	16.3746	2.55	17.0327	2.19	16.1964	2.70
15	17.2957	2.59	17.9588	2.23	17.1163	2.74
16	18.3877	2.63	19.0606	2.26	18.2058	2.78
17	19.3182	2.66	19.9958	2.29	19.1351	2.81
18	20.3989	2.70	21.0850	2.33	20.2137	2.84
19	21.3368	2.73	22.0272	2.36	21.1505	2.87
20	22.4087	2.76	23.1065	2.39	22.2205	2.90
21	23.3524	2.78	24.0543	2.41	23.1634	2.93
22	24.4172	2.81	25.1257	2.44	24.2265	2.95
23	25.3658	2.83	26.0781	2.46	25.1743	2.98
24	26.4247	2.86	27.1430	2.49	26.2317	3.00
25	27.3774	2.88	28.0992	2.51	27.1836	3.02
26	28.4314	2.90	29.1586	2.53	28.2363	3.04
27	29.3876	2.92	30.1181	2.56	29.1918	3.06
28	30.4374	2.94	31.1729	2.58	30.2404	3.08
29	31.3966	2.96	32.1351	2.60	31.1990	3.09
30	32.4429	2.98	33.1859	2.62	32.2441	3.11
31	33.4047	2.99	34.1506	2.63	33.2053	3.13
32	34.4478	3.01	35.1979	2.65	34.2475	3.14
33	35.4119	3.03	36.1647	2.67	35.2110	3.16
34	36.4523	3.04	37.2090	2.69	36.2505	3.17
35	37.4185	3.06	38.1777	2.70	37.2161	3.18
36	38.4565	3.07	39.2193	2.72	38.2533	3.19
37	39.4245	3.08	40.1896	2.73	39.2208	3.21
38	40.4603	3.10	41.2288	2.75	40.2558	3.22
39	41.4299	3.11	42.2007	2.76	41.2249	3.23
40	42.4638	3.12	43.2378	2.78	42.2582	3.24
41	43.4348	3.13	44.2110	2.79	43.2287	3.25
42	44.4671	3.14	45.2461	2.80	44.2604	3.26
43	45.4394	3.15	46.2205	2.82	45.2323	3.27
44	46.4701	3.17	47.2539	2.83	46.2624	3.28
45	47.4437	3.18	48.2294	2.84	47.2355	3.29
46	48.4729	3.19	49.2613	2.825	48.2642	3.30
47	49.4477	3.20	50.2378	2.87	49.2385	3.31
48	50.4756	3.21	51.2682	2.88	50.2660	3.32
49	51.4514	3.21	52.2457	2.89	51.2413	3.33
50	52.4781	3.22	53.2748	2.90	52.2676	3.33
51	53.4547	3.23	54.2531	2.91	53.2439	3.34
52	54.4804	3.24	55.2810	2.92	54.2691	3.35
53	55.4579	3.24	56.2601	2.93	55.2463	3.35
54	56.4826	3.25	57.2868	2.94	56.2705	3.36
55	57.4609	3.26	58.2667	2.95	57.2485	3.37

(Continued)

TABLE C.3 (CONTINUED)
External Gears: 14½° Pressure Angle

	1.728 Wire Diameter		Alternate 1.92 Wire Diameter		Alternate 1.68 Wire Diameter	
No. of Teeth	***M***	K_m	***M***	K_m	***M***	K_m
56	58.4847	3.27	59.2924	2.96	58.2719	3.37
57	59.4637	3.28	60.2729	2.97	59.2506	3.38
58	60.4866	3.28	61.2977	2.98	60.2731	3.39
59	61.4664	3.29	62.2788	2.99	61.2526	3.39
60	62.4884	3.30	63.3027	3.00	62.2743	3.40
61	63.4689	3.31	64.2844	3.01	63.2545	3.40
62	64.4902	3.31	65.3075	3.01	64.2755	3.41
63	65.4712	3.32	66.2898	3.02	65.2562	3.42
64	66.4918	3.32	67.3121	3.03	66.2765	3.42
65	67.4734	3.33	68.2949	3.04	67.2579	3.43
66	68.4933	3.34	69.3165	3.05	68.2775	3.43
67	69.4755	3.34	70.2998	3.05	69.2594	3.44
68	70.4948	3.35	71.3206	3.06	70.2785	3.44
69	71.4775	3.35	72.3044	3.07	71.2609	3.45
70	72.4963	3.36	73.3247	3.07	72.2794	3.45
71	73.4795	3.37	74.3089	3.08	73.2623	3.46
72	74.4977	3.37	75.3285	3.09	74.2803	3.46
73	75.4813	3.38	76.3131	3.10	75.2636	3.46
74	76.4990	3.38	77.3322	3.10	76.2811	3.47
75	77.4830	3.39	78.3172	3.11	77.2649	3.47
76	78.5002	3.39	79.3357	3.12	78.2819	3.48
77	79.4847	3.40	80.3211	3.12	79.2661	3.48
78	80.5014	3.40	81.3391	3.13	80.2827	3.49
79	81.4863	3.41	82.3249	3.13	81.2673	3.49
80	82.5026	3.41	83.3423	3.14	82.2834	3.50
81	83.4877	3.42	84.3285	3.15	83.2684	3.50
82	84.5037	3.42	85.3455	3.15	84.2841	3.50
83	85.4892	3.42	86.3319	3.16	85.2694	3.51
84	86.5047	3.43	87.3485	3.16	86.2847	3.51
85	87.4906	3.43	88.3352	3.17	87.2704	3.51
86	88.5057	3.44	89.3515	3.17	88.2854	3.52
87	89.4919	3.44	90.3384	3.18	89.2714	3.52
88	90.5067	3.44	91.3543	3.18	90.2860	3.52
89	91.4932	3.45	92.3415	3.19	91.2723	3.52
90	92.5076	3.45	93.3570	3.19	92.2866	3.53
91	93.4944	3.45	94.3445	3.20	93.2732	3.53
92	94.5085	3.45	95.3596	3.20	94.2872	3.54
93	95.4956	3.46	96.3474	3.21	95.2741	3.54
94	96.5094	3.46	97.3621	3.21	96.2877	3.54
95	97.4967	3.46	98.3502	3.22	97.2749	3.54
96	98.5102	3.46	99.3646	3.22	98.2882	3.55
97	99.4978	3.47	100.3529	3.23	99.2757	3.55
98	100.5110	3.47	101.3670	3.23	100.2887	3.55
99	101.4988	3.47	102.3555	3.24	101.2764	3.55
100	102.5118	3.48	103.3693	3.24	102.2892	3.56
101	103.4998	3.49	104.3580	3.24	103.2771	3.56
102	104.5125	3.49	105.3715	3.25	104.2897	3.56
103	105.5008	3.49	106.3604	3.25	105.2778	3.57
104	106.5132	3.50	107.3737	3.26	106.2901	3.57
105	107.5017	3.50	108.3628	3.26	107.2785	3.57
106	108.5139	3.50	109.3758	3.27	108.2905	3.57

(*Continued*)

TABLE C.3 (CONTINUED)
External Gears: 14½° Pressure Angle

	1.728 Wire Diameter		Alternate 1.92 Wire Diameter		Alternate 1.68 Wire Diameter	
No. of Teeth	***M***	K_m	***M***	K_m	***M***	K_m
107	109.5026	3.50	110.3651	3.27	109.2791	3.58
108	110.5035	3.51	111.3778	3.27	110.2910	3.58
109	111.5035	3.51	112.3672	3.28	111.2798	3.58
110	112.5152	3.51	113.3798	3.28	112.2914	3.58
111	113.5044	3.52	114.3695	3.29	113.2804	3.59
112	114.5159	3.52	115.3817	3.29	114.2918	3.59
113	115.5052	3.52	116.3716	3.29	115.2809	3.59
114	116.5165	3.53	117.3835	3.30	116.2921	3.59
115	117.5060	3.53	118.3736	3.30	117.2815	3.59
116	118.5171	3.53	119.3853	3.30	118.2925	3.60
117	119.5068	3.53	120.3756	3.31	119.2821	3.60
118	120.5177	3.53	121.3871	3.31	120.2929	3.60
119	121.5075	3.53	122.3775	3.31	121.2826	3.60
120	122.5182	3.54	123.3888	3.32	122.2932	3.60
121	123.5082	3.54	124.3793	3.32	123.2831	3.61
122	124.5188	3.54	125.3905	3.32	124.2936	3.61
123	125.5089	3.54	126.3811	3.33	125.2836	3.61
124	126.5193	3.55	127.3921	3.33	126.2939	3.61
125	127.5096	3.55	128.3829	3.33	127.2841	3.61
126	128.5198	3.55	129.3937	3.34	128.2941	3.61
127	129.5103	3.55	130.3846	3.34	129.2846	3.62
128	130.5203	3.55	131.3952	3.34	130.2945	3.62
129	131.5109	3.56	132.3863	3.34	131.2851	3.62
130	132.5208	3.56	133.3967	3.35	132.2948	3.62
131	133.5115	3.56	134.3879	3.35	133.2855	3.62
132	134.5213	3.56	135.3982	3.35	134.2951	3.62
133	135.5121	3.57	136.3895	3.36	135.2859	3.63
134	136.5217	3.57	137.3996	3.36	136.2594	3.63
135	137.5127	3.57	138.3911	3.36	137.2863	3.63
136	138.5221	3.57	139.4010	3.37	138.2957	3.63
137	139.5133	3.57	140.3926	3.37	139.2867	3.63
138	140.5226	3.57	141.4023	3.37	140.2960	3.63
139	141.5139	3.58	142.3940	3.37	141.2871	3.64
140	142.5230	3.58	143.4037	3.38	142.2962	3.64
141	143.5144	3.58	144.3955	3.38	143.2875	3.64
142	144.5234	3.58	145.4050	3.38	144.2965	3.64
143	145.5149	3.58	146.3969	3.39	145.2879	3.64
144	146.5238	3.58	147.4062	3.39	146.2967	3.64
145	147.5154	3.59	148.3982	3.39	147.2883	3.64
146	148.5242	3.59	149.4074	3.39	148.2970	3.64
147	149.5159	3.59	150.3996	3.40	149.2887	3.65
148	150.5246	3.59	151.4087	3.40	150.2972	3.65
149	151.5164	3.59	152.4009	3.40	151.2890	3.65
150	152.5250	3.59	153.4098	3.40	152.2974	3.65
151	153.5169	3.60	154.4022	3.41	153.2893	3.65
152	154.5254	3.60	155.4110	3.41	154.2977	3.65
153	155.5174	3.60	156.4034	3.41	155.2897	3.65
154	156.5257	3.60	157.4121	3.41	156.2979	3.65
155	157.5179	3.60	158.4046	3.41	157.2900	3.66
156	158.5261	3.60	159.4132	3.42	158.2981	3.66
157	159.5183	3.60	160.4058	3.42	159.2903	3.66

(*Continued*)

TABLE C.3 (CONTINUED)
External Gears: 14½° Pressure Angle

	1.728 Wire Diameter		Alternate 1.92 Wire Diameter		Alternate 1.68 Wire Diameter	
No. of Teeth	***M***	K_m	***M***	K_m	***M***	K_m
158	160.5264	3.61	161.4143	3.42	160.2983	3.66
159	161.5188	3.61	162.4070	3.42	161.2906	3.66
160	162.5267	3.61	163.4153	3.43	162.2985	3.66
161	163.5192	3.61	164.4081	3.43	163.2909	3.66
162	164.5270	3.61	165.4164	3.43	164.2987	3.66
163	165.5196	3.61	166.4092	3.43	165.2912	3.67
164	166.5273	3.61	167.4174	3.43	166.2989	3.67
165	167.5200	3.62	168.4103	3.44	167.2915	3.67
166	168.5276	3.62	169.4184	3.44	168.2990	3.67
167	169.5204	3.62	170.4114	3.44	169.2917	3.67
168	170.5279	3.62	171.4194	3.44	170.2992	3.67
169	171.5208	3.62	172.4125	3.44	171.2920	3.67
170	172.5282	3.62	173.4203	3.45	172.2994	3.67
171	173.5212	3.62	174.4135	3.45	173.2922	3.67
172	174.5286	3.62	175.4212	3.45	174.2996	3.68
173	175.5215	3.63	176.4145	3.45	175.2926	3.68
174	176.5288	3.63	177.4221	3.45	176.2998	3.68
175	177.5219	3.63	178.4155	3.45	177.2928	3.68
176	178.5291	3.63	179.4230	3.46	178.3000	3.68
177	179.5223	3.63	180.4165	3.46	179.2930	3.68
178	180.5294	3.63	181.4239	3.46	180.3002	3.68
179	181.5226	3.63	182.4174	3.46	181.2933	3.68
180	182.5297	3.63	183.4248	3.46	182.3003	3.68
181	183.5230	3.63	184.4183	3.47	183.2936	3.68
182	184.5299	3.64	185.4256	3.47	184.3005	3.68
183	185.5233	3.64	186.4193	3.47	185.2938	3.69
184	186.5302	3.64	187.4265	3.47	186.3006	3.69
185	187.5236	3.64	188.4202	3.47	187.2940	3.69
186	188.5304	3.64	189.4273	3.47	188.3008	3.69
187	189.5239	3.64	190.4210	3.48	189.2942	3.69
188	190.5307	3.64	191.4281	3.48	190.3010	3.69
189	191.5243	3.64	192.4219	3.48	191.2944	3.69
190	192.5310	3.64	193.4289	3.48	192.3011	3.69
200	202.5321	3.66	203.4326	3.50	202.3018	3.70
201	203.5260	3.66	204.4267	3.50	203.2957	3.70
300	302.5395	3.72	303.4573	3.60	302.3063	3.75
301	303.5355	3.72	304.4534	3.61	303.3022	3.75
400	402.5434	3.75	403.4706	3.66	402.3087	3.78
401	403.5404	3.75	404.4676	3.66	403.3056	3.78
500	502.5458	3.78	503.4788	3.70	502.3101	3.80
505	503.5433	3.78	504.4763	3.70	503.3076	3.80
∞	(N + 2).5558	2.87	(N + 3).5145	2.87	(N + 2).3159	3.87

Note: Measurements over wires and change factors for 1 diametral pitch external standard-addendum 14½° pressure angle spur gears with 1.728, 1.92, and 1.86 diameter wires. For any other diametral pitch, divide measurements given in the table by the diametral pitch. Change factors are the same for all diametral pitches. For 1 diametral pitch, arc tooth thickness is 1.57080 in. For 1 module, arc tooth thickness is 1.57080 mm.

TABLE C.4
External Gears: 20° Pressure Angle

	1.728 Wire Diameter		Alternate 1.92 Wire Diameter		Alternate 1.68 Wire Diameter	
No. of Teeth	***M***	K_m	***M***	K_m	***M***	K_m
5	7.0153	1.67	7.5271	1.48	6.8800	1.75
6	8.3032	1.83	8.8449	1.62	8.1600	1.91
7	9.1260	1.88	9.6702	1.67	8.9822	1.96
8	10.3271	1.94	10.8907	1.72	10.1783	2.01
9	11.1905	1.98	11.7573	1.77	11.0410	2.06
10	12.3445	2.01	12.9252	1.81	12.1914	2.10
11	13.2332	2.05	13.8173	1.84	13.0795	2.13
12	14.3578	2.09	14.9525	1.87	14.2013	2.16
13	15.2639	2.12	15.8618	1.91	15.1068	2.19
14	16.3683	2.14	16.9748	1.93	16.2091	2.21
15	17.2871	2.17	17.8964	1.96	17.1273	2.24
16	18.3768	2.19	18.9934	1.98	18.2154	2.26
17	19.3053	2.21	19.9244	2.01	19.1432	2.28
18	20.3840	2.23	21.0091	2.03	20.2205	2.29
19	21.3200	2.25	21.9475	2.05	21.1561	2.31
20	22.3900	2.26	23.0227	2.07	22.2249	2.33
21	23.3321	2.28	23.9670	2.08	23.1665	2.34
22	24.3952	2.29	25.0346	2.10	24.2286	2.35
23	25.3423	2.30	25.9837	2.12	25.1754	2.36
24	26.3997	2.32	27.0450	2.13	26.2318	2.38
25	27.3511	2.33	27.9982	2.15	27.1828	2.39
26	28.4036	2.34	29.0543	2.16	28.2346	2.40
27	29.3586	2.35	30.0109	2.17	29.1892	2.41
28	30.4071	2.36	31.0626	2.19	30.2371	2.42
29	31.3652	2.37	32.0222	2.20	31.1948	2.43
30	32.4102	2.38	33.0701	2.21	32.2392	2.43
31	33.3710	2.39	34.0323	2.22	33.1997	2.44
32	34.4130	2.40	35.0768	2.23	34.2412	2.45
33	35.3761	2.41	36.0413	2.24	35.2041	2.45
34	36.4155	2.41	37.0830	2.25	36.2430	2.46
35	37.3807	2.42	38.0495	2.26	37.2079	2.47
36	38.4178	2.43	39.0886	2.27	38.2445	2.48
37	39.3849	2.43	40.0569	2.28	39.2115	2.48
38	40.4198	2.44	41.0938	2.29	40.2460	2.49
39	41.3886	2.45	42.0636	2.29	41.2147	2.49
40	42.4217	2.45	43.0986	2.30	42.2473	2.50
41	43.3920	2.46	44.0699	2.31	43.2174	2.50
42	44.4234	2.46	45.1030	2.32	44.2485	2.51
43	45.3951	2.47	46.0756	2.32	45.2200	2.51
44	46.4250	2.47	47.1071	2.33	46.2496	2.52
45	47.3980	2.48	48.0809	2.34	47.2224	2.52
46	48.4265	2.48	49.1109	2.34	48.2506	2.53
47	49.4007	2.49	50.0858	2.35	49.2246	2.53
48	50.4279	2.49	51.1144	2.35	50.2516	2.53
49	51.4031	2.50	52.0903	2.36	51.2266	2.54
50	52.4292	2.50	53.1177	2.37	52.2525	2.54
51	53.4053	2.50	54.0945	2.37	53.2284	2.54
52	54.4304	2.51	55.1208	2.38	54.2533	2.55
53	55.4074	2.51	56.0985	2.38	55.2302	2.55
54	56.4315	2.52	57.1237	2.39	56.2541	2.55
55	57.4093	2.52	58.1022	2.39	57.2318	2.56

(*Continued*)

TABLE C.4 (CONTINUED)
External Gears: 20° Pressure Angle

No. of Teeth	1.728 Wire Diameter M	1.728 Wire Diameter K_m	Alternate 1.92 Wire Diameter M	Alternate 1.92 Wire Diameter K_m	Alternate 1.68 Wire Diameter M	Alternate 1.68 Wire Diameter K_m
56	58.4325	2.52	59.1265	2.40	58.2548	2.56
57	59.4111	2.53	60.1057	2.40	59.2333	2.56
58	60.4335	2.53	61.1291	2.41	60.2555	2.56
59	61.4128	2.53	62.1090	2.41	61.2347	2.57
60	62.4344	2.54	63.1315	2.41	62.2561	2.57
61	63.4144	2.54	64.1121	2.42	63.2360	2.57
62	64.4352	2.54	65.1338	2.42	64.2567	2.57
63	65.4159	2.54	66.1150	2.43	65.2372	2.58
64	66.4361	2.55	67.1360	2.43	66.2572	2.58
65	67.4173	2.55	68.1177	2.43	67.2383	2.58
66	68.4369	2.55	69.1381	2.44	68.2577	2.58
67	69.4186	2.55	70.1203	2.44	69.2394	2.59
68	70.4376	2.56	71.1401	2.44	70.2582	2.59
69	71.4198	2.56	72.1228	2.45	71.2405	2.59
70	72.4383	2.56	73.1420	2.45	72.2587	2.59
71	73.4210	2.56	74.1252	2.45	73.2414	2.59
72	74.4390	2.57	75.1438	2.46	74.2592	2.60
73	75.4221	2.57	76.1274	2.46	75.2423	2.60
74	76.4396	2.57	77.1455	2.46	76.2596	2.60
75	77.4232	2.57	78.1295	2.47	77.2432	2.60
76	78.4402	2.57	79.1471	2.47	78.2600	2.60
77	79.4242	2.58	80.1316	2.47	79.2440	2.60
78	80.4408	2.58	81.1486	2.48	80.2604	2.61
79	81.4252	2.58	82.1336	2.48	81.2448	2.61
80	82.4413	2.58	83.1501	2.48	82.2607	2.61
81	83.4262	2.58	84.1354	2.48	83.2456	2.61
82	84.4418	2.58	85.1516	2.49	84.2611	2.61
83	85.4271	2.59	86.1372	2.49	85.2463	2.61
84	86.4423	2.59	87.1529	2.49	86.2614	2.61
85	87.4279	2.59	88.1389	2.49	87.2470	2.62
86	88.4428	2.59	89.1542	2.50	88.2617	2.62
87	89.4287	2.59	90.1405	2.50	89.2476	2.62
88	90.4433	2.59	91.1555	2.50	90.2620	2.62
89	91.4295	2.60	92.1421	2.50	91.2482	2.62
90	92.4437	2.60	93.1567	2.50	92.2624	2.62
91	93.4303	2.60	94.1436	2.51	93.2489	2.62
92	94.4441	2.60	95.1579	2.51	94.2626	2.63
93	95.4310	2.60	96.1450	2.51	95.2494	2.63
94	96.4445	2.60	97.1590	2.51	96.2629	2.63
95	97.4317	2.60	98.1464	2.52	97.2500	2.63
96	98.4449	2.61	99.1601	2.52	98.2632	2.63
97	99.4323	2.61	100.1477	2.52	99.2506	2.63
98	100.4453	2.61	101.1611	2.52	100.2635	2.63
99	101.4329	2.61	102.1490	2.53	101.2511	2.63
100	102.4456	2.61	103.1621	2.53	102.2638	2.63
101	103.4335	2.61	104.1502	2.53	103.2516	2.64
102	104.4460	2.61	105.1631	2.53	104.2640	2.64
103	105.4341	2.61	106.1514	2.53	105.2520	2.64
104	106.4463	2.62	107.1640	2.53	106.2642	2.64
105	107.4346	2.62	108.1526	2.53	107.2525	2.64
106	108.4466	2.62	109.1649	2.54	108.2644	2.64

(*Continued*)

TABLE C.4 (CONTINUED)
External Gears: 20° Pressure Angle

	1.728 Wire Diameter		Alternate 1.92 Wire Diameter		Alternate 1.68 Wire Diameter	
No. of Teeth	*M*	K_m	*M*	K_m	*M*	K_m
107	109.4352	2.62	110.1537	2.54	109.2529	2.64
108	110.4469	2.62	111.1658	2.54	110.2645	2.64
109	111.4357	2.62	112.1548	2.54	111.2533	2.64
110	112.4472	2.62	113.1666	2.54	112.2647	2.64
111	113.4362	2.62	114.1558	2.54	113.2537	2.65
112	114.4475	2.62	115.1675	2.55	114.2649	2.65
113	115.4367	2.63	116.1568	2.55	115.2541	2.65
114	116.4478	2.63	117.1683	2.55	116.2651	2.65
115	117.4372	2.63	118.1578	2.55	117.2544	2.65
116	118.4481	2.63	119.1690	2.55	118.2653	2.65
117	119.4376	2.63	120.1587	2.55	119.2548	2.65
118	120.4484	2.63	121.1698	2.55	120.2655	2.65
119	121.4380	2.63	122.1597	2.56	121.2552	2.65
120	122.4486	2.63	123.1705	2.56	122.2656	2.65
121	123.4384	2.63	124.1605	2.56	123.2555	2.65
122	124.4489	2.63	125.1712	2.56	124.2658	2.65
123	125.4388	2.63	126.1614	2.55	125.2558	2.65
124	126.4491	2.64	127.1719	2.56	126.2660	2.65
125	127.4392	2.64	128.1622	2.56	127.2562	2.66
126	128.4493	2.64	129.1725	2.56	128.2661	2.66
127	129.4396	2.64	130.1630	2.57	129.2565	2.66
128	130.4496	2.63	131.1732	2.57	130.2663	2.66
129	131.4400	2.64	132.1638	2.57	131.2568	2.66
130	132.4498	2.64	133.1738	2.57	132.2664	2.66
131	133.4404	2.64	134.1646	2.57	133.2571	2.66
132	134.4500	2.64	135.1744	2.57	134.2666	2.66
133	135.4408	2.63	136.1653	2.57	135.2574	2.66
134	136.4502	2.64	137.1750	2.57	136.2667	2.66
135	137.4411	2.64	138.1661	2.58	137.2577	2.66
136	138.4504	2.64	139.1756	2.58	138.2669	2.66
137	139.4414	2.64	140.1668	2.58	139.2579	2.66
138	140.4506	2.65	141.1761	2.58	140.2670	2.66
139	141.4418	2.65	142.1674	2.58	141.2582	2.66
140	142.4508	2.65	143.1767	2.58	142.2671	2.66
141	143.4421	2.65	144.1681	2.58	143.2584	2.67
142	144.4510	2.65	145.1772	2.58	144.2672	2.67
143	145.4424	2.65	146.1688	2.58	145.2587	2.67
144	146.4512	2.65	147.1777	2.58	146.2674	2.67
145	147.4427	2.65	148.1694	2.59	147.2589	2.67
146	148.4513	2.65	149.1782	2.59	148.2675	2.67
147	149.4430	2.65	150.1700	2.59	149.2591	2.67
148	150.4515	2.65	151.1787	2.59	150.2676	2.67
149	151.4433	2.65	152.1706	2.59	151.2594	2.67
150	152.4516	2.65	153.1792	2.59	152.2677	2.67
151	153.4435	2.65	154.1712	2.59	153.2596	2.67
152	154.4518	2.65	155.1797	2.59	154.2678	2.67
153	155.4438	2.65	156.1718	2.59	155.2598	2.67
154	156.4520	2.66	157.1801	2.59	156.2679	2.67
155	157.4440	2.66	158.1723	2.60	157.2600	2.67
156	158.4521	2.66	159.1806	2.60	158.2680	2.67
157	159.4443	2.66	160.1729	2.60	159.2602	2.67

(*Continued*)

TABLE C.4 (CONTINUED)
External Gears: 20° Pressure Angle

	1.728 Wire Diameter		Alternate 1.92 Wire Diameter		Alternate 1.68 Wire Diameter	
No. of Teeth	***M***	K_m	***M***	K_m	***M***	K_m
158	160.4523	2.66	161.1810	2.60	160.2681	2.67
159	161.4445	2.66	162.1734	2.60	161.2604	2.67
160	162.4524	2.66	163.1814	2.60	162.2682	2.67
161	163.4448	2.66	164.1739	2.60	163.2606	2.68
162	164.4526	2.66	165.1818	2.60	164.2683	2.68
163	165.4450	2.66	166.1744	2.60	165.2608	2.68
164	166.4527	2.66	167.1822	2.60	166.2684	2.68
165	167.4453	2.66	168.1749	2.60	167.2610	2.68
166	168.4528	2.66	169.1826	2.60	168.2685	2.68
167	169.4455	2.66	170.1754	2.60	169.2611	2.68
168	170.4529	2.66	171.1830	2.61	170.2686	2.68
169	171.4457	2.66	172.1759	2.61	171.2613	2.68
170	172.4531	2.66	173.1834	2.61	172.2687	2.68
171	173.4459	2.66	174.1763	2.61	173.2615	2.68
172	174.4532	2.66	175.1838	2.61	174.2688	2.68
173	175.4461	2.66	176.1768	2.61	175.2617	2.68
174	176.4533	2.67	177.1842	2.61	176.2689	2.68
175	177.4463	2.67	178.1773	2.61	177.2619	2.68
176	178.4535	2.67	179.1846	2.61	178.2690	2.68
177	179.4465	2.67	180.1777	2.61	179.2621	2.68
178	180.4536	2.67	181.1849	2.61	180.2691	2.68
179	181.4467	2.67	182.1781	2.61	181.2622	2.68
180	182.4537	2.67	183.1852	2.61	182.2691	2.68
181	183.4469	2.67	184.1785	2.61	183.2623	2.68
182	184.4538	2.67	185.1856	2.62	184.26.92	2.68
183	185.4471	2.67	186.1789	2.62	185.2625	2.68
184	186.4539	2.67	187.1859	2.62	186.2693	2.68
185	187.4473	2.67	188.1793	2.62	187.2627	2.68
186	188.4540	2.67	189.1862	2.62	188.2694	2.68
187	189.4474	2.67	190.1797	2.62	189.2628	2.68
188	190.4541	2.67	191.1865	2.62	190.2694	2.68
189	191.4476	2.67	192.1801	2.62	191.2629	2.69
190	192.4542	2.67	193.1868	2.62	192.2694	2.69
200	202.4548	2.68	203.1883	2.63	202.2698	2.69
201	203.4487	2.68	204.1822	2.63	203.2636	2.69
300	302.4579	2.70	303.1977	2.66	302.2719	2.71
301	303.4538	2.70	304.1937	2.66	303.2678	2.71
400	402.4596	2.71	403.2026	2.68	402.2730	2.72
401	403.4565	2.71	404.1996	2.68	403.2699	2.72
500	502.4606	2.72	503.2056	2.70	502.2736	2.72
505	503.4581	2.72	504.2032	2.70	503.2711	2.72
∞	$(N + 2)$.4646	2.75	$(N + 3)$.2180	2.75	$(N + 2)$.2762	2.75

Note: Measurements over wires and change factors for 1 diametral pitch external standard-addendum 20° pressure angle spur gears with 1.728, 1.92, and 1.86 diameter wires. For any other diametral pitch, divide measurements given in the table by the diametral pitch. Change factors are the same for all diametral pitches. For 1 diametral pitch, arc tooth thickness is 1.57080 in. For 1 module, arc tooth thickness is 1.57080 mm.

is measured with three wire sizes of 0.096, 0.0864, and 0.084 in. By using Tables C.2 and C.4, the following simple computations are made:

Wire size $1P_d$ (in.)	1.92	1.728	1.68
Wire size $20P_d$ (in.)	0.096	0.0864	0.084
Measurement $1P_d$ (in.)	23.0227	22.3900	22.2249
Measurement $20P_d$ (in.)	1.1511	1.1195	1.1112
Change factor	2.07	2.26	2.33
Teeth to be thin (in.)	0.004	0.004	0.004
Reduction in measurement for thin teeth 0.004 in. × K_m (in.)	0.0083	0.0090	0.0093
Measurement with teeth 0.004 in. thin (in.)	1.1428	1.1105	1.1019

In using three sizes of wires in this case, it is possible to measure the tooth thickness at the pitch diameter and also to obtain a check on the involute profile. (If all three wire measurements indicate the *same* pitch line tooth thickness, the involute curve going through the points touched by the wires must be a *true* involute.)

Example C.3

An odd-tooth internal gear of 51 teeth, 8 diametral pitch, 20° pressure angle, is measured with 0.180 in. diameter wires, which, when placed in the tooth space of the gear, are not diametrically opposite by one-half tooth interval. From Table C.5, second column, the measurement between wires for $1P_d$ is 49.6404 in. For $8P_d$ the measurement is 49.6404 in./8, or 6.20505 in. The change factor K_m from Table C.5 is 2.71, and this, when multiplied by the amount the teeth are to be thin, will indicate the amount on increase in measurement between wires.

C.1.2 Interpretation of Results

The approximate amount the teeth are thick or thin may be determined by the change factors K_m and the amount the actual measurement differs from the computed measurement. To determine the variations in tooth thickness the following relationship is noted:

$$\Delta t = \frac{\Delta M}{K_m}, \qquad \text{(C.4)}$$

where

Δt—the amount the teeth are thick or thin

ΔM—the difference between computed and actual measurements

K_m—the change factor taken from the appropriate table

Tables C.3 through C.7 are based on the arc tooth thickness being exactly half the circular pitch. For 1 diametral pitch this value is 1.57080 in. For other diametral pitches the value is *1.57080 divided by the diametral pitch*.

When metric dimensioning is used, the arc tooth thickness value for 1 module is 1.57080 mm. For other modules the arc tooth thickness is *1.57080 multiplied by the module*.

When teeth are thinned to get backlash, the actual design tooth thickness is the thickness calculated (from the diametral pitch or the module) *minus* the amount of tooth thinning used in Equation C.4 for external teeth—or *plus* the amount of tooth thinning use in Equation C.4 for internal gears. The change in the M value is subtractive for external teeth and additive for internal gears.

C.1.3 Odd-Tooth Gears

It should be noted in Tables C.3 through C.7 that the measurement values for odd numbers of teeth represent dimensions over two wires that are not diametrically opposite to each other by one-half tooth interval. Thus, to determine the dimension from the top of one wire to the *center of the gear*, the following computation should be used:

$$\text{Dimension over wires} - \text{wire diameter} = \text{dimension between wire centers} \qquad \text{(C.5)}$$

$$\frac{\text{Dimension between wire centers}}{\cos(90°/N)} \times 0.5 + \frac{\text{wire diameter}}{2} = \text{radius from top of one wire to center of gear.} \qquad \text{(C.6)}$$

C.1.4 Special Pressure Angles and Number of Teeth

Because of the relative simplicity of the exact formulas, it is not recommended that interpolation be made from the tables in the case of special pressure angles. Where the number of teeth is greater than is to be found in the appropriate table, an interpolation may easily be made. Thus, from Table C.4 the measurement over 1.728 in. diameter wires for a 1-diametral-pitch, 335-tooth, 20° external gear would be

$$303.4538 + \left(\frac{34}{100} \times 100.0027\right) = 337.4547. \qquad \text{(C.7)}$$

C.1.5 Measurement of Racks

For 1.728 in. wires the decimal values of Tables C.3, C.4, and C.7 show the amount the measurement over wires exceeds the outside diameter of standard even-tooth gear. Therefore, if the decimal value given in nity (∞) is divided by 2, the amount the 1.728 in. wire projects above the top of the 1-pitch rack is obtained. These values are as follows:

14.5°	20°	25°
0.2779″	0.2323″	0.2241″

TABLE C.5
Internal Gears: 20° Pressure Angle

No. of Teeth	1.44 Wire Diameter		Alternate 1.68 Wire Diameter		No. of Teeth	1.44 Wire Diameter		Alternate 1.68 Wire Diameter	
	M	K_m	*M*	K_m		*M*	K_m	*M*	K_m
5	3.4090	2.30	1.68 wires too large						
6	4.6595	2.46	below 15 teeth		106	104.6650	2.73	103.7102	2.88
7	5.4823	2.43			107	105.6535	2.73	104.6989	2.88
8	6.6608	2.52			108	106.6650	2.73	105.7105	2.87
9	7.5230	2.50			109	107.6537	2.73	106.6994	2.87
10	8.6617	2.56			110	108.6651	2.73	107.7107	2.87
11	9.5490	2.55			111	109.6539	2.73	108.6998	2.87
12	10.6623	2.59			112	110.6651	2.73	109.7110	2.87
13	11.5669	2.58			113	111.6541	2.73	110.7002	2.87
14	12.6627	2.61			114	112.6651	2.73	111.7112	2.87
15	13.5801	2.60	12.4222	8.58	115	113.6543	2.73	112.7006	2.87
16	14.6630	2.63	13.5459	5.95	116	114.6651	2.73	113.7114	2.86
17	15.5902	2.62	14.5026	5.09	117	115.6545	2.73	114.7010	2.86
18	16.6633	2.64	15.5901	4.65	118	116.6651	2.73	115.7117	2.86
19	17.5981	2.63	16.5414	4.35	119	117.6547	2.73	116.7014	2.86
20	18.6635	2.65	17.6147	4.15	120	118.6651	2.73	117.7119	2.86
21	19.6045	2.64	18.5668	3.98	121	119.6548	2.73	118.7018	2.86
22	20.6636	2.66	19.6311	3.87	122	120.6651	2.73	119.7121	2.86
23	21.6099	2.65	20.5854	3.76	123	12.6550	2.73	120.7022	2.86
24	22.6638	2.67	21.6429	3.69	124	122.6651	2.73	121.7123	2.86
25	23.6143	2.66	22.5997	3.61	125	123.6552	2.73	122.7026	2.86
26	24.6639	2.67	23.6520	3.56	126	124.6651	2.73	123.7125	2.85
27	25.6181	2.67	24.6112	3.50	127	125.6554	2.73	124.7029	2.85
28	26.6640	2.67	25.6591	3.46	128	126.6651	2.73	125.7127	2.85
29	27.6214	2.67	26.6206	3.42	129	127.6556	2.73	126.7032	2.85
30	28.6641	2.68	27.6649	3.38	130	128.6652	2.73	127.7129	2.85
31	29.6242	2.68	28.6285	3.35	131	129.6557	2.73	128.7036	2.85
32	30.6642	2.68	29.6699	3.32	132	130.6652	2.73	129.7130	2.85
33	31.6267	2.68	30.6353	3.29	133	131.6559	2.73	130.7039	2.85
34	32.6642	2.69	31.6739	3.27	134	132.6652	2.73	131.7132	2.85
35	33.6289	2.69	32.6411	3.25	135	133.6560	2.73	132.7042	2.85
36	34.6643	2.69	33.6773	3.23	136	134.6652	2.73	133.7134	2.85
37	35.6310	2.69	34.6462	3.21	137	135.6561	2.73	134.7045	2.84
38	36.6643	2.69	35.6804	3.20	138	136.6652	2.73	135.7135	2.84
39	37.6327	2.69	36.6507	3.18	139	137.6563	2.73	136.7047	2.84
40	38.6644	2.70	37.6831	3.16	140	138.6652	2.73	137.7137	2.84
41	39.6343	2.69	38.6547	3.15	141	139.6564	2.73	138.7050	2.84
42	40.6644	2.70	39.6855	3.14	142	140.6652	2.73	139.7139	2.84
43	41.6357	2.70	40.6582	3.13	143	141.6565	2.73	140.7053	2.84
44	42.6645	2.70	41.6975	3.11	144	142.6652	2.73	141.7140	2.84
45	43.6371	2.70	42.6614	3.10	145	143.6566	2.73	142.7055	2.84
46	44.6645	2.70	43.6893	3.09	146	144.6652	2.73	143.7141	2.84
47	45.6383	2.70	44.6644	3.08	147	145.6568	2.73	144.7058	2.84
48	46.6646	2.70	45.6910	3.08	148	146.6652	2.73	145.7143	2.84
49	47.6394	2.70	46.6670	3.07	149	147.6569	2.73	146.7061	2.84
50	48.6646	2.71	47.6926	3.06	150	148.6652	2.73	147.7144	2.84
51	49.6404	2.71	48.6694	3.05	151	149.6570	2.73	148.7063	2.84
52	50.6646	2.71	49.6940	3.04	152	150.6652	2.73	149.7145	2.83
53	51.6414	2.71	50.6716	3.04	153	151.6571	2.73	150.7065	2.83
54	52.6647	2.71	51.6953	3.03	154	152.6652	2.73	151.7146	2.83
55	53.6422	2.71	52.6737	3.02	155	153.6572	2.73	152.7068	2.83
56	54.6647	2.71	53.6965	3.02	156	154.6652	2.73	153.7148	2.83
57	55.6431	2.71	54.6756	3.01	157	155.6573	2.73	154.7070	2.83

(*Continued*)

TABLE C.5 (CONTINUED)
Internal Gears: 20° Pressure Angle

No. of Teeth	1.44 Wire Diameter		Alternate 1.68 Wire Diameter		No. of Teeth	1.44 Wire Diameter		Alternate 1.68 Wire Diameter	
	M	K_m	M	K_m		M	K_m	M	K_m
58	56.6648	2.71	55.6975	3.01	158	156.6652	2.73	155.7149	2.83
59	57.6438	2.71	56.6774	3.00	159	157.6574	2.73	156.7072	2.83
60	58.6648	2.71	57.6985	3.00	160	158.6652	2.73	157.7150	2.83
61	59.6445	2.71	58.6789	2.99	161	159.6575	2.73	158.7074	2.83
62	60.6648	2.71	59.6994	2.99	162	160.6652	2.73	159.7151	2.83
63	61.6452	2.71	60.6805	2.98	163	161.6576	2.73	160.7076	2.83
64	62.6648	2.71	61.7003	2.98	164	162.6652	2.73	161.7152	2.83
65	63.6458	2.71	62.6819	2.97	165	163.6577	2.73	162.7078	2.83
66	64.6649	2.72	63.7011	2.97	166	164.6652	2.73	163.7153	2.83
67	65.6464	2.72	64.6832	2.96	167	165.6578	2.73	164.7080	2.83
68	66.6649	2.72	65.7018	2.96	168	166.6652	2.73	165.7154	2.83
69	67.6469	2.72	66.6844	2.96	169	167.6579	2.73	166.7082	2.83
70	68.6649	2.72	67.7025	2.95	170	168.6652	2.73	167.7156	2.82
71	69.6475	2.72	68.6856	2.95	171	169.6580	2.73	168.7084	2.82
72	70.6649	2.72	69.7032	2.95	172	170.6652	2.73	169.7157	2.82
73	71.6480	2.72	70.6867	2.94	173	171.6581	2.73	170.7086	2.82
74	72.6649	2.72	71.7038	2.94	174	172.6652	2.73	171.7158	2.82
75	73.6484	2.72	72.6878	2.94	175	173.6582	2.73	172.7087	2.82
76	74.6649	2.72	73.7044	2.94	176	174.6652	2.73	173.7158	2.82
77	75.6489	2.72	74.6888	2.93	177	175.6583	2.73	174.7089	2.82
78	76.6649	2.72	75.7049	2.93	178	176.6652	2.73	175.7159	2.82
79	77.6493	2.72	76.6897	2.93	179	177.6584	2.73	176.7090	2.82
80	78.6649	2.72	77.7054	2.92	180	178.6652	2.73	177.7160	2.82
81	79.6497	2.72	78.6905	2.92	181	179.6584	2.73	178.7092	2.82
82	80.6649	2.72	79.7059	2.92	182	180.6652	2.73	179.7161	2.82
83	81.6501	2.72	80.6914	2.92	183	181.6585	2.73	180.7094	2.82
84	82.6649	2.72	81.7064	2.91	184	182.6652	2.73	181.7162	2.82
85	83.6505	2.72	82.6922	2.91	185	183.6586	2.73	182.7095	2.82
86	84.6650	2.72	83.7068	2.91	186	184.6652	2.73	183.7162	2.82
87	85.6508	2.72	84.6929	2.91	187	185.6587	2.73	184.7097	2.82
88	86.6650	2.72	85.7072	2.91	188	186.6652	2.73	185.7163	2.82
89	87.6511	2.72	86.6936	2.90	189	187.6588	2.73	186.7098	2.82
90	88.6650	2.72	87.7076	2.90	190	188.6652	2.73	187.7164	2.82
91	89.6514	2.72	88.6943	2.90	200	198.6652	2.74	197.7168	2.81
92	90.6650	2.72	89.7080	2.90	201	199.6591	2.74	198.7107	2.81
93	91.6517	2.72	90.6950	2.90	300	298.6654	2.74	297.7192	2.79
94	92.6650	2.72	91.7-84	2.89	301	299.6612	2.74	298.7151	2.79
95	93.6520	2.72	92.6956	2.89					
96	94.6650	2.73	93.7087	2.89	400	398.6654	2.74	397.7203	2.78
97	95.6523	2.73	94.6962	2.89	401	399.6623	2.74	398.7172	2.78
98	96.6650	2.73	95.7090	2.89	500	498.6654	2.74	497.7210	2.77
99	97.6526	2.73	96.6968	2.89	501	499.6629	2.74	498.7185	2.77
100	98.6650	2.73	97.7093	2.88	∞	$(N-2)$.6652	2.75	$(N-3)$.7238	2.75
101	99.6528	2.73	98.6974	2.88					
102	100.6650	2.73	99.7096	2.88					
103	101.6531	2.73	100.6979	2.88					
104	102.6650	2.73	101.7099	2.88					
105	103.6533	2.73	102.6984	2.88					

Note: Measurements between wires and change factors for 1 diametral pitch internal standard-addendum 20° pressure angle with 1.44 and 1.68 in. diameter wires. For any other diametral pitch, divide measurements given in table by the diametral pitch. Change factors are the same for all diametral pitches. For 1 diametral pitch, arc tooth thickness is 1.57080 in. For 1 module, arc tooth thickness is 1.57080 mm.

TABLE C.6
Internal Gears: 14.5 Pressure Angle

No. of Teeth	1.44 Wire Diameter		Alternate 1.68 Wire Diameter		No. of Teeth	1.44 Wire Diameter		Alternate 1.68 Wire Diameter	
	M	K_m	*M*	K_m		*M*	K_m	*M*	K_m
5	3.5517	2.38	1.68 wires too large						
6	4.8157	2.61	below 15 teeth		106	104.6650	2.73	103.7102	2.88
7	5.6393	2.63			107	105.6535	2.73	104.6989	2.88
8	6.8262	2.77			108	106.6650	2.73	105.7105	2.87
9	7.6894	2.79			109	107.6537	2.73	106.6994	2.87
10	8.8337	2.89			110	108.6651	2.73	107.7107	2.87
11	9.7219	2.91			111	109.6539	2.73	108.6998	2.87
12	10.8394	2.99			112	110.6651	2.73	109.7110	2.87
13	11.7449	3.01			113	111.6541	2.73	110.7002	2.87
14	12.8438	3.07			114	112.6651	2.73	111.7112	2.87
15	13.7620	3.09			115	113.6543	2.73	112.7006	2.87
16	14.8474	3.13			116	114.6651	2.73	113.4114	2.86
17	15.7752	3.15			117	115.6545	2.73	114.7010	2.86
18	16.8504	3.19			118	116.6651	2.73	115.7117	2.86
19	17.7858	3.20			119	117.6547	2.73	116.7014	2.86
20	18.8529	3.24			120	118.6651	2.73	117.7119	2.86
21	19.7945	3.25			121	119.6548	2.73	118.7018	2.86
22	20.8550	3.27			122	120.6651	2.73	119.7121	2.86
23	21.8017	3.29			123	121.6550	2.73	120.7022	2.86
24	22.8569	3.31			124	122.6651	2.73	121.7123	2.86
25	23.8078	3.32			125	123.6552	2.73	122.7026	2.86
26	24.8585	3.34			126	124.6651	2.73	123.7125	2.85
27	25.8130	3.35			127	125.6554	2.73	124.7029	2.85
28	26.8599	3.37			128	126.6651	2.73	125.7127	2.85
29	27.8176	3.38			129	127.6556	2.73	126.7032	2.85
30	28.8612	3.40			130	128.6652	2.73	127.7129	2.85
31	29.8216	3.40			131	129.6557	2.73	128.7036	2.85
32	30.8623	3.42	29.4774	9.11	132	130.6652	2.73	129.7130	2.85
33	31.8251	3.43	30.4589	8.18	133	131.6559	2.73	130.7039	2.85
34	32.8633	3.44	31.5089	7.57	134	132.6652	2.73	131.7132	2.85
35	33.8282	3.45	32.4867	7.14	135	133.6560	2.73	132.7042	2.85
36	34.8642	3.46	33.5314	6.81	136	134.6652	2.73	133.7134	2.85
37	35.8311	3.46	34.5071	6.55	137	135.6561	2.73	134.7045	2.84
38	36.8650	3.47	35.5472	6.34	138	136.6652	2.73	135.7135	2.84
39	37.8336	3.48	36.5229	6.15	139	137.6563	2.73	136.7047	2.84
40	38.8658	3.49	37.5599	6.00	140	138.6652	2.73	137.7137	2.84
41	39.8359	3.50	38.5357	5.86	141	139.6564	2.73	138.7050	2.84
42	40.8665	3.51	39.5702	5.75	142	140.6652	2.73	139.7139	2.84
43	41.8380	3.51	40.5463	5.65	143	141.6565	2.73	140.7053	2.84
44	42.8672	3.52	41.5788	5.56	144	142.6652	2.73	141.7140	2.84
45	43.8399	3.52	42.5556	5.48	145	143.6566	2.73	142.7055	2.84
46	44.8678	3.53	43.5861	5.41	146	144.6652	2.73	143.7141	2.84
47	45.8416	3.54	44.5635	5.34	147	145.6568	2.73	144.7058	2.84
48	46.8683	3.54	45.5924	5.28	148	146.6652	2.73	145.7143	2.84
49	47.8432	3.55	46.5704	5.22	149	147.6569	2.73	146.7061	2.84
50	48.8688	3.56	47.5979	5.17	150	148.6652	2.73	147.7144	2.84
51	49.8447	3.56	48.5765	5.13	151	149.6570	2.73	148.7063	2.84
52	50.8692	3.57	49.6027	5.08	152	150.6652	2.73	149.7145	2.83
53	51.8461	3.57	50.5820	5.04	153	151.6571	2.73	150.7065	2.83
54	52.8697	3.58	51.6070	5.00	154	152.6652	2.73	151.7146	2.83
55	53.8474	3.58	52.5869	4.97	155	153.6572	2.73	152.7068	2.83
56	54.8701	3.58	53.6109	4.93	156	154.6652	2.73	153.7148	2.83

(*Continued*)

TABLE C.6 (CONTINUED)
Internal Gears: 14.5 Pressure Angle

No. of Teeth	1.44 Wire Diameter		Alternate 1.68 Wire Diameter	
	M	K_m	M	K_m
57	55.8486	3.59	54.5913	4.90
58	56.8705	3.59	55.6144	4.87
59	57.8497	3.60	56.5953	4.85
60	58.8709	3.60	57.6175	4.82
61	59.8508	3.60	58.5990	4.80
62	60.8712	3.61	59.6204	4.77
63	61.8517	3.61	60.6024	4.75
64	62.8715	3.61	61.6230	4.73
65	63.8526	3.62	62.6055	4.71
66	64.8718	3.62	63.6254	4.69
67	65.8535	3.63	64.6083	4.67
68	66.8721	3.63	65.6277	4.65
69	67.8543	3.63	66.6110	4.64
70	68.8724	3.63	67.6297	4.62
71	69.8551	3.63	68.6135	4.60
72	70.8727	3.64	69.6316	4.59
73	71.8558	3.64	70.6158	4.58
74	72.8729	3.65	71.6334	4.56
75	73.8565	3.65	72.6179	4.55
76	74.8731	3.65	73.6351	4.54
77	75.8572	3.65	74.6199	4.52
78	76.8734	3.65	75.6366	4.51
79	77.8678	3.66	76.6218	4.50
80	78.8736	3.66	77.6381	4.49
81	79.8584	3.66	78.6236	4.48
82	80.8738	3.66	79.6395	4.47
83	81.8590	3.67	80.6253	4.46
84	82.8740	3.67	81.6408	4.45
85	83.8595	3.67	82.6270	4.44
86	84.8742	3.67	83.6420	4.43
87	85.8600	3.67	84.6285	4.42
88	86.8743	3.68	85.6431	4.41
89	87.8605	3.68	86.6299	4.41
90	88.8745	3.68	87.6442	4.40
91	89.8610	3.68	88.6313	4.39
92	90.8747	3.68	89.6452	4.38
93	91.8614	3.69	90.6326	4.38
94	92.8749	3.69	91.6462	4.37
95	93.8619	3.69	92.6338	4.36
96	94.8750	3.69	93.6472	4.36
97	95.8623	3.69	94.6350	4.35
98	96.8752	3.69	95.6481	4.34
99	97.8627	3.70	96.6361	4.34
100	98.8753	3.70	97.6489	4.33
101	99.8631	3.70	98.6372	4.33
102	100.8754	3.70	99.6497	4.32
103	101.8635	3.70	100.6382	4.31
104	102.8756	3.70	101.6505	4.31
105	103.8638	3.70	102.6392	4.30
157	155.6573	2.73	154.7070	2.83
158	156.6652	2.73	155.7149	2.83
159	157.6574	2.73	156.7072	2.83
160	158.6652	2.73	157.7150	2.83
161	159.6575	2.73	158.7074	2.83
162	160.6652	2.73	159.7151	2.83
163	161.6576	2.73	160.7076	2.83
164	162.6652	2.73	161.7152	2.83
165	163.6577	2.73	162.7078	2.83
166	164.6652	2.73	163.7153	2.83
167	165.6578	2.73	164.7080	2.83
168	166.6652	2.73	165.7154	2.83
169	167.6579	2.73	166.7082	2.83
170	168.6652	2.73	167.7156	2.82
171	169.6580	2.73	168.7084	2.82
172	170.6652	2.73	169.7157	2.82
173	171.6581	2.73	170.7086	2.82
174	172.6652	2.73	171.7158	2.82
175	173.6582	2.73	172.7087	2.82
176	174.6652	2.73	173.7158	2.82
177	175.6583	2.73	174.7089	2.82
178	176.6652	2.73	175.7159	2.82
179	177.6584	2.73	176.7090	2.82
180	178.6652	2.73	177.7160	2.82
181	179.6584	2.73	178.7092	2.82
182	180.6652	2.73	179.7161	2.82
183	181.6585	2.73	180.7094	2.82
184	182.6652	2.73	181.7162	2.82
185	183.6586	2.73	182.7095	2.82
186	184.6652	2.73	183.7162	2.82
187	185.6587	2.73	184.7097	2.82
188	186.6652	2.73	185.7163	2.82
189	187.6588	2.73	186.7098	2.82
190	188.6652	2.73	187.7164	2.82
200	198.6652	2.74	197.7168	2.81
201	199.6591	2.74	198.7107	2.81
300	298.6654	2.74	297.7192	2.79
301	299.6612	2.74	298.7151	2.79
400	398.6654	2.74	397.7203	2.78
401	399.6623	2.74	398.7172	2.78
500	498.6654	2.74	497.7210	2.77
501	499.6629	2.74	498.7185	2.77
∞	$(N-2)$.6655	2.75	$(N-3)$.7238	2.75

Note: Measurements between wires and change factors for 1 diametral pitch internal standard-addendum 14.5°-pressure angle with 1.44 in. and 1.68 in. diameter wires. For any other diametral pitch divide measurements given in table by the diametral pitch. Change factors are the same for all diametral pitches. For 1 diametral pitch, arc tooth thickness is 1.57080 in. For 1 module, arc tooth thickness is 1.57080 mm.

TABLE C.7
External Gears: 25° Pressure Angle

No. of Teeth	1.728 Wire Diameter		Alternate 1.68 Wire Diameter		No. of Teeth	1.728 Wire Diameter		Alternate 1.68 Wire Diameter	
	M	K_m	*M*	K_m		*M*	K_m	*M*	K_m
5	7.0472	1.47	6.9202	1.59					
6	8.3340	1.60	8.2003	1.64	56	58.4287	2.03	58.2726	2.05
7	9.1536	1.64	9.0199	1.68	57	59.4071	2.03	59.2509	2.05
8	10.3533	1.67	10.2155	1.72	58	60.4293	2.04	60.2730	2.05
9	11.2142	1.70	11.0762	1.75	59	61.4084	2.04	61.2521	2.05
10	12.3667	1.73	12.2260	1.77	60	62.4299	2.04	62.2735	2.05
11	13.2536	1.75	13.1126	1.79	61	63.4097	2.04	63.2532	2.06
12	14.3768	1.77	14.2338	1.81	62	64.4304	2.04	64.2739	2.06
13	15.2814	1.79	15.1381	1.83	63	65.4109	2.04	65.2543	2.06
14	16.3846	1.81	16.2397	1.85	64	66.4309	2.04	66.2742	2.06
15	17.3021	1.83	17.1570	1.86	65	67.4120	2.05	67.2553	2.06
16	18.3908	1.84	18.2445	1.88	66	68.4314	2.05	68.2746	2.06
17	19.3181	1.85	19.1716	1.89	67	69.4130	2.05	69.2562	2.06
18	20.3959	1.86	20.2483	1.90	68	70.4319	2.05	70.2749	2.06
19	21.3310	1.88	21.1832	1.91	69	71.4140	2.05	71.2571	2.06
20	22.4002	1.88	22.2515	1.92	70	72.4323	2.05	72.2752	2.07
21	23.3415	1.89	23.1926	1.92	71	73.4150	2.05	73.2579	2.07
22	24.4038	1.90	24.2542	1.93	72	74.4327	2.05	74.2755	2.07
23	25.3502	1.91	25.2005	1.94	73	75.4159	2.06	75.2586	2.07
24	26.4069	1.92	26.2566	1.95	74	76.4331	2.06	76.2758	2.07
25	27.3576	1.93	27.2071	1.95	75	77.4167	2.06	77.2594	2.07
26	28.4096	1.93	28.2586	1.96	76	78.4335	2.06	78.2761	2.07
27	29.3640	1.94	29.2128	1.96	77	79.4175	2.06	79.2601	2.07
28	30.4120	1.94	30.2603	1.97	78	80.4339	2.06	80.2763	2.07
29	31.3695	1.95	31.2177	1.97	79	81.4183	2.06	81.2607	2.07
30	32.4141	1.95	32.2619	1.98	80	82.4342	2.06	82.2766	2.08
31	33.3743	1.96	33.2220	1.98	81	83.4190	2.06	83.2614	2.08
32	34.4159	1.96	34.2632	1.99	82	84.4345	2.07	84.2768	2.08
33	35.3786	1.97	35.2258	1.99	83	85.4196	2.07	85.2620	2.08
34	36.4176	1.97	36.2664	2.00	84	86.4348	2.07	86.2771	2.08
35	37.3824	1.98	37.2292	2.00	85	87.4203	2.07	87.2625	2.08
36	38.4191	1.98	38.2655	2.00	86	88.4351	2.07	88.2773	2.08
37	39.3858	1.98	39.2323	2.01	87	89.4209	2.07	89.2631	2.08
38	40.4205	1.99	40.2666	2.01	88	90.4354	2.07	90.2775	2.08
39	41.3889	1.99	41.2349	2.01	89	91.4215	2.07	91.2636	2.08
40	42.4217	2.00	42.2675	2.02	90	92.4357	2.07	92.2777	2.08
41	43.3917	2.00	43.2374	2.02	91	93.4221	2.07	93.2641	2.08
42	44.4228	2.00	44.2683	2.02	92	94.4359	2.07	94.2779	2.08
43	45.3942	2.00	45.2396	2.02	93	95.4227	2.07	95.2646	2.08
44	46.4239	2.01	46.2690	2.02	94	96.4362	2.07	96.2780	2.08
45	47.3965	2.01	47.2417	2.02	95	97.4232	2.08	97.2650	2.09
46	48.4248	2.01	48.2697	2.03	100	102.4369	2.08	102.2785	2.09
47	49.3986	2.02	49.2435	2.03	101	103.4247	2.08	103.2663	2.09
48	50.4257	2.02	50.2740	2.03	200	202.4424	2.11	202.2825	2.11
49	51.4006	2.02	51.2452	2.03	201	203.4363	2.11	203.2764	2.11
50	52.4265	2.02	52.2710	2.04	300	302.4443	2.12	302.2839	2.12
51	53.4024	2.02	53.2468	2.04	301	303.4402	2.12	303.2798	2.12

(*Continued*)

TABLE C.7 (CONTINUED)
External Gears: 25° Pressure Angle

No. of Teeth	1.728 Wire Diameter		Alternate 1.68 Wire Diameter		No. of Teeth	1.728 Wire Diameter		Alternate 1.68 Wire Diameter	
	M	K_m	M	K_m		M	K_m	M	K_m
52	54.4273	2.02	54.2716	2.04	400	402.4453	2.13	402.2845	2.13
53	55.4041	2.02	55.2483	2.04	401	403.4422	2.13	403.2815	2.13
54	56.4280	2.02	56.2721	2.05	500	502.4458	2.13	502.2850	2.13
55	57.4056	2.03	57.2497	2.05	∞	$(N + 2).4482$	2.14	$(N - 2).2866$	2.14

Note: Measurements between wires and change factors for 1 diametral pitch internal standard-addendum 25° pressure angle with 1.44 and 1.68 in. diameter wires. For any other diametral pitch divide measurements given in table by the diametral pitch. Change factors are the same for all diametral pitches. For 1 diametral pitch, arc tooth thickness is 1.57080 in. For 1 module, arc tooth thickness is 1.57080 mm.

Since the addendum of a 1-pitch gear is 1.0000 in., the amount the wire projects above the pitch line is 1.0000 plus the above values.

For any other diametral pitch P_d, divide the values by P_d and use wires 1.728 in./P_d in diameter.

C.1.6 Relation between Depth of Cut and Tooth Thickness

If the generation gear cutter is fed into the blank an amount exactly equal to the whole dept of tooth, the gear is cut without backlash. To provide backlash, the teeth must be made thinner by an equal amount Δt (increment of t). The formulas for nding the excess depth of cut E from Δt and vice versa are as given in Table C.8. The relationships are exact for hobs or rack-shaped tools. They are approximate for gear shaper cutters.

Example C.4

A total backlash of 0.016 in. is desired between a 30-tooth, 14½° pinion and a 100-tooth, 14½° gear; and 0.008 in. backlash will be taken on the pinion and 0.008 in. on the gear. From Table C.3, the wire measurement for the pinion should be 2.98 × 0.008 in. = 0.0238 in. under the computed values, and for the gear it should be 3.48 × 0.008 in. = 0.0278 in. undersize. If after a trial cut, the wire measurement on the gear is only 0.010 in. under the computed value, the backlash provided is 0.010/3.48 = 0.0029 in. This is 0.008 in. − 0.0029 in. = 0.0051 in.—too little—and, from Table C.8, the cutter must be fed in deeper by 1.93 × 0.0051 in. = 0.0098 in.

TABLE C.8
Change in Depth of Cut versus Change in Tooth Thickness

Pressure Angle of Gear (°)	E, Extra Depth of Cut to Thin Depth (Δt)	Δt, Amount Teeth Are Cut Too Thin due to Excess of Cut E
14½	$E = 1.93 \times \Delta t$	$\Delta t = \frac{E}{1.93} = 0.52E$
17½	$E = 1.59 \times \Delta t$	$\Delta t = \frac{E}{1.59} = 0.63E$
20	$E = 1.37 \times \Delta t$	$\Delta t = \frac{E}{1.37} = 0.73E$
25	$E = 1.07 \times \Delta t$	$\Delta t = \frac{E}{1.07} = 0.93E$
30	$E = 0.87 \times \Delta t$	$\Delta t = \frac{E}{0.87} = 1.15E$

C.1.7 Helical Gears

The pin measurement to helical gears may be computed either using conventional approach, an approximate value of it can be computed utilizing for this purpose the spur gear measurement tables (see Table C.9).

Even-tooth helical gears should be measured with two wires. Odd-tooth helical gears should not be measured with two wires but may be measured with three wires under certain conditions, or indirectly with one wire. Three wires may be used, provided the face width and the helix angle of the gear will permit the arrangement of two wires in adjacent tooth spaces on one side of the gear and a third wire in a tooth space on the opposite side, so that all three wires will automatically adjust themselves to line contact with a micrometer's measuring surfaces. In cases where three wires cannot be used, the gear should be mounted on an arbor and a radial measurement made from the top of one wire to the axis of the gear. See Table C.10.

It should be emphasized that the practice of using two wires for measuring odd-tooth helical gears and correcting the results by the application of a cos (90°/N) factor is not valid and may result in serious errors.

C.2 TRIGONOMETRIC FUNCTIONS

Table C.11 lists some trigonometric functions for general reference. Those doing an extensive amount of gear calculating will nd it necessary to have a whole book of seven-place trigonometric tables. With modern computer equipment for engineering work, trigonometric values are immediately available from the computer.

TABLE C.9
Wire Measurement of Standard Helical Gear

Date: Month, Day, Year Drawing No.: N 00-0000

For A–Z Gear Co. Data from Drawing		**Functions of ψ (psi)**	
1. Number of teeth	$N = 14$	cos 40°	= 0.7660444
2. Diametral pitch in plane of rotation	$P_d = 18.3850656$	$\dfrac{1}{\cos 40°}$	= 1.305407
3. Pitch diameter	$D = 07615''$	$\dfrac{1}{(\cos 40°)^3}$	= 2.224529
4. Helix angle (psi) with axis ($\tan\psi = \pi D/L$)	cos 40°		
5. Normal pressure angle (phi sub n)	$\phi_n = 14.5°$		
6. Normal tooth thickness	$T_n = 0.0654''$		

Computations

7. Find normal diametral pitch (may show on drawing)

$$P_n = P_d \times \frac{1}{\cos\psi} = 18.38 \times 1.305407 = 24$$

8. Find wire diameter

$$G = \frac{1.728''}{P_n} = \frac{1.728''}{24} = 0.072''$$

9. Find number or teeth in equivalent spur gear

$$N_e = N \times \frac{1}{(\cos 40°)^3} = 14 \times 2.224529 = 31.143406$$

Data from Tables (see Table C.4)

10. Refer to the proper table and using *even-tooth values* only interpolate for values ($M - D$) for N_e teeth (for 1 diametral pitch gears, $D = N$):

	1 Diametral Pitch Values				
	N	***D***	***M***	**(*M* − *D*)**	**Diff.**
Next lower (even)	(a) 30	(c) 30″	(d) 32.4429″	(e) 2.4429″	(f) 0.0049″
Next higher (even)	(b) 32	32″	34.4478″	2.4478″	

N_e value (9) = 31.143406
N value (a) = 30
Diff. (g) = 1.1434

$$(M - D) = \overset{(e)}{2.4429''} + \overset{(g)\ (f)}{\frac{1.1434}{2} \times 0.0049''} = 2.2457''$$

11. Find ($M - D$) for speci ed normal diametral pitch

$$(M - D)_1 = \frac{(M - D)}{P_n} = \frac{2.4457''}{24} = 0.1019''$$

12. Find measurement for odd-tooth gear

$$M_1 = (M - D)_1 + D = 0.1019'' + 0.7615'' = 0.8634''$$

13. Measurement for odd-tooth gear

$M_1/2$ = (odd-tooth helical gears should be mounted on an arbor and a radial measurement should be made from one wire to the center of the gear)

14. Reduction in measurement over wires to thin teeth 0.002″ in normal plane (Table C.10)

$$\Delta M = \frac{K \times t_n}{\cos\psi} = \frac{2.33 \times 0.002''}{0.7660444} = 0.0061''$$

15. Measurement over wires for teeth 0.002″ thin in normal plane

$$M_2 = M_1 - \Delta M = 0.8634'' - 0.0061'' = 0.8573''$$

Note: For use when normal pressure angle is 14½°, 20°, or 25°.

C.3 INVOLUTE FUNCTIONS

The involute function is widely used in gear calculations. A short table of involute functions for general reference is given in this section. The involute functions of an angle is called inv ϕ or simply θ; inv ϕ equals θ in radians. See Table C.12.

C.4 ARC AND CHORD DATA

Gear teeth are often measured by vernier calipers. The addendum setting for the calipers has to be made a slight amount larger than the gear addendum. Conversely, the chordal tooth thickness value is a slight amount smaller than the arc tooth thickness of the part. Table C.13 illustrates the rise of arc h_c

TABLE C.10

Change Factors *K* for 1.728 in. Wires Used on External Helical Gears

	Normal Pressure Angle = 14.5°							Normal Pressure Angle = 20°						
No. of Teeth	**ψ = 15°**	**ψ = 20°**	**ψ = 25°**	**ψ = 30°**	**ψ = 35°**	**ψ = 40°**	**ψ = 45°**	**ψ = 15°**	**ψ = 20°**	**ψ = 25°**	**ψ = 30°**	**ψ = 35°**	**ψ = 40°**	**ψ = 45°**
6	2.07	2.06	2.05	2.03	2.00	1.96	1.91	1.81	1.79	1.77	1.73	1.69	1.64	1.58
10	2.34	2.33	2.30	2.27	2.23	2.17	2.10	2.02	2.00	1.96	1.92	1.85	1.78	1.69
15	2.56	2.56	2.54	2.49	2.42	2.37	2.25	2.14	2.12	2.07	2.01	1.93	1.84	1.75
20	2.72	2.69	2.65	2.59	2.52	2.44	2.32	2.22	2.18	2.13	2.07	1.99	1.90	1.79
25	2.81	2.77	2.73	2.66	2.58	2.49	2.35	2.28	2.24	2.18	2.11	2.03	1.93	1.81
30	2.89	2.84	2.80	2.71	2.63	2.53	2.39	2.33	2.28	2.22	2.14	2.05	1.95	1.83
35	2.95	2.90	2.86	2.76	2.68	2.56	2.42	2.36	2.31	2.25	2.17	2.08	1.97	1.84
40	3.01	2.96	2.91	2.81	2.72	2.59	2.44	2.40	2.34	2.28	2.20	2.10	1.98	1.85
45	3.07	3.01	2.96	2.85	2.75	2.62	2.46	2.42	2.36	2.30	2.21	2.12	2.00	1.86
50	3.12	3.06	3.00	2.88	2.78	2.64	2.48	2.44	2.38	2.32	2.23	2.13	2.01	1.87
60	3.20	3.13	3.07	2.95	2.83	2.69	2.52	2.47	2.41	2.34	2.25	2.15	2.03	1.89
70	3.27	3.20	3.12	3.00	2.88	2.72	2.55	2.49	2.43	2.36	2.27	2.16	2.04	1.90
80	3.32	3.25	3.16	3.05	2.92	2.76	2.58	2.51	2.44	2.38	2.28	2.17	2.05	1.90
90	3.37	3.29	3.20	3.08	2.95	2.79	2.61	2.53	2.46	2.39	2.29	2.18	2.06	1.91
100	3.40	3.33	3.23	3.12	2.98	2.81	2.63	2.54	2.47	2.40	2.30	2.19	2.06	1.91
120	3.45	3.37	3.26	3.16	3.02	2.84	2.65	2.56	2.49	2.42	2.31	2.20	2.07	1.91
140	3.48	3.40	3.29	3.18	3.04	2.86	2.67	2.57	2.50	2.42	2.32	2.21	2.07	1.91
160	3.51	3.42	3.31	3.20	3.06	2.87	2.68	2.58	2.51	2.43	2.32	2.22	2.07	1.91
180	3.53	3.45	3.33	3.22	3.07	2.88	2.69	2.59	2.52	2.44	2.33	2.23	2.07	1.91
200	3.55	3.46	3.35	3.23	3.08	2.89	2.69	2.60	2.52	2.44	2.34	2.23	2.07	1.92
250	3.59	3.50	3.38	3.25	3.09	2.90	2.70	2.61	2.53	2.45	2.35	2.23	2.08	1.92
300	3.61	3.52	3.40	3.26	3.10	2.91	2.70	2.62	2.54	2.46	2.37	2.23	2.09	1.93
350	3.63	3.53	3.41	3.27	3.11	2.92	2.70	2.62	2.54	2.46	2.37	2.23	2.09	1.93
400	3.64	3.54	3.42	3.28	3.11	2.92	2.71	2.63	2.55	2.47	2.37	2.24	2.09	1.94
450	3.65	3.55	3.43	3.29	3.12	2.93	2.71	2.64	2.56	2.47	2.37	2.24	2.09	1.94
500	3.66	3.56	3.44	3.30	3.12	2.93	2.71	2.64	2.56	2.47	2.37	2.24	2.09	1.94

Note: $K = \dfrac{\cos\phi}{\sin\phi_w}$; Reduction in measurement = K × amount teeth are to be thin (on normal plane) ÷ cos ψ, or reduction in measurement = K × amount teeth are to be thin (on the plane of rotation of the gear); The amount of reduction is the same for all diametral pitches for a given tooth-thickness reduction.

TABLE C.11
Trigonometric Functions

ϕ (°)	arc ϕ	sin ϕ	tan ϕ	cot ϕ	cos ϕ	sec ϕ	cosec ϕ
1	0.01745	0.01745	0.01746	57.2900	0.99985	1.0002	57.2990
2	0.03491	0.03490	0.03492	28.6360	0.99939	1.0006	28.6540
3	0.05236	0.05234	0.05241	19.0810	0.99863	1.0014	19.1070
4	0.06981	0.06976	0.06993	14.3010	0.99756	1.0024	14.3360
5	0.08727	0.08716	0.08749	11.4300	0.99619	1.0038	11.4740
6	0.10472	0.10453	0.10510	9.5144	0.99452	1.0055	9.5668
7	0.12217	0.12187	0.12278	8.1444	0.99255	1.0075	8.2055
8	0.13963	0.13917	0.14054	7.1154	0.99027	1.0098	7.1853
9	0.15708	0.15643	0.15838	6.3138	0.98769	1.0125	6.3925
10	0.17453	0.17365	0.17633	5.6713	0.98481	1.0154	5.7588
11	0.19199	0.19081	0.19438	5.1446	0.98163	1.0187	5.2408
12	0.20944	0.20791	0.21256	4.7046	0.97815	1.0223	4.8097
13	0.22680	0.22495	0.23087	4.3315	0.97437	1.0263	4.4454
14	0.24435	0.24192	0.24933	4.0108	0.97030	1.0306	4.1336
15	0.26180	0.25882	0.26795	3.7321	0.96593	1.0353	3.8637
16	0.27925	0.27564	0.28675	3.4874	0.96126	1.0403	3.6280
17	0.29671	0.29237	0.30573	3.2709	0.95630	1.0457	3.4203
18	0.31416	0.30902	0.32492	3.0777	0.95106	1.0515	3.2361
19	0.33161	0.32557	0.34433	2.9042	0.94552	1.0576	3.0716
20	0.34907	0.34202	0.36397	2.7475	0.93969	1.0642	2.9238
21	0.36652	0.35837	0.38386	2.6051	0.93358	1.0711	2.7904
22	0.38397	0.37461	0.40403	2.4751	0.92718	1.0785	2.6695
23	0.40143	0.39073	0.42447	2.3559	0.92050	1.0864	2.5593
24	0.41888	0.40674	0.44523	2.2460	0.91355	1.0946	2.4586
25	0.43633	0.42262	0.46631	2.1445	0.90631	1.1034	2.3662
26	0.45379	0.43837	0.48773	2.0503	0.89879	1.1126	2.2812
27	0.47124	0.45399	0.50953	1.9626	0.89101	1.1223	2.2027
28	0.48869	0.46947	0.53171	1.8807	0.88295	1.1326	2.1301
29	0.50615	0.48481	0.55431	1.8040	0.87462	1.1434	2.0627
30	0.52360	0.50000	0.57735	1.7321	0.86603	1.1547	2.0000
31	0.54105	0.51504	0.60086	1.6643	0.85717	1.1666	1.9416
32	0.55851	0.52992	0.62487	1.6003	0.84805	1.1792	1.8871
33	0.57596	0.54464	0.64941	1.5399	0.83867	1.1924	1.8361
34	0.59341	0.55919	0.67451	1.4826	0.82904	1.2062	1.7883
35	0.61087	0.57358	0.70021	1.4281	0.81915	1.2208	1.7434
36	0.62832	0.58779	0.72654	1.3764	0.80902	1.2361	1.7013
37	0.64577	0.60182	0.75355	1.3270	0.79864	1.2521	1.6616
38	0.66323	0.61566	0.78129	1.2799	0.78801	1.2690	1.6243
39	0.68068	0.62932	0.80978	1.2349	0.77715	1.2868	1.5890
40	0.69813	0.64279	0.83910	1.1918	0.76604	1.3054	1.5557
41	0.71559	0.65606	0.86929	1.1504	0.75471	1.3250	1.5243
42	0.73304	0.66913	0.90040	1.1106	0.74314	1.3456	1.4945
43	0.75049	0.68200	0.93252	1.0724	0.73135	1.3673	1.4663
44	0.76794	0.69466	0.96569	1.0355	0.71934	1.3902	1.4396
45	0.78540	0.70711	1.00000	1.0000	0.70711	1.4142	1.4142
46	0.80285	0.71934	1.03550	0.96569	0.69466	1.4396	1.3902
47	0.82030	0.73135	1.07240	0.93252	0.68200	1.4663	1.3673
48	0.83776	0.74314	1.11060	0.90040	0.66913	1.4945	1.3456
49	0.85521	0.75471	1.15040	0.86929	0.65606	1.5243	1.3250
50	0.87266	0.76604	1.19180	0.83910	0.64279	1.5557	1.3054

TABLE C.12
Involute Functions (inv ϕ = tan ϕ − arc ϕ)

ϕ (°)	inv ϕ	Diff.	ϕ (°)	inv ϕ	Diff.	ϕ (°)	inv ϕ	Diff.	ϕ (°)	inv ϕ	Diff.
0.0	.0000000	0	10.0	.0017941	548	20.0	.0149044	2325	30.0	.0537515	5841
0.1	00000	0	10.1	18489	559	20.1	151369	2350	30.1	543356	5889
0.2	00000	0	10.2	19048	571	20.2	153719	2375	30.2	549245	5936
0.3	00000	1	10.3	19619	582	20.3	156094	2401	30.3	555181	5983
0.4	00001	1	10.4	20201	594	20.4	158495	2427	30.4	561164	6032
0.5	00002	2	10.5	20795	605	20.5	160922	2453	30.5	567196	6080
0.6	00004	2	10.6	21400	617	20.6	163375	2479	30.6	573276	6129
0.7	00006	3	10.7	22017	629	20.7	165854	2505	30.7	579405	6177
0.8	00009	4	10.8	22646	642	20.8	168359	2532	30.8	585582	6227
0.9	00013	5	10.9	23288	653	20.9	170891	2558	30.9	591809	6277
1.0	.0000018	6	11.0	.0023941	666	21.0	.0173449	2585	31.0	.0598086	6326
1.1	00024	7	11.1	24607	678	21.1	176034	2612	31.1	604412	6376
1.2	00031	8	11.2	25285	690	21.2	178646	2640	31.2	610788	6427
1.3	00039	10	11.3	25975	703	21.3	181286	2667	31.3	617215	6477
1.4	00049	11	11.4	26678	716	21.4	183953	2694	31.4	623692	6529
1.5	00060	13	11.5	27394	729	21.5	186647	2722	31.5	630221	6580
1.6	00073	14	11.6	28123	742	21.6	189369	2750	31.6	636801	6631
1.7	00087	16	11.7	28865	755	21.7	192119	2778	31.7	643432	6684
1.8	00103	19	11.8	29620	769	21.8	194897	2806	31.8	650116	6735
1.9	00122	20	11.9	30389	782	21.9	197703	2835	31.9	656851	6789
2.0	.0000142	22	12.0	.0031171	795	22.0	.0200538	2863	32.0	.0663640	6841
2.1	00164	25	12.1	31966	809	22.1	203401	2892	32.1	670481	6895
2.2	00189	27	12.2	32775	823	22.2	206293	2922	32.2	677376	6948
2.3	00216	29	12.3	33598	836	22.3	209215	2950	32.3	684324	7002
2.4	00245	32	12.4	34434	851	22.4	212165	2980	32.4	691326	7057
2.5	00277	35	12.5	35285	865	22.5	215145	3009	32.5	698383	7110
2.6	00312	37	12.6	36150	879	22.6	218154	3039	32.6	705493	7166
2.7	00349	40	12.7	37029	894	22.7	221193	3069	32.7	712659	7221
2.8	00389	44	12.8	37923	908	22.8	224262	3099	32.8	719880	7277
2.9	00433	46	12.9	38831	923	22.9	227361	3130	32.9	727157	7332
3.0	.0000479	50	13.0	.0039754	938	23.0	.0230491	3160	33.0	.0734489	7389
3.1	00529	52	13.1	40692	952	23.1	233651	3191	33.1	741878	7446
3.2	00581	57	13.2	41644	968	23.2	236842	3221	33.2	749324	7502
3.3	00638	60	13.3	42612	983	23.3	240063	3253	33.3	765826	7559
3.4	0698	63	13.4	43595	998	23.4	243316	3284	33.4	764385	7618
3.5	00761	67	13.5	44593	1014	23.5	246600	3316	33.5	772003	7675
3.6	00828	71	13.6	45607	1029	23.6	249916	3347	33.6	779678	7733
3.7	00899	75	13.7	46636	1045	23.7	253264	3378	33.7	787411	7793
3.8	00974	79	13.8	47681	1061	23.8	256642	3411	33.8	795204	7851
3.9	01053	83	13.9	48742	1077	23.9	260053	3444	33.9	803055	7911
4.0	.0001136	88	14.0	.0049819	1093	24.0	.0263497	3476	34.0	.0810966	7970
4.1	01234	92	14.1	50912	1109	24.1	266973	3508	34.1	818936	8031
4.2	01316	96	14.2	52021	1126	24.2	270481	3542	34.2	826967	8091
4.3	01412	101	14.3	53147	1142	24.3	274023	3575	34.3	835058	8152
4.4	01513	106	14.4	54289	1159	24.4	277598	3608	34.4	843210	8214
4.5	01619	110	14.5	55448	1176	24.5	281206	3642	34.5	851424	8275
4.6	01729	116	14.6	56624	1193	24.6	284848	3675	34.6	859699	8337
4.7	01845	120	14.7	57817	1210	24.7	288523	3709	34.7	868036	8399
4.8	01965	126	14.8	59027	1227	24.8	292232	3744	34.8	876435	8463
4.9	02091	131	14.9	60254	1244	24.9	295976	3777	34.9	884898	8525
5.0	.0002222	136	15.0	.0061498	1262	25.0	.0299753	3813	35.0	.0893423	8589
5.1	02358	142	15.1	62760	1279	25.1	303566	3847	35.1	902012	8653
5.2	02500	147	15.2	64039	1298	25.2	307413	3882	35.2	910665	8717

(*Continued*)

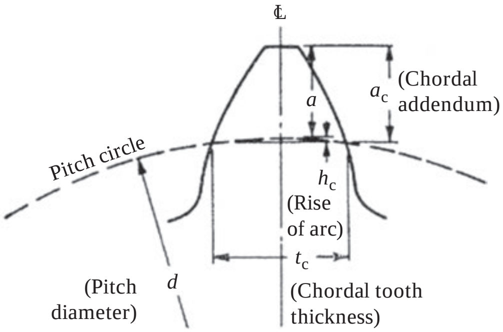
TABLE C.12 (CONTINUED)
Involute Functions (inv ϕ = tan ϕ − arc ϕ)

ϕ (°)	inv ϕ	Diff.	ϕ (°)	inv ϕ	Diff.	ϕ (°)	inv ϕ	Diff.	ϕ (°)	inv ϕ	Diff.
5.3	02647	154	15.3	65337	1315	25.3	311295	3918	35.3	919382	8783
5.4	02801	158	15.4	66652	1333	25.4	315213	3953	35.4	928165	8847
5.5	02959	165	15.5	67985	1352	25.5	319166	3988	35.5	937012	8913
5.6	03124	171	15.6	69337	1369	25.6	323154	4025	35.6	945925	8979
5.7	03295	177	15.7	70706	1389	25.7	327179	4060	35.7	954904	9045
5.8	03472	183	15.8	72095	1406	25.8	331239	4097	35.8	963949	9112
5.9	03655	190	15.9	73501	1426	25.9	335336	4134	35.9	973061	9179
6.0	.0003845	196	16.0	.0074927	1445	26.0	.0339470	4170	36.0	.0982240	9247
6.1	04041	203	16.1	76372	1463	26.1	343640	4207	36.1	991487	9315
6.2	04244	209	16.2	77835	1483	26.2	347847	4245	36.2	1000802	9383
6.3	04453	216	16.3	79318	1502	26.3	352092	4282	36.3	1010185	0452
6.4	04669	223	16.4	80820	1522	26.4	356374	4320	36.4	1019637	9522
6.5	04892	230	16.5	82342	1541	26.5	360694	4357	36.5	1029159	9591
6.6	05122	237	16.6	83883	1561	26.6	365051	4396	36.6	1038750	9662
6.7	05359	245	16.7	85444	1581	26.7	369447	4434	36.7	1048412	9732
6.8	05604	252	16.8	87025	1601	26.8	373881	4473	36.8	1058144	9803
6.9	05856	259	16.9	88626	1621	26.9	378354	4512	36.9	1067947	9875
7.0	.0006115	267	17.0	.0090247	1642	27.0	.0382866	4550	37.0	.1077822	9947
7.1	06382	275	17.1	91889	1662	27.1	387416	4590	37.1	1087769	10019
7.2	06657	282	17.2	93551	1683	27.2	392006	4630	37.2	1097788	10092
7.3	06939	291	17.3	95234	1703	27.3	396636	4670	37.3	1107880	10166
7.4	07230	298	17.4	96937	1725	27.4	401306	4709	37.4	1118046	10239
7.5	07528	307	17.5	98662	1745	27.5	406015	4750	37.5	1128285	10314
7.6	07835	315	17.6	.0100407	1767	27.6	410765	4790	37.6	1138599	10388
7.7	08150	323	17.7	102174	1789	27.7	415555	4832	37.7	1148987	10464
7.8	08473	332	17.8	103963	1810	27.8	420387	4872	37.8	1159451	10539
7.9	08805	340	17.9	105773	1831	27.9	425259	4913	37.9	119990	10615
8.0	.0009145	349	18.0	.0107604	1854	28.0	.0430172	4956	38.0	.1180605	10692
8.1	09494	358	18.1	109458	1875	28.1	435128	4996	38.1	1191297	10769
8.2	09852	367	18.2	111333	1898	28.2	440124	5039	38.2	1202066	10847
8.3	10219	376	18.3	113231	1920	28.3	445163	5082	38.3	1212913	10925
8.4	10595	385	18.4	115151	1943	28.4	450245	5134	38.4	1223838	11004
8.5	10980	395	18.5	117094	1965	28.5	455369	5166	38.5	1234842	11082
8.6	11375	404	18.6	119059	1989	28.6	460535	5210	38.6	1245924	11163
8.7	11779	413	18.7	121048	2011	28.7	465745	5253	38.7	1257087	11242
8.8	12192	423	18.8	123059	2034	28.8	470998	5297	38.8	1268329	11323
8.9	12615	433	18.9	125093	2058	28.9	476295	5341	38.9	1279652	11404
9.0	.0013048	443	19.0	.0127151	2081	29.0	.0481636	5384	39.0	.1291056	11486
9.1	13491	453	19.1	129232	2104	29.1	487020	5430	39.1	1302542	11568
9.2	13944	463	19.2	131336	2129	29.2	492450	5474	39.2	1314110	11651
9.3	14407	473	19.3	133465	2152	29.3	497924	5518	39.3	1325761	11734
9.4	14880	483	19.4	135617	2177	29.4	503442	5564	39.4	1337495	11818
9.5	15363	494	19.5	137794	2200	29.5	509006	5610	39.5	1349313	11903
9.6	15857	505	19.6	139994	2226	29.6	514616	5655	39.6	1361216	11987
9.7	16362	515	19.7	142220	2250	29.7	520271	5702	39.7	1373203	12072
9.8	16877	526	19.8	144470	2274	29.8	525973	5748	39.8	1385275	12159
9.9	17403	538	19.9	146744	2300	29.9	531721	5794	39.9	1397434	12245
10.0	.0017941		20.0	.0149044		30.0	.0537515		40.0	.1409679	

Note: For any value of ϕ not given in the table, special working tables may be easily constructed. Thus, inv 46.6 = tan 46.6 − arc 46.6 = 1.0574704 − 0.8133234 = 0.2441470; inv 46.7 = tan 46.7 − arc 46.7 = 1.0611742 − 0.8150688 = 0.2461054; and inv 46.8 = tan 46.8 − arc 46.8 = 1.0648918 − 0.8168141 = 0.2480777.

TABLE C.13

Chordal Tooth Thickness and Chordal Addendum for Gears of 1 Diametral Pitch (Divide Values by Diametral Pitch for Other Pitches)



No. of Teeth N	Chordal Thickness t_c	Chordal Addendum a_c	Rise of Arc h_c	No. of Teeth N	Chordal Thickness t_c	Chordal Addendum a_c	Rise of Arc h_c
8	1.56072	1.07686	0.07686	58	1.57060	1.01063	0.01063
9	1.56283	1.06836	0.06836	59	1.57061	1.01045	0.01045
10	1.56434	1.06158	0.06158	60	1.57062	1.01028	0.01028
11	1.56546	1.05597	0.05597	61	1.57062	1.01011	0.01011
12	1.56631	1.05133	0.05133	62	1.57062	1.00995	0.0095
13	1.56697	1.04733	0.04733	63	1.57063	1.00979	0.00979
14	1.56750	1.04401	0.04401	64	1.57064	1.00964	0.00964
15	1.56793	1.04109	0.04109	65	1.57064	1.00949	0.00949
16	1.56827	1.03852	0.03852	66	1.57064	1.00934	0.00934
17	1.56856	1.03623	0.03623	67	1.57065	1.00921	0.00921
18	1.56880	1.03425	0.03425	68	1.57065	1.00907	0.00907
19	1.56901	1.03245	0.03245	69	1.57065	1.00884	0.00884
20	1.56918	1.03083	0.03083	70	1.57066	1.00881	0.00881
21	1.56933	1.02936	0.02936	71	1.57066	1.00869	0.00869
22	1.56946	1.02803	0.02803	72	1.57066	1.00857	0.00857
23	1.56957	1.02681	0.02681	73	1.57067	1.00845	0.00845
24	1.56967	1.02569	0.02569	74	1.57067	1.00834	0.00834
25	1.56976	1.02466	0.02466	75	1.57068	1.00822	0.00822
26	1.56984	1.02372	0.02372	76	1.57068	1.00812	0.00812
27	1.56991	1.02284	0.02284	77	1.57068	1.00801	0.00801
28	1.56997	1.02202	0.02202	78	1.57068	1.00791	0.00791
29	1.57003	1.02126	0.02126	79	1.57068	1.00781	0.00781
30	1.57008	1.02056	0.02056	80	1.57068	1.00771	0.00771
31	1.57012	1.01989	0.01989	81	1.57068	1.00762	0.00762
32	1.57016	1.01927	0.01927	82	1.57069	1.00752	0.00752
33	1.57020	1.01869	0.01869	83	1.57069	1.00743	0.00743
34	1.57024	1.01814	0.01814	84	1.57069	1.00734	0.00734
35	1.57027	1.01762	0.01762	85	1.57069	1.00725	0.00725
36	1.57030	1.01713	0.01713	86	1.57070	1.00717	0.00717
37	1.57032	1.01667	0.01667	87	1.57070	1.00713	0.00713
38	1.57035	1.01623	0.01623	88	1.57070	1.00701	0.00701
39	1.57037	1.01581	0.01581	89	1.57070	1.00693	0.00693
40	1.57039	1.01541	0.01541	90	1.57070	1.00685	0.00685
41	1.57041	1.01504	0.01504	91	1.57071	1.00678	0.00678
42	1.57043	1.01468	0.01468	92	1.57071	1.00671	0.00671
43	1.57045	1.01435	0.01435	93	1.57071	1.00664	0.00664
44	1.57046	1.01401	0.01401	94	1.57071	1.00657	0.00657
45	1.57048	1.01371	0.01371	95	1.57072	1.00649	0.00649
46	1.57049	1.01318	0.01318	96	1.57072	1.00641	0.00641

(*Continued*)

TABLE C.13 (CONTINUED)
Chordal Tooth Thickness and Chordal Addendum for Gears of 1 Diametral Pitch (Divide Values by Diametral Pitch for Other Pitches)

No. of Teeth N	Chordal Thickness t_c	Chordal Addendum a_c	Rise of Arc h_c	No. of Teeth N	Chordal Thickness t_c	Chordal Addendum a_c	Rise of Arc h_c
47	1.57050	1.01312	0.01312	97	1.57072	1.00634	0.00634
48	1.57051	1.01285	0.01285	98	1.57072	1.00631	0.00631
49	1.57052	1.01259	0.01259	99	1.57072	1.00623	0.00623
50	1.57053	1.01234	0.01234	100	1.57073	1.00617	0.00617
51	1.57053	1.01209	0.01209	110	1.57074	1.00561	0.00561
52	1.57055	1.01186	0.01186	120	1.57075	1.00514	0.00514
53	1.57056	1.01164	0.01164	Rack	1.57080	1.00000	0.00000
54	1.57057	1.01142	0.01142				
55	1.57058	1.01121	0.01121				
56	1.57059	1.01102	0.01102				
57	1.57059	1.01082	0.01082				

Source: The Fellows Gear Shaper Co., *The Involute Curve and Involute Gearing*, The Fellows Gear Shaper Co., Spring eld, Vermont. With permission.

Note: For notations, see the illustration.

Notation:
a_c—chordal addendum
t_c—chordal thickness
h_c—rise of arc
a—addendum
d—pitch diameter

TABLE C.14
Hardness Testing Apparatuses and Applications for Gears

Instrument	Shape and Type of Intender	Loading	Recommended Use
Brinell	10 mm steel or tungsten carbide ball	3000 kg	For large gears and shafts in range of hardness from 100 to 400 HB. (If gears have a hard case, the case depth must be suf cient and the core strength adequate to support the area under test. Rockwell C test may be used to reveal any error from such irregular conditions.) Brinell tests, when fairly representative of the general hardness, are a good measure of the ultimate tensile strength of the material.
Rockwell C	Diamond Brale penetrator (120° diamond cone)	150 kg	For medium to large gears. Range approximately 25 to 68 HRC.
Rockwell A	Diamond Brale penetrator (120° diamond cone)	60 kg	For small gears and tips of large gear teeth. Range 62 to 85 HRA.
Rockwell 30-N	Diamond Brale penetrator (120° diamond cone, special indenter)	30 kg	For small parts and shallow case-hardened parts.
Rockwell 15-N	Diamond Brale penetrator (120° diamond cone, special indenter)	15 kg	Lightest Rockwell load. For testing very small parts and checking the working sides of teeth. Check for very thinly case-hardened parts. Rockwell 15-N is used 72-93.
Vickers, pyramid	136° diamond pyramid	50 kg	All applications where piece will not be too heavy for machine. For use in testing hardness of shallow cases, etc.
Scleroscope	Does not indent surface. Diamond-tipped tup bounced on specimen		For applications permitting no damage or indentation to surfaces (results not always comparable with indentation hardness tests).
Tukon	Knoop indenter	500–1000 g for practical use	A laboratory instrument used only for nding the hardness of material on pieces of cross sections, that is, hardness from surface inward, every 0.05 mm (0.002 in.), or hardness of individual micro-constituents. All test areas must have at mirror-polished surface. An extremely delicate and precise test. Used for any hardness.

TABLE C.15
Approximate Relation between Hardness Test Scales

Brinell 3000 kg, 10 mm	Rockwell				Vickers Pyramid	Scleroscope (Shore)	Tukon (Knoop)
	C	A	30-N	15-N			
	70	86.5	86.0	94.0	1076		
	65	84.0	82.0	92.0	820	90	846
	63	83.0	80.0	91.5	763	87	799
614	60	81.0	77.5	90.0	695	81	732
587	58	80.0	75.5	89.3	655	79	690
547	55	78.5	73.0	88.0	598	74	630
522	53	77.5	71.0	87.0	562	71	594
484	50	76.9	68.5	85.5	513	67	542
460	48	74.5	66.5	84.5	485	64	510
426	45	73.0	64.0	83.0	446	61	466
393	42	71.5	61.5	81.5	413	56	426
352	38	69.5	57.5	79.5	373	51	380
305	33	67.0	53.0	76.5	323	45	334
250	24	62.5	45.0	71.5	257	37.5	
230	20	60.5	41.5	69.5	236	34	
	Rockwell						
	B		30-T	15-T			
200	93		78.0	91.0	210	30	
180	89		75.5	89.5	189	28	
150	80		70.0	86.5	158	24	
100	56		54.0	79.0	105		
80[a]	47		47.7	75.7			
70[a]	34		38.5	71.5			

[a] Based on 500 kg load and 10 mm ball.

and the chordal thickness t_c. Tabulated values are given for 1 diametral pitch. The chordal thicknesses given have no allowance for backlash. The values shown should be divided by the pitch and then given a proper allowance for backlash and an appropriate tolerance.

The values given in Table C.13 may be read in millimeters for 1 module. For other sizes of teeth, *multiply* the table value by the module.

C.5 HARDNESS TESTING DATA

Almost all drawings for gear parts will need a hardness testing speci cation. The method of hardness testing will depend on the material and the size and shape of piece. Table C.14 gives general recommendations for the more commonly used hardness test methods. Table C.15 shows the approximate conversion between the different hardness test scales.

Appendix D: On the Concept of Novikov Gearing and on the Inadequacy of the Terms Wildhaber–Novikov Gearing and W-N Gearing

> The beginning of wisdom is to call things by their right names.
>
> **An old Chinese proverb**

Novikov gearing was invented early 1950s by Dr. Mikhail L. Novikov. An increased power density through a gear train is the main advantage of Novikov gearing.

The concept of Novikov gearing had been disclosed by Dr. Novikov in three main sources. His doctoral dissertation (1955) is the first source in which the novel concept of gearing is discussed. Second, the Soviet Union patent (Novikov, 1956) had been granted to Dr. Novikov on his invention of the novel gear system. Third, a comprehensive discussion on the concept of Novikov gearing can be found in the monograph by Dr. Novikov (1958).

The kinematics and geometry of Novikov gearing is disclosed in detail in these three sources (Novikov, 1955; 1956; 1958). The following discussion is mostly based on the original works by Dr. Novikov.

D.1 KINEMATICS AND GEOMETRY OF NOVIKOV GEARING

An attempt to demonstrate the principal features of the novel system of gearing, namely, of Novikov gearing, had been undertaken by the author (Radzevich, 2016), where additional information on the issue can be found.

D.1.1 Preamble

For a long while, gear experts in Western countries had very limited access to original sources in which the concept of Novikov gearing is discussed in detail. In Western countries, gear experts who were deeply involved in research and development of Novikov gearing often had no access even to Soviet Union (S.U.) Patent No. 109113 (Novikov, 1956) granted on Novikov gearing. For example, Dyson et al. (1986) undertook an analysis of Novikov gearing in comparison to the helical gearing earlier proposed by Ernest Wildhaber (1926). When making the comparison, Dyson et al. mistakenly referred to S.U. Patent No. 109750* (Pizulin, 1957) and not to S.U. Patent No. 109113 (Novikov, 1956)—a very common mistake committed by many gear experts in Western countries. The books of Dooner and Seireg (1995) and Dooner (2012) are another good examples in this concern.

The aforementioned, along with numerous other cases of using wrong sources of information for the comparison, makes it clear that Dyson et al. (1986), Dooner and Seireg (1995), and Dooner (2012), as well as many others, were not familiar with the main sources (Novikov, 1955; 1956; 1958); moreover, they were not familiar with the main patent (Novikov 1956). This has resulted in a poorly made comparison of the two inventions by Novikov (1956) and Wildhaber (1926).

Chironis (1967) approached very close to understanding and correctly interpreting the concept of Novikov gearing. But even he failed in making correct conclusions from the analysis that he undertook.

It should be mentioned here that in his work, Chironis (1967) quoted Wildhaber's opinion: "All the characteristics of the Novikov gearing are completely anticipated by my patent. My gearing never had a real test here, although a pair of gears was made in the 1920's. It would be wrong to dismiss the Russian development as old and known." The following discussion reveals that Dr. Wildhaber was mistaken when he made the statement "all the characteristics of the Novikov gearing are completely anticipated by my patent." Moreover, it is easy to show that the helical gearing by Ernest Wildhaber (U.S. Patent No. 1,601,750) is *not* workable in nature. For this purpose, it is sufficient to just recall the result earlier obtained by Leonhard Euler in the 18th century on involute tooth profile for mating gears.

D.1.2 Main Features of Novikov Gearing

Novikov gearing had been developed with the intent to increase the contact strength of the gear teeth. Gearing of this kind features higher contact strength due to favorable curvatures of interacting tooth flanks. Under equivalent contact stress, similar dimensions, and comparable rest of the design parameters, greater circular forces are permissible by the proposed gearing.

Novikov gearing was developed for but not limited to parallel axis gear trains. However, gear pairs featuring intersecting axes, as well as gear pairs having crossing axes of the rotations of the gears, can be designed on the basis of the concept proposed by Novikov.

Possible geometries of tooth profiles of the Novikov gears are schematically shown in Figure D.1. Here, a section of the tooth flanks by a plane that is perpendicular to the instant axis of relative rotation is shown. The axis is through the current point of contact of the tooth flanks.

* Pisulin, P.F., A water sprayer, S.U. Pat. No. 109750, National Cl. 36, d, 28b, 801. Filed: January 2, 1957.

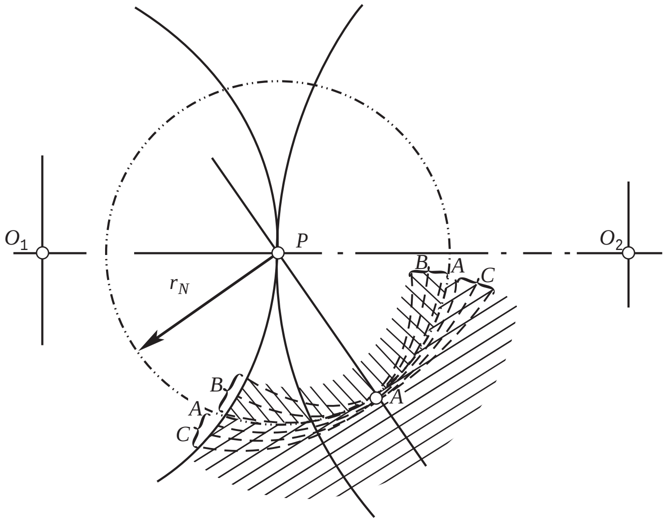


FIGURE D.1 The only schematic used in S.U. Patent No. 109113 to explain the concept of Novikov gearing.

In Figure D.1, the point of intersection of the planar section by the axis of instant relative rotation is denoted by *P*. The points of intersection of the planar section by the axes of the gear and of the pinion are designated as O_1 and O_2. Point *A* is the point of meshing (in its current location). The line of action is denoted as *PA*. Ultimately, Д*А*Д is the circle centering at point *P*. The circle corresponds to the limit case of the tooth profiles (in the case the profiles are aligned to each other).

Multiple curves *BAB* illustrate examples of possible tooth profiles of one of the mating gears. All the curves *BAB* are arbitrary smooth regular curves, which are located inside of the limit circular arc Д*А*Д (that is, the arcs *BAB* are situated within the bodily side of the limit tooth flank of one of the gears). All the tooth profiles *BAB* feature high rate of conformity to the limit circular arc Д*А*Д.

Multiple curves *CAC* illustrate examples of possible tooth profiles of the second set of the mating gears. All the curves *CAC* are arbitrary smooth regular curves, which are located outside of the limit circular arc Д*А*Д (that is, the arcs *CAC* are located within the bodily side of the limit tooth flank of another two gears). All the curves *CAC* feature high rate of conformity to the circular arc Д*А*Д.

The location and orientation either of straight line of meshing or of smooth curved line of meshing is specified in a space in which location and orientation of the axes of rotations of the gear and of the pinion are given. The line of meshing is located reasonably close to the axis of instant relative rotation of the gears. Either constant or time-dependent (smoothly varying in time) speed of motion of the point of contact along the line of meshing is assigned. A coordinate system is associated with the gear, and a corresponding coordinate system is associated with the pinion. In the coordinate systems, the moving meshing point traces contact lines. One of the contact lines is associated with the gear and another one is associated with the pinion. Certain smooth regular surfaces through the meshing lines can be employed as tooth flanks of the gear and of the pinion. The following requirements should be fulfilled in order so the surfaces could be used as the tooth flanks of Novikov gearing:

- At every location of the point of contact, the tooth flanks should have a common perpendicular and, thus, the requirements of the main theorem of meshing should be satisfied.
- Curvatures of tooth profiles should correspond to each other.
- No tooth flank interference is allowed within the working portions of the surfaces.

Once two surfaces are generated by one of the moving curves *BAB* and by one of the moving curves *CAC*, then the listed requirements are fulfilled and the surfaces can be employed as tooth flanks of Novikov gearing.

Consider a plane through the current meshing point, which is perpendicular to the instant axis of relative rotation. Construct two circular arcs that center at points within the straight line through the pitch point and the meshing point. The arc centers are located within the line of action and close to the pitch point. The constructed circular arcs can be considered as examples of tooth profiles of the gear and of the pinion. Tooth flanks are generated as loci of tooth profiles that are constructed for all possible locations of the contact point. The working portion of one of the two tooth flanks is convex, while the working portion of another tooth flank is concave (in the direction toward the axis of instant relative rotation). In a particular case, the radii of tooth profiles could be of the same magnitude and equal to the distance from the meshing point to the axis of instant relative rotation. The centers of both profiles in this particular case are located at the axis of instant relative rotation. Under such a scenario, point kind of contact is reduced to a special line kind of contact. This would require an extremely high accuracy of the center distance and its independence from operation conditions, which is impractical. Point contact is preferred when designing tooth profiles. A small difference between the radii of curvature of tooth profiles is desired. It should be born in mind that under run-in period, the point meshing of the gear teeth will be transformed to the earlier mentioned line meshing of the tooth profiles. However, the theoretical point contact of the tooth flanks will be retained.

Generally speaking, tooth profiles need not be of circular arc shapes. Tooth profiles of other geometries (those always passing through the meshing point) should be located (for one gear) within the interior of the abovementioned circular arc profile that centers at the point within the axis of instant relative rotation as shown in Figure D.1. For the other gear, the tooth profile should be located outside the circular arc.

The earlier discussion is will be easier to understand when a boundary circle of radius r_N having a pitch point *P* as the center is drawing through the point of contact point *K*. (In Figure D.1, this circle is added by the author to the original drawing by Dr. Novikov.) It is proposed to call this circle as *boundary Novikov circle* or just *boundary N-circle* for simplicity.

Novikov himself did not use the concept of the boundary circle (the concept of the boundary circle was introduced later by Radzevich at around 2008). This concept has proven to be convenient in the theory of high-conformity gearing and in Novikov gearing in particular. As an example, Figure D.2 illustrates the tooth ank of a gear *G*, which makes contact at the point *K* with tooth ank of the mating pinion *P*. Circular-arc teeth pro les *G* and *P* center at points o_g and o_p, accordingly. The centers are chosen to ful ll the necessary condition for magnitudes ρ_g and ρ_p for radii of curvature of the teeth pro les *G* and *P* at the point of tangency *K* ($\rho_g > \rho_p$). However, as the circular arcs *G* and *P* intersect the boundary N-circle, gearing of this kind is not feasible. As it will be later shown, helical gearing by Ernst Wildhaber (1926) features such unfavorable con guration of circular-arc teeth pro les, which makes Wildhaber's helical gearing (1926) not workable in nature.

The law of motion of the meshing point (that is, speed of the point and its trajectory) should be chosen to minimize the friction and the wear loses. Friction and wear losses are proportional to the relative sliding velocity in the gear mesh. Therefore, it is desired to reduce the sliding velocity as much as possible. For this purpose, the line of meshing should not be too far from the axis of instant relative rotation. On the other hand, the location of the line of meshing that is too close to the axis of instant relative rotation is also not desired as the contact strength of the gear tooth anks is reduced due to that. In addition, it is recommended to ensure favorable angles between the common perpendicular (along which tooth anks of one of the gears act against the tooth anks of another gear) and between the axes of rotations of the gears.

Opposite sides of tooth pro les are designed in a way similar to that just discussed. Tooth thicknesses and the tooth pitch are assigned to ensure the required bending strength of the teeth.

The face width of the gear or the length of the gear teeth should correlate to their pitch to ensure the required value of the face contact ratio (m_F). Gear pairs can feature either one point of contact (when working portions of the tooth ank contact each other in just one point, excluding the phases of the teeth reengagement), or multiple contact points when tooth anks simultaneously contact each other at several points.

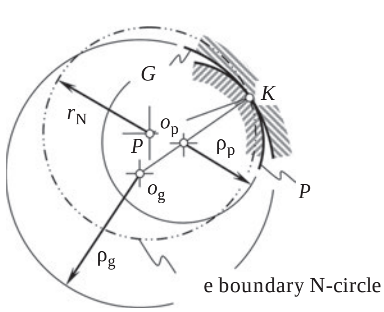


FIGURE D.2 Use of the concept of the boundary N-circle has been proven helpful to distinguish feasible and not feasible circular-arc tooth pro les for Novikov gearing.

For parallel axis gear pairs, it is preferred to employ a straight line as the line of meshing. The straight line is parallel to the axes of rotations of the gear and of the pinion. The speed of motion of the contact point along the straight line of meshing may be of constant value. In this particular case, the radii of curvature of tooth pro les in all sections by planes are equal to each other. Tooth anks in this case are a kind of regular screw surfaces. Gears that feature tooth anks of such geometry are easy for manufacture, and they can be cut on machine tools available on the market.

An example of parallel axis gearing with limit geometry of tooth pro les is illustrated in Figure D.1. The point contact of the teeth anks in this particular case is transformed to line contact. The curved contact line is located across the tooth pro le. When axial thrust in the gear pair is strongly undesired, herringbone gears can be used instead.

The kinematics and geometry of Novikov gearing is different from that of involute gearing as well as that of gearing of other designs.

Referring to Figure D.3, consider a parallel-axis Novikov gear pair that is composed of driving pinion and of driven gear.

The gear is rotated about the axis O_g, while the pinion is rotated about the axis O_p. The axes of rotations O_g and O_p are at a certain center distance apart of one another. The rotation of the gear ω_g and the rotation of the pinion ω_p are synchronized with each other in a proper manner.

The pitch circle of the gear is of radius R_g and the pitch circle of the pinion is of radius R_p accordingly. The pitch circles (R_g and R_p) are tangent to one another. The point of tangency of the pitch circles is the pitch point *P* of the gear pair. A line

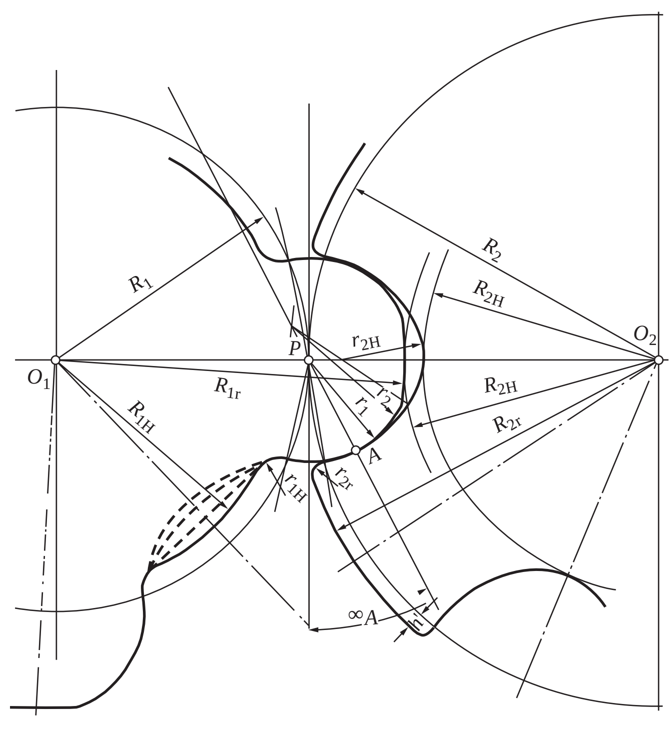


FIGURE D.3 A schematic that illustrates the concept of Novikov gearing in detail.

L_ϕ is a straight line through the pitch point P at a certain transverse pressure angle (ϕ_t) in relation to the perpendicular to the centerline O_g–O_p.

The point of contact K of tooth anks of the gear and of the pinion is a point within the straight line L_ϕ. The farther the contact point K is situated from the pitch point P, the more freedom there is in selecting the radii of curvature of the tooth pro les. At that same time, the farther the contact point K is situated from the pitch point P, the higher losses there are on friction between the tooth anks and on wear of tooth anks. Ultimately, the actual location of the contact point K is a kind of tradeoff between the two just mentioned factors.

Further, let us assume that the pinion is stationary and the gear is performing instant rotation in relation to the pinion. The axis P_{ln} of the instant rotation ω_{pl} is the straight line through the pitch point P. The axis of the instant rotation P_{ln} is parallel to the axes O_g and O_p of the rotations ω_g and ω_p. When the pinion is motionless, the contact point K traces a circle of limit radius r_{lim} centering at P.

The pinion tooth pro le P can either align with a circular arc of the limit circle (r_{lim}) or be relieved in the bodily side of the pinion tooth. As a consequence, the location of the center of curvature (c_p) of the convex pinion tooth pro le P within the straight line L_ϕ is limited just to the straight-line segment P_K. The pitch point is included into the interval as it is shown in Figure D.3, while the contact point K is not.

On other hand, the location of the center of curvature (c_g) of the concave gear tooth pro le G within the straight line L_ϕ is limited to the open interval $P \to \infty$. Theoretically, the pitch point P can be included in that interval for K. However, this is completely impractical, and the center of curvature c_g is situated beyond the pitch point P. Due to that, radius of curvature (r_p) of the convex pinion tooth pro le P is smaller than that (r_g) of the concave gear tooth pro le G ($r_p < r_g$).

It should be mentioned here that there are no physical constraints to designing a gear pair having convex gear tooth pro le, while the pinion tooth pro le is concave.

Both the pinion and the gear are helical and they are of opposite hands. No spur Novikov gearing is feasible in nature. Because the gears are helical and of opposite hands, the point of contact will axially move along the gears while remaining at the same radial position on both gear and pinion teeth. It is, therefore, fundamental to the operation of the gears that contact nominally occurs at a point and that the point of contact axially moves across the full face width of the gears during rotation. It is clearly a condition of operation that in a given pro le, the tooth surfaces should not interfere before or after culmination, when rotated at angular speeds that are in the gear ratio.

Transverse contact ratio (m_p) of the Novikov gear pair is zero ($m_p \equiv 0$). Face contact ratio (m_F) of the gear pair is always greater than 1 ($m_F > 1$).

For parallel axes' con guration, the line of action LA is a straight line through the contact point K. This line forms the transverse pressure angle ϕ_t perpendicular to the centerline.

In the transverse section of the gear pair, the contact point K is motionless. Because of this, the length of the path of contact P_c at every transverse cross section is zero ($P_c = 0$). The in nite number of zero length paths of contact P_c comprise the pseudopath of contact P_{pc}. The pseudopath of contact P_{pc} is parallel to the axes O_g and O_p. The length of the pseudopath of contact P_{pc} is equal to the active face width of the gear pair.

D.1.3 Construction of the Boundary N-Circle

The boundary N-circle for a Novikov gearing can be constructed in the way brie y outlined as follows.

Consider two axes of rotations O_g and O_p of a parallel-axis Novikov gearing as it is schematically depicted in Figure D.4. The axes O_g and O_p are at a certain center distance C. The gear and the pinion are rotating about the axes O_g and O_p, and the rotations are labeled ω_g and ω_p, correspondingly. The gear ratio of the Novikov gearing is equal to $u = \omega_g/\omega_p$.

The center distance C is subdivided by a point P on two segments O_gP and O_pP, correspondingly. The ratio of lengths of the straight-line segments O_gP and O_pP is reciprocal to the gear ratio u of Novikov gear pair. Once the straight-line segments O_gP and O_gP become the pitch radii $O_gP = r_g$ and $O_pP = r_p$ of the Novikov gear pair, then the equality $r_g/r_p = u$ is observed. The point P is the pitch point of the Novikov gearing.

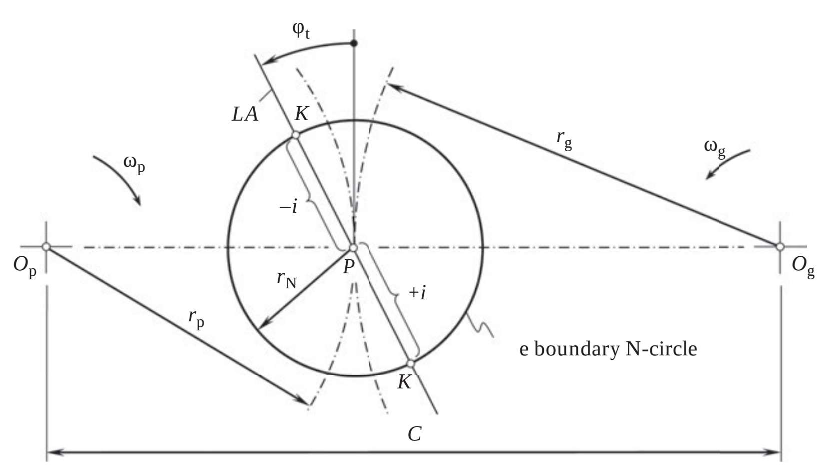


FIGURE D.4 Construction of the boundary N-circle for Novikov gearing.

A straight line LA through the pitch point P is at the transverse pressure angle ϕ_t with respect to perpendicular to the centerline O_gO_p. Two points K are within the straight line LA and are displaced from the pitch point P a certain distance $\pm l$ from the pitch point P. The lines of action are the two straight lines through the points K that are parallel to the rotation axes O_g and O_p. This distance, namely, the displacement l of the line of action, is one of the important geometrical parameters of Novikov gearing. The strength of gear teeth and the performance of a gear pair strongly depend on value of the displacement l.

The line of action, which is located beyond the pitch point P (in the direction of rotation of the gears), features positive displacement $\pm l$. A conformal gear mesh of this kind is referred to as *BY*-mesh of Novikov gear pair. The line of action, which is located before the pitch point (in the direction of rotation of the gears), features negative displacement $-l$. A conformal gear mesh of this kind is referred to as *BF*-mesh of Novikov gear pair.

In order to avoid violation of the conditions of meshing, as well as targeting wear reduction and reduction of friction losses, the contact lines are displaced at a reasonably short distance from the axis of instant rotation P_{ln}.

Let us assume that the pinion is motionless. Then the contact point K traces a circle within the corresponding transverse section of the gear pair. The circle centers at P. Similarly, the gear can be assumed stationary. Then the contact point K traces a circle within that same transverse section of the gear pair. This circle also centers at P. It is clear from the consideration how the boundary circle of radius l can be constructed.

A transverse section of a Novikov gear pair is subdivided by a circle of radius $r_N = |l|$ onto two areas. The area within the circle of radius r_N (including points that are within the circle itself) represents the area of possible shapes of tooth profiles of one of the mating gears, and the area outside the circle of radius r_N (including points that are within the circle itself) represents the area of possible shapes of tooth profiles of another of two mating gears.

D.1.4 Possible Geometries of Teeth Flanks for Novikov Gearing

Prior to designing mating tooth profiles of a Novikov gear pair, the N-circle should be drawn. Refer to Figure D.5, where the N-circle of radius r_N is constructed for the pinion tooth

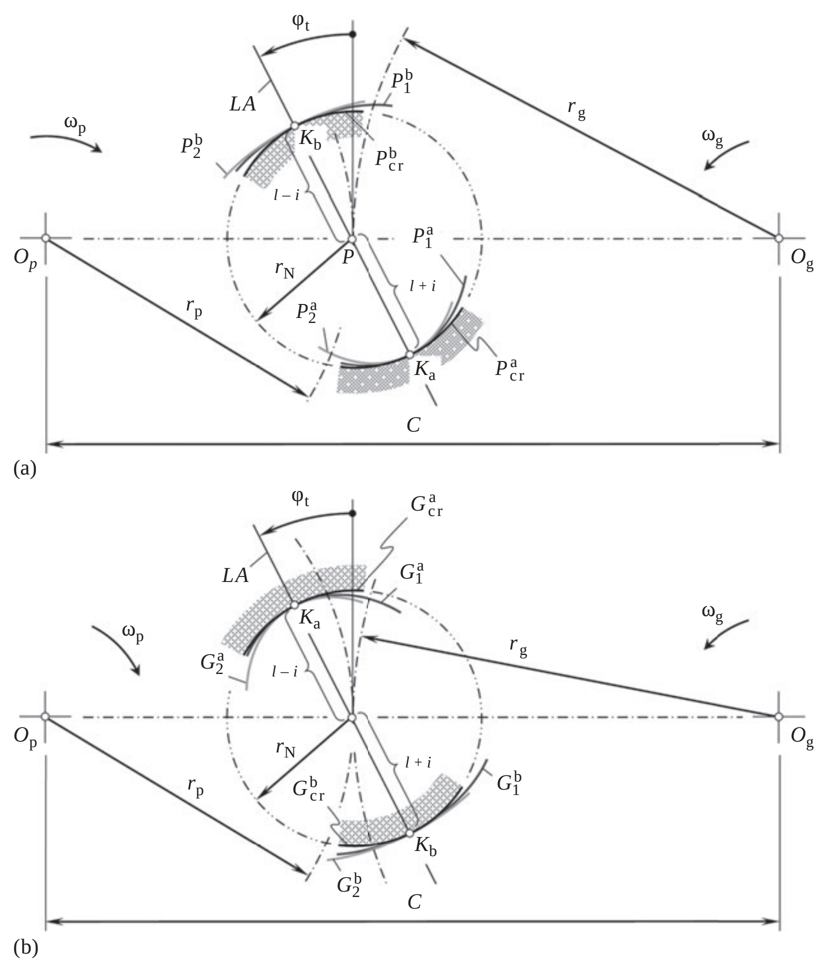


FIGURE D.5 Examples of possible tooth flank geometries for Novikov gearing: possible shapes of teeth flanks (a) of a pinion and (b) of a mating gear.

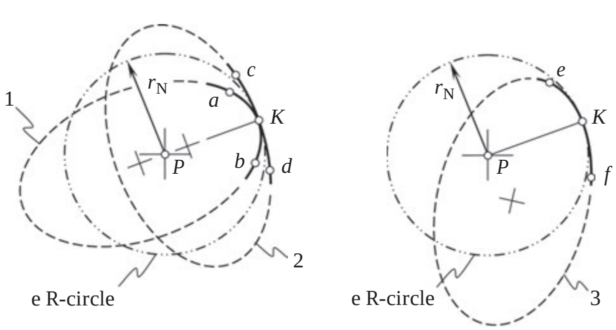
profile (Figure D.5a), and for the mating gear tooth profile (Figure D.5b) of a Novikov gear pair.

The displacement l is of positive value ($l > 0$) for the pinion addendum. The tooth profile of the pinion addendum is a convex segment of a smooth regular curve P_i^{a} ($i = 1, 2, \ldots$) through the contact point K_{a}. The radius of curvature R_{p} of the addendum profile is equal to or less than the radius r_{N} of the N-circle ($R_{\mathrm{p}} \leq r_{\mathrm{N}}$). The case of equality $R_{\mathrm{p}} = r_{\mathrm{N}}$ is the limit case, which is mostly of theoretical interest. Geometrically, the profile of the pinion addendum can be shaped in the form of a circular arc of the radius r_{N}. This case of profile of the pinion addendum is the limit one of theoretical importance.

It should be stressed here that none of the feasible profiles P_i^{a} of a pinion addendum intersects the N-circle. The pinion addendum profile is entirely located within the N-circle. Therefore, no arc of a smooth regular curve can be used as tooth profile of the pinion addendum. Circular arcs, arcs of ellipse at one of its apexes, cycloidal profiles containing an apex, etc., are examples of applicable kinds of curves for the addendum tooth profiles. Spiral curves (involute of a circle, Archimedean spiral, logarithmic spiral, and so forth) are examples of smooth regular curves no arc of which can be used in designing a pinion tooth addendum. This is due to the radius of curvature of a spiral curve (as well as of many others curves) steadily changing when a point is traveling along the curve. This is schematically illustrated in Figure D.6. Shown in Figure D.6a is ellipse-arc *ab* that is entirely located within the N-circle. Ellipse-arc *ab* can be selected as tooth addendum profile of a high-conformity gear pair. An ellipse-arc *cd* (Figure D.6a) is entirely located outside the N-circle. The ellipse-arc *cd* can be selected as tooth dedendum profile of a high-conformity gear pair. Ultimately, an ellipse-arc *ef* (Figure D.6b) intersects the N-circle. The ellipse-arc *ef* cannot be used as tooth profile of a high-conformity gear pair. The same is valid for most of spiral curves, and so forth.

Therefore, at the point of tangency K, spiral curves intersect the corresponding N-circle, which is prohibited. Ultimately, it should be clear that a variety of smooth regular curves can be used in designing the tooth profile of Novikov gearing. The variety of curves is not limited to circular arc only.

The displacement l is of negative value ($l < 0$) for the pinion dedendum (Figure D.5a). The tooth profile of the pinion



(a) (b)

FIGURE D.6 Examples of (a) feasible and (b) not feasible ellipse-arc tooth profiles for Novikov gearing.

dedendum is a concave segment of a smooth regular curve P_i^{b} ($i = 1, 2, \ldots$), through the contact point K_{b}. The radius of curvature R_{p} of the dedendum profile is equal to or exceeds the radius r_{N} of the N-circle ($R_{\mathrm{p}} \geq r_{\mathrm{N}}$). The case of equality $R_{\mathrm{p}} = r_{\mathrm{N}}$ is the limit case, which is mostly of theoretical interest. Geometrically, the profile of the pinion addendum can be shaped in the form of a circular arc of the radius r_{N}. This case of profile of the pinion addendum is the limit one of theoretical importance.

Constraints that are imposed on tooth profile geometry of the pinion dedendum are similar to that imposed on tooth profile of the pinion addendum. The dedendum profile is entirely located outside of the N-circle, shares a point with N-circle (the contact point K_{b}), and does not intersect the N-circle. Smooth regular curves not of all kinds can be implemented in design of the pinion tooth dedendum.

An analysis much similar to that performed earlier with regard to the pinion tooth profile can be performed with regard to the gear tooth profile as well. The analysis is illustrated in Figure D.5b. The gear tooth addendum G_i^{a} is entirely located within the boundary N-circle, while the gear tooth dedendum G_i^{b} is entirely located outside the boundary N-circle. Both the profile of the gear tooth addendum G_i^{a} and the profile of the gear tooth dedendum G_i^{b} share a common point with the boundary N-circle (the point K_{a} in the first case and the point K_{b} in the second). No intersection of tooth profiles G_i^{a} and G_i^{b} is allowed within the tooth height of the gear and of the pinion.

The importance of the concept of N-circle for gear engineers is as follows.

The boundary N-circle of a Novikov gear pair is a kind of constraint imposed on the gear tooth profile and on the pinion tooth profile. The gear engineer is free to select an arc of any smooth curve to shape the tooth addendum profile if the arc is entirely located within the N-circle. The gear engineer is also free to select an arc of any smooth curve to shape the tooth dedendum profile if the arc is entirely located outside the N-circle.

The possibility of a Novikov gear pair simultaneously having two contact points K' and K'' inspired Fed'akin to propose (1955) a kind of Novikov gear pair that features not one contact line CL, as Novikov gear system does, but two lines of action instead. The invention by Fed'akin is schematically illustrated in Figure D.7. Two paths of contact, namely, $P_{\mathrm{pc.BF}}$ and $P_{\mathrm{pc.BY}}$, are straight lines parallel to the axis of the instant rotation of the gears. The paths of contact $P_{\mathrm{pc.BF}}$ and $P_{\mathrm{pc.BY}}$ pass through the points K' and K''. They are at distances $+l$ and $-l$ from the pitch point P, respectively. As Novikov gears are helical, the contact points K' and K'' are displaced in axial direction in relation to one another at a distance ΔZ. This distance can be computed from the formula

$$\Delta Z = 2\frac{l}{\tan\psi}. \tag{D.1}$$

The axial displacement of contact points results in smoother rotation of the driven shaft of the Novikov gear pair.

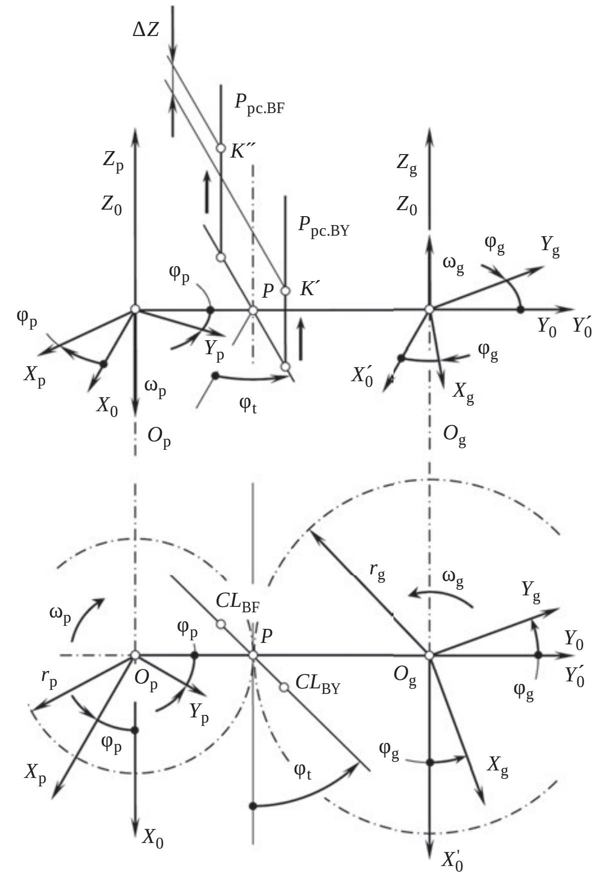


FIGURE D.7 On the concept of Novikov gearing having two lines of contact that is proposed by Fed'akin.

The average number of contact points between the gear and the pinion teeth flanks is doubled in the Novikov gear pair of this design.

When designing Novikov gears, the gear designer is free to pick a favorable smooth curve to shape tooth profile of the gear and of the pinion. An arc of the curve must be entirely located inside the N-circle for the tooth addendum, and a corresponding arc of the dedendum must be entirely located outside the N-circle of radius r_N.

D.1.5 Contact of Teeth Flanks for Novikov Gearing

The possibility of ensuring favorable conditions of contact of teeth flanks of Novikov gearing is the major advantage of gear system of this design. In order to systematically describe favorable conditions of contact of the teeth flanks, design parameters of a Novikov gear pair that influence the geometry of contact of the teeth flanks G and P should be considered.

Consider the configuration of the interacting teeth flanks at the point of culmination. Figure D.8 shows a section on the transverse plane. The pinion, which has an LH helix, is rotating ω_p about the axis O_p in a clockwise direction and is driving the gear. The latter is rotating ω_g about the axis O_g. The point of contact K moves in a direction at right angles to and into the plane of the paper in Figure D.8. The pinion and

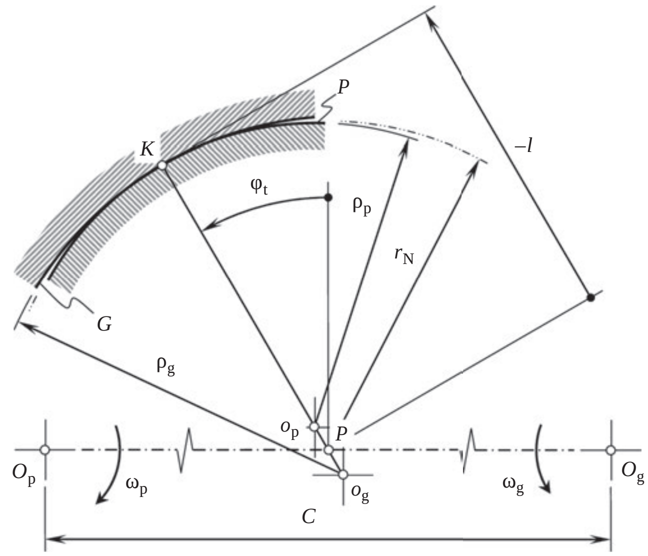


FIGURE D.8 Design parameters of a Novikov gear pair that influence the geometry of contact of the teeth flanks G and P.

the gear have working pitch radii of r_p and $r_g = u \cdot r_p$, respectively, where u is the gear ratio. The basic condition that the angular velocity ratio is equal to the gear ratio requires that the common normal at the point of contact between the teeth passes through the pitch point P. The angle ϕ_t is the transverse pressure angle. With teeth of involute form, this condition is maintained as the gears rotate with the teeth in contact. With circular arc teeth, however, the condition occurs at only one instance in any one transverse plane as the pitch circles roll together. Immediately before, as well as immediately after the configuration that is shown in Figure D.8, there is no contact in that particular plane between the teeth shown. French (1965) proposed to refer to the instantaneous contact of profiles in a transverse section as the *culminating condition*.

When the gears are loaded, due to elastic deformation of the gear materials, the contact point spreads over a certain area of contact, which results in a finite contact period.

Contact lines on the gear tooth flank G and on the pinion tooth flank P are helices of the opposite hands. If the screw parameter p_p of the pinion tooth flank (reduced pitch of the pinion) P is given, then for the computation of the screw parameter p_g of the gear tooth flank G (reduced pitch of the gear), the expression $p_g = p_p/u$ can be used. This means that for Novikov, helical gears, which are in point contact, will transform rotation with a constant gear ratio if their screw parameters p_g and p_p are related as follows:

$$\frac{p_g}{p_p} = \frac{\phi_g}{\phi_p}, \tag{D.2}$$

where $p_g = r_g \tan \lambda_g$, and λ_g is the lead angle, and r_g is the pitch radius of the gear. Similarly, $p_p = r_p \tan \lambda_p$, and λ_p is the lead angle, and r_p is the pitch radius of the pinion.

Because Novikov gears are helical and of opposite hands, the point of teeth flanks contact will move axially along the

gears while remaining at the same radial position on both gear and pinion teeth. It is, therefore, fundamental to the operation of the Novikov gears that contact nominally occurs at a point and that the point of contact moves axially across the full face width of the gears during rotation. It is clearly a condition of operation that in a given profile, the tooth surfaces should not interfere before or after culmination, when rotated and at angular speeds that are in the gear ratio.

D.2 MAIN FEATURES OF HELICAL GEARING BY WILDHABER

In 1926, the famous American gear engineer Dr. Ernst Wildhaber had been granted U.S. Patent No. 1,601,750 on helical gearing. The helical gearing by Dr. Wildhaber features circular-arc tooth profile. The main features of gearing of this design are illustrated in Figure D.9.

The invention relates to the tooth shape of gears, which run on parallel axes, and may be applied to helical gears, such as single-helical gears and double-helical gears or herringbone gears. To provide accurate gearing of circular arc profile is one of the purposes of the helical gearing (Wildhaber, 1926). No other tooth profiles except of circular arc profile are proposed in this invention.

Referring to Figure D.9, 1 denotes a helical gear having teeth 2 in contact with the teeth 3 of a mating pinion 4. As it is customary, the helical gearing is analyzed with reference to a normal section, i.e., line 2–2 in the upper portion of Figure D.9 being normal to the helix of the pitch circle. The lower portion of Figure D.9 illustrates the said normal section 2–2 for both pinion 4 and gear 1.

As an example, it has been assumed that the tooth profiles 6 of gear 1 are circular arcs of radii 7 and centers 8, in the normal section shown. Centers 8 are situated close to the pitch circle 9 of the gear. The location of centers 8 in relation to the line of action is not specified in the invention. The corresponding teeth of pinion 4 are shaped so as to allow rolling of pitch circles 9 and 10 on each other, as well known to those skilled in the art. So no freedom in choosing the pinion tooth profile is allowed in the invention.

When gear tooth 2 is in the position shown in Figure D.9 and its center at 8, then it contacts with tooth 3 at point 11, which may be determined by a perpendicular to tooth 2 through point 12, point 12 being the contact point between the two pitch circles 9 and 10. Commonly, point 12 is referred to as *pitch point*. The said perpendicular is in the present case connecting the line between pitch point 12 and center 8 of the tooth profile.

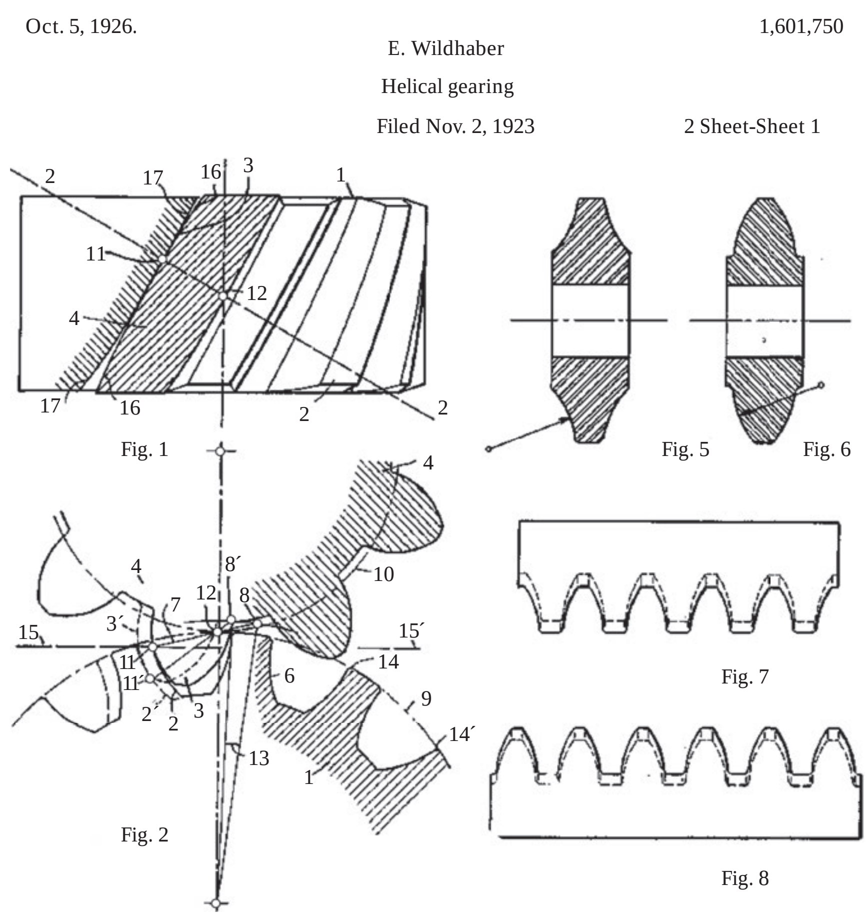


FIGURE D.9 Schematic of helical gearing. (From Wildhaber, E., Helical gearing, U.S. Patent No. 1,601,750, 1926, filed November 2, 1923, and published October 5, 1926.)

Another position 2′ of the gear tooth and 3′ of the corresponding pinion tooth are shown in dotted lines in Figure D.9. The tooth profiles contact here at point 11′, which can be determined like point 11. It will be noted that the contact point has traveled from 11 to 11′ during a small angular motion of the gears.* A certain line of action *LA* is passing through points 11 and 11′. The contact point has practically passed over the whole active profile during a turning angle 13 of the gear, which the angle corresponds to a fraction only of the normal pitch 14, 14′. The said normal pitch is equal to the circular pitch of the normal section shown. In omitting numerous inconsistencies and discrepancies between the design parameters of the gear pair, it is of critical importance to stress here that traveling of the contact point within a transverse section[†] of the gear pair indicates that the transverse contact ratio m_p of the helical gearing (Wildhaber, 1926) is larger than zero ($m_p > 0$).

In the gearing according to the invention of Wildhaber (1926), the contact point between two normal profiles passes over the whole active profile during a turning angle, which corresponds to less than one-half the normal pitch, and usually to much less than that.

It is then claimed that the helical gearing (Wildhaber, 1926) is capable of ensuring better contact between the teeth of the gear and of the pinion in a direction perpendicular to the contact line between two mating teeth. Therefore, it is expected that the proposed helical gearing features a line contact of tooth flanks of the gear and of the pinion.

The gearing according to the invention of Wildhaber (1926) is strictly a gearing for helical teeth. It would not be advisable on straight teeth, on account of the explained short duration of contact between tooth profiles. It should be pointed out here that in the invention of Wildhaber (1926), it is anticipated that there will be a short duration of contact and not instant contact between tooth profiles.

The working profiles of the gear are concave and circular, and their centers are substantially situated on the pitch circle of the gear. The convex working profiles of the pinion are also circular. Their radii are substantially the same as the radii of the mate tooth profiles. The centers of these profiles are similarly situated on pitch circle of the pinion. Because the centers of the teeth profiles are situated within the corresponding pitch circles, the centers cannot be situated within the line of action.

The performed analysis reveals that the helical gearing (Wildhaber, 1926) is a kind of helical gearing having noninvolute tooth profile and featuring transverse contact ratio that exceeds zero ($m_p > 0$). According to the results of the research undertaken earlier by Euler (in the 18th century), gear pairs of this particular nature are not physically feasible.[‡] In other words, the results of the research earlier obtained by Euler reveal that helical gearing by Dr. Wildhaber (1926) is not workable in nature.

The infeasibility of Wildhaber's helical gearing (1926) along with the principal features of the Novikov gearing (to be considered later) make it possible to conclude that these two kinds of gearing cannot be combined into a common gearing. They must be considered individually and separately from one another.

D.3 THE PRINCIPAL DIFFERENCES BETWEEN NOVIKOV GEARING AND HELICAL GEARING BY WILDHABER

The helical gearing by Dr. Wildhaber feature circular-arc tooth profile. This particular feature of Wildhaber's invention was confusing for some gear engineers in Western countries. Taking into account that both Novikov gearing and Wildhaber gearing are kinds of helical gears, less experienced gear engineers loosely decided to combine both gear systems into a common system and to refer to Novikov gearing as *Wildhaber–Novikov gearing* or just *W-N gearing* for simplicity. This combination is incorrect as outlined later. Therefore, the two gear systems cannot be combined in a common system, and they should be considered separately from each other. The above-mentioned terms *Wildhaber–Novikov gearing* and *W-N gearing* are meaningless. Both of them should be eliminated from gear engineering vocabulary as adopted by proficient gear experts.

The main differences between two gear systems are as follows:

- Transverse contact ratio m_p for Novikov gearing is always equal to zero ($m_p = 0$). This requirement is a must for Novikov gearing. Total contact ratio m_t in this case is equal to face contact ratio m_F; i.e., the expression $m_t = m_F > 1$ is valid for Novikov gearing.
- Transverse contact ratio m_p for Wildhaber gearing always exceeds zero ($m_p > 0$) as the contact point is traveling within transverse section (". . . the contact point has passed practically over the whole active profile during a turning angle 12 of the gear . . ."). Thus, total contact ratio m_t in this case is equal to the sum of transverse contact ratio m_p and of face contact ratio m_F; i.e., the expression $m_t = m_p + m_F > 1$ is valid for Wildhaber gearing.

It is clear that no gearing of any kind can simultaneously feature two different values of transverse contact ratio, namely, the transverse contact ratio $m_p = 0$ and the transverse contact ratio $m_p > 0$. This inconsistency makes clear the impossibility of combining of Novikov gearing and Wildhaber gearing into a certain common gear system.

As a consequence of the equality $m_p = 0$, Novikov gearing can be designed to have reasonably small difference between curvatures of convex tooth profile of one member and concave tooth profile of mating member of a Novikov gear pair. Wildhaber gearing does not allow that.

* The ability of the contact point to travel over the tooth profile is mentioned several times in the patent description.

[†] Once the contact point is traveling within the normal section 2–2, then the projection of the contact point onto the transverse section is traveling within the transverse section.

[‡] It should be stressed here that helical gearing (Wildhaber, 1926) is a kind of mistake committed by Dr. Wildhaber. Unfortunately, this mistake became widespread within the gear engineering community.

D.4 NOVIKOV GEARING AS A REDUCED CASE OF INVOLUTE GEARING

Actually, Novikov gearing is a reduced case of involute gearing.

D.4.1 A Desired Line of Contact in a Parallel-Axis Gearing

The tooth ank of the gear □ and that of the pinion □ make contact along the line of contact *LC*. The line of contact is a planar curve of a reasonable geometry that is entirely located within the plane of action *PA*. The teeth anks □ and □ interact only within the active portion of the plane of action shown in Figure D.10.

Referring to Figure D.10a, $N_g N_p$ is the total length of the plane of action. In reality, the active portion of the plane of action *PA* is of smaller length Z_{PA} (Figure D.10b). (In a case of Novikov gearing, the equality $Z_{PA} = 0$ is observed.)

In involute helical P_a gearing, the desired line of contact *LC* between the tooth ank of the gear □ and the pinion □

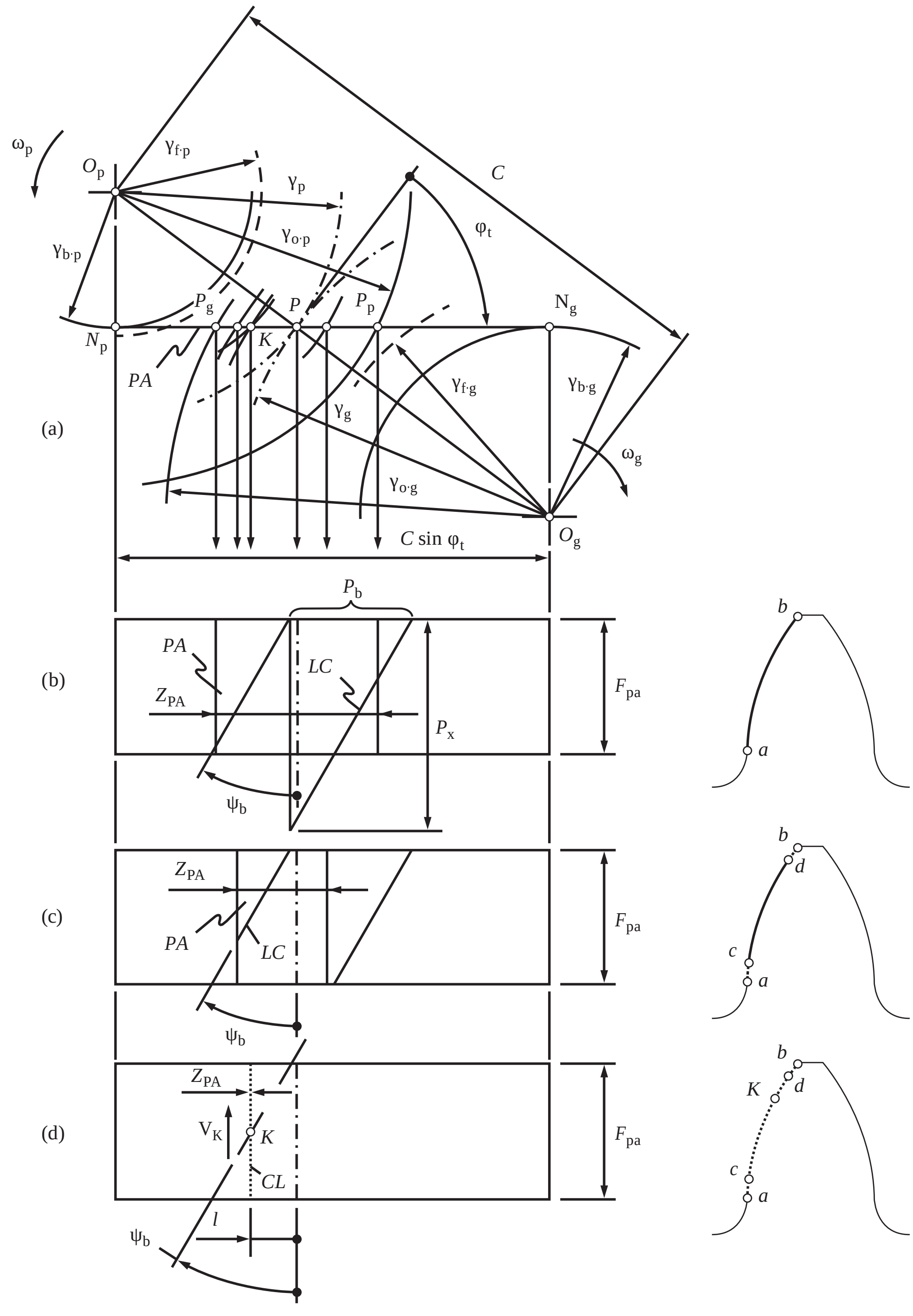


FIGURE D.10 (a–d) Elements of a parallel-axis gear featuring zero transverse ratio ($m_P = 0$). (a) Schematic of a parallel-axes gear pair with (b) a nominal, (c) a reduced, and (d) and zero length of the active portion of the line of action.

(remember that the tooth flanks □ and □ are not constructed yet) is a straight-line segment that forms a base helix angle ψ_b with the axis of instant rotation P_{ln}.

The total contact ratio m_t can be expressed in terms of the transverse contact ratio m_p and the face contact ratio m_F.

$$m_t = m_p + m_F, \tag{D.3}$$

where $m_p = Z_{PA}/p_b$ and $m_F = F_{pa} \tan \psi_b / p_b$.

The inequality $m_t \geq 0$ must be observed for any and all parallel-axis gear pairs.

When the base cylinders of diameters $d_{b.g}$ and $d_{b.p}$ rotate, the desired line of contact *LC* travels (together with the plane of action *PA*) in relation to the reference systems, one of which is associated with the gear, and another one is associated with the pinion. In such a motion, the tooth flank of the gear □ (as well as the tooth flank of the pinion □) can be interpreted as a family of consecutive positions of the desired line of contact in the corresponding reference system.

In the example (Figure D.10b), the active portion *ab* of the involute tooth profile is specified by the radii of the outer cylinders of the gear and of the pinion, $r_{o.g}$ and $r_{o.p}$, respectively. Point *a* corresponds to the start-of-active-profile point (SAP point), while point *b* corresponds to the end-of-active-profile point (EAP point).

For both members of a gear pair, that is, for the gear and the pinion, the radius r_{EAP} of the EAP circle can be smaller than the outer radius of the gear $r_{o.g}$ (or than the outer radius of the pinion $r_{o.p}$), for example, due to chamfering. Under such a scenario (Figure D.10c), the active portion of the plane of action becomes narrower. The SAP point *c* and the EAP point *d* become closer to one another: the active portion *cd* of the involute tooth profile is shorter than that of *ab* (Figure D.10b). This gives a certain freedom when selecting the geometry of nonactive portions *ac* and *bd* of the tooth profile. As these portions of the tooth profile do not interact with one another, the geometry of the segments *ac* and *bd* is not restricted by the conditions of meshing of the tooth profiles (which is a must for the active portion *cd*).

In the extreme case, the EAP circles of the gear and of the pinion can pass through a certain point *K* within the straight-line segment P_gP_p. Because of this, the length Z_{PA} of the active portion of the plane of action becomes zero ($Z_{PA} = 0$), and the active portion of the involute tooth profile shrinks to point *K*. The nonactive portions *aK* and *bK* of the tooth profile meet each other at point *K*. These portions are not subject to conditions of meshing of tooth profiles; thus, this gives a certain freedom when selecting the geometry of nonactive portions *aK* and *bK* of the tooth profile (Figure D.10d).

As the width of the active portion of the plane of action is zero ($Z_{PA} = 0$) and the involute tooth profile is shrunk to a point, the transverse contact ratio m_p becomes zero. In order to meet the inequality $m_t \geq 0$, the following inequality must be satisfied:

$$m_t = m_p + m_F = 0 + m_F = m_F > 0. \tag{D.4}$$

The point system of P_a gearing (Figure D.10d) gives much freedom when designing nonactive portions of tooth profiles of the gear and the pinion as the geometry of these portions is free of constraints imposed by conditions of the meshing of two conjugate tooth profiles.

The discussion reveals that Novikov gearing is a kind of reduced involute gearing, while helical gearing by Wildhaber is *not* a kind of involute gearing.

D.4.2 Design Features of Novikov Gearing

The concept of Novikov gearing is based on the schematic depicted in Figure D.10d. For this case (Figure D.10d), Novikov proposed to replace convex-to-convex contact of the teeth profiles of the gear and the pinion with convex-to-concave contact. Such a replacement becomes possible only in a case when the active portion of the involute tooth profile is shrunk to a point (and it is infeasible in cases when the active portion of the involute tooth profile is of certain length [Radzevich, 2012]).

D.5 POSSIBLE ROOT CAUSES FOR THE LOOSELY USED TERMS WILDHABER–NOVIKOV GEARING AND W-N GEARING

From the author's standpoint, unfamiliarity of Western engineers with the original publications (Novikov, 1955; 1957; 1958) is the root cause of the incorrect interpretation of the concept of Novikov gearing. The absence of access to the original documents (Novikov, 1955; 1957; 1958), some of which were classified for a long period, along with the so-called Iron Curtain, created a significant barrier for Western engineers that were interested in the novel system of gearing.*

Because of that, in Western countries, attention was wrongly focused on the similarity of circular-arc tooth profiles (which is of secondary importance for Novikov gearing), and not on the zero transverse contact ratio ($m_p = 0$), which is of critical importance for Novikov gearing. Once the equality $m_p = 0$ is valid for a gear pair, then the gear engineer gets much freedom in designing teeth profiles of mating gears, making them maximally conformal to one another.

It should be mentioned here that as early as November 13–16, 1957, an all-union scientific conference, "Practice of Implementation of Novikov Gearing," was held in Moscow. A decision to call the novel system of gearing as Novikov gearing had been adopted by the conference. This decision honors Dr. Novikov as the inventor of novel system of gearing, and it should be acknowledged by gear experts all around the world.

* No scientific publications by Western gear engineers who quoted the original publications by Novikov (1955, 1958) are known to the author. The other original publication by Dr. Novikov (1957) is often mistakenly quoted as S.U. Patent No. 109750 (Pisulin, 1957).

D.6 A BRIEF BIOGRAPHICAL SKETCH OF DR. MIKHAIL L. NOVIKOV (1915–1957)

Not too much is known about Dr. Novikov. It is known that he was born on March 25, 1915, in the city of Ivanovo (Russia). He was born to a lower-class family—his parents were workers. At the age of 15 he began working as an apprentice at a machine-building factory.

Beginning 1934, Novikov attended Moscow Bauman Technical University (MBTU). After 2 years of his study at MBTU was over, he switched to the Military Aircraft Engineering Academy (MAEA), bearing the name of the famous scientist Professor N. I. Zhukovsky. It is likely this transition was due to his significant achievements in education. After graduating from MAEA in 1940, he was offered to work there with one of the special departments. In a short period, he did pass from a position of assistant professor to the chair of the department having a military rank of colonel.

In parallel to teaching engineering courses for the academy students, which was a must for him, Novikov was involved in an intensive research. He had been granted patents on numerous inventions, most of which were of critical importance for aviation. In particular, Dr. Novikov was concerned with the problem of increasing the bearing capacity of gear pairs. The research undertaken in this particular area of mechanical engineering ended with the development of a novel system of gearing later called Novikov gearing in his honor.

Mikhail Novikov became very well known in the international engineering community for the invention of a novel system of gearing. His idea was that he could overcome the barrier caused by the relations between the curvatures of the contacting surfaces when the gear tooth surfaces are in line contact. For this purpose, he proposed to reduce the transverse contact ratio m_p of a helical gear pair to zero ($m_p = 0$). Under such a scenario, the total contact ratio m_t of the gear pair is equal to the face contact ratio m_F and, thus, the expression $m_t = m_F > 1$ is valid. For gearing of this kind, the gear designer has more freedom in setting curvatures of mating tooth flanks of a gear pair.

Evolving this concept, Professor Novikov proposed that arcs of smooth regular curves be chosen as the profiles of helical gear (Novikov, 1956). These arcs are constructed on a plane that is perpendicular to the axis of instant rotation of the gears (perpendicular to the pitch line of a gear pair). In particular (but *not* mandatory!), circular arcs could be used in designing the teeth profiles. The difference between the curvatures of the convex tooth profile of one of the gears and the concave tooth profile of the mate is kept small to absorb the manufacturing errors, as well as the teeth flank displacements under the load, and so forth.

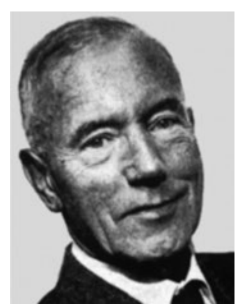
Nonproficient researchers, having the wrong understanding of the kinematics and geometry of Novikov gearing, mistakenly identified the Novikov gearing with the helical gearing earlier proposed by Wildhaber (1926). For Wildhaber's helical gearing, the expression $m_t = m_F > 1$ is *not* valid, as for this gearing, another relation among the design parameters is observed ($m_P > 0$, $m_F > 0$, and $m_t = m_P + m_F > 1$). They loosely call Novikov gearing as Wildhaber–Novikov gearing or, for simplicity, W-N gearing. Proficient gear engineers realize the inadequacy of these two last terms and avoid using them.

Novikov's invention became very popular in the former Union of Soviet Socialist Republics and later in Russia, and he was awarded a prestigious national prize. Lots of research on Novikov gearing had been carried out in the United States as well as in other Western countries. His invention inspired many researchers, which resulted in valuable contributions being made to the theory of gearing.

Through all of his life, Novikov was a modest person regardless of his fame and power he had.

An intensive research work was ceased with his sudden death at a young age, a catastrophe for his family, colleagues, and students.

D.7 A BRIEF BIOGRAPHICAL SKETCH OF DR. ERNST WILDHABER (1893–1979)



Ernst Wildhaber is one of the most famous inventors in the field of gear manufacture and design. He received 279 patents, some of which have a broad application in the gear industry because of his work as an engineering consultant for Gleason Works. Dr. Wildhaber's most famous inventions are (a) the hypoid gear drive, which is still used in cars, and (b) the Revacycle method, which is a very productive way to generate straight bevel gears.

Dr. Wildhaber graduated from the Technische Hochschule of Zurich University in Switzerland and then came to the United States in 1919. In 1924, he went to work for Gleason Works, where he began the most successful period of his career as a creative engineer and inventor.

Some of Wildhaber's former colleagues from Gleason Works have recounted how deeply impressed they were with his creativity, imagination, almost legendary intuition, and alacrity. This last characteristic was even the source of a colleague's complaint that Wildhaber was not as patient as university professors are while giving explanations. Could it be that he expected his coworkers to comprehend at the same speed as his thoughts? They remembered that he even sped up the stairs, taking two steps at a time. Wildhaber's inventions reveal signs of his originality. He proposed different pressure angles for the driving and the coast tooth sides of a hypoid gear, which allowed him to provide constancy of the tooth top land.

Wildhaber's invention milestone was the Revacycle and the unusual shape of the tool based on the location of blades on a spatial curve. His theoretical developments (Wildhaber, 1946a; 1946b; 1946c; 1956) also display his originality; for example, he found the solution to avoiding singularities and undercutting in hypoid gear drives.

Ernst Wildhaber's talent was recognized not only in the United States but also by the world engineering community and particularly by his alma mater, Zurich University, which awarded him an honorary doctorate in engineering in 1962.

D.8 GEAR PAIRS AND CAM MECHANISMS HAVING POINT SYSTEM OF MESHING BY M. L. NOVIKOV (S.U. PATENT NO. 109,113)

Novikov's patent (S.U. Patent No. 109,113 of 1956) is a rare publication which is not available to most gear experts. No translation of the patent from Russian to English is available for the public. This causes problems in the proper understanding and in the interpretation of signi cance of this milestone invention.

Because of this and for the readers' convenience, an invention disclosure of the Novikov gearing along with its translation from Russian to English is given for free discussion and for a comparison with Wildhaber's gearing.

Класс 47h, 6 № 109113

СССР

ОПИСАНИЕ ИЗОБРЕТЕНИЯ

К АВТОРСКОМУ СВИДЕТЕЛЬСТВУ

М. Л. Новиков

ЗУБЧАТЫЕ ПЕРЕДАЧИ, А ТАКЖЕ КУЛАЧКОВЫЕ МЕХАНИЗМЫ С ТОЧЕЧНОЙ СИСТЕМОЙ ЗАЦЕПЛЕНИЯ

Заявлено 19 апреля 1956 г. за № 550525 в Комитет по делам изобретений и открытий при Совете Министров СССР

Известные зубчатые передачи с точечной системой зацепления обладают низкой контактной прочностью и широкого практического применения не получили.

Существующие зубчатые передачи с линейчатыми системами зацепления, в том числе широко распространенные эвольвентные, также обладают ограниченной контактной прочностью.

Предлагаемая зубчатая передача имеет более высокую контактную прочность, что обусловлено более выгодными кривизнами сопряженных поверхностей зубцов. Передаваемые окружные усилия могут быть в несколько раз большими по сравнению с эвольвентным зацеплением при одинаковых контактных напряжениях, одинаковых габаритных размерах и прочих равных условиях. Другим преимуществом их является меньшая чувствительность к неточностям изготовления и деформациям деталей зубчатых передач.

Предлагаемые зубчатые передачи могут быть выполнены с параллельными, пересекающимися и перекрещивающимися осями, с внешним и внутренним зацеплением, с постоянным и переменным передаточным числом и могут быть применены для кулачковых механизмов.

На чертеже изображены возможные профили зубцов, получающиеся при пересечении их рабочих поверхностей, плоскостью, перпендикулярной к мгновенной оси относительно вращения-скольжения и проходящей через текущее положение точки зацепления.

Здесь P — точка пересечения с мгновенной осью относительного вращения-скольжения плоскости профилей, перпендикулярной мгновенной оси.

O_1 и O_2 — точки пересечения плоскости профилей с осями зубчатых колес.

A — точка зацепления (текущее положение).

PA — линия давления.

$ДАД$ — круговая дуга с центром в точке P, являющаяся предельным случаем профилей зубцов (совпадающих друг с другом).

(a)

Кривые *ВАВ* — кривые произвольной плавной формы, находящиеся внутри круговой дуги *ДАД* (т. е. уходящие в «тело» зубцов одного из колес) и расположенные в непосредственной близости от нее, являются профилями зубцов одного из колес.

Кривые *САС*—кривые произвольной плавной формы, находящиеся вне круговой дуги *ДАД* (уходящие в «тело» другого зубца) и расположенные в непосредственной близости от *ДАД*, являются профилями зубцов другого колеса.

Сущность изобретения заключается в следующем.

В пространстве, в котором зафиксированы оси вращения зубчатых колес, задается линия зацепления в виде прямой или плавной кривой, проходящая в непосредственной близости от мгновенной оси относительного вращения-скольжения. Вдоль линии зацепления назначается движение точки зацепления с постоянной или плавно изменяющейся скоростью. Движущаяся точка зацепления описывает в пространствах, связанных с вращающимися зубчатыми колесами, контактные линии. Через эти линии можно провести ряд некоторых поверхностей, которые могут быть сопряженными рабочими поверхностями зубцов, если они будут иметь общую нормаль в каждом текущем положении точки зацепления и будет удовлетворяться основная теорема зацепления, если кривизны поверхностей будут удовлетворять заданному передаточному отношению и если, наконец, будет отсутствовать взаимное пересечение поверхностей в пределах их рабочих участков.

Предлагаемые виды рабочих поверхностей зубцов удовлетворяют поставленным условиям и обеспечивают высокую контактную прочность зубцов.

В плоскостях, перпендикулярных мгновенной оси относительного вращения-скольжения и проходящих через текущее положение точки зацепления, проводятся дуги окружностей из центров, находящихся на прямой, проходящей через точку зацепления и мгновенную ось, и в непосредственной близости от этой оси. Дуги окружностей можно считать профилями зубцов.

Непрерывная совокупность профилей для всех текущих положений точки зацепления образует сопряженные поверхности зубцов, при этом рабочая поверхность для одного из колес будет выпуклой, а для другого—вогнутой (по направлению, перпендикулярному к мгновенной оси). В частном случае радиусы профилей зубцов могут быть одинаковыми по величине и равными расстоянию от точки зацепления до мгновенной оси. Центры обоих профилей при этом будут лежать на мгновенной оси. В этом случае точечное зацепление вырождается в особый вид линейчатого, однако при реализации его требуется весьма высокая точность и абсолютная стабильность межосевого расстояния, что практически неосуществимо. При конструировании зубцов следует отдать предпочтение точечному зацеплению с малой разницей в величинах радиусов профилей, имея в виду, что практически при технологической, а также при естественной приработке зубцов точечное зацепление будет приближаться к упомянутому линейчатому, хотя теоретический контакт при этом будет точечным.

Профили зубцов могут иметь форму, отличающуюся от дуги окружности, однако кривые профилей другой формы (проходя всегда через точку зацепления) должны находиться внутри упомянутого кругового профиля с центром в точке, находящейся на мгновенной оси (должны уходить в «тело» зубцов).

Закон движения точки зацепления (т .е. скорость и траектория ее движения) выбирается так, чтобы потери на трение и износ были незначительными. Потери на трение и износ пропорциональны относительной скорости скольжения в зацеплении, следовательно необходимо стремиться к уменьшению последней, а для этого линию зацепления не следует значительно удалять от мгновенной оси. Однако чрезмерное

(b)

приближение линии зацепления к мгновенной оси нецелесообразно, так как связано с уменьшением контактной прочности. Кроме того, при выборе закона движения точки зацепления следует предусматривать благоприятные с конструктивной точки зрения углы между общей нормалью (вдоль которой расположена сила взаимодействия зубцов) и осями колес, а также технологические условия.

Замыкающие (тыловые) стороны зубцов образуются аналогичным вышеизложенному методом. Толщина зубцов (и шаг их) устанавливается в соответствии с необходимой прочностью зубцов на изгиб и сдвиг.

Ширина обода зубчатых колес или длина зубцов должна находиться в таком соотношении с шагом их, при котором обеспечивался бы заданный коэффициент перекрытия при пересопряжении пар зубцов. Зубчатые передачи могут иметь одноточечное зацепление, т. е. в работе может участвовать только одна пара зубцов (за исключением периода пересопряжения), и могут быть передачи с многоточечным зацеплением, когда в одновременной работе находится несколько пар зубцов.

В случае зубчатых передач с параллельными осями удобнее всего, исходя из конструктивных и технологических соображений, выбрать линию зацепления в виде прямой, параллельной осям колес, а скорость движения точки зацепления принять постоянной. При этом радиусы профилей зубцов во всех плоскостях, перпендикулярных осям, будут одинаковы, рабочие поверхности зубцов будут представлять собой правильные винтовые поверхности. Изготовление таких зубцов возможно на всех современных зуборезных станках с применением специального инструмента.

На чертеже представлена зубчатая передача с параллельными осями с предельным случаем точечного контакта, переходящего в линейчатый, с криволинейной контактной линией, располагающейся поперек зубцов. В случае нежелательности осевых компонентов сил, нагружающих подшипники, можно применять зубчатые колеса шевронного типа.

Предмет изобретения

Зубчатые передачи, а также кулачковые механизмы с точечной системой зацепления, **отличающиеся** тем, что, с целью увеличения передаваемых окружных усилий, профили зубцов, получающиеся при пересечении их рабочих поверхностей плоскостью, перпендикулярной к мгновенной оси относительного вращения-скольжения и проходящей через текущее положение точки зацепления, являются дугами окружностей или другими плавными кривыми, приближающимися по форме и по величине радиуса кривизны к дуге окружности с центром, совпадающим с точкой пересечения рассматриваемой плоскости с мгновенной осью, а линия зацепления, представляющая геометрическое место точек зацепления в пространстве, в котором зафиксированы оси вращения колес, является прямой или плавной кривой.

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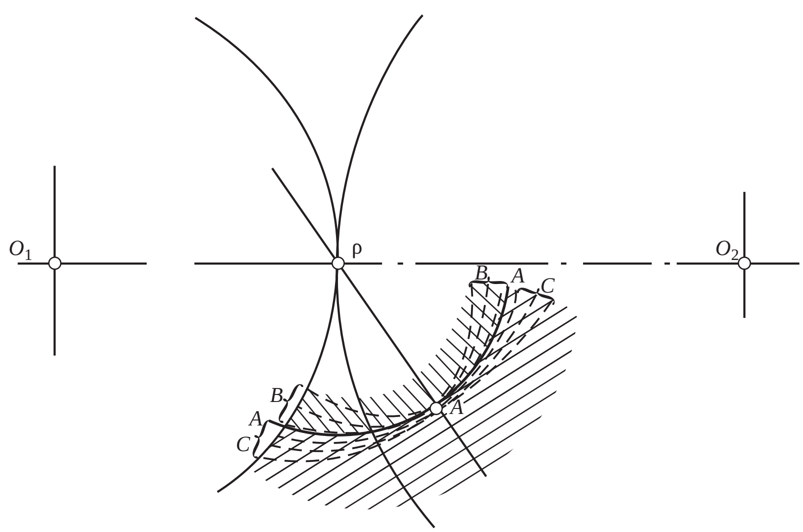

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Classification 47h, 6 №109113

USSR

INVENTION DISCLOSURE
to the Certificate on Invention

M.L. Novikov

GEAR PAIRS AND CAM MECHANISMS HAVING POINT SYSTEM OF MESHING

Filed: April 19, 1956, application №550525 to Committee on Inventions and Discoveries at Council of Ministries of the USSR

Known designs of gearing, those featuring point system of meshing, featuring low contact strength and are not widely used in practice.

The contact strength of known designs of gearing, those having line system of meshing, including the widely used involute gearing, is limited as well.

The proposed gearing features higher contact strength due to favorable curvatures of interacting tooth flanks. Under equivalent contact stress, similar dimensions and comparable rest of the design parameters, greater circular forces are permissible by the proposed gearing. Lower sensitivity to manufacturing errors and to deflections under the load is another advantage of the proposed gearing.

The proposed gearing can be designed either with parallel, or with intersecting, or with crossing axes of rotations of the gears. External gearing as well as internal gearing of the proposed system of meshing is possible. The tooth ratio of the proposed gearing either can be f constant value or can be variable and time dependent. The propose concept of gearing can be utilized in design of cam mechanisms.

Possible tooth profiles in a cross section of tooth flanks by a plane that is perpendicular to the instant axis of relative rotation through the current point of contact are illustrated in the Figure.

Here, the point of intersection of the planar cross section by the axis of instant relative rotation is denoted by P.

O_1 and O_2 are the points of intersection of the planar cross section by the axes of the gear and of the pinion.

A is the point of meshing (in its current location).

PA denotes the line of action.

ДАД is the circle centering at the point P which corresponds to the limit case of the tooth profiles (in the case the profiles are aligned to each other).

Several curves *BAB* represent examples of tooth profiles of one of the mating gears. The curves *BAB* are arbitrary smooth curves, which are located inside of the circular arc *ДАД* (i.e., the arcs are located within the bodily side of the limit tooth flank of one of the gears). The curves *BAB* are located close to the circular arc *ДАД* and they feature high rate of conformity to the circular arc.

Several curves *CAC* represent examples of tooth profiles of the second of the mating gears. The curves *CAC* are arbitrary smooth curves, which are located outside of the circular arc *ДАД* (i.e., the arcs are located within the bodily side of the limit tooth flank of another of two gears). The curves *CAC* are also located close to the circular arc *ДАД* and they feature high rate of conformity to the circular arc.

The entity of the invention is disclosed below in detail.

The location and orientation of either straight line of meshing or of smooth curved line of meshing is specified in a space in which the location and orientation of axes of rotations of the gear and of the pinion are given. The line of meshing is located reasonably close to the axis of instant relative rotation of the gears. Either constant or time-dependent (smoothly varying in time) speed of motion of the point of meshing along the line of meshing is assigned. A coordinate system is associated with the gear, and a corresponding coordinate system is associated with the pinion. In the coordinate systems the moving meshing point traces contact lines. One of the contact lines is associated with the gear and another one is associated with the pinion. Certain smooth regular surfaces through the meshing lines can be employed as tooth flanks of the gear and of the pinion. The following requirements should be fulfilled so that the surfaces could be used as the tooth flanks:

- At every location of the point of meshing, the tooth flanks should have a common perpendicular and, thus, the requirements of the main theorem of meshing should be satisfied;
- The curvatures of tooth profiles should correspond to each other; and, finally,
- No tooth flanks interference is occurred within the working portions of the surfaces.

The proposed kinds of tooth flanks fulfill the above listed requirements and allow for high contact strength of the gear teeth.

Consider a plane through the current meshing point, that is perpendicular to the instant axis of relative rotation. Construct two circular arcs centering at points within the straight line through the pitch point and the meshing point. The arc centers are located close to the pitch point. The constructed circular arcs can be considered as examples of the tooth profiles of the gear and of the pinion. Tooth flanks are generated as loci of tooth profiles constructed for all possible locations of the meshing point. The working portion of one of two tooth flanks is convex, while the working portion of another tooth flank is concave (in the direction toward the axis of instant relative rotation). In a particular case the radii of tooth profiles could be of the same magnitude and equal to the distance from the meshing point to the axis of instant relative rotation. The centers of both profiles in this particular case are located at the axis of instant relative rotation. Under such a scenario, point kind of meshing is reduced to a special kind of line kind of meshing. This would require an extremely high accuracy of the center distance and its independence from operation conditions, which is impractical. Point meshing is preferred when designing tooth profiles. A small difference between the radii of curvature of tooth profiles is desired. It should be kept in mind that under run-in period, point meshing of the gear teeth will be transforming to the mentioned above line meshing of the tooth profiles. However, the theoretical point contact of the tooth flanks will be retained.

Tooth profiles can differ from the circular arcs. However, tooth profiles of other geometries (those always passing through the meshing point) should be located (for one gear) within the interior of the above-mentioned circular arc profile that is centering at the point within the axis of instant relative rotation as shown in the Figure. For another gear, the tooth profile should be located outside the circular arc.

The law of motion of the meshing point (i.e., speed of the point and its trajectory) should be chosen so as to minimize the friction and wear loses. Friction and wear loses are proportional to the relative sliding velocity in the gear mesh. Therefore, it is desired to reduce the sliding velocity as much as possible. For this purpose the line of meshing should not be remote too far from the axis of instant relative rotation. On the other hand, too close location of the line of meshing to the axis of instant relative rotation is also not desired as that reduces the contact strength of the gear tooth flanks. In addition, it is recommended to ensure favorable angles between the common perpendicular (along which tooth flanks of one of the gears acts against the tooth flank of another gear) and between the axes of rotations of the gears.

Opposite sides of tooth profiles are designed in the way similar to that just discussed. Tooth thicknesses and pitch are assigned to ensure the required bending tooth strength.

The face width of the gear or length of the gear teeth should correlate to their pitch to ensure the required value of the face contact ratio. Gear pairs can feature either one point of contact (when working portions of the tooth flank contact each other just in one point, excluding the phases of the teeth reengagement), or they can feature multiple contact points when tooth flanks contact each other at several points simultaneously.

For parallel axis gear pairs it is preferred to employ a straight line as the line of meshing, which is parallel to the axes of rotations of the gear and of the pinion. The speed of the meshing point along the straight line of meshing can be of constant value. In this particular case, the radii of curvature of tooth profiles in all cross sections by planes are equal to each other. Tooth flanks in this case are a kind of regular screw surfaces. Gears that feature tooth flanks of such geometry are easy to manufacture, and they can be cut on machine tools available on the market.

An example of parallel axis gearing with limit geometry of tooth profiles is illustrated in the Figure. Point contact of the tooth flanks in this particular case is transformed to line contact. The curved contact line is located across the tooth profile. When axial thrust in the gear pair is strongly undesired, herringbone gears can be used instead.

SUBJECT OF THE INVENTION

Gear pairs as well as cam mechanisms having point system of engagement differ from known designs in the following: (a) Tooth profiles are created as the lines of intersection of the tooth flanks by planes, which are perpendicular to the axis of instant relative rotation that is passing through the point of meshing in its current location and are circular arcs or other smooth regular curves conformal to the radii of curvature of the circular arc centering at the point of intersection of the instant axis of rotation by the plane; while the line of action, that is, the loci of points of meshing in space (within which the configuration of the axes of rotations of the gear and of the pinion are specified), is a straight line or a smooth regular curve.

D.9 *HELICAL GEARING* BY ERNST WILDHABER (U.S. PATENT NO. 1,601,750)

With the greatest respect to Dr. Ernst Wildhaber and to what had been done by him in the field of gearing and gear machining, it should be mentioned here that his helical gearing (U.S. Patent No. 1,601,750, of 1926) is a kind of mistake. Dr. E. Wildhaber's mistake could be forgiven—we all make mistakes from time to time.

Unfortunately, Dr. Wildhaber's mistake significantly affected further developments in the field of gearing, and ultimately it resulted in the wide usage of the completely wrong term *Wildhaber–Novikov gearing* or simply *W-N gearing.*

The combination of Wildhaber's gearing with the gearing proposed by Dr. M. L. Novikov is incorrect, and, thus, it should be eliminated from the scientific vocabulary. These two completely different kinds of gearing, namely, the one proposed by Dr. E. Wildhaber and another one proposed by Dr. M. L. Novikov, must be considered individually, and can *not* be combined into the wrong term *Wildhaber–Novikov* gearing.

For this purpose and for the readers' convenience, an invention disclosure of Wildhaber's gearing is placed in the following for free discussion and for a comparison with Novikov's gearing.

Patented Oct. 5, 1926. **1,601,750**

UNITED STATES PATENT OFFICE.

ERNEST WILDHABER, OF BROOKLYN, NEW YORK.

HELICAL GEARING.

Application filed November 2, 1923. Serial No. 672,254.

My invention relates to the tooth shape of gears, which run on parallel axes, and may be applied to helical gears, such as single helical gears and double helical gears or herringbone gears.

One purpose of my invention is to provide helical gearing with improved tooth contact, so as to lessen surface stresses and wear.

A further purpose of the invention is to provide helical gearing, which is capable of rapid and accurate production, and which may be ground without difficulty, if so desired.

A still further purpose of the invention is to provide accurate gearing of circular tooth profile.

My invention is illustratively exemplified in the accompanying drawings, in which, Figure 1 is a side elevational view of my improved gear showing parts thereof in section; Figure 2 is a normal sectional view of Figure 1, taken on the lines 2—2 of the latter figure; Figure 3 is a side elevational view of a pair of gears constructed in accordance with my invention; Figure 4 is a sectional view taken through a pair of gears; Figures 5 and 6 are sectional views of milling cutters used in the manufacture of my improved gears; Figures 7 and 8 are elevational views of corresponding tools of rack shape, to be used in reciprocating machines for cutting helical gears in accordance with my invention; Figures 9 and 10 are side elevational views of my improved gear showing a pair of grinding wheels in different operating positions, the wheels being set to grind opposite tooth surfaces; Figure 11 is a view of a gear taken in normal section and showing the grinding wheels in operating position; Figure 12 is a view of a mate pinion showing the grinding wheels in operating position; Figure 13 is a view of modified form of gear made in accordance with my invention; Figure 14 is a sectional view taken through an internal gear and its pinion; Figure 15 is a normal section through helical teeth of composite outline, constructed from my invention; Figure 16 is a view of a reciprocating tool of rack shape in operating position; and Figure 17 is a view of a modified type of reciprocating tools, in position to start a cut on a herringbone gear.

Referring to the drawings, and particularly to Figures 1 and 2, 1 denotes a helical gear having teeth 2 in contact with the teeth 3 of a mating pinion 4. In order to clearly illustrate the degree of contact between the teeth of the gear and pinion the tooth 4 is shown in section in Figure 1.

It is customary to analyze helical gearing with reference to a normal section, i. e. line 2—2 of Figure 1, line 2—2 being normal to the helix of the pitch circle. Figure 2 illustrates the said normal section 2—2 for both pinion 4 and gear 1.

It has been assumed as an example, that the tooth profiles 6 of gear 1 are circular arcs of radii 7 and centers 8, in the shown normal section. Centers 8 are situated close to the pitch circle 9 of the gear. The corresponding teeth of pinion 4 are so shaped as to allow rolling of the pitch circles 9 and 10 on each other, as well known to those skilled in the art.

When the gear tooth 2 is in the position shown, in Figures 1 and 2, and its center at 8, then it contacts with tooth 3 at point 11, which may be determined by a perpendicular to tooth 2 through point 12, point 12 being the contact point between the two pitch circles 9 and 10. The said perpendicular is in the present case the connecting line between point 12 and center 8 of the tooth profile.

Another position 2′ of the gear tooth, and 3′ of the coresponding pinion tooth are shown in dotted lines in Figure 2. The tooth profiles contact here at a point 11′, which can be determined like point 11. It will be noted that the contact point has traveled from 11 to 11′ during a small angular motion of the gears. The contact point has passed practically over the whole active profile during a turning angle 13 of the gear, which angle corresponds to a fraction only of the normal pitch 14, 14′. The said normal pitch equals the circular pitch of the shown normal section.

In gearing now in use, however, the tooth outline and the tooth proportions are so selected, that the contact of corresponding normal profiles lasts for an angle, which, as a rule, corresponds to more than one full pitch.

In gearing according to my invention, the contact point between two normal profiles passes over the whole active profile during a turning angle, which corresponds

(a)

to less than one half the normal pitch, and usually to much less than that.

I have found, that gearing designed according to my invention, allows the teeth to come into better contact with each other, inasmuch as the tooth surfaces remain much closer to each in a direction perpendicular to the contact line between two mating teeth. This is illustrated by a section taken in direction of lines 15, 15′ of Figure 2. In Figure 1 the lateral profile 16 of tooth 3 and profile 17 of tooth 2 of said section are shown to contact at point 11, and to remain close to each other on their whole length. The same holds true for other sections, taken parallel to section 15, 15′.

Close contact between teeth is well known to reduce wear and to improve the efficiency of the gears.

Although a circular arc is shown as the normal tooth profile of gear 1, in Figure 2, it will be understood, that this is not the only shape to effect the stated purpose, of increasing the speed, at which the contact point travels over the tooth profile of a normal section. As a rule, however, the shape can be approximated by a circle, whose center is close to the pitch center.

The gearing according to the present invention is strictly a gearing for helical teeth. It would not be advisable on straight teeth, on account of the explained short duration of contact between tooth profiles. This would cause intermittant action, whereas on helical gears similar parts of the teeth are always in contact, on account of the twisted nature of the tooth surfaces.

Figure 3 may be considered as a view taken in the direction of the axes of a pair of gears. The tooth profiles are then circles in a section, which is perpendicular to the axes. The gear is provided with helical teeth, with working faces below the pitch circle 20, while the pinion teeth have working faces above the pitch circle 21 only. The working profiles 22 of the gear are concave and circular, and their centers are substantially situated on the pitch circle 20. The convex working profiles 23 of the pinion are also of circular shape. Their radii 24 are substantially the same as the radii 25 of the mate profiles. The centers 26, 26′, 26″ are similarly situated on pitch circle 21. Profile centers 27, 27′, 27″ of pitch circle 20, and profile centers 26, 26′, 26″ of pitch circle 21 correspond to each other. They coincide during the mesh, which takes place on the whole tooth profile at once.

Figure 3 can also be considered as a section perpendicular to the helical teeth, and shows then the normal tooth profiles.

Figure 4 shows a refinement of the preferred embodiments of my invention. It is a normal section through the helical teeth, but can also be considered as a section perpendicular to the axes. Corresponding profiles 30 and 31 are circular, as in Figure 3, but in this case the radius of the concave circular profile 30 is made a trifle larger than the radius of the convex circular profile 31. Consequently the profile centers 32 and 33 do not exactly coincide during the mesh. The radii 34 and 35 of the circles 36 and 37, constituted by the profile centers 32 and 33 respectively, are not accurately identical with the pitch radii 38 and 39 of the two gears. The sum of the radii 34 and 35 is a trifle larger than the sum of the pitch radii. The radii 34 and 35 are so selected, that the main tooth pressure runs about in a direction 33, 40.

The slight difference of the radii of profiles 30 and 31 facilitates the tooth contact, and allows for small errors in making and assembling.

Figures 5 and 6 show a pair of milling cutters for milling conjugate teeth. The cutters may be applied in the usual manner, their axes being inclined in correspondence with the tooth inclination, i. e. with the helix angle of teeth. It will be found that the cutters are to be inclined for an angle, which is a trifle smaller than the helix angle in the pitch circle, for producing most accurate results.

In Figures 7 and 8 I have shown a pair of rack shaped cutters, for use in a reciprocating machine. The teeth of these tools are relieved inwardly, in the usual manner, as evident by the dotted lines.

The convex grinding wheels shown in Figure 9 are illustrated in their operating positions, in a view which is taken perpendicular to the axis of the gear blank as well as to the axis of the grinding wheels, i. e. in a view along the gear radii 41, 41′ of Figure 11. The wheels, which are to produce concave circular teeth profiles in a normal section, are of convex circular profile, its radius 42 being the same as the radius of the concave circular profile. The grinding wheels are inclined for an angle 43, which equals the helix angle of the teeth, in the pitch circle. The wheels grind along their profiles indicated in dotted lines 44 and 44′, which are located in a normal section. As shown in Figure 9, the two grinding wheels are coaxially arranged with respect to each other.

The device shown in Figure 10 corresponds to that shown in Figure 9, with the exception that the grinding wheels 45 and 46 are not coaxially arranged. Although the arrangement shown in Figure 9 imposes certain restrictions on the tooth design, it is frequently preferred. The arrangement of Figure 10 is advantageous, when grinding wheels are not free to run out, for instance when they must clear against a shoulder, or in the case of herringbone teeth.

(b)

Referring particularly to Figure 11, a normal section is illustrated and taken along lines N, N′ of Figure 9. In this view the axis of the coaxially arranged grinding wheels is situated in the said normal section. The wheels grind along the profiles 44 and 44′ of the shown normal section, while the blank performs a translatory motion in the direction of its axis, and, in timed relation thereto, a turning motion about its axis. In other words, the blank is screwed past the grinding wheels.

Figure 12 discloses a normal section through the teeth of the mating gear or pinion. Grinding wheels 50 and 50′ are provided with concave circular profiles 52 and 52′ with which they grind the convex gear teeth.

It will be understood, that milling cutters might be used instead of the grinding wheels shown in Figures 9 to 12; and also that grinding wheels of a shape shown in Figures 5 and 6 might be used, if so desired.

The teeth ground according to Figures 9, 11 and 12 are preferably so designed, that the centers of opposite tooth arcs 44 and 44′, 52 and 52′, respectively, in Figure 12 coincide. In Figures 11 and 12 the tooth arcs of every third tooth side have a common center.

The tooth arcs of every fifth tooth side have a common center in the normal section shown in Figure 13.

In Figure 11 the common center of opposite tooth arcs of alternate teeth is situated on the center line of the intermediate tooth. The corresponding pinion shows convex circular profiles, of which opposite tooth sides of adjacent teeth have common centers in the middle of the intermediate tooth space.

The normal section shown in Figure 14 shows an internal gear and its mate pinion, constructed in accordance with my invention. It will be noted that the internal gear is preferably provided with the concave tooth profiles. In external gears similarly preference is given to providing the larger gear with concave tooth profiles.

The normal section through a pair of helical gears shown in Figure 15, discloses opposite tooth profiles, the addendum being convex and the dedendum concave.

A rack shaped planing tool is illustrated in operating position in Figure 16. Tools of this kind have been shown in another view in the Figures 7 and 8. The reciprocatory tool 60 moves in the direction 61, at an inclination, which equals the helix angle of the teeth. Gear 62, with its axis 63, is shown in dotted and dash lines. In order to cut the proper tooth shape, gear blank 62 after every cut is slightly fed in a rolling generating motion with respect to a rack which is embodied by tool 60.

Another reciprocatory tool 64 is shown in Figure 17, the tool in this case being provided with stepped teeth 65, 65′, 65″ which allow it to clear shoulders, and herringbone teeth. The tool moves in direction 66 of the helical teeth, which it cuts.

Other ways of producing gearing according to my invention, i. e. hobbing, planing with a pinion cutter, rolling and casting, may be contemplated, but it is not deemed necessary at this time to give a detailed explanation of the mechanism used in connection therewith.

Briefly stated my invention consists in providing helical gearing of such profile, that the tooth contact passes rapidly over the normal profile of the teeth. This has been found to result in close contact between helical mate teeth. In a direction at right angles to the contact line, the mate teeth recede from each other only slightly, and thus provide a tooth contact, which is not very far from surface contact.

What I claim and desire to secure by Letters Patent is:—

1. Helical gear teeth with an active profile of approximately the form of a single circular arc, in a section laid perpendicularly to the tooth direction, the said profile being so positioned with respect to the pitch circle of that gear that the tooth contact with a mate gear passes over the said profile during a turning angle, which corresponds to less than one half of the normal pitch.

2. Helical gear teeth containing an active profile of the form of a single-circular arc, in a section which is laid through the radius of the gear, the center of said profile being located substantially on the pitch circle of the gear.

3. Helical gear teeth with an active profile of the form of a single circular arc shape in a section, which is laid perpendicularly to the tooth direction, the center of said profile being located practically in the pitch circle of the gear.

4. Helical gear teeth containing only a concave working face, substantially situated below the pitch circle, the profile of said working face in a normal section being circular, and the center of said profile being located on the pitch circle.

5. Helical gear teeth of concave, substantially circular working profile, the center of said profile being located practically on the pitch circle, and outside of the center of the tooth space to which it belongs.

6. Helical gear teeth with active profiles of circular shape in a normal section, the centers of said profiles being located practically on the pitch circle and so disposed that centers of two opposite sides of different teeth coincide.

7. Helical gear teeth containing only a convex working face situated above the pitch circle, the profile of said working face

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being circular in a normal section, and the center of said profile being located practically on the pitch circle.

8. Helical gear teeth of convex, substantially circular working profile, in normal section, the center of said profile being located practically on the pitch circle, and outside of the center line of the tooth, to which it belongs.

9. Helical gear teeth provided with working faces of convex circular profile, the center of said profile being located on the pitch circle, and its radius being larger than one quarter of the normal pitch.

10. Helical gear teeth provided exclusively with working faces of concave circular profile, the center of said profile being located on the pitch circle, and its radius amounting to one half up to one and one half times the normal pitch of the teeth.

11. Helical teeth of a pair of mate gears, the teeth of one gear being provided exclusively with convex working faces of substantially circular profile, the teeth of the other gear having exclusively concave working faces of substantially the same profile.

12. Helical teeth of a pair of gears, the teeth of the pinion being provided exclusively with convex working faces of substantially circular profile in normal section, the center of said profile being located close to the pitch circle, the teeth of the gear having concave working faces of substantially the same profile.

13. A pair of gears, having teeth extending across the faces of said gears along lines inclined to the generatrices of the respective pitch surfaces the tooth profiles of one gear being exclusively convex circular arcs, and the mate tooth profiles of the other gear of said pair being exclusively concave circular arcs of substantially the same radius.

14. A pair of gears, having teeth extending across the faces of said gears along lines inclined to the straight generatrices of the respective pitch surfaces, the pinion having a convex and substantially circular tooth profile in a section which is perpendicular to its pitch surface, the center of said tooth profile being approximately situated in the pitch surface, the mate gear having a concave mate tooth profile which is substantially a circular arc of the same radius.

15. A pair of gears, having teeth extending across the faces of said gears along lines inclined to the generatrices of the respective pitch surfaces, the pinion having a convex and substantially circular tooth profile in a section laid perpendicularly to the direction of a tooth, the center of said profile being situated on the pitch surface of said pinion, the mate gear having a concave mate tooth profile, which is substantially a circular arc of the same radius, said arc having its center on the pitch surface of the gear.

16. A pair of gears, having teeth extending across the faces of said gears along lines inclined to the straight generatrices of the respective pitch surfaces, one of said gears having convex tooth profiles and the other of said gears having concave tooth profiles, said profiles being substantially circular arcs and being substantially the same all along a tooth side.

17. A pair of gears having teeth extending across their faces along lines inclined to the generatrices of their respective pitch surfaces, one of said gears having a tooth profile which is convex in a section laid perpendicular to the direction of a tooth and the other of said gears having a tooth profile which is concave in a section laid perpendicular to the direction of a tooth, the mate profiles being substantially equal circular arcs and each profle being the same all along a tooth side.

18. A pair of gears, having teeth extending across the faces of said gears along lines inclined to the generatrices of the respective pitch surfaces, said gears having complementary tooth profiles in a section laid perpendicularly to the direction of a tooth, mate profiles being convex and concave circular arcs. the radii of said arcs being larger than one half of the normal pitch.

19. A gear having teeth extending across its face along lines inclined to the generatrices of its pitch surface, said gear having side tooth surfaces whose working portions are in the form of single circular arcs whose centers are located substantially on the pitch surface of the gear.

20. A gear having teeth extending across its face along lines inclined to the generatrices of its pitch surface, said gear having side tooth surfaces whose working portions are in the form of single circular arcs whose centers lie outside the teeth and are located substantially on the pitch surface of the gear.

21. A pair of gears provided with teeth which extend across their faces along lines inclined to the generatrices of their respective pitch surfaces, one of said gears being provided with active tooth surfaces which are exclusively convex circular arcs and the other of said gears having active tooth surfaces which are exclusively concave circular arcs.

22. A gear having teeth extending across it face along lines inclined to the generatrices of its pitch surface, said gear having teeth whose active tooth surfaces have profiles in the form of single circular arcs the centers of the profiles of a tooth lying on opposite sides of said tooth and being located substantially on the pitch surface of said gear.

23. A gear having teeth extending across its face along lines inclined to the gener-

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atrices of its pitch surface, said gear having teeth whose active tooth surfaces have profiles in the form of single circular arcs, the centers of the profiles of a tooth lying on opposite sides of said tooth and outside of said tooth and being located substantially on the pitch surface of said gear.

24. A pair of gears provided with teeth which extend across their faces along lines inclined to the generatrices of their respective pitch surfaces, said gears having teeth with active tooth surfaces the profiles of which are single circular arcs, mate profiles having substantially the same radius.

25. A pair of gears provided with teeth which extend across their faces along lines inclined to the generatrices of their respective pitch surfaces, the teeth of said gears having active tooth surfaces the profiles of which are single circular arcs the centers of which are located outside of the respective teeth, mate tooth profiles having substantially the same radius.

26. A pair of gears provided with teeth which extend across their faces along lines inclined to the generatrices of their respective pitch surfaces, one of said gears having active tooth surfaces whose profiles are exclusively convex circular arcs and the other of said gears having active tooth surfaces whose profiles are exclusively concave circular arcs, the centers of the tooth surfaces of each gear being located outside of the respective teeth of such gear.

27. A pair of gears provided with teeth which extend across their faces along lines inclined to the generatrices of their respective pitch surfaces, each of said gears being provided with active tooth surfaces whose profiles are single circular arcs, the active tooth surfaces of one gear being situated outside the pitch surface of said gear and the active tooth surfaces of the other gear being situated inside the pitch surface of said gear.

28. A pair of gears provided with teeth which extend across their faces along lines inclined to the generatrices of their respective pitch surfaces, said gears having complementary tooth profiles, mate profiles being respectively convex and concave circular arcs of substantially equal radii whose centers lie on the pitch surfaces of the respective gears.

In testimony whereof I affix my signature.

ERNEST WILDHABER.

(e)

Oct. 5, 1926 1,601,750

E. Wildhaber

Helical gearing

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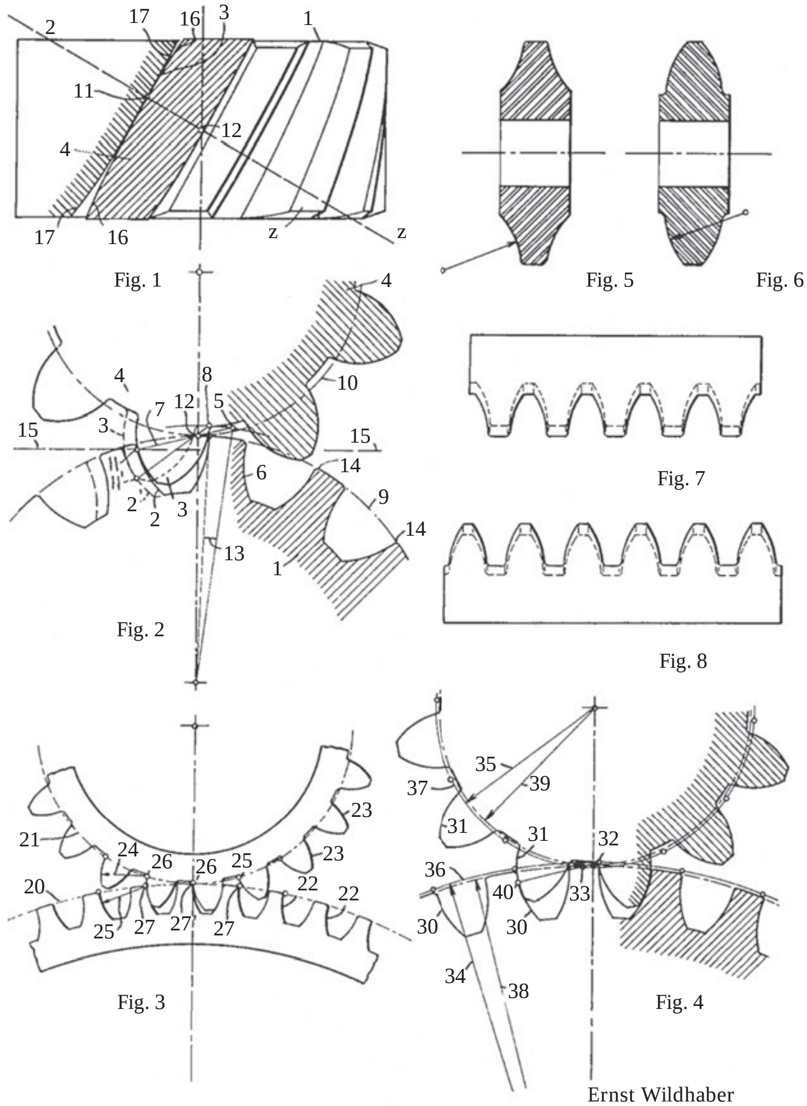

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Oct. 5, 1926 1,601,750

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Helical gearing

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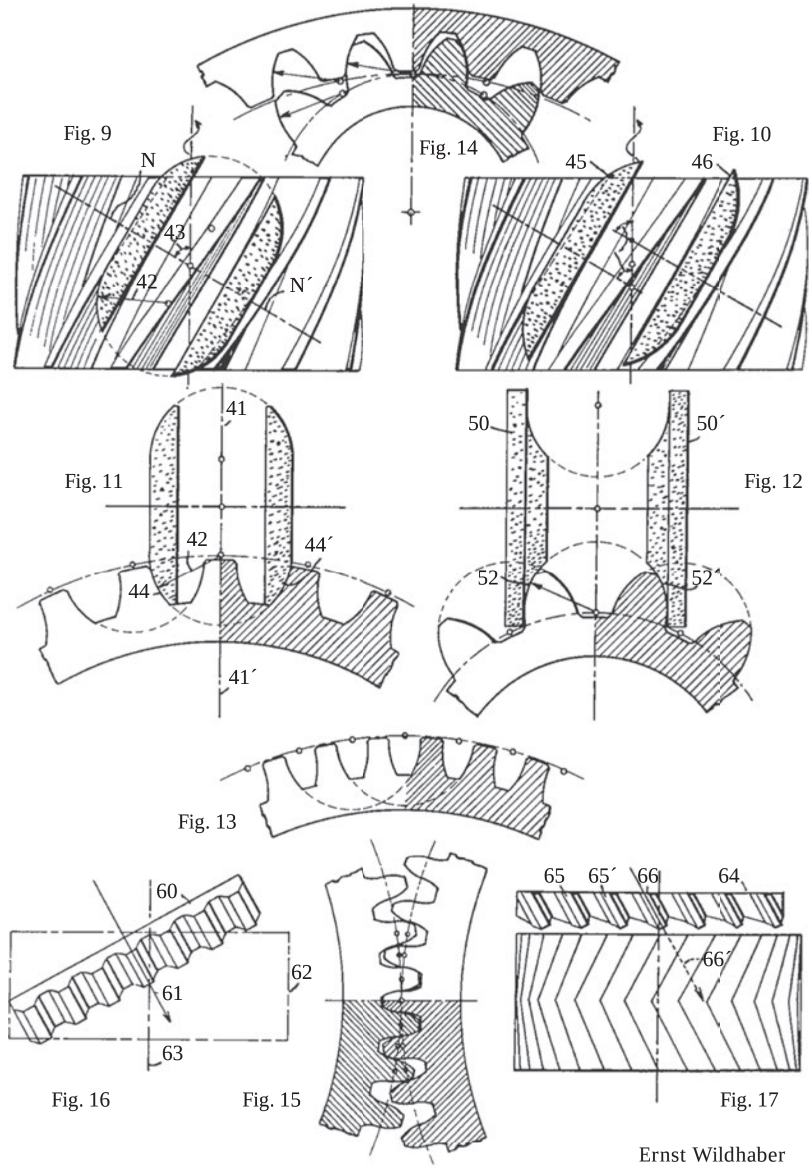

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Appendix E: An Improved Load-Equalizing Means for Planetary Pinions*

This section of the book deals with the load-equalizing means in planetary gearboxes. More generally, the topic of the section relates to the load-equalizing means in multiflow gear trains, that is, in gear trains that feature two or more flows of the power being transmitting.

Because of manufacturing errors, the displacements under operating load, and for other reasons, all the planetary pinions share the power unequally. The larger the displacements, the larger the difference in power being transmitting by each particular planetary pinion. This entails a reduction in power density that the planetary gearbox is capable of transmitting. An increase in the accuracy of machining of the components is a straightforward way to equalize the load share in planetary gearboxes. The accuracy of machining can be increased only to a certain extent, because the higher the accuracy of machining of the components, the higher the cost of the planetary gearbox, and vice versa. Therefore, load-equalizing means for planetary pinions need to be developed.

E.1 A BRIEF OVERVIEW OF APPROACHES TO EQUAL POWER SHARING IN PLANETARY GEARBOXES

Planetary gearboxes are widely used in the industry. Examples can be easily found in the automotive and aerospace industries, as well as in other industries. An example of a five-pinion planetary gearbox is shown in Figure E.1.

Planetary gearboxes with multiple planetary pinions enable the achievement of a substantial reduction in the dimensions and weight of the gear drive, particularly when the number of planetary pinions is large enough. However, this is true only under the condition that the *transmitted load is equally shared among all the planet pinions*.

Epicyclic gear systems have typically been equipped with straddle-mounted planetary pinions with pins supported on the input and output sides of the carrier. The torsional windup of the carrier, the position accuracy of the pins, the machining tolerances of the planetary gear system components, and bearings clearances can all contribute to poor load sharing among the planetary pinions as well as misaligned gear contacts in the deflected state.

In the traditional epicyclical gearing system, where the distance between planetary pinion centerlines is specified by the design to be within a fixed range, it is widely recognized that the load sharing is not equal among the planetary gear meshes. Similarly, stress is distributed variably at mesh points. Load sharing and stress distribution at each mesh point are heavily influenced by global design configuration, backlash tolerance, component design tolerances, manufacturing accuracy, component deflection and thermal distortion. Figure E.2 shows in exaggerated form that contact is made at the mesh point $K_{sg.p}$ of the planet pinion before any contact is made at the mesh points of the other planets (it is assumed that the ring gear makes contact with all the pinions at points $K_{rg.p}$). In a rigid system, this condition imposes unbalanced loading among the planetary pinions.

To share torque equally among all the planetary pinions, application of flexible components in the design of a planetary gearbox sounds promising. For example, the *Stoeckicht* system (circa 1940) solves this problem by making the annulus ring flexible while allowing it and the sun gear to float without bearings so that they are supported by their respective mesh points.

Alternatively, designers have applied a number of novel designs with various levels of success to build epicyclical gearing systems that help to distribute load among the planet pinions more evenly, thereby increasing power density. In general, such improvements use components in the gear train that are elastically compliant and are intended to compensate for clearance variations without imparting any negative operating characteristics, including the following:

- Flexible ring gears have been applied, but the effectiveness of this approach is not universal because radial deflections of the ring gear are not enough to compensate for clearance (backlash) variations present at the various mesh points.
- Floating ring gear system (used in some off-highway applications).
- Floating sun gear.
- Floating planet carrier.
- Double-helical gear with floating members.
- Floating planetary pinion, also called flexible pin or abbreviated to *flexpin*.

There are a few more approaches to be mentioned. The interested reader is referred to Radzevich (2012a), Tkachenko (2003), and Radzevich (2012b), where a brief overview of known load-equalizing means for planetary pinions in a planetary gearbox can be found.

For the discussion in this section, it is convenient to begin with gearboxes featuring flexpins.

* This section of the book is based on the article by the author titled as "An Improved Load Equalizing Means for Planetary Pinions." The article was solicited by *Gear Technology* magazine, and was submitted to the magazine in September/October 2014. For some reason publication of the article has been postponed.

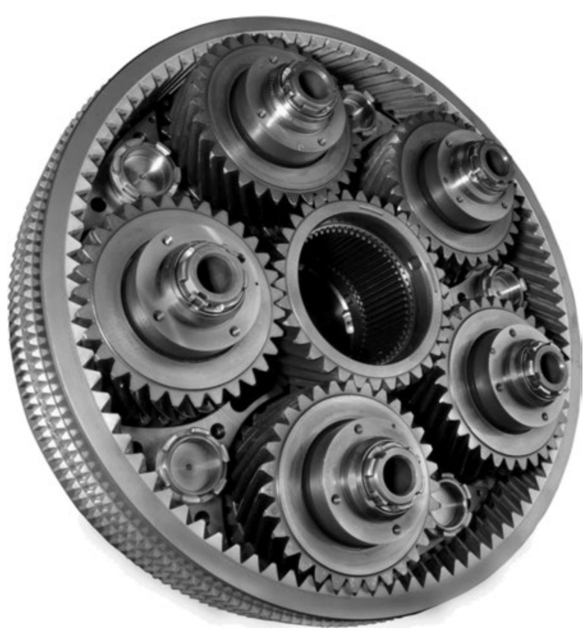

FIGURE E.1 An example of ve-pinion planetary gearbox.

E.2 PLANETARY GEAR DRIVES WITH FLEXIBLE PINS

The application of gearboxes with exible pins is based on the ideas of the British inventor Raymond J. Hicks (Hicks, 1967). In 1964, Hicks developed a method of providing load sharing between the planet pinions of an epicyclic gearbox, the exible pin, which has been applied to a large variety of industrial, aerospace, and marine gearboxes from 1964 onward.

Hicks's original invention is illustrated in Figure E.3. Here, in Figure E.3, a fragmentary and diagrammatic part-sectional elevation of an epicyclic gear with exure grossly exaggerated for the purpose of illustration is shown.

Referring rstly to Figure E.3, which illustrates the invention diagrammatically, the epicyclic gear broadly comprises a sun wheel 10, an annulus gear ring 11, and a plurality of plates 12, which mesh with both sun and annulus. The planets are supported on spindles 13 fast with a carrier 14. The effect of the gear depends on whether the sun, the annulus, or the carrier is the input or output, and which of these three is xed either permanently or optionally. In any event, center 15 of the planet teeth, measured axially, is at an equal distance from points 16 and 17, which lie on the planes contacting the point of emergence of spindle 13 from the carrier and the planet, respectively, so that the couples will be equal as hereinbefore explained.

Likewise, for the reasons hereinafter explained, the planet does not skew if misaligned but de ects the shaft, as shown in the gure, until each of the planets is equally loaded. Annular gap 18 permits this. It will be appreciated that in practice the de ection involved will be relatively slight.

Planet 12 may seat directly on spindle 13, or as shown in Figure E.3, may seat and be journalled on a sleeve 19 which provides the gap. In this case the sleeve is fast with the spindle.

The spindle is pressed or shrink- tted in carrier 14 and possibly in sleeve 19 but a system of circlips 20 is also used as a precaution against damage through t relaxation.

The use of exible pin eliminates the need for straddle mounting and thereby enables the maximum possible number of planet pinions to be used subject to tip-to-tip clearance for

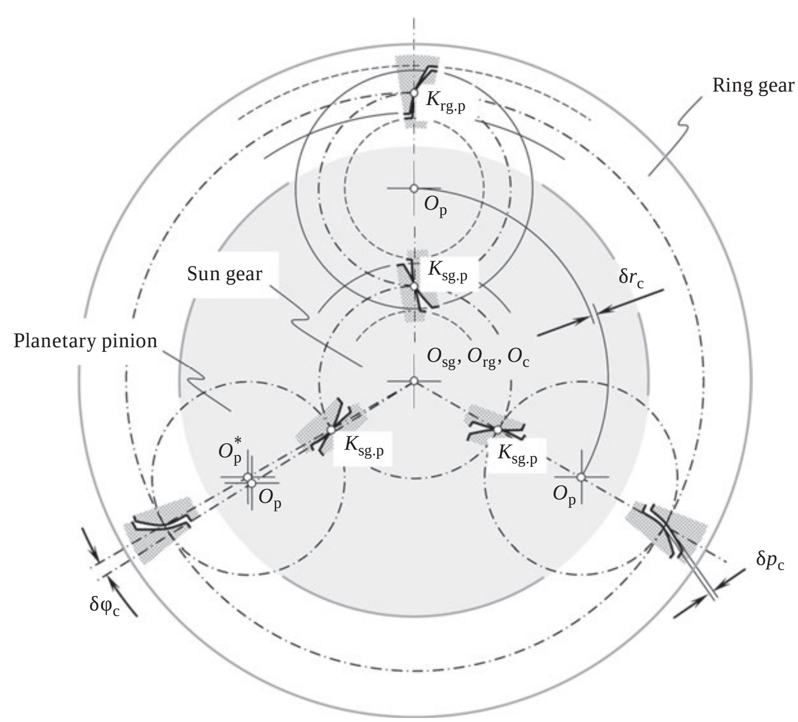

FIGURE E.2 Deviation of the actual con guration of the planet pinion axis of rotation O_p^* from its desired con guration O_p.

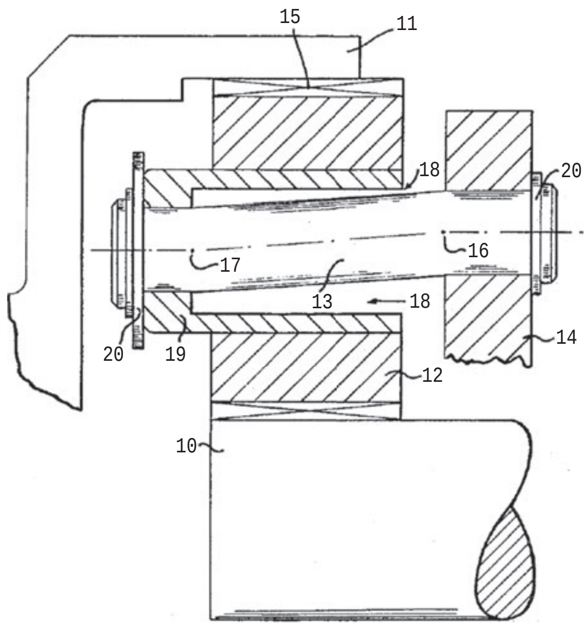

FIGURE E.3 A fragmentary and diagrammatic part-sectional elevation of an epicyclic gear with flexure grossly exaggerated. (After Hicks, R. J., Load equalizing means for planetary pinions, U.S. Patent 3,303,713, filed February 8, 1965, and issued February 14, 1967.)

any particular gear ratio. The number of planet pinions varies with the ratio between the annulus and sun gear tooth numbers.

Load sharing is achieved by ensuring that deflection of the planet pinion spindle under its normal load is considerably greater than the manufacturing errors that cause maldistribution; that is, if one planet pinion tends to take more load than the others, it will deflect until the others take their share.

Later on, Hicks's flexpin concept was enhanced (Hicks, 1969).

Figure E.4a shows a typical planet gear supported by a planet spindle mounted on a flexible pin cantilevered from a simple carrier plate. The two ends of the pin are fitted to the carrier plate and the spindle, whereas the latter is counterbored to allow the pin to deflect freely.

Figure E.4b shows that a uniform tooth load, where the centroid is symmetrical with the teeth length of the flexible pin, exerts equal and opposite moments on the built-in ends so that they remain parallel during deflection.

Figure E.4c shows that a point load concentrated on either end of the tooth face produces a relative angular deflection of that end, with respect to the other, which is six times the finite deflection that occurs when it is loaded at the center.

The deflections shown are theoretical values, which assume that the built-in portions of the pin at either end are supported so rigidly that they have zero slopes. However, static tests have shown that elastic deflections in the joints between the spindle and the carrier plane give complementary finite slopes such that the effective flexibility of the pin is more than doubled without affecting the parallel movement of the spindle.

With the proportions shown, the relative rigidity of the spindle is such that its own cantilever deflection in terms of the total is so small that it has virtually no effect on tooth load distribution. If a thinner planet spindle is used with a significant flexibility, it is possible to compensate for this by reducing the length of the counterbore.

An important feature of the design is that because the planet spindle and flexible pin are coaxial, it is capable of deflection about two axes, which makes it virtually self-aligning. This means that the pin is influenced by radial as well as tangential tooth loads, and it is able to compensate for helix errors of different magnitudes or sense at the sun and annulus mesh points. It is therefore capable of compensating for the torsional deflection of the sun gear, which takes place in gearboxes of large tooth ratio. If the resultant load of the sun and annulus mesh points is not on the same plane as the midpoint of the unsupported portion of the flexible pin, there are two restoring effects:

- The offset tangential load tips the spindle on the tangential plane in a manner that tends to offset the respective load points an equal amount to either side of the midpoint of the pin.
- The radial couple resulting from the offset radial loads tilts the spindle on the radial plane until the residual couple is reduced to an amount compatible with the angular flexibility of the spindle assembly.

In short, there is a complex movement on two planes as the spindle takes up a position of minimum strain energy. This complex movement is in fact beneficial since it promotes a slight crowing effect as a result of the skewed or nonparallel axes.

If, on the other hand, the planet pinion is cross cornered so that the resultant tangential load is on the same plane as the midpoint of the pin, there is still a radial tilting couple to provide a restoring action.

When a gearbox has a rotating planet carrier, additional radial loads and deflections are imposed on the flexible pin assembly due to the centrifugal weights of the planet pinion, the spindle, and the pin.

The flexible pin eliminates the need for straddle mounting and, therefore, enables the maximum possible number of planet pinions to be used subject to tip-to-tip clearance for any particular epicyclic ratio. Load sharing is achieved by ensuring that deflection of the planet spindle under its normal load is considerably greater than the manufacturing errors which cause maldistribution, that is, if one planet tends to take more load than the others, it will deflect until the others take their share.

To put it simply, the flexible pin is designed to use high deflections to provide uniform tooth loads between planet pinions and across sun-to-planet and planet-to-annulus tooth face widths. An added benefit of producing equal loads across the tooth contact face widths is the occurrence of equal loading along the planet pinion bearings, which is the most critical element of a high-capacity low-speed epicyclic gear.

Conversely, the industrial design of epicyclic gear requires high carrier rigidity relative to the gear tooth stiffness, which is impractical and leads to maldistribution of load across the teeth and bearing, leading to premature failure.

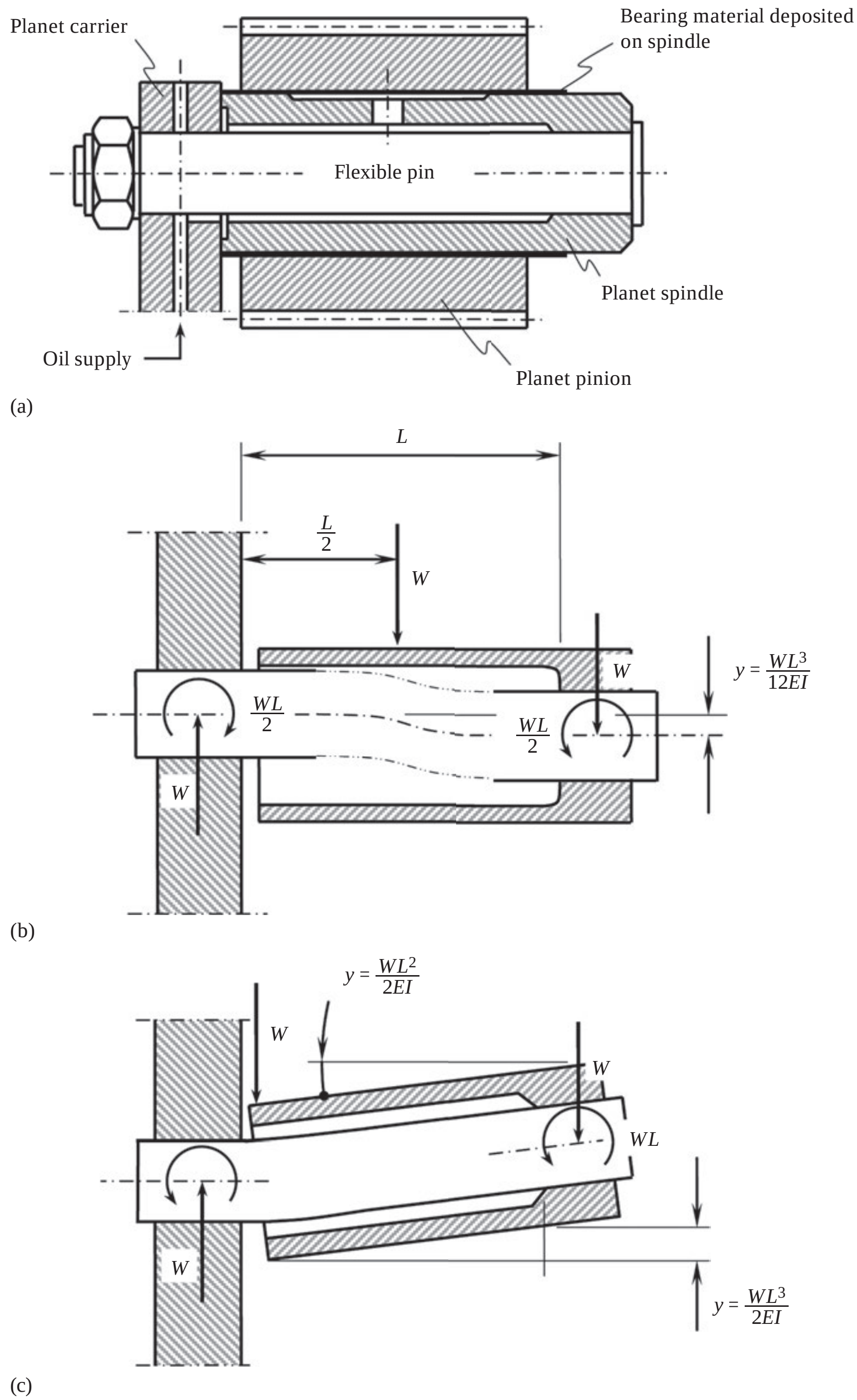

FIGURE E.4 Compact orbital gear's exible pin: (a) planet pinion rotates on a exibly mounted spindle, (b) planet spindle is loaded at center, and (c) planet spindle loaded at end. W is applied load, L is the length of the expin, and E is Young's modulus of elasticity. (After Hicks, R. J., *Proc. Inst. Mech. Engrs.*, 184, 85–94, 1969.)

Because a supporting shaft of the planet gear is of a exible double cantilever construction (exible pin system), a planet pinion that receives more load moves in parallel due to sagging of the pinion, so that all the planet pinions receive equal load. Consequently, an excellent equal sharing effect is shown in such cases and the whole system is of a smaller size.

Due to the exible pin system, the shock-absorbing effect for torque variation of a prime mover or a load is expected.

If the load is distributed evenly among the teeth faces, it is the same as when a concentrated load is applied to the center; the pins ex as double cantilever beams, and parallelism relative to other planetary pinions is not lost. If there is any error

in relative positioning between flexible pins, due to errors in machining or assembly, the planetary pinion positioned here receives more load than the others and the flexible pin supporting that gear flexes further to absorb the error. Thus, the uniform load distribution mechanism keeps load distribution even.

If an eccentric load is applied to the left end of a tooth face, the flexible pin flexes as shown in Figure E.4c and the load on the right side of the tooth face increases, mitigating the eccentric distribution of the load across the width of the tooth. The effect of gear tooth trace errors, gear casing deformation, misalignment, and other problems can be absorbed and mitigated.

However, for just about all equipment types, economics dictate the need for increased power density and improved reliability. A common approach is attempting to build in more planets, thereby reducing forces and stresses at each mesh point. But as planets are added, so is the uncertainty about just how much power each planet is transmitting.

Instead of fixing the angular positions of the planet pinions, the flexible pins were designed so that they deflect independently in a circumferential direction, which ultimately helps equalize the force distribution among the planets while transmitting torque at various levels. This feature is henceforth referred to as *torsional compliancy.*

Torsional compliancy is achieved by applying the double-cantilevered-beam design that is illustrated in Figure E.4b. Simply stated, when two tangential forces are applied to the flexpin pinion, the angular deflection caused by the bending of the pin cantilevered from a carrier wall can be offset in the opposite direction by the angular deflection caused by bending of the sleeve cantilevered from the other end of the pin. If sections of the pin and the sleeve are carefully designed with that goal in mind, deflection at each gear contact follows a circumferential translation, which means that the axis of the gear contact does not tip from side to side angular positioning inaccuracy nor lead from torsional windup of the carrier.

Flexible pins have been designed into various types of equipment and the designs have typically included assembly of separable components including gears, pins, mounting sleeves, backing plates, cap screws, and various type of rolling-element bearing races and bushings.

Such a design achieves the objective of creating a torsionally compliant system. Additionally, since gears are less prone to be tipped off axis because the single-sided planetary carrier can no longer wind up, it can be argued that gear contacts have a much higher probability of remaining centered at all meshes. It follows then that the flexible pin permits the designer to specify narrower gears and still avoid stress concentration at the ends of the face. Power density is therefore improved in the axial direction.

An elastic deformation of the flexpin allows for accommodation of the manufacturing errors, the displacements under operating load, and so forth. The elastic deformation must be large enough to accommodate the manufacturing errors, and so forth, and does not exceed this particular value. When zero torque is applied to the driving member of the planetary gearbox, no force is exerted from the flexpin and no deformation of the flexpin is observed. When the maximum torque is applied, the maximum force is exerted from the flexpin and the maximum deformation of the flexpin occurs. Because only elastic deformations of the flexpin are considered, that is, Young's law is valid, therefore, the loaded diagram is represented by a linear function as shown in Figure E.5. Huge displacements *y* of the flexpin are necessary to attain the operating load that acts

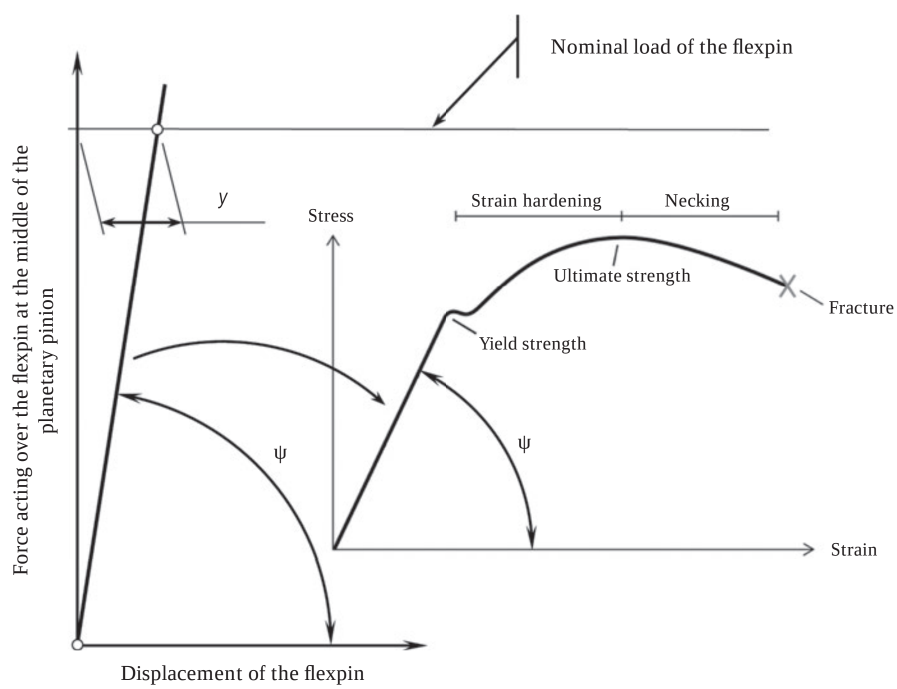

FIGURE E.5 Stress–strain diagram for the flexpin: the displacement *y* is shown in Figure E.4. (After Hicks, R. J., Load equalizing means for planetary pinions, U.S. Patent 3,303,713, filed February 8, 1965, and issued February 14, 1967.)

against the expin. It is desirable to keep the displacements y as small as possible; however, the displacements need to be suf cient to accommodate the manufacturing errors, and so forth.

The expin concept is considered in detail to make a correct comparison of this concept with the concept of the so-called *elastic absorbers of manufacturing errors* (or just *EAMEs*, for simplicity), discussed in the following.

E.3 ELASTIC ABSORBERS OF MANUFACTURING ERRORS

The purpose of the EAMEs is twofold:

- To reduce the required minimum displacement of the planetary pinions
- To make a gear train with split torque insensitive to manufacturing errors as well as displacements of other kinds

The capability of an elastic absorber to accommodate the manufacturing errors strongly depends on its stiffness. For most of the materials used in the production of gears and gear units, the relationship between the applied load and the displacement caused by the load is linear, as illustrated in Figure E.5.

In the worst-case scenario, the accuracy A of load sharing among the planet pinions can be calculated from the following formula (Figure E.6):

$$A = \left[1 - \frac{1 + (1-k)(n_{pp} - 1)}{n_{pp}}\right] \times 100\% . \quad \text{(E.1)}$$

In Equation E.1

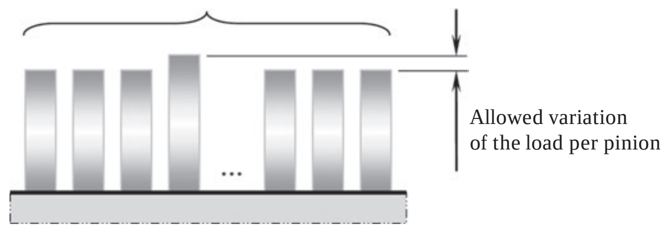

FIGURE E.6 Schematic of a worst-case scenario of load sharing among the planet pinions of a multi ow gear train.

k—the allowed variation in the load per planet pinion
n_{pp}—the total number of planet pinions

The calculations reveal that in the worst-case scenario for a planetary gearbox that has eight planet pinions (n_{pp} = 8) and allowed variation in the load per planet pinion k = 0.1, deviation of the transmitted load from the desired value does not exceed 8.75%. For a planetary gear drive with three planet pinions (n_{pp} = 3) and allowed variation of the load per planet pinion k = 0.05, deviation of the transmitted load from the desired value does not exceed 3.33%. The actual deviations are less than those calculated for the worst-case scenario.

If the preloaded elastic absorber is loaded by a precalculated value, the actual displacements of the pinions do not exceed the prescribed tolerance for the displacement of the pinions.

Consider a planetary gearbox for which the permissible range of variation in load share among the planetary pinions is equal to $\pm OR_l$ as shown in the left upper corner in Figure E.7. The operating range of the deformation of the elastic absorber, OR_d, needs to be the smallest possible; however, it must be large enough to accommodate the manufacturing

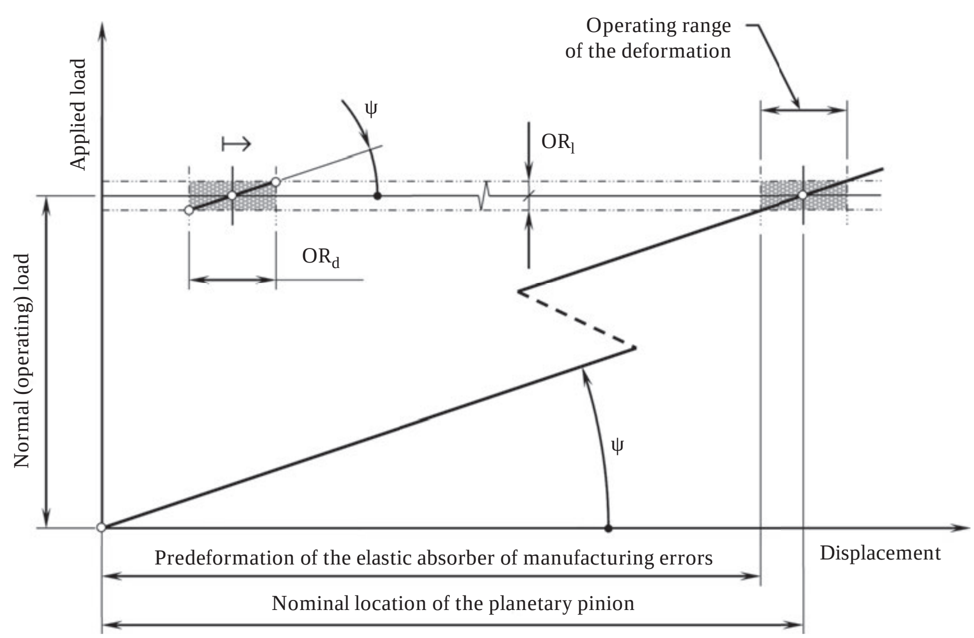

FIGURE E.7 Determination of the principal design parameters of the EAME. (Courtesy of New Venture Gear, Syracuse, New York, circa 2000.)

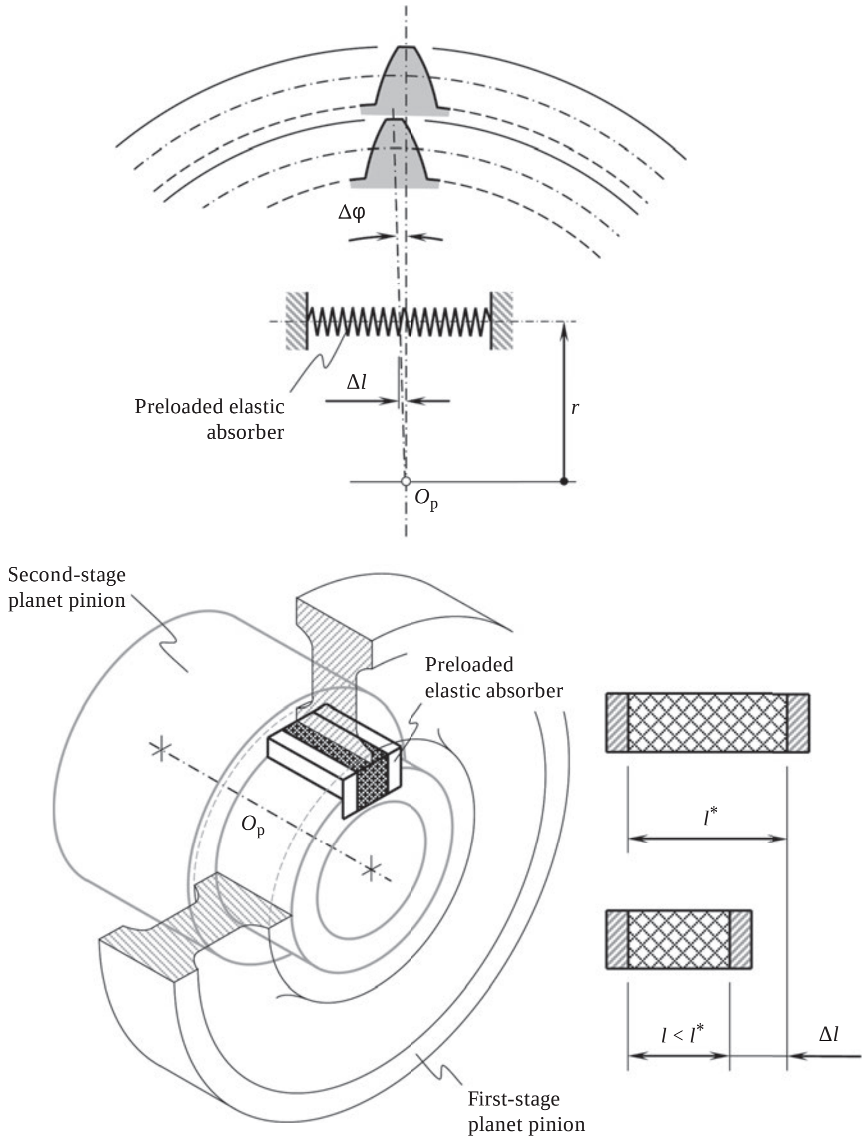

FIGURE E.8 Application of a preloaded EAME in the design of a cluster planet pinion (as proposed by S. P. Radzevich around 2000). (Courtesy of New Venture Gear, Syracuse, New York.)

errors and the planetary pinions displacements under operating load. The two ranges $\pm OR_l$ and OR_d specify a rectangle. In Figure E.7, a diagonal of this rectangle forms an angle ψ with the horizontal axis. The desirable stiffness c of the elastic absorber needs to be equal to (or less than)

$$c \le \tan \psi. \tag{E.2}$$

Once the stiffness c is determined, then a straight line through the origin can be constructed. This line is at the angle ψ with respect to the horizontal axis. The point of interception of the constructed straight line and the straight line of the nominal (operating) load, NL_{op}, speci es the desirable predeformation of the EAME (Figure E.7). The predeformation PD_{ea} of the elastic absorber is calculated from the equation

$$PD_{ea} = (NL_{op} - OR_l) \cot \psi. \tag{E.3}$$

Calculation of the design parameters of the EAME is based on two parameters, that is, $\pm OR_l$ and OR_d.

E.4 AN ILLUSTRATIVE EXAMPLE OF APPLICATION OF THE PRELOADED EAME

In a two-stage planetary reducer, the preloaded EAME can be placed between the rst-stage planet pinion and the second-stage planet pinion,* as schematically depicted in Figure E.8.

It is common practice to hob both the planet pinions of the cluster planet pinion. For this purpose, it is convenient to assemble the cluster planet pinion comprising two planet pinions. Proper phasing of the pieces in relation to one another while assembling the cluster planet pinion is a critical issue in this case. Misphasing error of planet pinions is not allowed.

* Radzevich, S. P., A planetary reducer, Invention disclosure, led to New Venture Gear, Inc., Syracuse, New York, on October 30, 2001, patent pending.

The preloaded EAME is installed between the two planet pinions of the cluster planet pinion (Figure E.8).

For equal torque sharing among the planetary pinions, the misphasing $\Delta\varphi$ must be zero. As the misphasing $\Delta\varphi$ cannot be eliminated, it must be absorbed.

For this purpose, it is necessary to introduce an additional degree of freedom for one of the planetary pinions in relation to another and in this way to make the planet pinions self-aligning. Self-alignment of the planet pinions can be ensured by implementation of the preloaded EAME.

An angular displacement $\Delta\varphi$ to be absorbed by the elastic absorber can be eliminated when the linear displacement Δl is equal to

$$\Delta l = \Delta\varphi \times r \times \frac{\pi}{180^\circ}. \tag{E.4}$$

In Equation E.4, the radial location of the preloaded elastic absorber is speci ed by the distance r.

The deformation Δl of an elastic body under load usually (but not necessarily) relates to the applied load T linearly or (at least) almost linearly, $\Delta = c \times T$ (c is a proportionality factor equal to the rigidity of the preloaded elastic absorber). In such a case, the angle ϕ can be calculated from the formula

$$\phi = \tan^{-1}(c). \tag{E.5}$$

In the general case, when c is not constant, the current value of c is equal to $c = dT/d(\Delta l)$.

The variation interval for the applied load should be known for the calculation of the design parameter of the preloaded EAME.

The advantage of the EAME over the expin concept is clearly illustrated in Figure E.9. When the expin concept is used, only a small fraction $\Delta(OR_d)$ is used to accommodate the unfavorable displacements of the planetary pinions under operating load (Figure E.9a). This is because the expins are not preloaded, and, therefore, the stiffness angle φ_{fp} is relatively small. When the concept of the EAME is used, the entire OR_d is used to accommodate for the unfavorable displacements of the planetary pinions under operating load (Figure E.9b). Because of this, the stiffness angle φ_{ea} is large. Generally speaking, the inequality φ_{ea} φ_{fp} is always observed. This advantage of the EAME is signi cant as it makes possible a higher power density being transmitted through the planetary gearbox.

Evidently, the disclosed concept of the EAME can be used for the improvement of power density in any and all gear trains that feature split torque.

E.5 CONCLUSION

A brief overview of known approaches for equal power sharing in planetary gearboxes is presented. Planetary gear drives with exible pins (Hicks's approach) are discussed. A novel method for equal power sharing in planetary gearboxes is proposed. The method is based on use of the so-called EAME. An illustrative example of application of the preloaded EAME is provided.

Use of the concept of EAME allows a signi cant improvement in power density being transmitted through the planetary gearbox.

Numerous patents on inventions are granted in the United States and other countries all around the world on the inventions of gear trains with split torque that are based on the implementation of the EAMEs.

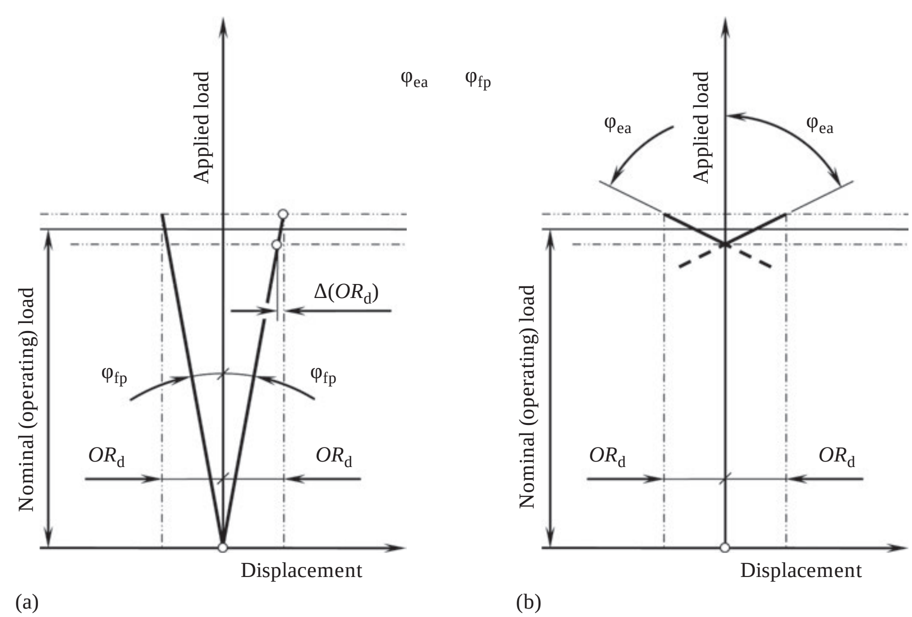

FIGURE E.9 Comparison of the load-versus-displacement diagrams (a) for the expin approach and (b) for the EAME.

Appendix F: Geometrically Accurate (Ideal or Perfect) Crossed-Axis Gearing with Line Contact between the Tooth Flanks of the Gear and the Pinion

This section of the book deals with gearing that features crossing axes of rotation of a gear and a mating pinion, that is, crossed-axis gearing, or just C_a *gearing*, for simplicity. The discussion begins with a brief historical overview of crossed-axis gearing. It is shown that all known gearings of this particular type of gearing either are approximate gearings or feature point contact between the tooth flanks of a gear and a mating pinion. Approximate C_a gearings are not capable of transmitting a rotation smoothly. An ideal C_a gearing of known design, that is, general spatial involute gearing, features point contact between the tooth flanks of a gear and a mating pinion. The point contact limits the power density transmitted by the gear pair because of the low bearing capacity of their tooth flanks. This means that the development of geometrically accurate (ideal) C_a gearing with line contact between the tooth flanks of the gear and the pinion is necessary. To develop a crossed-axis gearing with the desirable properties, the kinematics of C_a gearing is thoroughly investigated. It is shown that the tooth flanks of the gear and the mating pinion can be generated as loci of the desirable line of their contact considered in corresponding reference systems associated with the gear and the pinion. Crossed-axis gearing of the proposed design* is referred to as *R gearing*. *R* gearing is the only type of ideal (or perfect) C_a gearing with line contact between the tooth flanks of the gear and the pinion. Parallel-axis involute gearing (Euler, 1760) and intersected-axis gearing with the spherical involute tooth flank geometry (Grant, 1889) represent reduced cases of *R* gearing. The shaft angle is either zero or equals $\Sigma = 180°$ in the first case. In the second case, the center distance is zero. The capability to transmit a rotation smoothly and a high power density are the two main advantages of *R* gearing.

F.1 INTRODUCTION

Various types of gearing that feature skewed axis of rotation of a gear and a mating pinion are used in the industry. Early designs of crossed-axis gears can be found in Leonardo da Vinci's famous book *The Madrid Codices* (1493). Hypoid gearing, Spiroid gearing, and Helicon gearing, along with numerous types of worm gearings, represent perfect examples of gearing with skewed axis of rotation of a gear and a mating pinion that are used in the industry nowadays. In gearings of all of these types, it is vital to have line contact between the tooth flanks of the mating gears as this enables a higher bearing capacity of the interacting tooth flanks. The line contact is critical to improving the power density of the gearbox.

The kinematics and the geometry of the geometrically accurate crossed-axis gearing (or just C_a gearing, for simplicity) with line contact between the tooth flanks of the gear, □, and the mating pinion, □, are discussed in this appendix. The gearing of this type is the only type of C_a gearing capable of transmitting a rotation smoothly; that is, gearing of this type is the only type of C_a gearing with a constant angular velocity ratio $u_\omega = \omega_p/\omega_g$ = constant. (Here, rotations of the gear and the pinion are designated as ω_g and ω_p, correspondingly.) Gearings for which the angular velocity ratio u_ω is constant (u_ω = constant) are commonly referred to as the *ideal gearings*. To better understand the term *ideal gearing*, it is instructive to note here that in a case of parallel-axis gearing (or just P_a *gearing*, for simplicity), involute gearing is the only type of ideal gearing; similarly, in a case of intersected-axis gearing (or just I_a *gearing*, for simplicity), gearing that features spherical involute geometry is the only type of ideal gearing.

Several efforts were undertaken in the past aiming to investigate ideal crossed-axis gearing. The most significant results of the research in the field obtained so far are summarized and are generalized by Phillips (2003). It should be stressed here that general spatial involute gearing (Phillips, 2003) features point contact between the tooth flanks of the gear, □, and the mating pinion, □. The desirable line contact in spatial involute gearing is impossible at all, as in this particular type of gearing the tooth flanks of both the gear and the pinion are developed from the corresponding base cylinders. Because of this, neither the base pitch of the gear nor the base pitch of the mating pinion can be equal to the operating base pitch of the C_a gear pair, which is a must.

The claim in Corollary 2.2 of Stachel (2004a; p. 38), "If two helical involutes Φ_1, Φ_2 are placed such that they are in contact along a common generator and if their axis are kept fixed, then Φ_1 and Φ_2 serve as gear flanks for a spatial gearing with permanent straight line contact," is a mistake. In spatial involute gearing, line contact between the tooth flanks is possible when C_a gearing is reduced to a parallel-axis gearing. Only in this degenerate case the generating straight line of

* Patent pending.

one of two involute helicoids can be aligned with the straight generating line of the other involute helicoid.*

The interested reader may wish to go to other publications on spatial gearings (Figliolini et al., 2009; Stachel, 2000, 2004b).

To the best of our knowledge, geometrically accurate (ideal) crossed-axis gearing with line contact between the tooth flanks of the gear and the pinion is not known yet. The main goal of this appendix is to develop the kinematical and geometrical basics of geometrically accurate crossed-axis gearing with line contact between the interacting tooth flanks of the gear and the mating pinion. Ideal C_a gearing will feature a higher bearing capacity of the interacting tooth flanks, and, as a consequence, it will feature a higher power density that is vital in many applications. Helicopter transmissions and transmissions for rotorcraft are just few to be mentioned.

F.2 KINEMATICS OF CROSSED-AXIS GEARING

The transmission and transformation of a rotation from a driving shaft to a driven shaft is the main purpose of implementation of crossed-axis gears. Both the input rotation and the output rotation can be easily represented by the corresponding rotation vectors† ω_g and ω_p (Figure F.1). The vectors ω_g and ω_p are along the gear and the pinion centerlines O_g and O_p, which cross one another. The closest distance of approach between the lines of action of the rotation vectors ω_g and ω_p is denoted by C. This distance is along the centerline ℄ and is commonly referred to as the *center distance*.

A vector $\omega_{pl} = \omega_p - \omega_g$ analytically describes the instant rotation of the pinion in relation to the motionless gear. The axis of instant rotation P_{ln} (or the *pitch line*, in other words) is along the vector ω_{pl}.

It should be noted here that in the case of crossed axes of rotation of the driving shaft and the driven shaft, there is no freedom in choosing a configuration of the axis of instant rotation P_{ln} in relation to the rotation vectors ω_g and ω_p. Once the rotation vectors ω_g and ω_p and their relative location and orientation are specified, the configuration of the axis of instant rotation P_{ln} can be expressed in terms of the rotations ω_g and ω_p and of the center distance C.

The variety of all possible types of C_a gear pairs is limited to the total number of possible combinations of the rotation vectors ω_g and ω_p (a) of various magnitudes and (b) featuring different shaft angles Σ (Table F.1). (Remember that the shaft angle Σ is specified as the angle between the rotation vector ω_g of the gear and the rotation vector ω_p of the pinion; that is, $\Sigma = \angle(\omega_g, \omega_p)$).

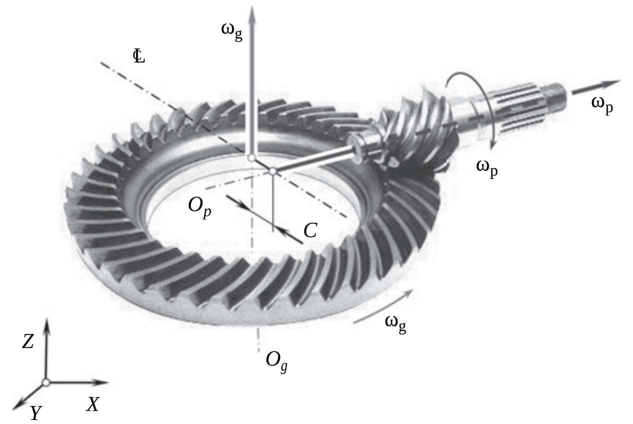

FIGURE F.1 Kinematics of crossed-axis gearing.

Any and all crossed-axis gear pairs meet one of the three expressions listed in Table F.1.

In particular cases, the centerlines of the driving shaft and the driven shaft cross each other at a right angle ($\Sigma = 90°$). This particular case is the most common in practice. Crossed-axis gear pairs of this particular type are referred to as *orthogonal crossed-axis gear pairs*. For gearing of this particular type, the cross product of the rotation vectors of the gear, ω_g, and the pinion, ω_p, is always zero ($\omega_g \times \omega_p = 0$).

The gear angle Σ_g can be expressed in terms of the shaft angle Σ and of the magnitudes ω_g and ω_p of the rotation vectors ω_g and ω_p:

$$\Sigma_g = \tan^{-1}\left(\frac{\sin\Sigma}{\omega_p/\omega_g + \cos\Sigma}\right). \quad \text{(F.1)}$$

For a shaft angle of 90°, Equation F.1 reduces to $\Sigma_g = \tan^{-1}(\omega_g/\omega_p)$.

Similar equations are valid for the calculation of the pinion angle Σ_p.

The center distance C can be interpreted as the summa of the pitch radii of the gear, r_g, and the pinion, r_p:

$$C = r_g + r_p. \quad \text{(F.2)}$$

This equation yields an expression for the calculation of the pitch radius r_g:

$$r_g = \frac{1 + \omega_p - \omega_g}{1 + \omega_p} \times C. \quad \text{(F.3)}$$

Then, the pinion pitch radius r_p equals $r_p = C - r_g$.

TABLE F.1
Analytical Criteria of Type of Crossed-Axis Gearing

Type of Crossed-Axis Gearing	Analytical Criterion [$C \neq 0$ and $\Sigma \neq 0$]
External crossed-axis gear pair	$\omega_g \cdot (\omega_p - \omega_g) < 0$
Rack-type crossed-axis gear pair	$\omega_g \cdot (\omega_p - \omega_g) = 0$
Internal crossed-axis gear pair	$\omega_g \cdot (\omega_p - \omega_g) > 0$

* It is worth noting here that all of (a) the geometry of both surfaces Φ_1 and Φ_2, (b) their contact geometry, and (c) *the kinematics of a relative motion* are required to be taken into consideration when making a conclusion whether or not "… Φ_1 and Φ_2 serve as gear flanks for a spatial gearing with permanent straight line contact." The *helical involutes* Φ_1 and Φ_2 are generated from the *base cylinders*, while the tooth flanks in a spatial C_a gearing are generated from the *base cones*. The *belt-and-pulley* analogies in the cases under consideration (for the helical involutes in the first case and in a spatial C_a gearing in the second case) are completely different from one another. This is the key to make clear that the above statement; that is, Corollary 2.2 is a mistake.

† It is instructive to note here that rotations are not vectors in nature. Therefore, special care is needed to be undertaken when treating rotations as vectors.

F.3 BASE CONES IN IDEAL CROSSED-AXIS GEAR PAIRS

A belt-and-pulley analogy can be constructed for a crossed-axis gearing similar to the belt-and-pulley analogy known for a parallel-axis gearing (Figliolini et al., 2009). For this purpose, two base cones are associated with the gear and the pinion in a C_a gearing. Smooth rotation of the base cones can be construed as a belt-and-pulley mechanism with the belt in the form of a round tape. This concept is schematically illustrated in Figure F.2. An orthogonal C_a gear pair is shown here for illustrative purposes only. That same approach is applicable with respect to angular bevel gears with a shaft angle $\Sigma \neq 90°$, namely, either an obtuse or an acute shaft angle Σ.

The schematic (Figure F.2) is constructed starting from the rotation vectors ω_g and ω_p of the gear and the pinion. The

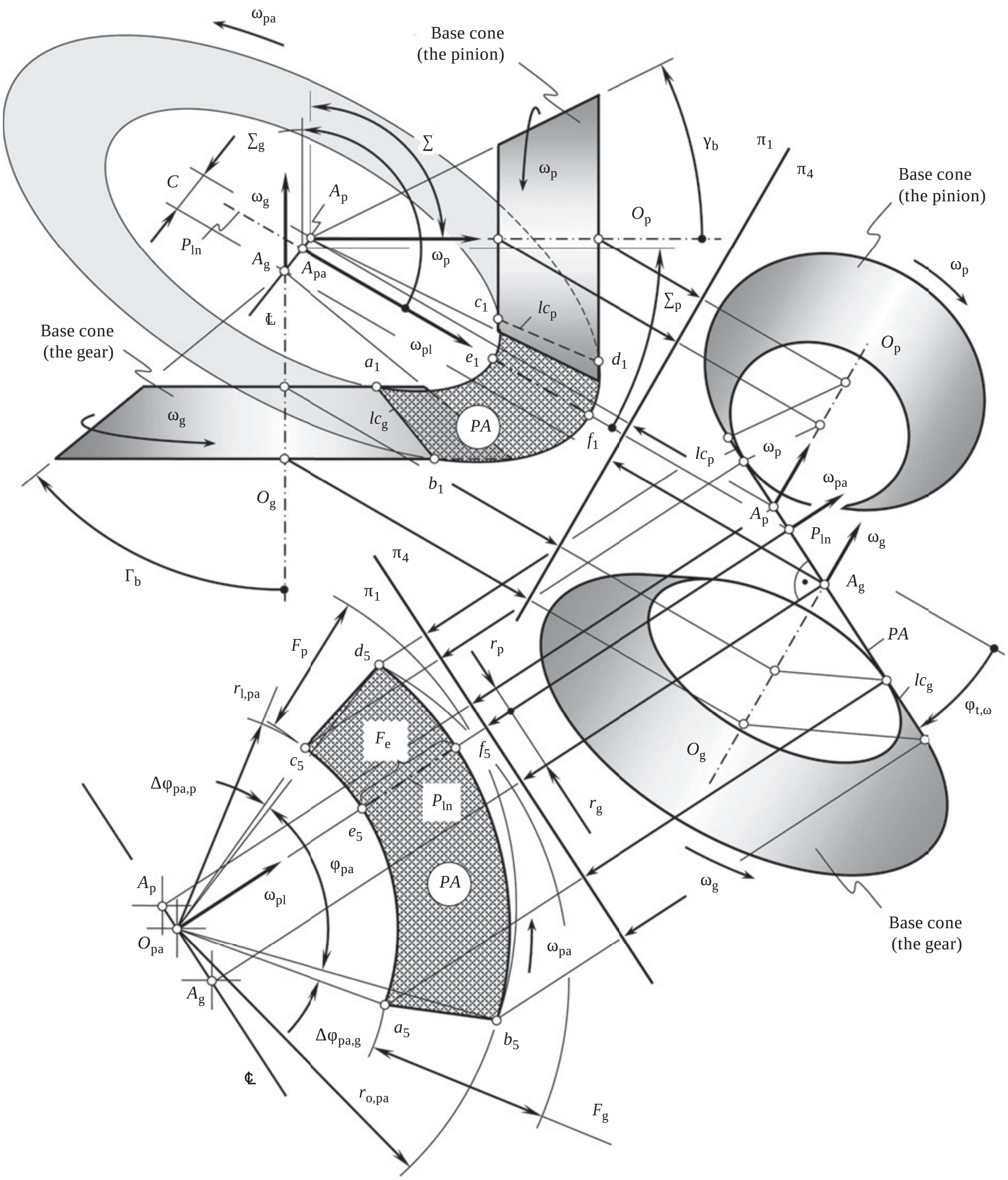

FIGURE F.2 Base cones and the plane of action *PA* in an orthogonal crossed-axis gear pair.

gear and its pinion rotate about their axes O_g and O_p, respectively. The rotation vectors ω_g and ω_p allow for the construction of the vector ω_{pl} of instant relative rotation. The rotation vector ω_{pl} meets the requirement $\omega_{pl} = \omega_p - \omega_g$. The axis of instant rotation P_{ln} is aligned with the vector of instant rotation ω_{pl}.

The vector of instant rotation ω_{pl} is the vector through an apex A_{pa} within the center distance C. The endpoints of the straight-line segment C are labeled A_g and A_p. The point A_g is the point of intersection of the centerline and the gear axis of rotation O_g. The point A_p is the point of intersection of the centerline and the pinion axis of rotation O_p.

For the convenience of further analysis, three principal planes are introduced in the following.

Consider the vector diagram for an arbitrary C_a gear pair given in Figure F.3. Points A_g and A_p are points of intersection of the gear axis of rotation O_g and the pinion axis of rotation O_p, respectively, with the centerline . The point A_g is referred to as the *gear apex*, and the point A_p is referred to as the *pinion apex*.

The vector of instant rotation ω_{pl} of the pinion in relation to the gear is a vector through the point A_{pa}. This point is located within the centerline . The point A_{pa} is referred to as the *plane-of-action apex*.

The *axis of instant rotation* P_{ln} is the straight line through the point A_{pa} along the vector of instant rotation ω_{pl}. This straight line is also referred to as pitch line.

Two straight lines through a common point uniquely specify a plane through these two lines. In the case under consideration, this is the plane through the axis of instant rotation P_{ln} and through the centerline . The plane is referred to as the *pitch-line plane*.

Definition F.1

The pitch-line plane (P_{ln} plane) in a crossed-axis gear pair is the reference plane through the centerline and the axis of instance rotation of the gear and the pinion.

In a case of parallel-axis gearing as well as intersected-axis gearing, the pitch-line plane reduces to the plane through the axis of rotation of the gear and the pinion. For intersected-axis gearing, as well as for parallel-axis gearing, the P_{ln} plane can be also de ned as the plane through the axis of rotation of the gear and the axis of rotation of the pinion.

Two more important planes can be introduced here. They are the so-called *centerline plane* (C_{ln} plane) and *normal plane* (N_{ln} plane).

Definition F.2

The centerline plane (C_{ln} plane) in a crossed-axis gear pair is the reference plane through the centerline perpendicular the axis of instance rotation of the gear and the pinion.

In a case of crossed-axis gearing, the centerline plane is perpendicular to the axis of instance rotation of the gear and the pinion.

Definition F.3

The normal plane (N_{ln} plane) in a crossed-axis gear pair is the referenced plane the axis of instance rotation of the gear and the pinion perpendicular to the centerline.

The three planes, that is, P_{ln} plane, C_{ln} plane, and N_{ln} plane, are of importance in the theory of gearing and in the theory

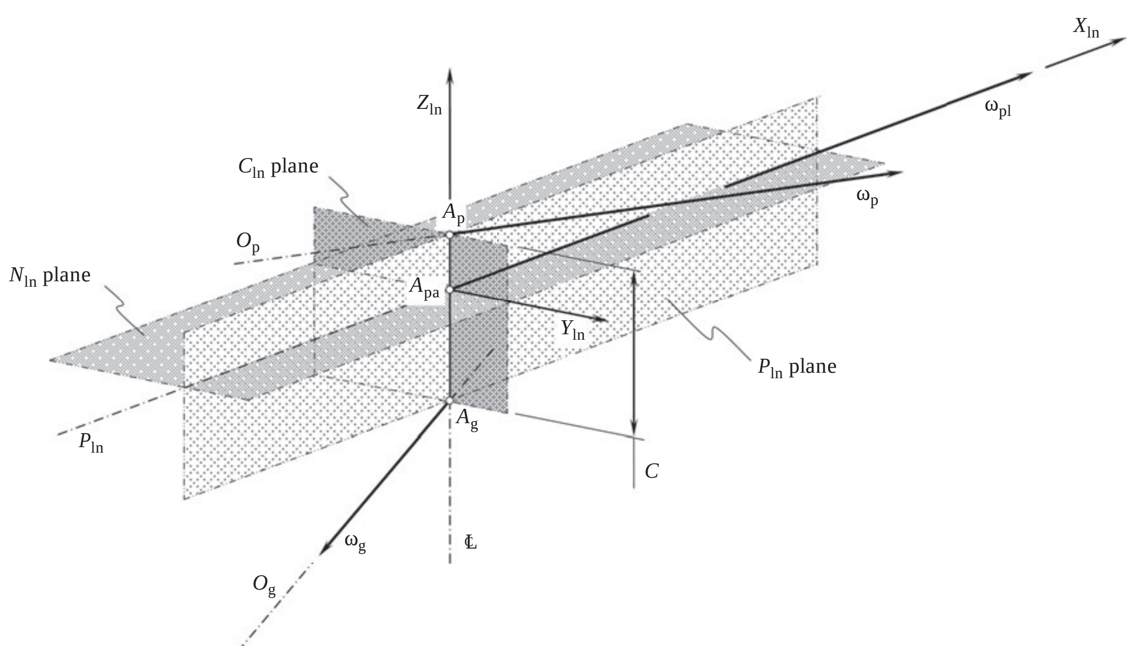

FIGURE F.3 On the de nition of the pitch-line plane (P_{ln} plane), centerline plane (C_{ln} plane), and normal plane (N_{ln} plane) in spatial gearing.

of high-conformal gearing in particular. They are referred to as the *fundamental planes* of a gear pair.

A fundamental *Cartesian* reference system $X_{ln}Y_{ln}Z_{ln}$ is associated with a gear pair. The axes of the reference system $X_{ln}Y_{ln}Z_{ln}$ are along the lines of intersection of the fundamental planes as shown in Figure F.3.

The plane-of-action apex A_{pa} is at a certain distance r_g from the axis of rotation O_g. At that same time, the point A_{pa} is at a certain distance r_p from the axis of rotation O_p. The following expression $r_g + r_p = C$, where the radii r_g and r_p are signed values is valid.

For a pair of rotation vectors ω_g and ω_p, the ratio tan Σ_g/tan Σ_p can be calculated as

$$\frac{r_p}{r_g} = \frac{\tan\Sigma_g}{\tan\Sigma_p}. \tag{F.4}$$

The plane of action *PA* is a plane through the axis of instant rotation P_{ln}. Let us assume that the plane of action rotates about the gear axis of rotation O_g. The base cone of the gear is an envelope to the consecutive positions of *PA* in such a rotation. Similarly, the base cone of the pinion is constructed.

The plane of action *PA* is tangent to the base cones of the gear and the pinion. Because of that, the plane of action *PA* makes a certain transverse pressure angle $\phi_{t.\omega}$ in relation to the P_{ln} plane. The pressure angle $\phi_{t.\omega}$ is measured within the C_{ln} plane.

The portion of the schematic plotted in the left upper corner in Figure F.2 is constructed within the plane of projections π_1. Two other planes of projections, π_2 and π_3, of the standard set of planes of projections $\pi_1\pi_2\pi_3$ are not used in this particular consideration. Therefore, these planes π_2 and π_3 are not shown in Figure F.2. Instead, two auxiliary planes of projections, namely, the plane of projections π_4 and π_5, are used. The axis of projections π_1/π_4 is constructed so as to be perpendicular to the axis of instant rotation P_{ln}. The axis of projections π_4/π_5 is constructed so as to be parallel to the trace of the plane of action *PA* within the plane of projections π_4. The plane of action *PA* is projected with no distortions onto the plane of projections π_4.

The plane of action can be interpreted as a flexible zero thickness film. The film is free to wrap or unwrap from and onto the base cones of the gear and the pinion. The plane of action *PA* is not allowed to be bent about an axis perpendicular to the plane *PA*. Under a uniform rotation of the gears, the plane of action *PA* rotates about the axis O_{pa}. The rotation vector ω_{pa} is along the axis O_{pa}. The rotation vector ω_{pa} is perpendicular to the plane of action *PA*.

As the axis of instant rotation P_{ln} and the axes of rotations of the gear, O_g, and the pinion, O_p, cross one another, the pure rolling of the base cones of the gear and of the pinion over the pitch plane *PA* is not observed, but rolling together with sliding of *PA* over the base cones is observed instead.

For C_a gearing, the plane of action *PA* can be understood as a round cone that has a cone angle of 90°. As sin 90° = 1, the magnitude ω_{pa} of the rotation vector ω_{pa} can be calculated from the formula

$$\omega_{pa} = \frac{\omega_g}{\sin\Gamma_b} = \frac{\omega_p}{\sin\gamma_b}, \tag{F.5}$$

where

Γ_b—the base cone angle of the gear
γ_b—the base cone angle of the pinion

In C_a gearing, the base cone angles Γ_b and γ_b vary within the intervals $0° < \Gamma_b < 180°$ and $0° < \gamma_b < (180° - \Gamma_b)$, respectively.

A desired working portion, or, in other words, a *functional portion of the plane of action PA*, can be constructed in the following way. Consider a straight-line segment *ef* within the axis of instant rotation P_{ln} (Figure F.2). When the gears rotate, the straight-line segment *ef* travels together with the plane of action *PA*. The point *f* traces a circular arc of radius $r_{o.pa}$, while the point *e* traces a circular arc of radius $r_{l.pa}$. The effective face width of the plane of action F_{eff} or, in other words, the working (functional) portion of the plane of action is located between two circles of radii $r_{o.pa}$ and $r_{l.pa}$. To get the desired face width of the plane of action F_{eff}, the face width of the gear F_g and the face width of the pinion F_p should be of values as shown in Figure F.2. The appropriate radii of the outer circles, $r_{o.g}$ and $r_{o.p}$, as well as of the inner circles, $r_{l.g}$ and $r_{l.p}$, should be of values under which both, the face width of the gear F_g and the face width of the pinion F_p overlap the face width F_{eff}. The radii $r_{o.g}$ and $r_{l.g}$ are centered at the gear apex A_g, while the radii $r_{o.p}$ and $r_{l.p}$ are centered at the pinion apex A_p. The inequalities $F_g > F_{eff}$ and $F_p > F_{eff}$ occur because the apexes A_g and A_p are not coincident with one another; thus, sliding in axial direction of the gear and the pinion is inevitable in crossed-axis gearing.

The straight-line segments lc_g and lc_p are along the corresponding lines of contact of the plane of action *PA* with the base cones of the gear and the pinion.

In the angular directions, the functional portion of the plane of action *PA* spans within the central angle

$$\Phi_{pa} = \varphi_{pa} + \Delta\varphi_{pa.g} + \Delta\varphi_{pa.p}. \tag{F.6}$$

The components $\Delta\varphi_{pa.g}$ and $\Delta\varphi_{pa.p}$ are because the gear axis of rotation O_g and the pinion axis of rotation O_p are the straight lines, which do not pass through the plane-of-action apex A_{pa}.

F.4 TOOTH FLANKS OF THE GEOMETRICALLY ACCURATE (IDEAL OR PERFECT) CROSSED-AXIS GEARS

Conjugate tooth flanks* of a gear and a mating pinion in a C_a gear pair are in line contact with one another. As the gears rotate, the line of contact travels with respect to several reference systems associated with (a) the gear, (b) the pinion, and (c) the housing. The tooth flank of the gear

* *Reversibly enveloping surfaces* (or just R_e *surfaces*, for simplicity) is the other term for conjugate tooth flanks (Radzevich, 2013).

can be construed as a locus of the consecutive positions of the desirable line of contact LC_{des} in its motion in relation to the reference system $X_gY_gZ_g$ associated with the gear. Similarly, the tooth ank of the pinion □ can be represented as a locus of the consecutive positions of that same line of contact LC_{des} in its motion in relation to the reference system $X_pY_pZ_p$ associated with the pinion. Ultimately, a locus of consecutive positions of that same line of contact LC_{des} in its motion in relation to a stationary reference system associated with the housing $X_hY_hZ_h$ represents the surface of action. Therefore, once the line of contact is known, the kinematics of a crossed-axis gearing (Figure F.2) can be employed for the derivation of an analytical representation of the tooth ank of the gear, □, and of the pinion, □. For this purpose, several auxiliary reference systems are commonly used. A Cartesian coordinate system $X_rY_rZ_r$ associated with the plane of action (Figure F.4) is one of the auxiliary reference systems.

In a local reference system $x_{lc}y_{lc}z_{lc}$ associated with the plane of action PA and centered at the center O_{lc} of the circular arc of the radius R_{lc}, the position vector of a point $\mathbf{r}_{des}^{(lc)}$ of the desirable line of contact LC_{des} can be analytically described by an expression in the form (Figure F.5)

$$\mathbf{r}_{des}^{(lc)}(\varphi_{lc}) = \begin{bmatrix} R_{lc}\cos\varphi_{lc} \\ R_{lc}\sin\varphi_{lc} \\ 0 \\ 1 \end{bmatrix}, \tag{F.7}$$

where φ_{lc} is the angular parameter of the desired line of contact LC_{des}.

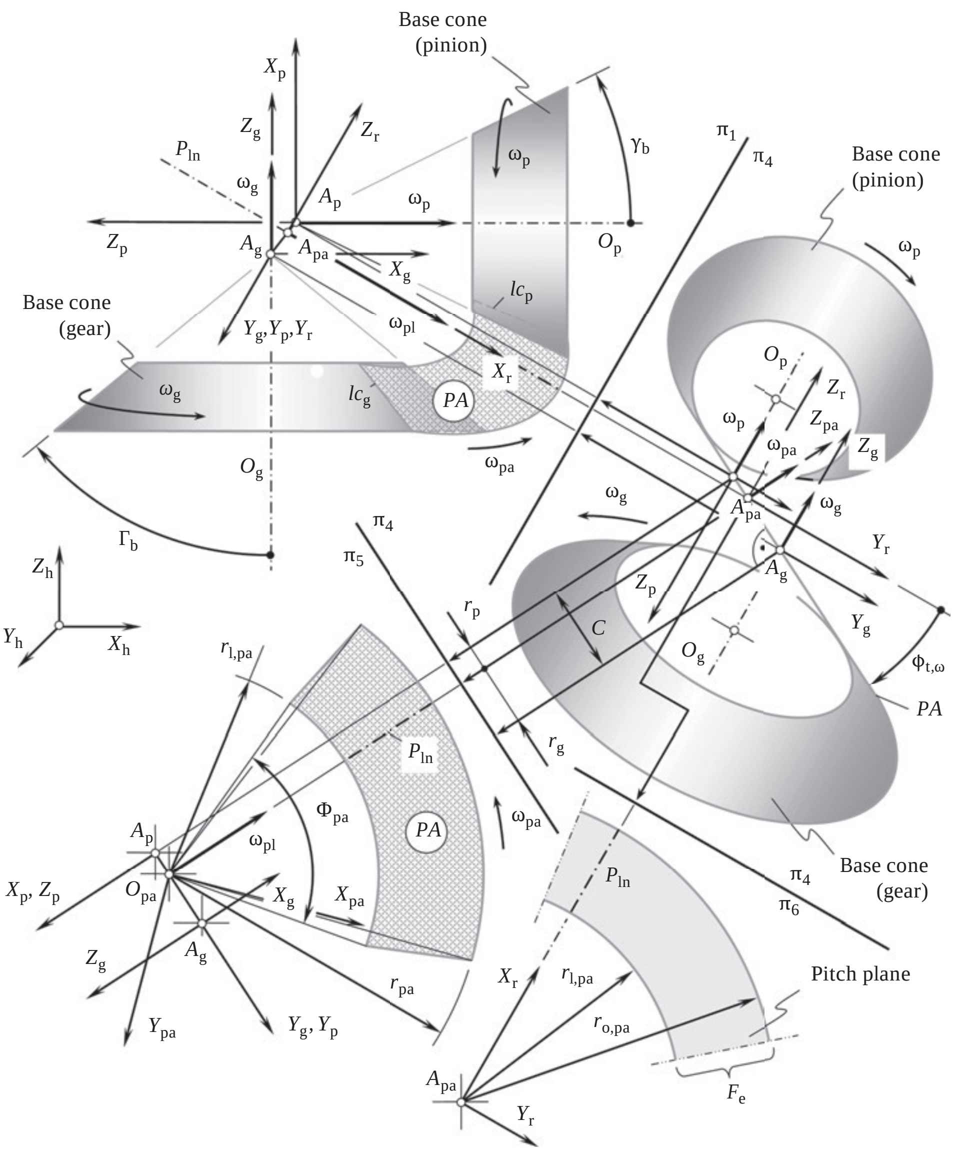

FIGURE F.4 Reference systems that are used for the derivation of an analytical expression for a gear tooth ank □ and a pinion tooth ank □ for a crossed-axis gear pair.

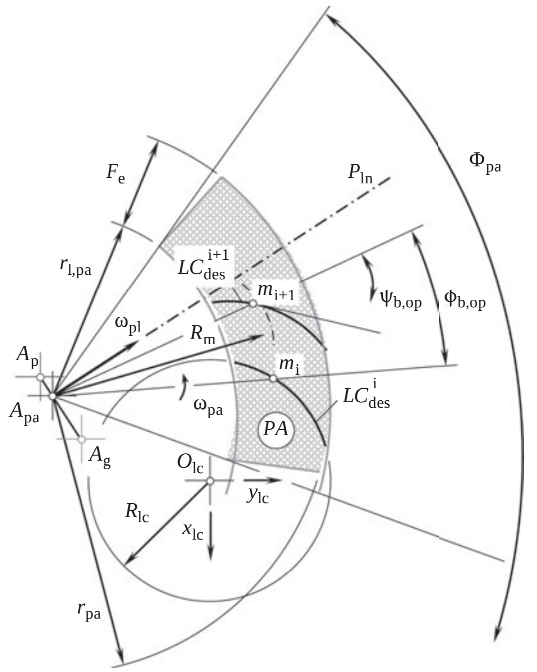

FIGURE F.5 Configuration of the desirable line of contact LC_{des} in the form of an arc of a circle within the plane of action PA.

In a reference system $X_{pa}Y_{pa}Z_{pa}$, the position vector of a point $\mathbf{r}_{des}^{(pa)}$ of the desirable line of contact LC_{des} can be analytically described by an equation:

$$\mathbf{r}_{des}^{(pa)}(\varphi_{lc}) = \underbrace{\mathbf{Tr}[(r_{w.pa} - R_{lc}\sin\psi_{lc}),\ X]\cdot\mathbf{Tr}(R_{lc}\cos\psi_{lc},\ Y)}_{\mathbf{Rs}(lc\mapsto pa)} \cdot \mathbf{r}_{des}^{(lc)}(\varphi_{lc}), \tag{F.8}$$

where $\mathbf{Tr}[(r_{w.pa} - R_{lc}\sin\psi_{lc}), X]$ and $\mathbf{Tr}(R_{lc}\cos\psi_{lc}, Y)$ are the standard operators of the translations along the coordinate axes X and Y, correspondingly. Standard formulas (Radzevich, 2013; 2014) are used for the calculation of these operators. The interested reader may wish to exercise on his/her own composing the operators of translation as well as the operators of rotation $\mathbf{Rt}(\varphi \mapsto X)$, used in the following. $\mathbf{Rs}(lc \mapsto pa)$ is the operator of the resultant coordinate system transformation, that is, the operator of the transition from $x_{lc}y_{lc}z_{lc}$ to $X_{pa}Y_{pa}Z_{pa}$.

Ultimately, it can be shown that in the reference system $X_{pa}Y_{pa}Z_{pa}$ associated with the plane of action, the position vector of a point $\mathbf{r}_{des}^{(pa)}$ can be written in the form

$$\mathbf{r}_{des}^{(pa)}(\varphi_{lc}) = \mathbf{Rs}(lc \mapsto pa)\cdot\mathbf{r}_{des}^{(lc)}(\varphi_{lc}) = \begin{bmatrix} R_{lc}\cos\varphi_{lc} + r_{w.pa} + R_{lc}\sin\psi_{lc} \\ R_{lc}\sin\varphi_{lc} - R_{lc}\cos\psi_{lc} \\ 0 \\ 1 \end{bmatrix}. \tag{F.9}$$

When the gears rotate, the desirable line of contact LC_{des}, travels with respect to a reference system $X_gY_gZ_g$ associated with the gear. Simultaneously, LC_{des} travels with respect to a reference system $X_pY_pZ_p$ associated with the pinion. Therefore, the gear tooth flank $\mathcal{G}$ can be viewed as a locus of consecutive positions of the line of contact LC_{des} in the reference system $X_gY_gZ_g$. Similarly, the pinion tooth flank $\mathcal{P}$ can be viewed as a locus of consecutive positions of the line of contact LC_{des} in the reference system $X_pY_pZ_p$. To derive equations for the position vectors of a point of the gear tooth flank, $\mathbf{r}_g$, and the pinion tooth flank, $\mathbf{r}_p$, the operators, $\mathbf{Rs}(pa \mapsto g)$ and $\mathbf{Rs}(pa \mapsto p)$ of transition from the reference system $X_{pa}Y_{pa}Z_{pa}$ to the reference systems $X_gY_gZ_g$ and $X_pY_pZ_p$ are used:

$$\mathbf{r}_g(\varphi_{lc}, \varphi_g) = \mathbf{Rs}(pa \mapsto g)\cdot\mathbf{r}_{des}^{(lc)}(\varphi_{lc}), \tag{F.10}$$

$$\mathbf{r}_p(\varphi_{lc}, \varphi_p) = \mathbf{Rs}(pa \mapsto p)\cdot\mathbf{r}_{des}^{(lc)}(\varphi_{lc}). \tag{F.11}$$

The operator of the resultant coordinate system transformation $\mathbf{Rs}(pa \mapsto g)$ is a function of the angle of rotation of the gear, φ_g, and of the angular parameter, φ_{lc}. The operator of the resultant coordinate system transformation $\mathbf{Rs}(pa \mapsto p)$ is a function of the angle of rotation of the pinion φ_p and of the angular parameter φ_{lc}.

Paths of contact P_c are circular arcs through points of the desirable line of contact LC_{des}. All the paths of contact are within the plane of action PA and are centered at the plane-of-action apex A_{pa}. It can be construed that paths of contact lie on spheres,* all of which are centered at the plane-of-action apex A_{pa}.

At every instant of time, the instant line of action LA_{inst} is a straight line tangent to the path of contact at corresponding its point. All the instant lines of action intersect the pitch line P_{ln}. In this way, the condition of conjugacy is met.

In a particular case, the tooth flanks of the gear and the pinion in a C_a gear pair can be designed so that the desirable line of contact LC_{des} of the teeth flanks $\mathcal{G}$ and $\mathcal{P}$ is aligned (at a certain instant of time) with a line through the apex A_{pa}. When the gears rotate, at a certain instant of time the line of contact aligns with the pitch line P_{ln}. The teeth flanks $\mathcal{G}$ and $\mathcal{P}$, which are generated by means of the line of contact LC_{des} of this particular type, are referred to as *pseudostraight tooth flanks*. The term *pseudostraight tooth flanks* reflects that the tooth flanks $\mathcal{G}$ and $\mathcal{P}$ are generated by a straight line. At a certain instant of time, the straight generating line aligns with the straight pitch line P_{ln}.

Pseudostraight crossed-axis gearing features a zero face contact ratio ($m_F = 0$).

The unit normal vector $\mathbf{n}_g$ to the gear tooth flank $\mathcal{G}$ can be calculated at every particular case of crossed-axis gears. The unit normal vector $\mathbf{n}_g$ and the straight line along the vector

* Because the paths of contact in crossed-axis gearings lie on spheres, crossed-axis gearings can be called *spherical gearings*. This causes confusion because engagement in mesh in intersected-axis gearings is also observed on spheres. Therefore, the terms *intersected-axis gearing* and *crossed-axis gearing* are preferred rather than the ambiguous term *spherical gearing*.

$\mathbf{n}_g$ are used to calculate deviations of a machined gear tooth flank from the tooth flank of the desired geometry.

With the position vector $\mathbf{r}_g(v, \theta_{pa})$ of a point of the gear tooth flank □ known, the unit normal vector $\mathbf{n}_g$ can be calculated from the following well-known formula:

$$\mathbf{n}_g(v, \theta_{pa}) = \frac{\dfrac{\partial \mathbf{r}_g}{\partial v} \times \dfrac{\partial \mathbf{r}_g}{\partial \theta_{pa}}}{\left| \dfrac{\partial \mathbf{r}_g}{\partial v} \times \dfrac{\partial \mathbf{r}_g}{\partial \theta_{pa}} \right|}(v, \theta_{pa}). \tag{F.12}$$

Calculation of the derivatives $\partial \mathbf{r}_g/\partial v$ and $\partial \mathbf{r}_g/\partial \theta pa$, followed by the formula transformation, is a drilling procedure. Calculation of the unit normal vector $\mathbf{n}_g$ can be significantly simplified if the vector $\mathbf{n}_g$ as well as the straight line along the vector $\mathbf{n}_g$ are determined in the reference system $X_{pa}Y_{pa}Z_{pa}$ (in this reference system, the unit normal vector $\mathbf{n}_g$ is identical to the unit normal vector $\mathbf{n}_{lc}$ to the desirable line of contact LC_{des}, and $\mathbf{n}_g$ is entirely located within the plane of action PA). Afterward, implementation of the operator **Rs**(PA □ of the resultant coordinate system transformation allows for representation of both the unit normal vector $\mathbf{n}_{lc}$ and the straight line along $\mathbf{n}_{lc}$ in the coordinate system $X_gY_gZ_g$ associated with the gear.

The discussed approach to the determination of the geometry of the gear tooth flank □ and the pinion tooth flank □ is based on the generation of the tooth flanks in the form of a family of consecutive positions of the line of contact LC that travels together with the plane of action PA. This approach does not require specification of the tooth flanks in the form of enveloping surfaces to consecutive positions of the generating basic rack. This means that the proposed method for the generation of the tooth flanks □ and □ does not require implementation of the elements of the theory of enveloping surfaces. This is a significant advantage of the disclosed method for the generation of tooth flank □ of the gear, and tooth flank □ of the pinion in an intersected-axis gearing.

The derived equations for the gear tooth flank □, as well as for the pinion tooth flank □, can be used as reference surfaces (datum surfaces) when (a) designing, (b) machining, and (c) inspecting gears for a C_a gearing that have line contact of the tooth flanks □ and □ of the gear and the pinion. Surfaces of this type are an equivalent to screw involute surfaces widely used for P_a gearing. They are constructed on the premise of describing (and not enveloping) principle of surface generation.

Crossed-axis gearing that have tooth flanks of the proposed geometry is the most general type of gearing that features *line contact* of the tooth flanks □ and □. In a particular case, when the center distance is reduced to zero ($C = 0$), the C_a gearing of the proposed geometry simplifies to I_a gearing that have line contact of the tooth flanks. Under another scenario, namely, when the crossed-axis angle is equal either 0 or π, the C_a gearing of the proposed geometry simplifies to P_a gearing that features line contact of the tooth flanks.

The desirable geometry of contact of teeth flanks □ and □ of the gear and the pinion in R gearing can be specified on the stage of analysis of the shape and configuration of the line of contact LC within the plane of action PA. The indicatrix of conformity Cnf_R(□/□) at a point of contact of the tooth flanks □ and □ (Radzevich, 2012a, 2013, 2014) can be expressed in terms of the shape and configuration of the line of contact. Ultimately, those parameters of the shape and configuration of the line of contact are selected under which the minimum diameter of the indicatrix of conformity Cnf_R(□/□) is as small as possible.

The crossed-axis gearing, for which the tooth flanks of the gear and the pinion are generated as loci of consequent positions of the line of contact LC that travels together with the plane of action PA, is a novel type of gearing. This novel type of gearing ensures line contact of the tooth flanks of the gear and of the pinion. This gearing is referred to as R gearing.

Parallel-axis involute gearing (Euler, 1760) and intersected-axis gearing with the spherical involute tooth flank geometry represent reduced cases of R gearing. The shaft angle is either zero or equals $\Sigma = 180°$ in the first case. In the second case, the center distance is zero.

F.5 CONCLUSION

This appendix deals with gearing that features crossing axes of rotation of a gear and a mating pinion, that is, with C_a gearing. A novel type of C_a gearing is proposed and is investigated in this appendix. The proposed design of the ideal crossed-axis gearing is referred to as R gearing. It is shown that R gearing is capable of transmitting a rotation smoothly and with the highest possible power density. The latter is of critical importance to many industrial applications including but not limited to the aerospace industry (helicopter transmissions), the automotive industry, electric-wind-power-station applications, and so forth.

It should be also mentioned here that parallel-axis involute gearing (Euler, 1760), as well as intersected-axis gearing with the spherical involute tooth flank geometry, represent reduced cases of R gearing. The shaft angle is either zero or equals $\Sigma = 180°$ in the first case. In the second case, the center distance is zero.

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